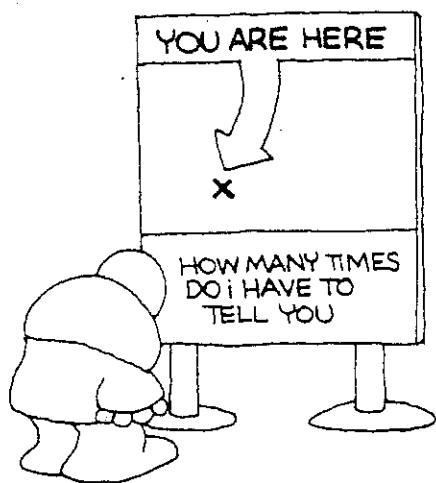


Friendship Junior High School  
Seventh Grade Advanced Math Packet



*Exploring  
Topics  
In Algebra*

Algebra Units 9-13

Foundation Skills  
Simplifying Expressions  
Radical Operations  
Equations & Inequalities  
Working With Monomials

# Foundation Skills

## 1. INTEGERS & EXPONENTS

### Demonstration

Adding and subtracting:

$$\begin{aligned} \text{A) } & (-6) + (-4) + (7) + (2) \\ & (-10) + (9) = -1 \end{aligned}$$

$$\begin{aligned} \text{B) } & (-8) - (-4) + (3) - (5) \\ & (-8) + (4) + (3) + (-5) \\ & (-13) + (7) = -6 \end{aligned}$$

### Demonstration

Multiplying and dividing:

$$\text{C) } (-4)(-2)(1)(-3) = -24$$

$$\text{D) } (-12) \div (-4) = 3$$

$$\text{E) } (-6)(+2)(-2)(+2) = 48$$

Note: An odd number of negatives results in a negative answer. An even number of negatives results in a positive answer.

Note: Multiply or divide without considering the sign. Then determine the sign of the answer.

### Demonstration

Exponents:

$$\text{F) } 2^3 = 2 \cdot 2 \cdot 2 \cdot 1 = 8$$

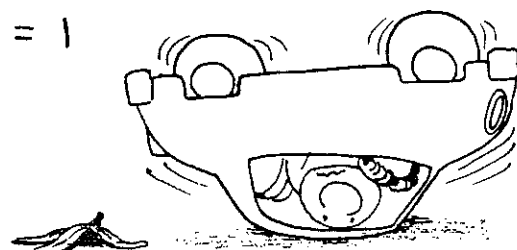
$$\text{G) } 3^4 = 3 \cdot 3 \cdot 3 \cdot 3 \cdot 1 = 81$$

$$\text{H) } 6^0 = 1$$

$$\text{I) } (-2)^3 = (-2)(-2)(-2)(1) = -8$$

$$\text{J) } (-4)^2 = (-4)(-4)(1) = 16$$

$$\text{K) } (-6)^0 = 1$$



### Problem Set 9.1

#### Integers & Exponents

$$\textcircled{1} (-8) + (-7)$$

$$\textcircled{7} (-6)(-4)$$

$$\textcircled{2} (-4) + (9)$$

$$\textcircled{8} (12)(-3)$$

$$\textcircled{3} (-7) - (-4)$$

$$\textcircled{9} (8) \div (-2)$$

$$\textcircled{4} (-12) - (8)$$

$$\textcircled{10} (27) \div (-3)$$

$$\textcircled{5} (6) - (-9)$$

$$\textcircled{11} (-18) \div (-9)$$

$$\textcircled{6} (-14) + (-3)$$

$$\textcircled{12} (-4)(5)$$

## UNIT 9

# Foundation Skills

$(13) (-8) + (9) + (-4) + (6) \quad (27) 2^4$

$(14) (-5) - (-3) + (-2) - (8) \quad (28) 1^6$

$(15) (-7) + (-9) - (-5) + (7) - (2) \quad (29) 3^3$

$(16) (-3) - (-5) + (-11) + (4) - (-1) \quad (30) 7^2$

$(17) (8) - (+4) - (-3) + (-5) \quad (31) 8^0$

$(18) (-3) - (-1) + (-7) - (-8) - (-4) \quad (32) (-2)^3$

$(19) (-1) + (-1) - (-1) + (-1) - (-1) \quad (33) (-4)^2$

$(20) (-2)(3)(-4)(2)(-1) \quad (34) (-1)^5$

$(21) (-5)(-1)(-1)(-3)(3) \quad (35) (-3)^4$

$(22) (4)(-2)(-3)(1)(-3) \quad (36) (-5)^0$

$(23) (-2)(-2)(-3)(-3)(-4) \quad (37) 10^4$

$(24) (2)(3)(1)(2)(-3)(2) \quad (38) (-2)^5$

$(25) (3)(-1)(-4)(5)(-2) \quad (39) (-2)^2$

$(26) (-1)(-3)(2)(-6)(-4) \quad (40) 5^3$

### 2. ORDER OF OPERATIONS

When evaluating an expression, do the calculations in this order:

1. Parenthesis
2. Exponents
3. Multiplication / Division
4. Addition / Subtraction

### Demonstration

A)  $6 - (3)(-4) + (-2)$

$6 - (-12) + (-2)$

$6 + (12) + (-2)$

$(18) + (-2) = 16$

B)  $(-8)(12-10) - (-12) \div (-3)$

$(-8)(2) - (-12) \div (-3)$

$(-16) - (4)$

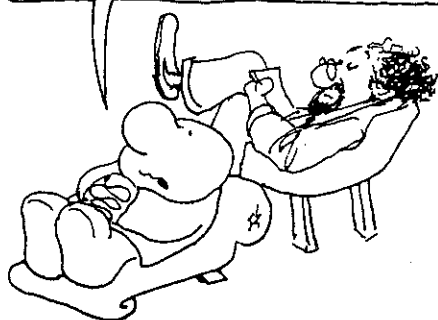
$(-16) + (-4) = -20$

### Demonstration

Signs inside of parenthesis are attached before exponents are evaluated.

When parenthesis are not present, exponents are applied before signs are attached.

... LATELY I'VE FOUND MYSELF CALLING MY ANSWERING MACHINE JUST TO FIND OUT WHERE I AM...



## UNIT 9

# Foundation Skills

C)  $(-2)^3 = -8$       E)  $-2^3 = -8$

D)  $(-2)^2 = 4$       F)  $-2^2 = -4$

G)  $-3^2 - (-2)^2$   
 $-9 - (4)$   
 $-9 + (-4) = -13$

### Demonstration

When there is a fraction bar in an expression, evaluate the numerator and denominator completely before applying the fraction bar.

H)  $\frac{-2^4 + 2^2}{(-3)^2 - 3}$

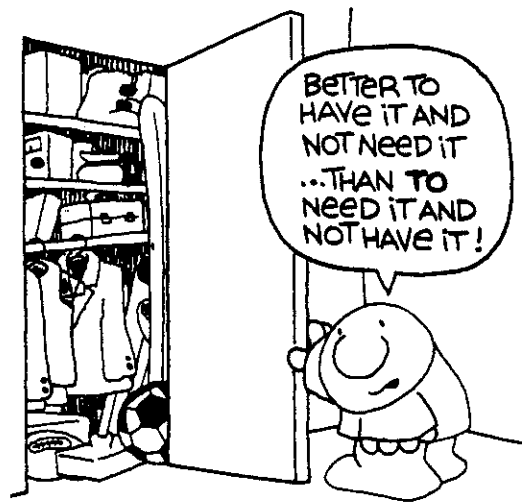
$$\frac{-16 + 4}{9 - 3} = \frac{-12}{6} = -2$$

Always be sure to reduce fractions:

I)  $\frac{(-3) + (-2)}{-2^4 + 3^0}$

$$\frac{(-3) + (-2)}{-16 + 1}$$

$$\frac{-5}{-15} = \frac{-1}{-3} = \frac{1}{3}$$



### Problem Set 9.2

#### Order Of Operations

Determine the value of each expression:

- ①  $(-3) + (-2)(-4)$
- ②  $(-8) - (6)(-2)$
- ③  $(-2) + (3 \times 4)$
- ④  $(-2) + (-8) \div (-2)$
- ⑤  $(-3) - (-6) \div (3)$
- ⑥  $(-4) + (-1)(5) - (2)$
- ⑦  $(-3) - (-4) \div (-2) - (-4)$
- ⑧  $(2) - (3 \times 2) \div (2)$
- ⑨  $(-3) \times (-2) + (-4) - (-1)$
- ⑩  $(6) \div (-3) - (-2)(-3)$

## UNIT 9

# Foundation Skills

$$\textcircled{11} (-2)^4 \quad \textcircled{17} -4^2 \quad \textcircled{23} (-4)^3$$

$$\textcircled{12} -2^4 \quad \textcircled{18} -5^0 \quad \textcircled{24} (-5)^2$$

$$\textcircled{13} -3^2 \quad \textcircled{19} (-5)^0 \quad \textcircled{25} -3^4$$

$$\textcircled{14} (-3)^2 \quad \textcircled{20} -2^5 \quad \textcircled{26} -1^8$$

$$\textcircled{15} (-3)^3 \quad \textcircled{21} (-2)^5 \quad \textcircled{27} (-1)^0$$

$$\textcircled{16} -3^3 \quad \textcircled{22} -7^2 \quad \textcircled{28} -1^0$$

$$\textcircled{29} (-3)^2 - 2^3$$

$$\textcircled{30} (-2)^0 - (-3)^2$$

$$\textcircled{31} (-3)^3 + (-2)^2$$

$$\textcircled{32} (-2)^4 - 2^2$$

$$\textcircled{33} (-2)^2 - (-3)^2 - 3^2$$

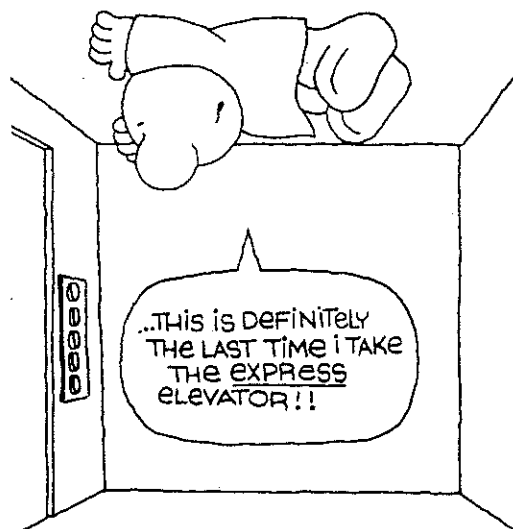
$$\textcircled{34} (-5)^0 - 4^0 - (-2)^5$$

$$\textcircled{37} \frac{(-2)^2 - (-3)^0}{-2^3}$$

$$\textcircled{38} \frac{(-2)^3 + (-1)}{-2^2 - 2}$$

$$\textcircled{39} \frac{-4^2 - (-2)^2 - (-5)^0}{-3}$$

$$\textcircled{40} \frac{-3 - (-2)^3 - (-2)^2}{3 - 3^2}$$



### Challenge Problems

$$\textcircled{35} \frac{(-2) - (-4)}{(-3) + (5)}$$

$$\textcircled{36} \frac{(-3) - (-3)(-4)}{(-3) - (-15)}$$

### 3. EVALUATING EXPRESSIONS

When evaluating expressions, it is important to use parenthesis when substituting values.

# Foundation Skills

## Demonstration

Evaluate each expression:

$$x = -1 \quad y = -2 \quad z = 2$$

A)  $x + 2y - z$

$$(-1) + 2(-2) - (2)$$

$$(-1) + (-4) + (-2) = -7$$

B)  $3xy - 2y^2$

$$3(-1)(-2) - 2(-2)^2$$

$$3(-1)(-2) - 2(4)$$

$$(6) - (8)$$

$$(6) + (-8) = -2$$

C)  $x^2y - (x + y)$

$$(-1)^2(-2) - ((-1) + (-2))$$

$$(-1)^2(-2) - (-3)$$

$$(1)(-2) - (-3)$$

$$(-2) - (-3)$$

$$(-2) + (3) = 1$$

D)  $\frac{3x^3y}{z-x}$

$$\frac{3(-1)^3(-2)}{(2) - (-1)} = \frac{3(-1)(-2)}{(2) - (-1)}$$

$$\frac{6}{(2) + (1)} = \frac{6}{3} = 2$$

## Problem Set 9.3

### Evaluating Expressions

Evaluate each expression:

$$a = -2 \quad b = -1 \quad c = 3$$

①  $a - 3b$

⑯  $3b^3 - abc$

②  $2b - 4c$

⑰  $4b^2 - 2a^3$

③  $3ab + c$

⑱  $4ac + 2b^2c$

④  $4ac - b$

⑲  $a^3 - b^2 - c^2$

⑤  $ab - ac$

⑳  $a^2b^2c$

⑥  $abc - b$

⑦  $3b + c - 2a$

⑧  $a + 3bc - b$

⑨  $2ab - c + b$

⑩  $3b - 2ab + c$

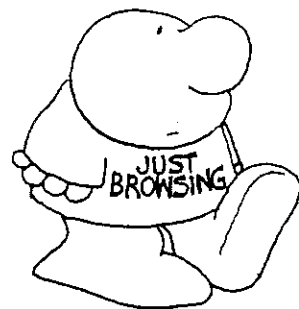
⑪  $a^2 - 2b^2$

⑫  $a^3 + b^2c$

⑬  $3b - 2a^2$

⑭  $ab^3 - c^2$

⑮  $2a^2b^2 - c$



### Challenge Problems

⑳  $2a - (b + c)$

㉑  $(a + b) - 2b^2$

continued

UNIT 9

# Foundation Skills

23  $\frac{a^2bc}{b+c}$

26  $\frac{-3(b+c)}{5a}$

24  $\frac{c^2-b}{8a-b}$

27  $\frac{ab^2c-a}{4b^2-ab}$

25  $\frac{-2(a+b)}{3c+b}$

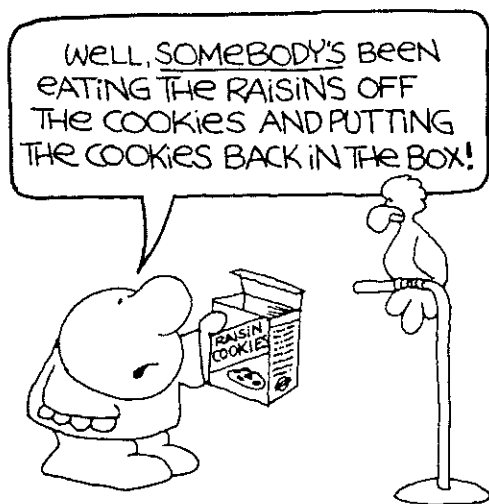
28  $\frac{a^2b^3-2c}{2(a+b)}$

11  $(-8) + (-5) - (-2) - (-7) - (-5)$

12  $(-2) \times (-1) \times (3) \times (-4)$

13  $(-3) \times (-2) \times (-2) \times (-2)$

14  $(8) - (-4) + (-6) - (-7) - (-2)$



## Exponents:

15  $4^2$       20  $(-3)^3$       25  $-5^2$

16  $-2^3$       21  $(-2)^4$       26  $(-8)^0$

17  $(-1)^4$       22  $7^0$       27  $-3^3$

18  $-3^2$       23  $7^2$       28  $(-2)^2$

19  $5^3$       24  $-1^4$       29  $-3^4$

## REVIEW & PRACTICE

### Integers:

1  $(-6) + (-8)$       6  $(-8) \times (7)$

2  $(-3) \times (-2)$       7  $(-9) - (-8)$

3  $(-4) - (-9)$       8  $(-7) - (-4)$

4  $(-3) - (-7)$       9  $(-3) + (11)$

5  $(12) \div (-4)$       10  $(9) \div (-3)$

### Order of operations:

30  $(-4) - (-7) \times (-4) - (8)$

31  $(6) - (-4)(-2) - (-12) \div (3)$

32  $(-2) + (-3) \times (-4) - (-6) + (-1)$

33  $(-10) \div (-2) - (-3) + (-8) \div (2)$

34  $(-3)^2 + (-2)^3$

35  $(-1)^3 - (-4)^0$

# Foundation Skills

36)  $(-2) - 3^2 - (-3)^3$

37)  $-6^0 - (-2)^4 - 2^3$

38)  $-2^2 - (-2)^2 - (-2)^0$

39)  $\frac{(-3)^2 - 2^3}{(-2)^2}$

40)  $\frac{(-2)(-3) - 3^2}{(-2) - (-1)^3}$

51)  $a^3 - b^2 + c^3$

52)  $a^2 b c^3$

## Challenge Problems

53)  $2a + 3b - (b+c)$

54)  $2(a+b) - c^3$

55)  $3(a+c) - a^2 + c^2$

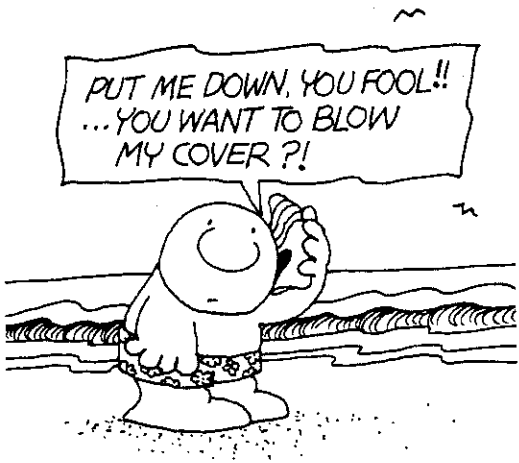
56)  $(a+b) + a^2 b$

57)  $\frac{a^2 b c}{a-b}$

58)  $\frac{2a - ac^2}{a^2 - 2c}$

59)  $a^2 + \frac{2b}{3a} - ac^2$

60)  $ab - \frac{c^3}{2a} + b^2$



Evaluating expressions:  
 $a = -1$   $b = -3$   $c = -2$

41)  $2a - b$

46)  $ac - abc$

42)  $3a + 2c$

47)  $a^2 + b^2$

43)  $ab - 3c$

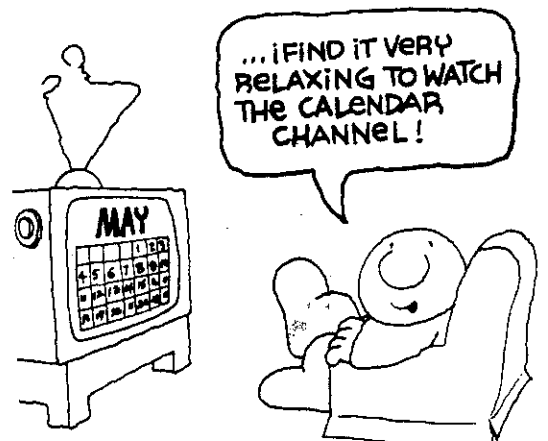
48)  $2a - c^3$

44)  $4a - b - c$

49)  $3ab^2 - 2a^2$

45)  $2ac + b$

50)  $a^2 c - 2b$





# Foundation Skills

## PRACTICE TEST #1

① Integers:  
 $(-12) - (+9)$

② Exponents:  
 $-3^4$

Order of operations:

③  $(-2) - (+5)(-3) - (-4)$

④  $-2^2 - (-2)^3 - (-5)^0$

Evaluating expressions:  
 $x = -3$   $y = -2$   $z = -1$

⑤  $3y^2 - 2xz^4$

⑥ 
$$\frac{yz - (x+z)}{y^3 - 2x}$$



## PRACTICE TEST #2

① Integers:  
 $(-14) - (-8)$

② Exponents:  
 $(-5)^2$

Order of operations:

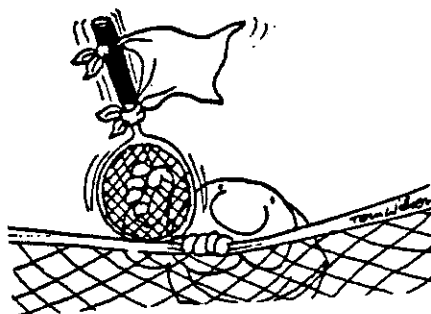
③  $(-4) - (-8) \div (-2) - (-4)(+3)$

④  $(-3)^2 - (-2)^0 - (-1)^4 - 2^2$

Evaluating expressions:  
 $a = -1$   $b = -2$   $c = -3$

⑤  $3a^2b^3 - 2ab$

⑥ 
$$\frac{2(b+c) - b^2}{-c^2 - b}$$



# Simplifying Expressions

## 1. COMBINING TERMS

Group all like terms together.  
Cross out terms as you group them.

### Demonstration

A)  $a + 3b + 2a + b$   
 ~~$a + 3b + 2a + b$~~  =  $3a + 4b$

B)  $4x - 3 + 2x + 7 - x$   
 ~~$4x - 3 + 2x + 7 - x$~~  =  $5x + 4$

Note: Each term includes the (+) or (-) sign.

Note:  $x^2$  and  $x$  are not like terms.  $a$  and  $ab$  are not like terms.

C)  $4a - 3ab + ab - 2b - a$   
 ~~$4a - 3ab + ab - 2b - a$~~   
 $3a - 2b - 2ab$

D)  $3x - 2xy + x^2 - 5x + xy$   
 ~~$3x - 2xy + x^2 - 5x + xy$~~   
 $-2x - xy + x^2$

E)  $a^2 - ab^2 + a^2b + ab^2 - 2a^2$   
 ~~$a^2 - ab^2 + a^2b + ab^2 - 2a^2$~~   
 $-a^2 + a^2b$

## Problem Set 10.1 Combining Terms

Simplify each expression:

- ①  $x + 3x + 2 + 5$
- ②  $3y + 7 + 2 + 4y$
- ③  $6x - 5 + 2x + 3 - x$
- ④  $4a + 3b - 2a + b - 7 + 3$
- ⑤  $3b - 4a - 2b - 1 - 5a$
- ⑥  $12 - 4b - 3a - b + 5a - 6$
- ⑦  $2a + 3ab - a - 2ab - 2a$
- ⑧  $5x + 2xy - x - 3xy - 2x$
- ⑨  $8n + 2mn - 4n + mn - 4n$
- ⑩  $3b - 2ab + b - 5 - ab + 5$



## UNIT 10

# Simplifying Expressions

⑪  $4x^2 + 2x - 3xy + x^2 - 3x$

⑫  $xy - 2x^2 + 3 - 4xy - x^2 + 2x$

⑬  $2ab - 3a + b + 4b - 2a - ab$

⑭  $7b - 2a - 3ab - a + 4ab - b$

⑮  $3a^2 - a + 2ab - 5a + 2a^2 - ab$

⑯  $a^2 - 2b + 5a^2 - 2a + 2b - 3a$

⑰  $4 - 2x - xy - 3x^2 + xy + x - 3$

⑱  $x^2 - 3y + 2x^2 - xy + y - 3x + 2xy$

⑲  $4m - 2n - 3mn + n - m + mn$

⑳  $4m - 3n - mn + 2n - mn - n + m$

㉑  $3a^2 + 2ab - a^2 + 5 - ab + 2$

㉒  $8 - 3ab + a^2 - 5 + ab + 2a^2 - 1$

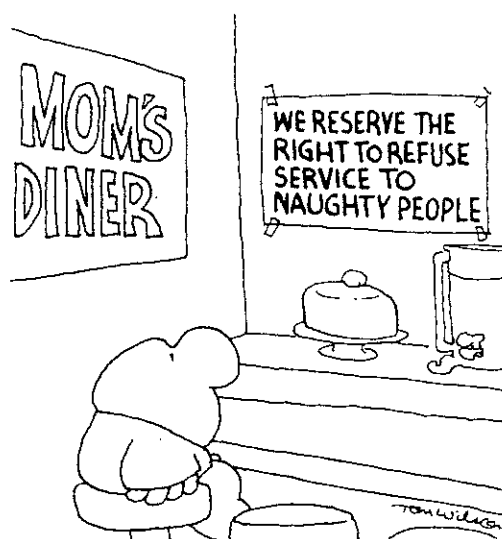
㉓  $4a - 2a + a - 3 + 5 + 1 - 3a$

㉔  $x - 2xy + 3x^2 - xy + x - x^2$

㉕  $6n^2 - n - 2n^3 - n^2 - 3n^3 - 5 - 2n^3$

㉖  $x^2y - xy^2 - 2xy^2 + 3x^2y - 2xy$

㉗  $4a - ab + 2abc - 3ab + 2a - abc$



### 2. DISTRIBUTIVE PROPERTY

Simplifying expressions that include use of the distributive property requires you to multiply terms.

### Demonstration

Multiply terms:

A)  $(4a)(3) = 12a$     C)  $(3x)(2x) = 6x^2$

B)  $(5n)(2) = 10n$     D)  $(5a)(a) = 5a^2$

### Challenge Problems

㉘  $3xy - 2x^2y + x^2 - xy - x^2y$

㉙  $5a^2b - ab^2 - 2 + a^2 - 3ab^2 - 2a^2b$

㉚  $4x - 2x^3 - x^2 - 3x^3 + x - 2x^2$

# Simplifying Expressions

E)  $(2a)(4b) = 8ab$     G)  $(a)(4ab) = 4a^2b$

F)  $(x)(3x^2) = 3x^3$     H)  $(2y)(3xy) = 6xy^2$

$$\begin{aligned} & 2(x-4y) - 3(2x+y) \\ & \cancel{2x} - \cancel{8y} - \cancel{6x} - \cancel{3y} \\ & -4x - 11y \end{aligned}$$

## Demonstration

Use the distributive property to multiply:

I)  $3(2x-y)$   
 $3(2x-y) = 6x-3y$

J)  $2a(4a-3b)$   
 $2a(4a-3b) = 8a^2-6ab$

K)  $-4x(3x-2)$   
 $-4x(3x-2) = -12x^2+8x$

## Demonstration

Simplify each expression:

L)  $4x + 3xy + 2y(x-3) + 2y$

Use the distributive property

$$4x + 3xy + \boxed{2y(x-3)} + 2y$$

$$\cancel{4x} + \cancel{3xy} + \cancel{2xy} - \cancel{6y} + \cancel{2y}$$

$$4x + 5xy - 4y$$

1)  $2(x-4y) - 3(2x+y)$

Remember to include the sign

N)  $2x^2 - 3x(4x-y) - 5xy$   
 $2x^2 - \boxed{3x(4x-y)} - 5xy$   
 $\cancel{2x^2} - \cancel{12x^2} + \cancel{3xy} - 5xy$   
 $-10x^2 - 2xy$

Remember: Do the distributive property before you combine like terms.



## Problem Set 10.2

### Distributive Property

Simplify each expression:

①  $2(x+y)$

②  $3(a+b)$

UNIT 10

# Simplifying Expressions

③  $4(3a-b)$

⑦  $-5(3x-2)$

④  $3(x-4y)$

⑧  $-2(a-7)$

⑤  $2a(a+b)$

⑨  $-x(3x-2)$

⑥  $3x(x-2y)$

⑩  $-b(4a+b)$

⑳  $3x(4x-3y) - 2y(3x+4y) - 2y^2$

㉑  $12a - 3ab - 2a(4-3b) - 5a + ab$

㉒  $14x - 2xy - x(3y-2) + xy - 2x$

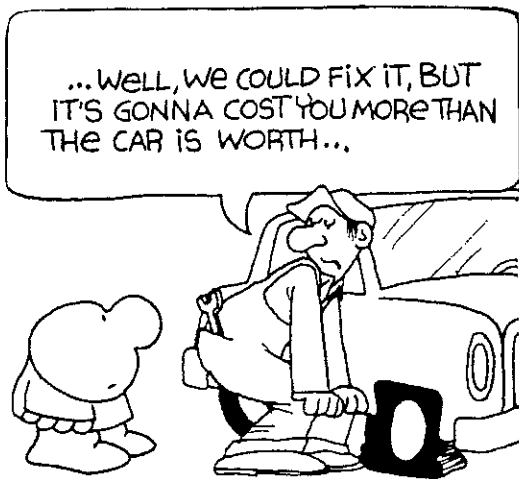
## Challenge Problems

㉓  $3xy(x-2y) - 4x(y^2+2xy) - 2x^2y$

㉔  $4ab(a+3b) - 2b(ab-2a^2) + ab^2$

㉕  $2a(a^2-3a) + a^2(4a-3) - 5a^3$

㉖  $2n(4n-n^2) - 2n^2(5-3n) + n^2$



⑪  $3(x-2y) - 2x + 3y$

⑫  $2(a+3b) + 3a + 2b$

⑬  $-3(x-2) - 8 + 3x - 4$

⑭  $-4(n+5) - 2n + 7 + n + 13$

⑮  $2x - 3(x-4) - 2x + 5$

⑯  $3 - 2(2x+1) - x + 7 - 3x$

⑰  $2x(x-3y) - 3(x^2-2) + xy$

⑱  $a(2a+b) - 3(ab+a^2) - ab$

㉑  $2a(3a+4b) - 2b(a+4)$

## REVIEW & PRACTICE

Simplify each expression:

①  $3a + 5 + 6a + 2$

②  $4 + 2n + 7 + 5n + 1$

③  $5x - 7 - 3x - 2$

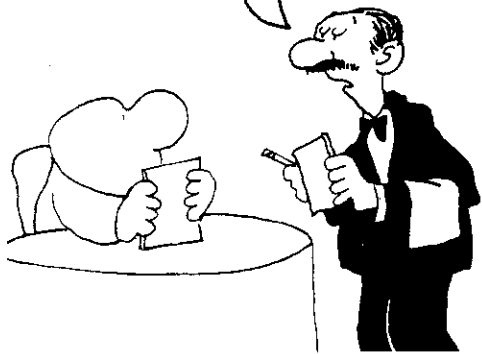
④  $8c + 4 - 12c - 6$

⑤  $8ab - 3a + 4a - 2ab$

⑥  $7x - 5xy + 3x + xy$

# Simplifying Expressions

WE HAD TO DISCONTINUE THE ALPHABET SOUP, SIR....  
...SOME SORT OF COPYRIGHT PROBLEM...



⑦  $3 - 2b + 5ab - 4 + b - 2a + ab$

⑧  $xy - 2y + 3x + 4 - 6xy + y - 1 + x$

⑨  $x^2 + 3xy + y^2 - 2xy - 4y^2 + 3x^2$

⑩  $-ab + b^2 + 2a^2 - 4ab - b^2 + 2a^2$

⑪  $12 - 3n + m - 4 - 2m + 5n$

⑫  $7x + y - 3 + 4y - 5 - 3x - 2$

⑬  $4a^2 + 2a - 3a^2 + a - 5 + 5a$

⑭  $11 - 2x^2 - 3 - 4x + 2x^2 - 3 + x$

⑮  $8mn - n^2 + 3m + 2n^2 - mn - 5m$

⑯  $2(x - y)$

⑰  $5(3a + 4b)$

⑱  $3(a + b)$

⑲  $-2x(x - y)$

⑳  $3(2x - 4y)$

㉑  $-3n(2m + n)$

㉒  $2(a - 3b) + 3(2a + b)$

㉓  $4(3x - y) - 5x + 4y$

㉔  $8a - 3b + 3(a - 2b)$

㉕  $12x - 2(3x + y) - 4y$

㉖  $8n - 4(2m - n) + 5n$

㉗  $2(3x - y) - 3(x + 4y)$

㉘  $3(2n + 4m) - 4(n + 2m)$

㉙  $a(2a - b) - b(3a + 2) - 4b$

㉚  $4x(x - y) - x(3x + 2y)$

㉛  $2x(3x - 5y) + 4(xy - 3x^2)$

㉜  $-3a(a - 2b) - 4(2ab + a^2)$

## Challenge Problems

㉝  $2x^2y - 3xy^2 - x^2y + 4x^2 + 5xy^2$

㉞  $2ab - 3a^2b + 4ab - 2ab^2 - a^2b$

㉟  $3x(xy - 2y^2) - 2y(y - 2x^2)$

㊱  $2a(4ab + b^2) - 3b(2b + 4a^2)$

㊲  $2ab(a^2 - 3b) - 4a^2(ab + 2) - a^2$

㊳  $3xy(2x - y) - y^2(x + 2) - x^2y$

# Simplifying Expressions

## PRACTICE TEST #1

Combining Terms:

- ①  $3a + 2b - 5ab + 4b - 5a + ab$
- ②  $8 - y^2 - 6 - 4y^2 + 2 + 3y^2$

Distributive Property:

- ③  $3(2x - 5y)$
- ④  $2(a - 3b) + 3(4a - 2b)$
- ⑤  $3a^2 - 2a(a - b) + 3ab$
- ⑥  $3x^2y - 2x(xy - x) + 3x^2$

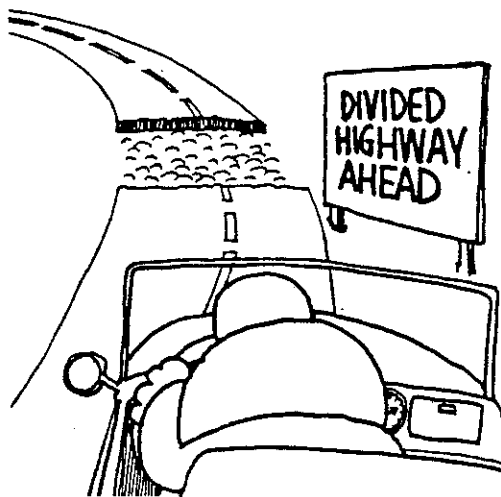
## PRACTICE TEST #2

Combining Terms:

- ①  $4x - 3xy + 8x - y + 5y + 4xy$
- ②  $12 - 4x^2 - 2 - x^2 + 5 + 3x^2$

Distributive Property:

- ③  $2(4a - b)$
- ④  $3(2x - 5y) - 4(x - 3y)$
- ⑤  $4x^2 - 2x(3x - y) - 2xy$
- ⑥  $5a^2b - a(3ab + a) - 4a^2$



# Radical Operations

## 1. RADICAL VALUES

The following terms are important to know:

### Definitions

A radical sign produces a value equal to the positive square root of the radicand.

$$\sqrt{64} = 8$$

radical sign ( $\sqrt{\quad}$ )      radicand (64)

The expression  $\sqrt{64}$  is read as: "radical sixty-four"

### Demonstration

Determine the value for each radical:

A)  $\sqrt{36} = 6$

B)  $\sqrt{100} = 10$

C)  $\sqrt{1} = 1$

Note: In each example above, the radicand is a perfect square. If the radicand is not a perfect square,

you can identify the value within a range of consecutive integers.

D)  $\sqrt{18}$        $\sqrt{16} < \sqrt{18} < \sqrt{25}$   
 $4 < \sqrt{18} < 5$

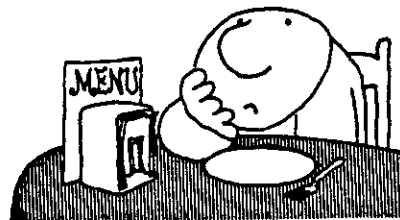
Between 4 and 5  
(closer to 4)

E)  $-\sqrt{34}$        $-\sqrt{36} < -\sqrt{34} < -\sqrt{25}$   
 $-6 < -\sqrt{34} < -5$

Between -6 and -5  
(closer to -6)

Note: Always use less than signs ( $<$ ) when placing the value between consecutive integers. Be sure to place the smaller value on the left of your compound expression. This is important to remember when dealing with negative values. (-6 is left of -5)

IT'S NOT EATING ALONE  
THAT I MIND SO MUCH  
...IT'S ALWAYS HAVING  
TO PICK UP THE CHECK!!



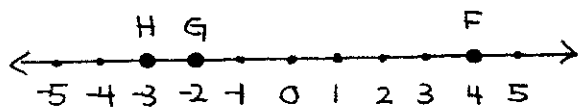


# Radical Operations

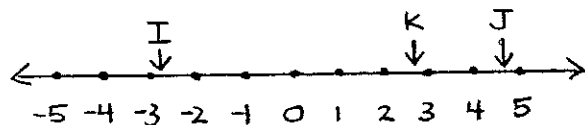
## Demonstration

Place each value on a number line:

F)  $\sqrt{16}$     G)  $-\sqrt{4}$     H)  $-\sqrt{9}$



I)  $-\sqrt{8}$     J)  $\sqrt{22}$     K)  $\sqrt{3}$



Write a compound sentence using less than signs to express each of the following within a range of consecutive integers:

⑮  $\sqrt{15}$     ⑲  $\sqrt{17}$     ⑳  $\sqrt{55}$

⑯  $\sqrt{37}$     ㉑  $\sqrt{85}$     ㉒  $\sqrt{2}$

⑰  $-\sqrt{11}$     ㉓  $-\sqrt{40}$     ㉔  $-\sqrt{20}$

⑱  $-\sqrt{26}$     ㉕  $-\sqrt{70}$     ㉖  $-\sqrt{105}$

Use a bold circle to indicate each value on a number line:

㉗  $A = \sqrt{9}$      $B = \sqrt{36}$      $C = -\sqrt{16}$

㉘  $D = -\sqrt{1}$      $E = -\sqrt{25}$      $F = \sqrt{4}$

Use an arrow to indicate each value on a number line:

㉙  $G = \sqrt{7}$      $H = -\sqrt{17}$      $I = \sqrt{40}$

㉚  $J = -\sqrt{10}$      $K = \sqrt{5}$      $L = \sqrt{28}$

## Problem Set 11.1

### Radical Values

Determine the value for each radical:

①  $\sqrt{49}$

⑧  $-\sqrt{121}$

②  $\sqrt{144}$

⑨  $\sqrt{169}$

③  $\sqrt{25}$

⑩  $\sqrt{400}$

④  $\sqrt{1}$

⑪  $\sqrt{16}$

⑤  $-\sqrt{1}$

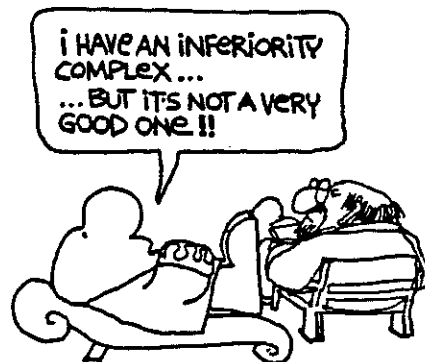
⑫  $-\sqrt{9}$

⑥  $-\sqrt{81}$

⑬  $-\sqrt{196}$

⑦  $-\sqrt{4}$

⑭  $\sqrt{225}$



# Radical Operations

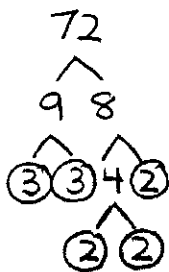
## 2. SIMPLIFYING RADICALS

If the radicand is not a perfect square, you can use a factor tree to determine the prime factorization. Then identify and pull out perfect squares to simplify.

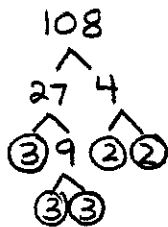
### Demonstration

Simplify each radical:

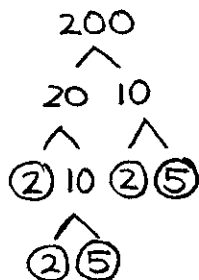
a)  $\sqrt{72}$   
 $\sqrt{2 \cdot 2 \cdot 2 \cdot 3 \cdot 3}$   
 $2 \cdot 3 \sqrt{2}$   
 $6\sqrt{2}$



b)  $3\sqrt{108}$   
 $3\sqrt{2 \cdot 2 \cdot 3 \cdot 3 \cdot 3}$   
 $3 \cdot 2 \cdot 3 \sqrt{3} = 18\sqrt{3}$



c)  $7\sqrt{200}$   
 $7\sqrt{2 \cdot 2 \cdot 2 \cdot 5 \cdot 5}$   
 $7 \cdot 2 \cdot 5 \sqrt{2}$   
 $70\sqrt{2}$



### Problem Set 11.2

### Simplifying Radicals

Simplify each radical:

- |                |                 |                 |
|----------------|-----------------|-----------------|
| ① $\sqrt{96}$  | ⑪ $\sqrt{12}$   | ⑳ $2\sqrt{250}$ |
| ② $\sqrt{54}$  | ⑫ $\sqrt{98}$   | ㉑ $7\sqrt{8}$   |
| ③ $\sqrt{120}$ | ⑬ $\sqrt{15}$   | ㉒ $9\sqrt{75}$  |
| ④ $\sqrt{216}$ | ⑭ $\sqrt{240}$  | ㉓ $2\sqrt{60}$  |
| ⑤ $\sqrt{50}$  | ⑮ $\sqrt{14}$   | ㉔ $3\sqrt{56}$  |
| ⑥ $\sqrt{112}$ | ⑯ $\sqrt{6}$    | ㉕ $3\sqrt{450}$ |
| ⑦ $\sqrt{48}$  | ⑰ $3\sqrt{40}$  | ㉖ $7\sqrt{288}$ |
| ⑧ $\sqrt{80}$  | ⑱ $2\sqrt{32}$  | ㉗ $4\sqrt{432}$ |
| ⑨ $\sqrt{90}$  | ㉑ $5\sqrt{18}$  | ㉘ $2\sqrt{200}$ |
| ⑩ $\sqrt{24}$  | ㉒ $3\sqrt{162}$ | ㉙ $5\sqrt{243}$ |



# Radical Operations

## 3. COMBINING RADICAL TERMS

When adding or subtracting radical terms, you can only combine like terms.

### Demonstration

Add/Subtract the following radical terms:

A)  $3\sqrt{5} + 2\sqrt{5} - \sqrt{5}$   
 $4\sqrt{5}$

B)  $4\sqrt{3} + 2\sqrt{7} - 3\sqrt{3}$   
 $\sqrt{3} + 2\sqrt{7}$

Sometimes terms can be combined after they are simplified.

C)  $\sqrt{27} - 3\sqrt{2} + 4\sqrt{3}$   
 $\sqrt{3 \cdot 3 \cdot 3} - 3\sqrt{2} + 4\sqrt{3}$   
 $3\sqrt{3} - 3\sqrt{2} + 4\sqrt{3}$   
 $7\sqrt{3} - 3\sqrt{2}$

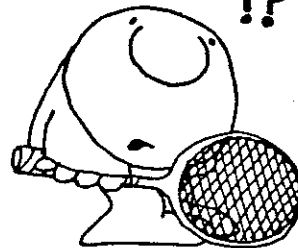
D)  $2\sqrt{8} - 3\sqrt{2} + \sqrt{125} - \sqrt{2}$   
 $2\sqrt{2 \cdot 2 \cdot 2} - 3\sqrt{2} + \sqrt{5 \cdot 5 \cdot 5} - \sqrt{2}$

$$4\sqrt{2} - 3\sqrt{2} + 5\sqrt{5} - \sqrt{2}$$

$$5\sqrt{5}$$

E)  $3\sqrt{12} - 2\sqrt{75} + \sqrt{27} + \sqrt{3}$   
 $3\sqrt{2 \cdot 2 \cdot 3} - 2\sqrt{3 \cdot 5 \cdot 5} + \sqrt{3 \cdot 3 \cdot 3} + \sqrt{3}$   
 $6\sqrt{3} - 10\sqrt{3} + 3\sqrt{3} + \sqrt{3}$   
 $0$

..HOW CAN SO MANY  
PEOPLE GET TURNED ON  
BY A GAME WHERE  
LOVE MEANS NOTHING  
??



### Problem Set 11.3 Combining Terms

Add/Subtract the following radical terms:

- |                           |                                      |
|---------------------------|--------------------------------------|
| ① $3\sqrt{7} - \sqrt{7}$  | ⑥ $4\sqrt{5} + 3\sqrt{7}$            |
| ② $2\sqrt{5} - 5\sqrt{5}$ | ⑦ $2\sqrt{7} - 3\sqrt{7}$            |
| ③ $3\sqrt{6} + 2\sqrt{6}$ | ⑧ $6\sqrt{2} - 3\sqrt{2}$            |
| ④ $4\sqrt{3} + 7\sqrt{3}$ | ⑨ $\sqrt{7} - 2\sqrt{7} + 3\sqrt{7}$ |
| ⑤ $3\sqrt{2} - 2\sqrt{3}$ | ⑩ $5\sqrt{3} - 2\sqrt{3} - \sqrt{3}$ |

# Radical Operations

⑪  $3\sqrt{3} - 2\sqrt{5} + 3\sqrt{5} - 3\sqrt{3}$

⑫  $2\sqrt{7} - 3\sqrt{5} + 4\sqrt{7} - 2\sqrt{7}$

⑬  $4\sqrt{3} + 3\sqrt{2} - 2\sqrt{3} - 6\sqrt{2}$

⑭  $3\sqrt{5} + \sqrt{6} - 2\sqrt{5} + \sqrt{5}$

⑮  $3\sqrt{10} - 2\sqrt{5} + 3\sqrt{2} - 4\sqrt{10}$

⑯  $3\sqrt{7} - 2\sqrt{3} + 2\sqrt{7} - 3\sqrt{3}$

## Challenge Problems

⑰  $3\sqrt{2} + \sqrt{18} - \sqrt{50}$

⑱  $4\sqrt{5} + \sqrt{20} - \sqrt{125}$

⑲  $4\sqrt{2} - 3\sqrt{6} - \sqrt{8}$

⑳  $\sqrt{32} - \sqrt{18} + 2\sqrt{50}$

㉑  $2\sqrt{27} + \sqrt{12} - 8\sqrt{3}$

㉒  $3\sqrt{3} - 2\sqrt{20} - \sqrt{5} + 2\sqrt{12}$

㉓  $2\sqrt{8} - 3\sqrt{12} + \sqrt{75}$

㉔  $3\sqrt{3} - 2\sqrt{45} + \sqrt{108} + 6\sqrt{5}$

㉕  $3\sqrt{18} - \sqrt{128} + 2\sqrt{8}$

㉖  $2\sqrt{6} - 3\sqrt{54} + \sqrt{12}$

㉗  $2\sqrt{8} - 3\sqrt{18} + \sqrt{50}$

## Review Problems

Indicate the value of each radical:

⑳  $\sqrt{49}$

㉑  $-\sqrt{81}$

Use arrows to indicate each radical on a number line:

㉒  $A = \sqrt{15}$     $B = -\sqrt{20}$     $C = \sqrt{5}$



## 4. MULTIPLYING RADICALS

When multiplying radical expressions, multiply the coefficients and multiply the radicands. It is not necessary to have like radicands when multiplying.

# Radical Operations

## Demonstration

Multiply and simplify:

A)  $(\sqrt{3})(\sqrt{6})$

$$\sqrt{18} = \sqrt{2 \cdot 3 \cdot 3} = 3\sqrt{2}$$

B)  $(2\sqrt{2})(5\sqrt{6})$

$$10\sqrt{12} = 10\sqrt{2 \cdot 2 \cdot 3} = 20\sqrt{3}$$

C)  $(\sqrt{5})^2$

$$\sqrt{25} = 5$$

D)  $(3\sqrt{2})^2$

$$9\sqrt{4} = 9 \cdot 2 = 18$$

E)  $3(\sqrt{2} - 2\sqrt{3})$

$$3\sqrt{2} - 6\sqrt{3}$$

F)  $2\sqrt{3}(\sqrt{6} - 5)$

$$2\sqrt{18} - 10\sqrt{3}$$

$$2\sqrt{2 \cdot 3 \cdot 3} - 10\sqrt{3}$$

$$6\sqrt{2} - 10\sqrt{3}$$



## Problem Set 11.4

### Multiplying Radicals

Multiply and simplify:

①  $(\sqrt{2})(\sqrt{5})$

⑩  $(3\sqrt{2})^2$

②  $(\sqrt{3})(\sqrt{2})$

⑪  $(4\sqrt{10})(3\sqrt{6})$

③  $(\sqrt{7})(\sqrt{3})$

⑫  $(3\sqrt{3})(6\sqrt{15})$

④  $(\sqrt{5})(\sqrt{3})$

⑬  $(2\sqrt{3})(\sqrt{2})(\sqrt{6})$

⑤  $(\sqrt{6})(\sqrt{2})$

⑭  $(\sqrt{5})(2\sqrt{3})(\sqrt{6})$

⑥  $(\sqrt{5})(\sqrt{10})$

⑮  $(3\sqrt{2})(4\sqrt{14})$

⑦  $(\sqrt{3})^2$

⑯  $(2\sqrt{5})(5\sqrt{10})$

⑧  $(\sqrt{6})^2$

⑰  $(\sqrt{6})(\sqrt{2})(\sqrt{10})$

⑨  $(2\sqrt{5})^2$

⑱  $(\sqrt{3})(2\sqrt{6})(\sqrt{2})$

# Radical Operations

## Challenge Problems

Multiply and simplify:

- (19)  $3(2\sqrt{5} - \sqrt{3})$   
 (20)  $2\sqrt{3}(\sqrt{2} - 1)$   
 (21)  $3\sqrt{5}(2\sqrt{10} - \sqrt{5})$   
 (22)  $\sqrt{6}(2\sqrt{3} - \sqrt{2})$   
 (23)  $2\sqrt{2}(3\sqrt{2} - \sqrt{18})$   
 (24)  $\sqrt{3}(4\sqrt{12} - 2\sqrt{3})$

## Review Problems

Use arrows to indicate each radical on a number line:

(25)  $A = \sqrt{29}$     $B = -\sqrt{19}$     $C = -\sqrt{10}$

Add / Subtract and simplify:

- (26)  $3\sqrt{12} + \sqrt{8} - 3\sqrt{2} - \sqrt{3}$   
 (27)  $4\sqrt{18} - \sqrt{125} + \sqrt{32} + \sqrt{5}$   
 (28)  $2\sqrt{24} - \sqrt{27}$   
 (29)  $5\sqrt{8} + \sqrt{128}$   
 (30)  $\sqrt{3}(\sqrt{6} - 2\sqrt{3}) - 3\sqrt{2}$   
 (31)  $5\sqrt{3} - 3\sqrt{2}(\sqrt{6} + \sqrt{12})$



## REVIEW & PRACTICE

Determine the value for each radical:

- (1)  $\sqrt{144}$    (3)  $-\sqrt{81}$    (5)  $-\sqrt{49}$   
 (2)  $-\sqrt{64}$    (4)  $\sqrt{169}$    (6)  $\sqrt{1}$

Write a compound sentence using less than signs to express each within a range of consecutive integers:

- (7)  $\sqrt{125}$    (8)  $-\sqrt{19}$    (9)  $-\sqrt{85}$

Use an arrow to indicate each value on a number line:

- (10)  $A = \sqrt{22}$     $B = -\sqrt{3}$     $C = -\sqrt{29}$   
 (11)  $D = -\sqrt{5}$     $E = \sqrt{14}$     $F = \sqrt{45}$

# Radical Operations

Simplify each radical:

- ⑫  $\sqrt{160}$     ⑮  $2\sqrt{96}$     ⑳  $3\sqrt{375}$   
 ⑬  $\sqrt{84}$     ⑰  $4\sqrt{360}$     ㉑  $4\sqrt{196}$   
 ⑭  $\sqrt{180}$     ⑱  $3\sqrt{12}$     ㉒  $2\sqrt{27}$   
 ⑮  $\sqrt{150}$     ⑲  $5\sqrt{126}$     ㉓  $3\sqrt{128}$

Add/Subtract:

- ㉔  $3\sqrt{5} + 2\sqrt{3} - 4\sqrt{5} - 3\sqrt{3}$   
 ㉕  $\sqrt{6} - 2\sqrt{3} + 3\sqrt{2} - \sqrt{2} + 2\sqrt{3}$   
 ㉖  $4\sqrt{5} - 3\sqrt{5} - 2\sqrt{7} + 3\sqrt{7}$   
 ㉗  $-\sqrt{3} - 3\sqrt{2} + \sqrt{2} - 4\sqrt{2}$   
 ㉘  $3\sqrt{5} - 2\sqrt{3} + \sqrt{5} + 2\sqrt{3} - 4\sqrt{5}$   
 ㉙  $5\sqrt{6} - \sqrt{7} - 3\sqrt{6} - 2\sqrt{7} + \sqrt{6}$

multiply and simplify:

- ㉚  $(\sqrt{3})(2\sqrt{5})$     ㉝  $(2\sqrt{5})^2$   
 ㉛  $(3\sqrt{2})(\sqrt{6})$     ㉞  $(3\sqrt{6})^2$   
 ㉜  $(\sqrt{5})(2\sqrt{15})$     ㉟  $(\sqrt{7})^2$   
 ㉟  $(3\sqrt{3})(2\sqrt{6})$     ㊱  $(\sqrt{6})(\sqrt{8})$   
 ㊲  $(3\sqrt{2})^2$     ㊳  $(3\sqrt{2})(4\sqrt{12})$

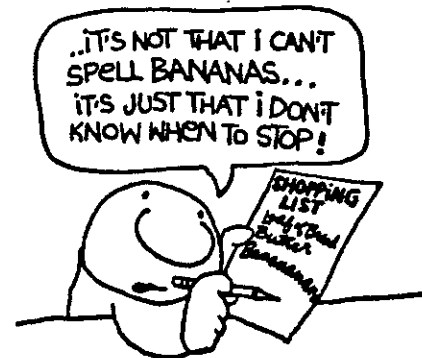
## Challenge Problems

Combine and simplify:

- ㉔⑩  $2\sqrt{12} - 3\sqrt{3} + 4\sqrt{75}$   
 ㉔⑪  $\sqrt{8} + 3\sqrt{18} - 4\sqrt{50}$   
 ㉔⑫  $3\sqrt{125} - 2\sqrt{8} + 3\sqrt{5}$   
 ㉔⑬  $4\sqrt{27} + 2\sqrt{45} - 3\sqrt{12}$   
 ㉔⑭  $\sqrt{18} - \sqrt{32} + \sqrt{98}$   
 ㉔⑮  $\sqrt{200} - 3\sqrt{108} + 2\sqrt{50}$

multiply and simplify:

- ㉔⑯  $\sqrt{3}(2\sqrt{6} - 3\sqrt{3})$   
 ㉔⑰  $\sqrt{2}(\sqrt{8} - 2\sqrt{2})$   
 ㉔⑱  $3\sqrt{3}(\sqrt{6} - 5\sqrt{2})$   
 ㉔⑲  $2\sqrt{5}(3\sqrt{5} + \sqrt{2})$



# Radical Operations

## PRACTICE TEST #1

Use an arrow to indicate the value on a number line:

①  $-\sqrt{11}$

Simplify each radical:

②  $\sqrt{180}$

③  $4\sqrt{400}$

Add / Subtract:

④  $4\sqrt{2} - \sqrt{3} + 2\sqrt{2} + 3\sqrt{3}$

Multiply and simplify:

⑤  $(3\sqrt{10})(5\sqrt{2})$

Combine and simplify:

⑥  $\sqrt{12} - 4\sqrt{3} - 3\sqrt{75}$

## PRACTICE TEST #2

Use an arrow to indicate the value on a number line:

①  $\sqrt{20}$

Simplify each radical:

②  $\sqrt{250}$

③  $3\sqrt{128}$

Add / Subtract:

④  $\sqrt{6} - 2\sqrt{2} - 3\sqrt{6} + 3\sqrt{2}$

Multiply and simplify:

⑤  $(\sqrt{3})(2\sqrt{6})(3\sqrt{5})$

Combine and simplify:

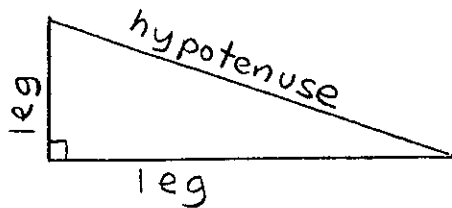
⑥  $\sqrt{125} - 4\sqrt{5} + 2\sqrt{45}$



# Radical Operations

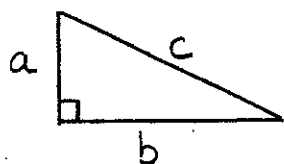
## THE PYTHAGOREAN THEOREM

Definition: In a right triangle, the sides adjacent to the right angle are called legs. The side opposite from the right angle is called the hypotenuse.



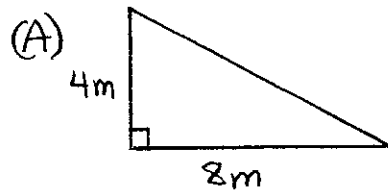
The hypotenuse is always the longest side.

**THE PYTHAGOREAN THEOREM:**  
In ancient Greece, a mathematician named Pythagoras discovered a relationship that exists in all right triangles. His theorem allows you to find the length of a side of a right triangle if only two sides are given.



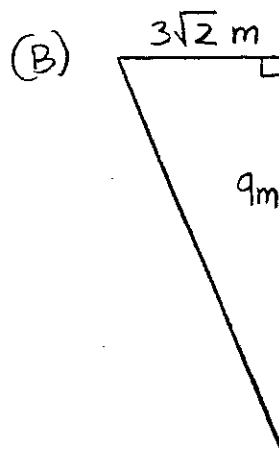
$$a^2 + b^2 = c^2$$

**DEMONSTRATION:** Use the Pythagorean Theorem to determine the unknown side of each right triangle. Be sure to simplify radicals.



$$\begin{aligned} a^2 + b^2 &= c^2 \\ (4)^2 + (8)^2 &= c^2 \\ 16 + 64 &= c^2 \\ 80 &= c^2 \\ \sqrt{80} &= c \end{aligned}$$

$$\begin{aligned} c &= \sqrt{80} = \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 5} \\ c &= 4\sqrt{5} \text{ m} \end{aligned}$$



$$\begin{aligned} a^2 + b^2 &= c^2 \\ (3\sqrt{2})^2 + (9)^2 &= c^2 \\ 18 + 81 &= c^2 \\ 99 &= c^2 \\ \sqrt{99} &= c \\ c &= \sqrt{3 \cdot 3 \cdot 11} \\ c &= 3\sqrt{11} \text{ m} \end{aligned}$$

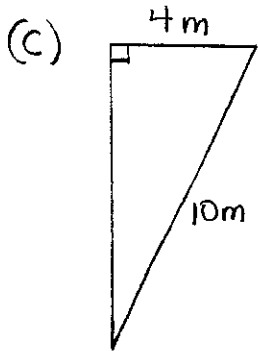
In each problem above, the missing side is the hypotenuse (opposite from the right angle).

In the following problems,

UNIT 11

# Radical Operations

the missing side is one of the legs.



$$a^2 + b^2 = c^2$$

$$(4)^2 + b^2 = (10)^2$$

$$16 + b^2 = 100$$

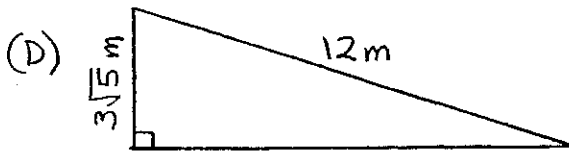
$$b^2 = 100 - 16$$

$$b^2 = 84$$

$$b = \sqrt{84}$$

$$b = \sqrt{2 \cdot 2 \cdot 3 \cdot 7}$$

$$b = 2\sqrt{21} \text{ m}$$



$$a^2 + b^2 = c^2$$

$$(3\sqrt{5})^2 + b^2 = (12)^2$$

$$45 + b^2 = 144$$

$$b^2 = 144 - 45$$

$$b^2 = 99$$

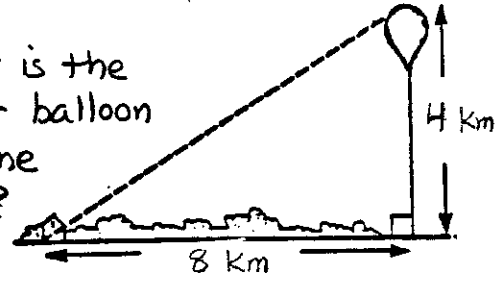
$$b = \sqrt{99} = \sqrt{3 \cdot 3 \cdot 11} = 3\sqrt{11} \text{ m}$$

THE WHOLE WORLD ISN'T AGAINST YOU....THERE ARE BILLIONS OF PEOPLE WHO DON'T CARE ONE WAY OR ANOTHER!

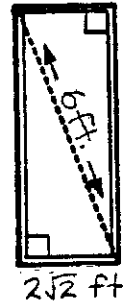


Use the Pythagorean Theorem to answer each of the following. Simplify all radicals.

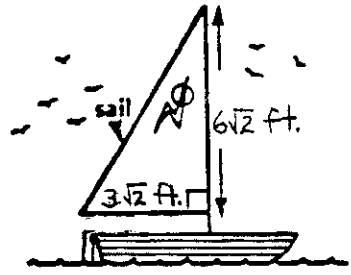
- ① How far is the weather balloon from the station?



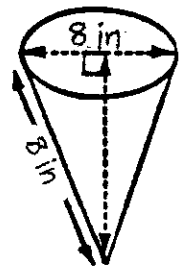
- ② How high is the window screen?



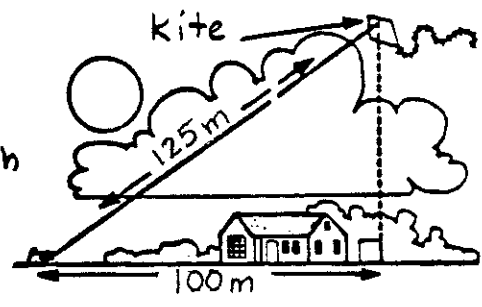
- ③ How long is the longest side of the sail?



- ④ How deep is the cone?



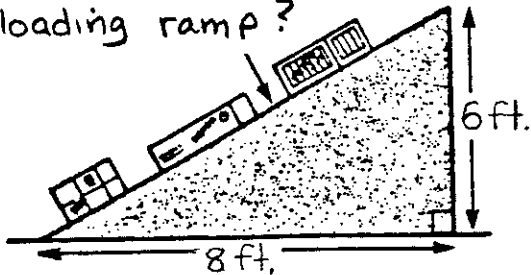
- ⑤ How high is the kite?



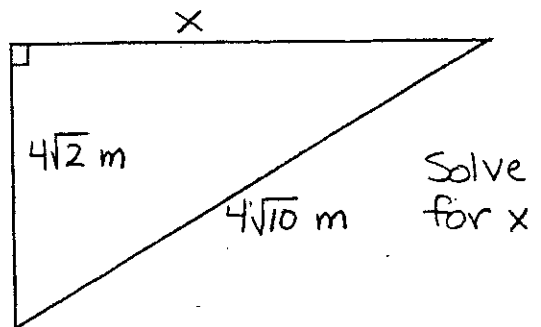
(use a calculator to solve)

# Radical Operations

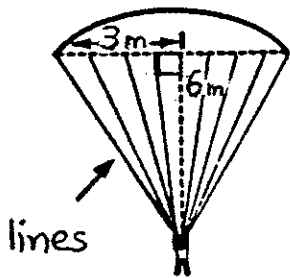
⑥ How long is the loading ramp?



⑪

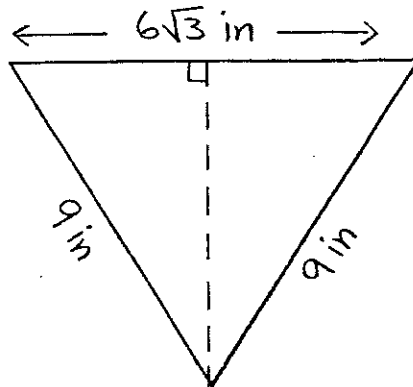


⑦



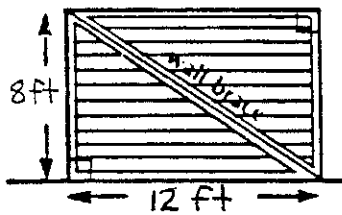
How long are the suspension lines of the parachute?

⑫

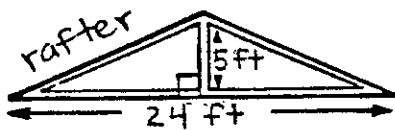


How long is the dotted line?

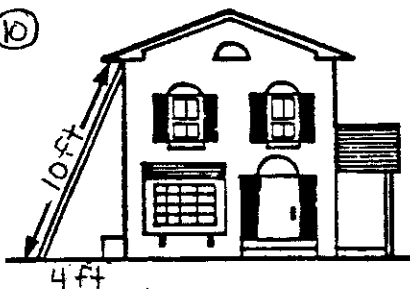
⑧ How long is the wall brace?



⑨ How long is the rafter?



⑩



At what height does the ladder touch the house?



# Cumulative Review

## REVIEW & PRACTICE

Integers:

- ①  $(-3) + (+7)$       ③  $(-7) - (-5)$   
 ②  $(-8) \times (-3)$       ④  $(-4) \div (+4)$   
 ⑤  $(-3) + (-4) - (-7) + (-6) - (+8)$   
 ⑥  $(-3) \times (-1) \times (+2) \times (-2)$

Exponents:

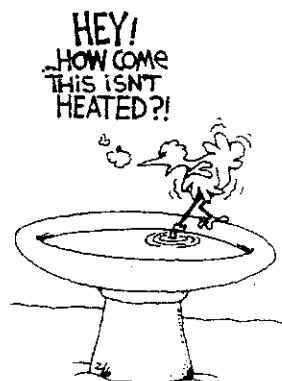
- ⑦  $-5^2$       ⑨  $(-4)^0$   
 ⑧  $(-5)^2$       ⑩  $(-3)^3$

Order of operations:

- ⑪  $(-6) - (-2) \times (-3) + (+4)$   
 ⑫  $(-4) \div (-1) - (-2) \times (+5)$   
 ⑬  $(-2)^3 - (-3)^2$   
 ⑭  $(-3)^0 - 3^2 + (-2)^4$   
 ⑮  $\frac{(-1)^2 - (-3)^3}{(-6)^2 - 2^5}$   
 ⑯  $\frac{(-1)(-3) - 4^2 + 5^0}{(-2)^2 - 2(-1)}$

Evaluating expressions:  
 $a = -3$     $b = -2$     $c = -1$

- ⑰  $2a - 3b + c$   
 ⑱  $ab^2 - 2bc^3$   
 ⑲  $-3(a+b) - c^2$   
 ⑳  $2ab - abc$   
 ㉑  $\frac{a^2 b^2 c}{2b}$   
 ㉒  $\frac{3a + 2b}{b^2 c^3}$



Simplifying expressions:

- ㉓  $4x - 3 + 2x + 7$   
 ㉔  $n - 3m + 4 - 2n + m - 3$   
 ㉕  $-a^2 - ab + 3ab - 2a^2 - 4ab$   
 ㉖  $n^2 - 2nm + 3n^2 - 4 + 5nm$   
 ㉗  $3(x - 2y) - 4(2x + y)$   
 ㉘  $2(a + b) - 3(2a - 3b)$   
 ㉙  $a(a - b) + 3a^2 - 2ab$

# Cumulative Review

③①  $2x(x-3y) + y(x-3y)$

③②  $2a(a-b^2) - b(ab-2)$

③③  $4(2x-y^2) - y(x+3y) + 2x$

Simplify each radical:

③④  $\sqrt{108}$

③⑤  $2\sqrt{27}$

③⑥  $\sqrt{40}$

③⑦  $-3\sqrt{96}$

③⑧  $\sqrt{245}$

③⑨  $2\sqrt{80}$

Multiply and simplify:

④①  $(\sqrt{3})(2\sqrt{6})$

④②  $(3\sqrt{2})(4\sqrt{10})$

④③  $(3\sqrt{3})^2$

④④  $(2\sqrt{5})^2$

④⑤  $2\sqrt{3}(4\sqrt{3} - \sqrt{6})$

④⑥  $\sqrt{2}(3\sqrt{10} - \sqrt{6})$

Combine and simplify:

④⑦  $\sqrt{3} + 2\sqrt{12} - 4\sqrt{75}$

④⑧  $3\sqrt{20} - 2\sqrt{27} + 2\sqrt{45}$

④⑨  $2\sqrt{32} - 3\sqrt{50} + \sqrt{63}$

④⑩  $\sqrt{125} + 5\sqrt{80} + 2\sqrt{81}$

④⑪  $3\sqrt{200} - 2\sqrt{98} - 3\sqrt{72}$

④⑫  $\sqrt{24} + 2\sqrt{54} - 5\sqrt{150}$

## PRACTICE TEST

Integers, exponents, and order of operations:

①  $(-3) + (-4) - (-5) - (+4)$

②  $(-20) \div (-4)$

③  $(-3)^2$

④  $-3^2$



# Cumulative Review

⑤  $(-4)^0 - (-2)^3$

⑥  $(-3)^2 - (-4)(+3) + (-2)$

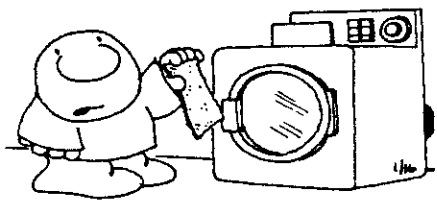
Evaluating expressions:  
 $a=3$   $b=-2$   $c=-1$

⑦  $-2a - 3b$

⑧  $a - b^3 + 2c^2$

⑨  $2a(b+c)^2$

...YOU KNOW YOUR LIFE  
 NEEDS MORE EXCITEMENT  
 ...WHEN YOUR GREATEST  
 CHALLENGE ALL WEEK IS  
 REMOVING THE LINT FROM  
 YOUR DRYER'S LINT-SCREEN  
 ALL IN ONE PIECE!



Simplifying expressions:

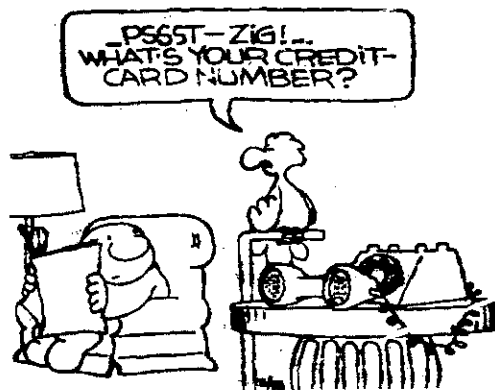
⑩  $3a + 2b - 4a - 3b$

⑪  $2a^2 - 3ab + 4 - 5ab - 3a^2$

⑫  $2(a+b) - 3a + 4b$

⑬  $3a(a-b) - 2b(b+3)$

⑭  $4a(a+b^2) - 2a^2(3-2b)$



Simplify each radical:

⑮  $\sqrt{60}$

⑯  $3\sqrt{72}$

Multiply, combine,  
 simplify:

⑰  $(3\sqrt{5})^2$

⑱  $(2\sqrt{3})(4\sqrt{21})$

⑲  $\sqrt{12} - 2\sqrt{108} + 2\sqrt{48}$

⑳  $3\sqrt{20} - \sqrt{45} + 2\sqrt{25}$

THE LAST TIME  
 I REALLY TRIED  
 TO PUSH MYSELF  
 ...I ENDED UP  
 GETTING INTO A  
 SHOVING MATCH!



# Equations & Inequalities

## 1. SOLVING EQUATIONS

This lesson is a review of equation work done during Unit 4 last year.

### Demonstration

A) Simple equation:

$$8 - n = 12$$

$$8^{-8} - n = 12^{-8}$$

$$-n = 4$$

$$n = -4$$

B) Two step equation:

$$3a - 4 = 11$$

$$3a - 4^{+4} = 11^{+4}$$

$$3a = 15$$

$$\left(\frac{1}{3}\right)(3a) = \left(\frac{1}{3}\right)(15)$$

$$a = 15/3$$

$$a = 5$$

C) Combining terms:

$$3n - 5 + n = 4 - n + 6$$

$$4n - 5 = 10 - n$$

$$4n^{+n} - 5 = 10 - n^{+n}$$

$$5n - 5^{+5} = 10^{+5}$$

$$5n = 15$$

$$\left(\frac{1}{5}\right)(5n) = \left(\frac{1}{5}\right)(15)$$

$$n = 15/5$$

$$n = 3$$

D) Distributive property:

$$4 - 3(2n - 1) = 13 - 4n$$

$$4 \boxed{-3(2n - 1)} = 13 - 4n$$

$$4 - 6n + 3 = 13 - 4n$$

$$7 - 6n^{+n} = 13 - 4n^{+n}$$

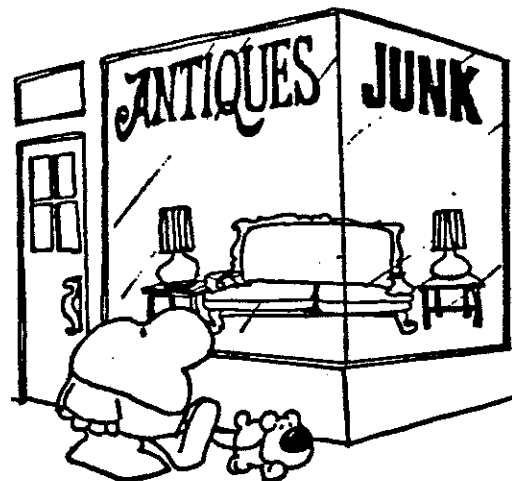
$$7^{-7} - 2n = 13^{-7}$$

$$-2n = 6$$

$$\left(-\frac{1}{2}\right)(-2n) = \left(-\frac{1}{2}\right)(6)$$

$$n = -6/2$$

$$n = -3$$



# Equations & Inequalities

## Problem Set 12.1

### Solving Equations

Solve each equation and show all steps:

- ①  $x - 3 = 12$
- ②  $12 + n = -5$
- ③  $7 = 15 - a$
- ④  $13 - n = -1$
- ⑤  $2n - 5 = 11$
- ⑥  $8 - 3n = 20$
- ⑦  $-14 = 5x - 4$
- ⑧  $4 - 6a = -8$
- ⑨  $4n - 3 - n + 7 = 2n + 3$
- ⑩  $9 - x + 3 + 5x = x - 4 + 7x - 4$
- ⑪  $a - 2a - 3a + 1 = 2 - 2a + 3$
- ⑫  $-3 + 4n - 5 - 7n = n + 7 - 2n - 1$
- ⑬  $2(4n - 3) = 5n + 15$
- ⑭  $10 - 3(x - 4) = x + 2$
- ⑮  $4 - 2(3a + 5) = 3(a - 4) - 3$

⑯  $2 - 3(n - 4) = 2(3n - 1) - 2$

⑰  $2(n + 4) - 3(2n - 4) = 8$

⑱  $3(4x - 2) - 2(5x + 3) = 6x$

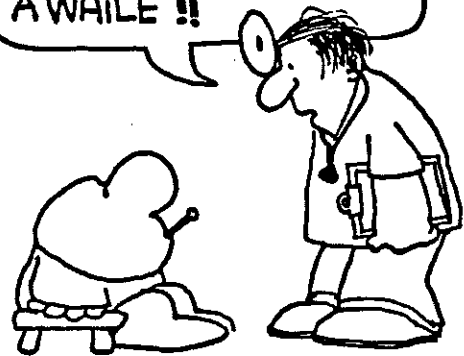
### Challenge Problems

In the following problems, the variable is not an integer. Keep the variable in fraction (or improper fraction) form.

⑲  $n - 2(4n - 5) = 2n - 3$

⑳  $7 - 3(4x - 2) + x = -3$

YOU SHOULD BE ALL RIGHT  
... JUST AVOID THE  
ENVIRONMENT FOR  
A WHILE !!



### 2. PROBLEM SOLVING

This lesson reviews work with integer problem solving from Unit 4.



# Equations & Inequalities

## Demonstration

Set up an equation to solve each problem:

- A) A number increased by six is negative two. Find the number.

$$n + 6 = -2$$

$$n + 6^{-6} = -2^{-6}$$

$$n = -8$$

-8

- B) Three times a number decreased by seven is three less than twice the number. Find the number.

$$3n - 7 = 2n - 3$$

$$3n^{-2n} - 7 = 2n^{-2n} - 3$$

$$n - 7^{+7} = -3^{+7}$$

$$n = 4$$

4

You also have to distribute the subtraction sign into the quantity.

- C) Four times a number decreased by five more than twice the number is nine. Find the number.

$$4n - (2n + 5) = 9$$

↑            ↑            ↑            ↑            ↑  
 four    decreased    five more    is    nine  
 times    by            than twice    the number  
 a  
 number

$$4n - (2n + 5) = 9$$

$$4n - 2n - 5 = 9$$

$$2n - 5^{+5} = 9^{+5}$$

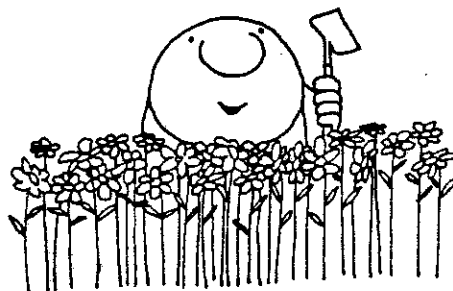
$$2n = 14$$

$$\left(\frac{1}{2}\right)(2n) = \left(\frac{1}{2}\right)(14)$$

$$n = 7$$

7

..FLOWERS HELP TO MAKE UP FOR ALL THE UGLY IN THE WORLD !!



## Demonstration

The following problems require you to recognize that a "quantity" must be enclosed in parenthesis.



# Equations & Inequalities

number. Find the number.

- ⑧ Five times a number decreased by three more than twice the number is nine. Find the number.
- ⑨ Three times a number decreased by two less than the number is negative two. Find the number.
- ⑩ Three more than twice a number is equal to fifteen less than four times the number. Find the number.
- ⑪ Five less than four times a number is equal to three more than six times the number. Find the number.
- ⑫ Two more than twice a number decreased by three less than four times the number is negative seven. Find the number.

## Challenge Problems

- ⑬ Two less than four times a number decreased by five more than the number is equal to eleven more than twice the number.

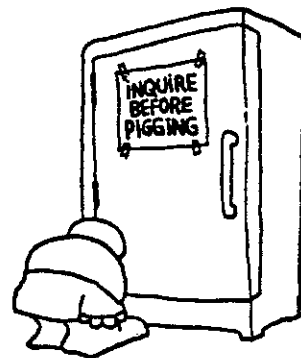
Find the number.

- ⑭ Six less than a number decreased by three less than three times the number is twelve more than the number. Find it.
- ⑮ Eight times a number decreased by six more than twice the number is equal to four times the number. Find the number.

### 3. INEQUALITIES

An equation has a single solution – one value for the variable that makes the equation a true statement.

An inequality can have an infinite number of values for the variable that make the inequality a true statement.



# Equations & Inequalities

## Demonstration

Solve each inequality:

A)  $3n - 5 < 7$

$$3n - 5^{+5} < 7^{+5}$$

$$3n < 12$$

$$(\frac{1}{3})(3n) < (\frac{1}{3})(12)$$

$$n < 4$$

Note: All values less than 4 make the statement true.

B)  $4(2x-3) \geq 5x+6$

$$4(2x-3) \geq 5x+6$$

$$8x^{-5x} - 12 \geq 5x^{-5x} + 6$$

$$3x^{-12} \geq 6^{+12}$$

$$3x \geq 18$$

$$(\frac{1}{3})(3x) \geq (\frac{1}{3})(18)$$

$$x \geq 6$$

Note: All values greater than or equal to 6 make the statement true.

Sometimes it is necessary to "flip" the inequality sign when solving. Here are some situations when

the sign must be "flipped":

C)  $12 - 3n < 15$

$$12^{-12} - 3n < 15^{-12}$$

$$-3n < 3$$

when both sides are multiplied by a negative value ( $-\frac{1}{3}$ ), flip the sign.

$$(-\frac{1}{3})(-3n) > (-\frac{1}{3})(3)$$

$$n > -1$$

D)  $3a - 4 \geq 4a + 7$

$$3a^{-4a} - 4 \geq 4a^{-4a} + 7$$

$$-a^{-4} - 4 \geq 7^{+4}$$

$$-a \geq 11$$

when a variable is changed from negative to positive, flip the sign. It is the same as multiplying both sides by  $-1$ .

$$-a \geq 11$$

$$a \leq -11$$

If you multiply both sides by the same negative, don't forget to flip the sign!

# Equations & Inequalities

$$E) -12 < x - 5$$

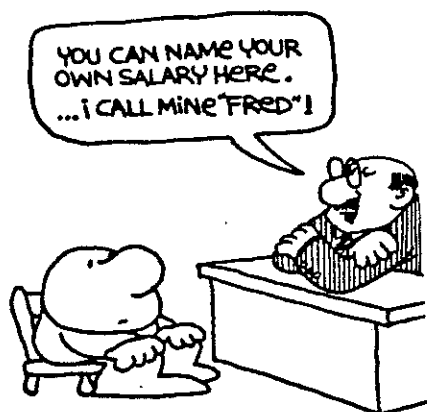
$$-12^{+5} < x - 5^{+5}$$

$$-7 < x$$

When the variable is isolated on the right (and must be moved to the left), flip the sign.

$$-7 < x$$

$$x > -7$$



## Problem Set 12.3 Inequalities

Solve each inequality and show all steps:

$$\textcircled{1} 2n - 3 < n - 9$$

$$\textcircled{2} 3x + 7 \geq x - 9$$

$$\textcircled{3} 2(x - 4) > -10$$

$$\textcircled{4} 3(4n - 1) \leq 9n + 6$$

$$\textcircled{5} 14 > 2(n - 3) - n$$

$$\textcircled{6} 6 - x \geq 15$$

$$\textcircled{7} n - 3 < 2n + 7$$

$$\textcircled{8} 2n > 3(2n + 4)$$

$$\textcircled{9} 3(n - 5) \leq 2(n + 2)$$

$$\textcircled{10} 5a - 7 > 9a - 11$$

$$\textcircled{11} 8 - 3x < -7$$

$$\textcircled{12} 17 \geq 4(n - 3) + 1$$

## Challenge Problems

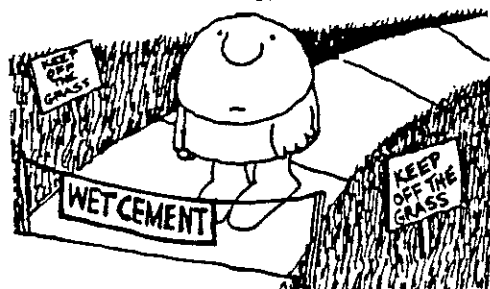
If the solution to a problem below involves a fraction, leave the answer in fraction or improper fraction form.

$$\textcircled{13} 2(n - 3) - 3(n + 2) \leq 7$$

$$\textcircled{14} 4x - 3(2x - 2) > x - 5$$

$$\textcircled{15} 3 + 2a - 5 \geq a - 4(a - 3)$$

$$\textcircled{16} 8n - 2(3n + 4) < 15 - n$$



# Equations & Inequalities

## 4. NUMBER LINE GRAPHING

The solution to an equation or inequality can be shown on a number line graph.

### Demonstration

Solve each equation or inequality and graph the solution:

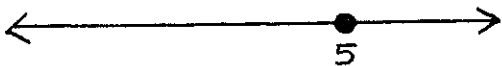
A)  $3n - 5 = 10$

$$3n - 5 = 10$$

$$3n = 15$$

$$(\frac{1}{3})(3n) = (\frac{1}{3})(15)$$

$$n = 5$$



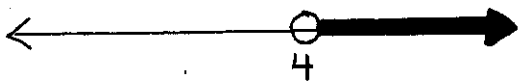
B)  $2n - 3 > 5$

$$2n - 3 > 5$$

$$2n > 8$$

$$(\frac{1}{2})(2n) > (\frac{1}{2})(8)$$

$$n > 4$$



Note: Put an "open" circle on 4 because 4 is not a solution. Be sure to shade in the arrow!

c)  $2(n-5) \geq 3n-6$

$$2(n-5) \geq 3n-6$$

$$2n-10 \geq 3n-6$$

$$-n-10 \geq -6$$

$$-n \geq 4$$

$$n \leq -4$$

Be sure to flip the sign!



Note: Put a "bold" circle on -4 because -4 is part of the solution. Don't forget to shade in the arrow!



Sometimes, when you are solving an equation or inequality, the variable "drops out."

Should this happen, there are two possibilities:

# Equations & Inequalities

● If the remaining statement is true, you have an identity. All solutions will work.

● If the remaining statement is false, you have a false equation (or inequality). No solutions will work.

## Demonstration

$$D) 2x - 4 = 2(x - 2)$$

$$2\overset{-2x}{x} - 4 = 2\overset{-2x}{x} - 4$$

$$-4 = -4 \text{ (true identity)}$$

All solutions



$$E) 2n - 6 < 3(n - 1) - n$$

$$2n - 6 < 3n - 3 - n$$

$$2\overset{-2n}{n} - 6 < 2\overset{-2n}{n} - 3$$

$$-6 < -3 \text{ (true identity)}$$

All solutions



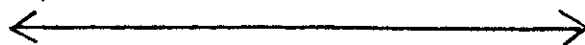
All possible values satisfy the original equation or inequality in the problems above.

$$F) 2(3x + 4) \geq 6x + 9$$

$$6\overset{-6x}{x} + 8 \geq 6\overset{-6x}{x} + 9$$

$$8 \geq 9 \text{ (false inequality)}$$

No Solutions



Note: The graph above is an empty number line because no solutions will satisfy the original inequality.

## Problem Set 12.4

### Number Line Graphing

#### PART I

Solve and graph the solution on a number line:

$$\textcircled{1} 4n - 3 = n - 12$$

$$\textcircled{2} 5x - 4 = 2(x + 1)$$

$$\textcircled{3} 3(n - 4) < 4n - 2$$

$$\textcircled{4} 2(3x - 1) \geq 8x - 10$$

$$\textcircled{5} 4(x - 3) - x \geq x - 6$$

$$\textcircled{6} 5n < 2n - 3 + n + 5$$

# Equations & Inequalities

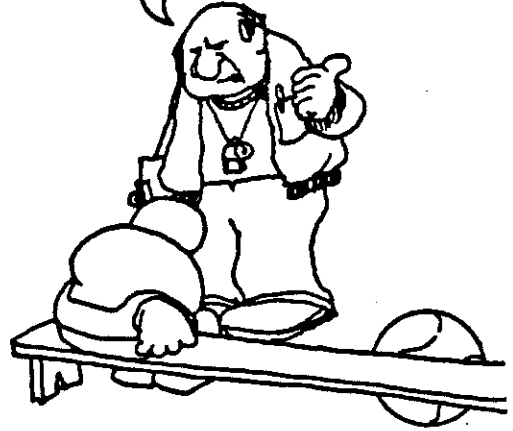
- ⑦  $4n - 3 = 8n - 11$   
 ⑧  $3(2x - 1) = 2(x - 4) - 7$   
 ⑨  $8 - 2(n - 3) < n - 1$   
 ⑩  $6 - 3(x + 1) \geq 5 - x$

## PART II

Solve and graph on a number line. If the variable drops out, indicate all or no solutions and show the graph:

- ⑪  $3(n - 1) > -6$   
 ⑫  $2x = 2(x - 1) + 2$   
 ⑬  $8 + 2n > 2(n + 3) + 4$   
 ⑭  $3n - 1 \leq 5n + 3$   
 ⑮  $2(3x + 1) = 6x - 2$   
 ⑯  $3(n - 1) \leq 2n - 1 + n$   
 ⑰  $x > 3(x - 2)$   
 ⑱  $4(n - 3) = 4n - 3$   
 ⑲  $2(2a + 4) > 5a + 17$   
 ⑳  $3x - 6 \leq 3(x - 2)$

OK ZIGGY...GET IN THERE AND GET FOULED !!



## REVIEW & PRACTICE

Solve each equation:

- ①  $3x - 2 = 4x - 5$   
 ②  $2(x - 4) = -12$   
 ③  $3n - 5 + 2n + 4 = n + 15$   
 ④  $n - 6 = 3(n - 2) + 4$   
 ⑤  $15 = 2(x - 3) + 1$   
 ⑥  $3n - 2 - n - 7 = n - 9$

Set up an equation to solve each problem:

- ⑦ A number increased by five is negative seven.



# Equations & Inequalities

Find the number.

- ⑧ Twice a number decreased by eight is two. Find the number.
- ⑨ Three more than three times a number decreased by two more than twice the number is one less than twice the number. Find the number.

- ⑩ Five times a number decreased by three less than twice the number is one less than the number. Find it.



THINGS TO DO TODAY	
1.	inhale
2.	exhale
3.	inhale
4.	exhale
5.	inhale
6.	exhale
7.	inhale
8.	exhale
9.	inhale
10.	exhale
11.	inhale
12.	exhale
13.	inhale
14.	exhale

- ⑪ Two more than four times a number decreased by one less than the number is one more than four times the number. Find the number.
- ⑫ Five times a number decreased by six times a number is equal to the

number increased by twelve. Find the number

Solve each inequality. Show all steps. Don't forget to flip the sign as necessary:

- ⑬  $2n < n - 7$
- ⑭  $3(2x - 4) \geq x + 3$
- ⑮  $a + 5 > 4(a - 1)$
- ⑯  $4x - 2 + 5 - 6x \leq 7$
- ⑰  $2n - 5 > 3(2n - 1) + 6$
- ⑱  $3(x - 3) - 2(x + 1) \leq 4$

Solve each equation or inequality and graph the solution on a number line:

- ⑲  $4x - 7 = 2x + 5$
- ⑳  $3(2n - 1) < 6n - 2$
- ㉑  $2(4x - 3) = 8x + 6$
- ㉒  $n + 3 \geq 3(n - 1)$
- ㉓  $16 < 2(x - 3)$
- ㉔  $3(n + 2) - n < 8$

# Equations & Inequalities

## PRACTICE TEST #1

Solve:

$$\textcircled{1} \quad 2(n-5) = 14$$

$$\textcircled{2} \quad 4x - 3 < 7x - 12$$

Set up an equation to solve:

- $\textcircled{3}$  Four times a number decreased by three less than twice the number is fifteen. Find the number.

Solve and graph on a number line:

$$\textcircled{4} \quad 3(4x-1) = 12x-1$$

$$\textcircled{5} \quad 5a \geq 3(3a-8)$$

Set up an equation to solve:

- $\textcircled{6}$  Three less than twice a number decreased by five more than four times the number equals twice the number. Find the number.

## PRACTICE TEST #2

Solve:

$$\textcircled{1} \quad 3(2x+7) = x+6$$

$$\textcircled{2} \quad 2n \geq 5(n-3)$$

Set up an equation to solve:

- $\textcircled{3}$  Three more than a number decreased by three times the number is nine more than the number. Find it.

Solve and graph on a number line:

$$\textcircled{4} \quad 2(n-5) = 3(n-7)$$

$$\textcircled{5} \quad a-5 < 3(a-3)$$

Set up an equation to solve:

- $\textcircled{6}$  Five more than a number decreased by two less than three times the number equals two decreased by three times the number. Find it.

# Working With Monomials

## 1. MULTIPLYING MONOMIALS

A monomial is a number, variable, or product of numbers and variables.

When you multiply monomials with the same base, you add the exponents:

$$a^x \cdot a^y = a^{x+y}$$

### Demonstration

Multiply these monomials:

A)  $x^3 \cdot x^2$

$$x^3 \cdot x^2 = x^5$$

B)  $(3a^2)(4a)$

$$(3a^2)(4a) = 12a^3$$

C)  $(a^3b^2)(a^2b^4)$

$$(a^3b^2)(a^2b^4) = a^5b^6$$

D)  $(-5x^2)(3x^3y^2)(2xy^4)$

$$(-5x^2)(3x^3y^2)(2xy^4) = -30x^6y^6$$

When you raise a power to a power, you multiply the exponents:

$$(a^x)^y = a^{xy}$$

### Demonstration

Multiply and simplify:

E)  $(2x^2y^4)^3$

$$(2x^2y^4)^3 = 8x^6y^{12}$$

F)  $(3a^4b)^2(2a^2)^3$

$$(9a^8b^2)(8a^6) = 72a^{14}b^2$$

G)  $(-ab^2)^3$

$$(-ab^2)^3 = -a^3b^6$$

Note: A negative value to an odd power (3) remains negative.

H)  $(-3xy^2)^2(x^3y)^3$

$$(9x^2y^4)(x^9y^3) = 9x^{11}y^7$$

Note: A negative value to an even power (2) become positive.

# Working With Monomials

## Problem Set 13.1

### Multiplying Monomials

multiply and simplify:

①  $(m^2)(m^3)$

⑩  $(2m^2)^3$

②  $(x^3)(x^4)$

⑪  $(-y^3)^6$

③  $(m^3n)(mn^2)$

⑫  $(-m^5)^2$

④  $(r^3t^4)(r^4t^4)$

⑬  $(a^3x^2)^4$

⑤  $(3a^2b)(2ab^5)$

⑭  $(2a^2b)^2$

⑥  $(4x^2y^3)(2xy^6)$

⑮  $(3x^2y)^2(2xy^2)^3$

⑦  $(-2x^2y)(-6x^4y^7)$

⑯  $(2a^2b^3)^4(a^3b)^2$

⑧  $(-3x^5y)(2x^4)$

⑰  $(-3a^3x^5)^2(ax^2)^3$

⑨  $(-2n^4y^3)(3ny^4)$

⑱  $(-4x^2y^3)^3(xy)^2$

⑩  $(ab)(ac)(bc)$

⑲  $(4ab)^2(a^3b)^4$

⑪  $(m^2n)(am)(an^2)$

⑳  $(-3x^2y)^2(-2x)^3$

⑫  $(3y^3z)(7y^4)$

㉑  $a^2(ab^2)^3(-a^2)^3$

⑬  $(x^2)^3$

㉒  $x^3(x^2y)^2(-xy)^2$

⑭  $(a^3)^4$

㉓  $2x^2(-xy^2)^3$

⑮  $(3y)^2$

㉔  $-3a^2(-ab^2)^4$



### 2. DIVIDING MONOMIALS

When you divide monomials with the same base, you subtract the exponents:

$$\frac{n^x}{n^y} = n^{x-y}$$

### Demonstration

Divide and simplify:

A)  $\frac{x^5}{x^2} = x^{5-2} = x^3$

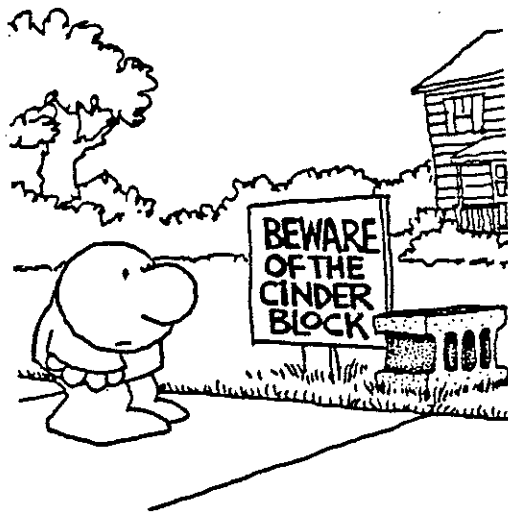
# Working With Monomials

$$B) \frac{a^4 b^3}{a b^2} = a^{4-1} b^{3-2}$$

$$\frac{a^4 b^3}{a b^2} = a^3 b$$

Note: when simplifying, you can "cancel" values:

$$\frac{a^4 b^3}{a b^2} = \frac{\cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{a} \cdot \cancel{b} \cdot \cancel{b} \cdot b}{\cancel{a} \cdot \cancel{b} \cdot \cancel{b}} = a^3 b$$



$$C) \frac{2r^3 n^5}{6r^7 n^3} = \frac{\cancel{2} \cdot \cancel{r} \cdot \cancel{r} \cdot \cancel{r} \cdot \cancel{n} \cdot \cancel{n} \cdot \cancel{n} \cdot n \cdot n}{\cancel{2} \cdot 3 \cdot \cancel{r} \cdot \cancel{r} \cdot \cancel{r} \cdot \cancel{r} \cdot \cancel{r} \cdot \cancel{r} \cdot \cancel{n} \cdot \cancel{n} \cdot \cancel{n}}$$

$$\frac{2r^3 n^5}{6r^7 n^3} = \frac{n^2}{3r^4}$$

$$D) \frac{12x^3 y^2 z}{8x^5 y z^3}$$

$$\frac{12x^3 y^2 z}{8x^5 y z^3} = \frac{3y}{2x^2 z^2}$$

$$E) \frac{3a^2 b^2 c}{12a^3 b^2 c^2}$$

$$\frac{3a^2 b^2 c}{12a^3 b^2 c^2} = \frac{1}{4ac}$$

Note: when you cancel all values in the numerator, 1 is still a factor.

## Problem Set 13.2

### Dividing Monomials

Divide and simplify:

①  $\frac{n^4}{n^3}$

⑥  $\frac{9a^4 bc^5}{6a^2 b^2 c}$

②  $\frac{x^7}{x^4}$

⑦  $\frac{12x^3 y^2 z}{6xy^3 z}$

③  $\frac{5n^2}{n^6}$

⑧  $\frac{20a^3 bc^4}{12ab^3 c^2}$

④  $\frac{x^2}{x^3}$

⑨  $\frac{3nm^2}{6n^2 m^3}$

⑤  $\frac{ab^2}{a^2 b^2}$

⑩  $\frac{4x^2 y}{2x^2 y}$

# Working With Monomials

⑪  $\frac{x^3y}{3xy^3}$

⑭  $\frac{x^3y^3z^3}{x^2y^5z^4}$

⑫  $\frac{4mn^2p^5}{12mnp}$

⑮  $\frac{2m^2n^5}{mn^3p^2}$

⑬  $\frac{8a^5b^2c^2}{6ab^3c^2}$

⑯  $\frac{abc}{a^2bc^3}$

Multiply and simplify:

⑰  $(x^2)(xy^3)$

⑱  $2a(a^2b)(b^3c)$

⑲  $(-3a^2b)^2$

⑳  $(-2xy^2)^3$

㉑  $(2ab^2c)^4(-a^2b)^2$

㉒  $(-3x^2y^3)^2(-xy)^3$

㉓  $(ab)(b^2c)(a^2c)^3$

㉔  $(-xy)(-x^2y)^3$

㉕  $-2a(a^2b)^3$

㉖  $3mn(m^2n)(n^3)^2$

㉗  $-2xy(x^2y)^2(-y)^2$



## 3. NEGATIVE EXPONENTS

It is possible for the value of an exponent to be negative, but in final simplified form, you cannot include negative exponents.

Negative exponents:

$$n^{-x} = \frac{1}{n^x} \quad \frac{1}{n^{-x}} = n^x$$

A method for simplifying monomials with negative exponents involves changing to a positive exponent and moving the value from the numerator to denominator or

# Working With Monomials

denominator to numerator.

## Demonstration

Simplify :

$$A) \frac{x^2 y^{-3}}{z}$$

$$\frac{x^2 y^{-3}}{z} = \frac{x^2}{z y^3}$$

$$B) \frac{a^2}{b^{-2} c^{-1}}$$

$$\frac{a^2}{b^{-2} c^{-1}} = \frac{a^2 b^2 c}{1} = a^2 b^2 c$$

$$C) \frac{3x^{-2}}{x^{-6}}$$

$$\frac{3x^{-2}}{x^{-6}} = \frac{3x^6}{x^2} = 3x^4$$

$$D) \frac{-2ab^{-1}c}{a^2 b^3 c^{-3}}$$

$$\frac{-2ab^3 c c^3}{a^2 b^1} = \frac{-2b^2 c^4}{a}$$

Note: Do not confuse negative

values (-2) with negative exponents. Negative constants and coefficients do not change to positive and switch positions in a fraction.

$$E) -3x^{-2}y$$

$$-3x^{-2}y = \frac{-3y}{x^2}$$

$$F) \frac{-4a^2 b^{-3} c d^{-2}}{8a^5 b^{-1} c^{-2} d^3}$$

$$\frac{-4a^2 b^1 c c^2}{8a^5 b^3 d^3 d^2} = \frac{-c^3}{2a^3 b^2 d^5}$$

$$G) (-3xy^{-2})^2 (xy^3)^{-2}$$

$$(9x^2 y^{-4})(x^{-2} y^{-6})$$

$$9x^0 y^{-10} = \frac{9}{y^{10}}$$

Note:  $x^0 = 1$ , therefore it drops out of the final simplified form.

Note: The problem above follows all rules for multiplying monomials and taking powers to powers.

# Working With Monomials

## Problem Set 13.3

### Negative Exponents

Simplify:

$$\textcircled{1} \frac{1}{a^{-3}}$$

$$\textcircled{9} \frac{-x^3 y^{-2}}{x^2 y}$$

$$\textcircled{2} \frac{x^2}{y^{-2}}$$

$$\textcircled{10} \frac{-8a^{-3}b}{6a^{-5}b^2}$$

$$\textcircled{3} \frac{n^2 m^{-3}}{p^3}$$

$$\textcircled{11} \frac{xy z^{-1}}{x^2 y^3}$$

$$\textcircled{4} \frac{3x^2 y^{-4}}{z}$$

$$\textcircled{12} \frac{12a^{-2}b^{-3}cd}{8ab^{-2}cd^{-1}}$$

$$\textcircled{5} \frac{2x^2 y^{-3}}{x^5 y}$$

$$\textcircled{13} \frac{-n^{-3}m^{-2}p^3}{nmp^{-1}}$$

$$\textcircled{6} \frac{-3a^5 bc^{-1}}{a^2 c^2}$$

$$\textcircled{14} \frac{4ab^{-1}c^{-2}}{a^{-3}b^2c}$$

$$\textcircled{7} \frac{-2mn^{-4}}{6m^{-3}n}$$

$$\textcircled{15} \frac{-3x^{-2}y^{-3}}{x^{-5}y^{-1}}$$

$$\textcircled{8} \frac{10a^{-1}b^{-2}}{15a^{-3}}$$

$$\textcircled{16} \frac{-5n^{-3}m^{-1}}{10n^{-4}m^{-3}}$$



## Challenge Problems

Simplify:

$$\textcircled{17} (xy^2)^{-2} (2xy^2)^3$$

$$\textcircled{18} (ab)^{-3} (a^2b)^3$$

$$\textcircled{19} (-2x^2y)^2 (x^{-3}y^2)^{-2}$$

$$\textcircled{20} (-a^2b^{-2})^3 (abc^2)^{-2}$$

$$\textcircled{21} -3x(xy^2)^{-1} (x^3y)$$

$$\textcircled{22} ab^{-2} (ab^{-3})^2$$

$$\textcircled{23} (nxy^{-2})^{-2} (2x)^3$$

$$\textcircled{24} abc(-3a^2b^{-3})^2$$

$$\textcircled{25} x^{-1}y(-2xy^{-2})$$

$$\textcircled{26} a^{-2}b^{-3}(-ab^2)^2$$



# Working With Monomials

## REVIEW & PRACTICE

Multiply / Divide  
and simplify:

①  $(x^3)(x^5)$

②  $(n^2)(n^3)(n^4)$

③  $(2ab)(3a^2b^3)$

④  $(-2a^2)(ab^3c)$

⑤  $(n^3)^2$

⑥  $(a^2)^5$

⑦  $(-2x^3)^3$

⑧  $(-3y)^2$

⑨  $(-x^2y)^3(2xy)^2$

⑩  $(-2ab^2c)^2(ab^2)^3$

⑪  $ab(-ab^2)^4(-b^2)^3$

⑫  $2n^2(-nm^2)^2$

⑬  $\frac{x^5}{x^2}$

⑭  $\frac{n^2}{n^3m^2}$

⑮  $\frac{8a^2b^4c}{6abc^2}$

⑯  $\frac{18x^5y^2}{6x^2y}$

⑰  $\frac{-2a^3bc^2}{8ab^3c^3}$

⑱  $\frac{-3x^4y^2z}{12xy^5z^3}$

⑲  $\frac{x^{-4}}{y^2}$

⑳  $\frac{a^{-3}b^2}{ab^{-3}}$

㉑  $\frac{-2x^{-2}y^{-3}z}{4x^{-4}yz^{-2}}$

㉒  $\frac{-9ab^{-3}c^{-5}}{6a^{-2}bc^2}$

㉓  $\frac{x^{-3}y^{-3}z}{x^{-2}yz^{-1}}$

㉔  $\frac{-8ab^{-2}c}{10ab^{-3}c^{-1}}$

㉕  $(ab)^{-1}(2a^{-2}b)$

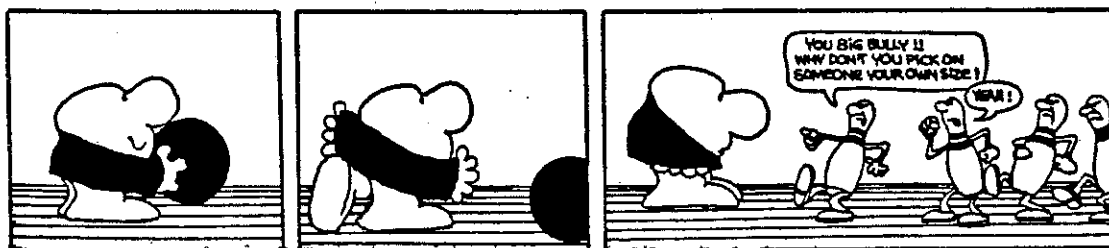
㉖  $x^2(-3xy^{-2})$

㉗  $-3a(ab^2c)^{-3}$

㉘  $-2xy(-x^{-2}y)^{-2}$

㉙  $abc(-a^2b)^{-3}$

㉚  $x^{-2}y^{-3}(-xy^2)^{-2}$



# Working With Monomials

## PRACTICE TEST #1

multiply / Divide  
and simplify :

①  $(x^5)(x^7)$

②  $(4x^3y)(2xy^2z)^3$

③  $(-3a^2b^4)^2$

④  $\frac{x^2y^3z^4}{x^5y^3z}$

⑤  $\frac{-14ab^{-2}c^{-1}d^3}{6a^2bc^{-3}d^2}$

⑥  $xy^{-3}(-x^{-2}y)^3$

## PRACTICE TEST #2

multiply / Divide  
and simplify :

①  $(n^5)(n^5)$

②  $(4a^2b)(2ab^3)^3$

③  $(-2xy^3z^2)^3$

④  $\frac{2ab^3cd^2}{a^2bc^4d^2}$

⑤  $\frac{18xy^{-3}z^{-2}}{9x^{-2}y^{-5}z}$

⑥  $3xy^{-2}(x^{-3}y^4)^2$

