Friendship Junior High School
Accelerated Math Program
Mr. Lavine (Room 102A)

A.T.I.M.

Advanced Topics In Mathematics

UNIT 1
Polynomials

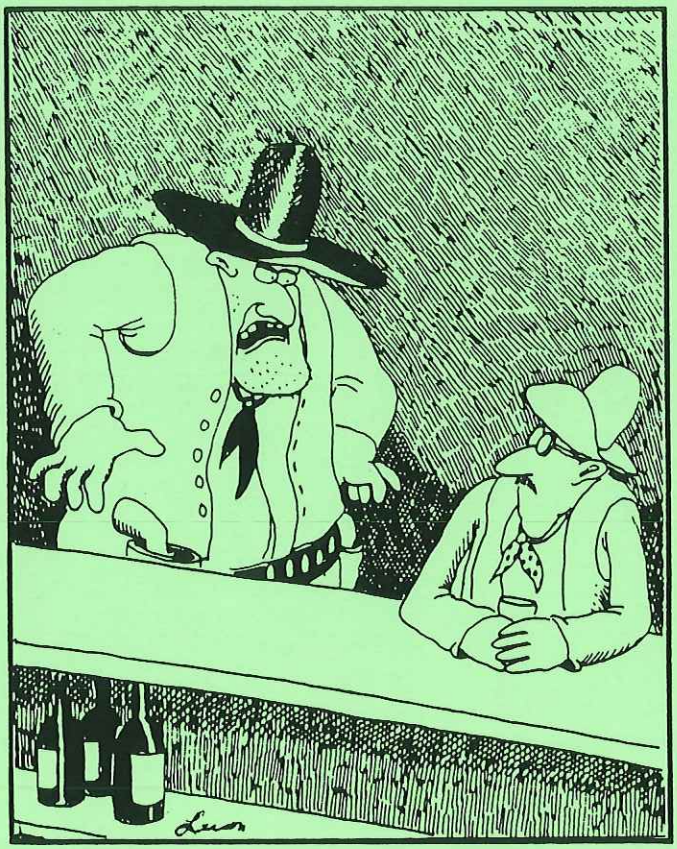
UNIT 2
Systems

UNIT 3
Problem Solving

UNIT 17
Sequence & Series

UNIT 18
Combinations & Permutations

UNIT 19
Mathematics of Chance



"I asked you a question, buddy . . . What's the square root of 5,248?"

Simplifying Polynomials

ANSWER KEY 1.1

① $(5a)(6a^2b)(3ab^3) + (4a^2)(3b^3)(2a^2b)$

$(90a^4b^4) + (24a^4b^4) = \boxed{114a^4b^4}$

② $(5mn^2)(m^3n)(-3p^2) + (8np)(3mp)(m^3n^2)$

$(-15m^4n^3p^2) + (24m^4n^3p^2) = \boxed{9m^4n^3p^2}$

③ $(2xy^2)^3 + (2xy^2)^2(6xy^2)$

$(8x^3y^6) + (24x^3y^6) = \boxed{32x^3y^6}$

④ $(3a)(a^2b)^3 + (2a)^2(-a^5b^3)$

$(3a)(a^6b^3) + (4a^2)(-a^5b^3)$

$(3a^7b^3) + (-4a^7b^3) = \boxed{-a^7b^3}$

⑤ $\dfrac{-2a^3b^6}{24a^2b^2} = \boxed{\dfrac{-ab^4}{12}}$

⑥ $\dfrac{3m^{-4}}{4^{-1}m^{-2}} = \dfrac{(3)(4)m^2}{m^4} = \boxed{\dfrac{12}{m^2}}$

⑦ $\dfrac{14a^{-2}bc^{-3}}{6a^3b^4c^{-1}} = \boxed{\dfrac{7}{3a^5b^3c^2}}$

⑧ $\dfrac{26x^{-3}y^4z^{-5}}{-18x^4y^{-3}z^{-2}} = \boxed{\dfrac{-13y^7}{9x^7z^3}}$

⑨ $\dfrac{(-2r^3)^2(r^{-2})^{-1}}{(r^2)^{-3}} = \dfrac{(4r^6)(r^2)}{r^{-6}} = \boxed{4r^{14}}$

⑩ $\dfrac{(-3x^{-2})^{-1}(x^2)^3}{-(-2x)^2} = \dfrac{(-3)^{-1}(x^2)(x^6)}{-(4x^2)}$

$\dfrac{x^8}{(-3)(-4x^2)} = \boxed{\dfrac{x^6}{12}}$

⑪ $\dfrac{x^{2n}}{x^{2n-3}} = x^{2n-(2n-3)} = \boxed{x^3}$

⑫ $\dfrac{3n^{2x-4}}{n^{x+3}} = 3n^{(2x-4)-(x+3)} = \boxed{3n^{x-7}}$

⑬ $\left(\dfrac{-3y^4}{2y^2}\right)^{-2} = \left(\dfrac{-3y^2}{2}\right)^{-2} = \left(\dfrac{2}{-3y^2}\right)^2 = \boxed{\dfrac{4}{9y^4}}$

⑭ $\left(\dfrac{3}{2x^{-2}}\right)^{-1} = \left(\dfrac{3x^2}{2}\right)^{-1} = \boxed{\dfrac{2}{3x^2}}$

⑮ $\left(\dfrac{x}{y^{-1}z^2}\right)^{-1} = \left(\dfrac{xy}{z^2}\right)^{-1} = \boxed{\dfrac{z^2}{xy}}$

⑯ $\left(\dfrac{-2y^3}{x^{-2}y^{-1}}\right)^{-2} = (-2x^2y^4)^{-2} = \left(\dfrac{1}{-2x^2y^4}\right)^2 = \boxed{\dfrac{1}{4x^4y^8}}$

⑰ $m^{-3}(m^2+m^4-m^{-1}) = m^{-1}+m^1-m^{-4}$

$\boxed{\dfrac{1}{m}+m-\dfrac{1}{m^4}}$

⑱ $a^3(a^{-2}+a^{-5}+a) = a+a^{-2}+a^4$

$\boxed{a+\dfrac{1}{a^2}+a^4}$

⑲ $4a^{-1}b^2(a^2b^{-1}+3a^3b^{-2}+4^{-2}ab^{-1})$

$4ab+12a^2+4^{-1}b = \boxed{4ab+12a^2+\dfrac{b}{4}}$

⑳ $y^2x^{-3}(yx^4+y^{-1}x^3+y^{-2}x^2)$

$y^3x+y+x^{-1} = \boxed{xy^3+y+\dfrac{1}{x}}$

Simplifying Polynomials

ANSWER KEY 1.1

(21) $(2n-3m)(2n+3m) = \boxed{4n^2-9m^2}$

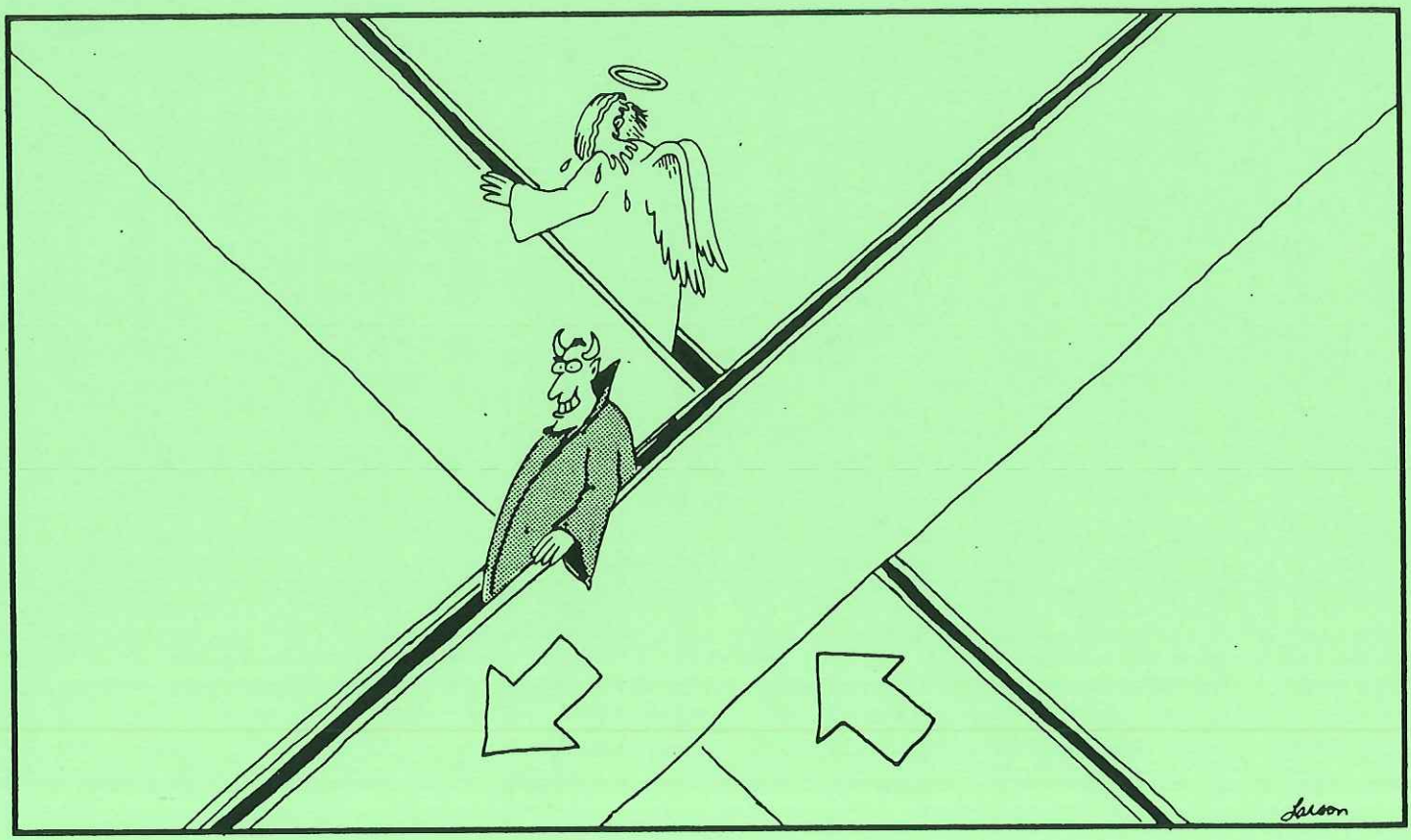
(22) $(3x-2y)^2 = \boxed{9x^2-12xy+4y^2}$

(23) $(2n^{x+1}+3m^{3x})^2 = \boxed{4n^{2x+2}+12n^{x+1}m^{3x}+9m^{6x}}$

(24) $(a^{n+1}+3b^{4n})(a^{n+1}-3b^{4n}) = \boxed{a^{2n+2}-9b^{8n}}$

(25) $(2x^{3a-1}+4y^{a})(2x^{3a-1}-4y^{a}) = \boxed{4x^{6a-2}-16y^{2a}}$

(26) $(5x^{3n+1}-2y^{n-2})^2 = \boxed{25x^{6n+2}-20x^{3n+1}y^{n-2}+4y^{2n-4}}$



Factoring Perfect Cubes

ANSWER KEY 1.2

① $5x^2y - 10xy^2 = \boxed{5xy(x-2y)}$

② $-15x^2-5x = \boxed{-5x(3x+1)}$

③ $r^2-9 = \boxed{(r+3)(r-3)}$

④ $x^2-49 = \boxed{(x+7)(x-7)}$

⑤ $k^2+12k+36 = \boxed{(k+6)^2}$

⑥ $4n^2-20np+25p^2 = \boxed{(2n-5p)^2}$

⑦ $3y^2+5y+2$
$3y^2+3y+2y+2$
$3y(y+1)+2(y+1) = \boxed{(y+1)(3y+2)}$

⑧ $4x^2+11x+6$
$4x^2+8x+3x+6$
$4x(x+2)+3(x+2) = \boxed{(x+2)(4x+3)}$

⑨ $(2a+b)^2-(a-3b)^2$
$[(2a+b)+(a-3b)][(2a+b)-(a-3b)]$
$\boxed{(3a-2b)(a+4b)}$

⑩ $(3x-2y)^2-(2x+y)^2$
$[(3x-2y)+(2x+y)][(3x-2y)-(2x+y)]$
$\boxed{(5x-y)(x-3y)}$

⑪ $a^2b^2-a^4+a^2b^2-b^4$
$a^2(b^2-a^2)+b^2(a^2-b^2)$
$a^2(b^2-a^2)-b^2(b^2-a^2)$
$(b^2-a^2)(a^2-b^2) = \boxed{(b+a)(b-a)(a+b)(a-b)}$

⑫ $x^8n^4-16x^8-n^4+16$
$x^8(n^4-16)-1(n^4-16) = (x^8-1)(n^4-16)$
$\boxed{(x^4+1)(x^2+1)(x+1)(x-1)(n^2+4)(n+2)(n-2)}$

⑬ $x^3-y^3 = \boxed{(x-y)(x^2+xy+y^2)}$

⑭ $n^3+27 = \boxed{(n+3)(n^2-3n+9)}$

⑮ $8b^3+27c^3 = \boxed{(2b+3c)(4b^2-6bc+9c^2)}$

⑯ $2y^3-16x^3 = \boxed{2(y-2x)(y^2+2xy+4x^2)}$

⑰ $y^6-n^3m^9 = \boxed{(y^2-nm^3)(y^4+y^2nm^3+n^2m^6)}$

⑱ $64x^3+27y^6 = \boxed{(4x+3y^2)(16x^2-12xy^2+9y^4)}$

⑲ $16x^4-196y^4 = 4(4x^4-49y^4)$
$\boxed{4(2x^2+7y^2)(2x^2-7y^2)}$

⑳ $(a+b)^2-b^2 = [(a+b)+b][(a+b)-b]$
$\boxed{(a+2b)(a)}$

㉑ $a^5-a^3b^2-a^2b^3+b^5$
$a^3(a^2-b^2)-b^3(a^2-b^2) = (a^2-b^2)(a^3-b^3)$
$\boxed{(a+b)(a-b)(a-b)(a^2+ab+b^2)}$

㉒ $a^6-a^3b^3-b^6+a^3b^3$
$a^3(a^3-b^3)-b^3(b^3-a^3)$
$a^3(a^3-b^3)+b^3(a^3-b^3) = (a^3b^3)(a^3+b^3)$
$\boxed{(a-b)(a^2+ab+b^2)(a+b)(a^2-ab+b^2)}$

Factoring Perfect Cubes

ANSWER KEY 1.2

(23) $x^3+9x^2+23x+15$

factors: $\pm1, \pm3, \pm5, \pm15$

$(-1)^3+9(-1)^2+23(-1)+15=0$

$(x+1)$

$$\begin{array}{r|l} x+1 & x^2+8x+15 \\ & \underline{x^3+x^2} \\ & \quad 8x^2+23x \\ & \quad \underline{8x^2+8x} \\ & \quad\quad 15x+15 \\ & \quad\quad \underline{15x+15} \end{array}$$

$(x+1)(x^2+8x+15)$

$\boxed{(x+1)(x+5)(x+3)}$

(24) x^3-2x^2-5x+6

factors: $\pm1, \pm2, \pm3, \pm6$

$(1)^3-2(1)^2-5(1)+6=0$

$(x-1)$

$$\begin{array}{r|l} x-1 & x^2-x-6 \\ & \underline{x^3-x^2} \\ & \quad -x^2-5x \\ & \quad \underline{-x^2+x} \\ & \quad\quad -6x+6 \\ & \quad\quad \underline{-6x+6} \end{array}$$

$(x-1)(x^2-x-6)$

$\boxed{(x-1)(x+2)(x-3)}$

(25) $n^3-2n^2-9n+18$

factors: $\pm1, \pm2, \pm3, \pm6, \pm9, \pm18$

$(2)^3-2(2)^2-9(2)+18=0$

$(n-2)$

$$\begin{array}{r|l} n-2 & n^2 \quad -9 \\ & \underline{n^3-2n^2} \\ & \quad -9n+18 \\ & \quad \underline{-9n+18} \end{array}$$

$(n-2)(n^2-9) = \boxed{(n-2)(n-3)(n+3)}$

(26) x^7+y^7

$\boxed{(x+y)(x^6-x^5y+x^4y^2-x^3y^3+x^2y^4-xy^5+y^6)}$

(27) a^5-243b^5

$(a-3b)\left[(a^4+a^3(3b)+a^2(3b)^2+a(3b)^3+(3b)^4\right]$

$\boxed{(a-3b)(a^4+3a^3b+9a^2b^2+27ab^3+81b^4)}$

(28) $x^{15}+32y^{10}$

$(x^3)^5+(2y^2)^5$

$(x^3+2y^2)\left[(x^3)^4-(x^3)^3(2y^2)+(x^3)^2(2y^2)^2-(x^3)(2y^2)^3+(2y^2)^4\right]$

$\boxed{(x^3+2y^2)(x^{12}-2x^9y^2+4x^6y^4-8x^3y^6+16y^8)}$

Grouping Polynomials

ANSWER KEY 1.3

① $k^3+4k^2-9k-36$
$k^2(k+4)-9(k+4)$
$(k+4)(k^2-9)$
$\boxed{(k+4)(k+3)(k-3)}$

② $a^2x-b^2x+a^2y-b^2y$
$x(a^2-b^2)+y(a^2-b^2)$
$(a^2-b^2)(x+y)$
$\boxed{(a+b)(a-b)(x+y)}$

③ $x^2-y^2+4y-4x$
$(x-y)(x+y)+4(y-x)$
$(x-y)(x+y)-4(x-y)$
$\boxed{(x-y)(x+y-4)}$

④ $a+b+3a^2-3b^2$
$(a+b)+3(a^2-b^2)$
$(a+b)+3(a+b)(a-b)$
$\boxed{(a+b)(1+3a-3b)}$

⑤ $b^2-y^2-p^2-2yp$
$b^2-y^2-2yp-p^2$
$b^2-(y^2+2yp+p^2)$
$b^2-(y+p)^2$
$\boxed{(b+y+p)(b-y-p)}$

⑥ $a^2-9+2ab+b^2$
$a^2+2ab+b^2-9$
$(a+b)^2-9$
$\boxed{(a+b+3)(a+b-3)}$

⑦ $2ab+2am+b^2-m^2$
$2a(b+m)+(b-m)(b+m)$
$\boxed{(b+m)(2a+b-m)}$

⑧ $3pq+3ps+q^2-s^2$
$3p(q+s)+(q-s)(q+s)$
$\boxed{(q+s)(3p+q-s)}$

⑨ $16m^3-2$
$2(8m^3-1)$
$\boxed{2(2m-1)(4m^2+2m+1)}$

⑩ $3r+81r^4$
$3r(1+27r^3)$
$\boxed{3r(1+3r)(1-3r+9r^2)}$

⑪ $x^3+y^3-x^2y-xy^2$
$(x+y)(x^2-xy+y^2)-xy(x+y)$
$(x+y)(x^2-xy+y^2-xy)$
$(x+y)(x^2-2xy+y^2)$
$\boxed{(x+y)(x-y)^2}$

⑫ $t^3+125+5t^2+25t$
$(t+5)(t^2-5t+25)+5t(t+5)$
$(t+5)(t^2-5t+25+5t)$
$\boxed{(t+5)(t^2+25)}$

⑬ x^4-13x^2+36
$(x^2)^2-13x^2+36$
$(x^2-9)(x^2-4)$
$\boxed{(x+3)(x-3)(x+2)(x-2)}$

⑭ y^4-14y^2+45
$(y^2)^2-14y^2+45$
$(y^2-9)(y^2-5)$
$\boxed{(y+3)(y-3)(y^2-5)}$

⑮ $r^2-rt-rt^2+t^3$
$r(r-t)-t^2(r-t)$
$\boxed{(r-t)(r-t^2)}$

⑯ $8ax-6x-12a+9$
$2x(4a-3)-3(4a-3)$
$\boxed{(4a-3)(2x-3)}$

⑰ $10x^2-14xy-15x+21y$
$2x(5x-7y)-3(5x-7y)$
$\boxed{(5x-7y)(2x-3)}$

⑱ $a^4-12a^3b+24a^2b^2-8ab^3$
$a(a^3-12a^2b+24ab^2-8b^3)$
$a(a^3-8b^3-12a^2b+24ab^2)$
$a[(a-2b)(a^2+2ab+4b^2)-12ab(a-2b)]$
$a[(a-2b)(a^2+2ab+4b^2-12ab)]$
$\boxed{a(a-2b)(a^2-10ab+4b^2)}$

⑲ $a^4-16a^2+3a^3-48a$
$a(a^3-16a+3a^2-48)$
$a[a(a^2-16)+3(a^2-16)]$
$a(a^2-16)(a+3)$
$\boxed{a(a+4)(a-4)(a+3)}$

⑳ $x^3y-3x^2y-6xy+8y$
$y(x^3-3x^2-6x+8)$
$y(x^3+8-3x^2-6x)$
$y(x+2)(x^2-2x+4)-3x(x+2)$
$y(x+2)(x^2-5x+4)$
$\boxed{y(x+2)(x-4)(x-1)}$

Polynomial Division & Synthetic Division

ANSWER KEY 1.4

① $10x-3 \;\overline{)\,80x^2+6x-4}$

$8x+3+\frac{5}{10x-3}$

$80x^2-24x$

$30x-4$

$30x-9$

$+5$

② $x+2 \;\overline{)\,x^3+4x-4}$

$x^2-2x+8-\frac{20}{x+2}$

x^3+2x^2

$-2x^2+4x$

$-2x^2-4x$

$8x-4$

$8x+16$

-20

③ $x^2-2x+2 \;\overline{)\,x^4+4}$

x^2+2x+2

$x^4-2x^3+2x^2$

$2x^3-2x^2$

$2x^3-4x^2+4x$

$2x^2-4x+4$

$2x^2-4x+4$

④ $a^2+a-1 \;\overline{)\,a^4-3a^2+2}$

$a^2-a-1+\frac{1}{a^2+a-1}$

$a^4+a^3-a^2$

$-a^3-2a^2$

$-a^3-a^2+a$

$-a^2-a+2$

$-a^2-a+1$

1

⑤ $(2x^3-3x^2+3x-4) \div (x-2)$

2	2	-3	3	-4
		4	2	10
	2	1	5	6

$\boxed{2x^2+x+5+\frac{6}{x-2}}$

⑥ $(2a^3+a^2-2a+3) \div (a+1)$

-1	2	1	-2	3
		-2	1	1
	2	-1	-1	4

$\boxed{2a^2-a-1+\frac{4}{a+1}}$

⑦ $(4x^4-5x^2+2x+3) \div (2x-1)$

$(2x^4-\frac{5}{2}x^2+x+\frac{3}{2}) \div (x-\frac{1}{2})$

$\frac{1}{2}$	2	0	$-\frac{5}{2}$	1	$\frac{3}{2}$
		1	$\frac{1}{2}$	-1	0
	2	1	-2	0	$\frac{3}{2}$

$(\frac{3}{2}) \times 2 = 3$

$\boxed{2x^3+x^2-2x+\frac{3}{2x-1}}$

Continued

Polynomial Division & Synthetic Division

ANSWER KEY 1.4

⑧ $(4y^4-5y^2-8y-10)\div(2y-3)$
$(2y^4-\frac{5}{2}y^2-4y-5)\div(y-\frac{3}{2})$

$\frac{3}{2}$ | 2 | 0 | $-\frac{5}{2}$ | -4 | -5
| | 3 | $\frac{9}{2}$ | 3 | $-\frac{3}{2}$
2 | 3 | 2 | -1 | $-\frac{13}{2}$ $\times 2 = -13$

$\boxed{2y^3+3y^2+2y-1-\frac{13}{2y-3}}$

⑨ $(x^5+32)\div(x+2)$

-2 | 1 | 0 | 0 | 0 | 0 | 32
| | -2 | 4 | -8 | 16 | -32
1 | -2 | 4 | -8 | 16 | 0

$\boxed{x^4-2x^3+4x^2-8x+16}$

⑩ $(6x^3-28x^2+19x+3)\div(3x-2)$
$(2x^3-\frac{28}{3}x^2+\frac{19}{3}x+1)\div(x-\frac{2}{3})$

$\frac{2}{3}$ | 2 | $-\frac{28}{3}$ | $\frac{19}{3}$ | 1
| | $\frac{4}{3}$ | $-\frac{16}{3}$ | $\frac{2}{3}$
2 | -8 | 1 | $\frac{5}{3}$ $\times 3 = 5$

$\boxed{2x^2-8x+1+\frac{5}{3x-2}}$

⑪ $\left(\frac{-x^2y^{-3}z}{2x^{-1}y^2z^3}\right)^{-3}=\left(\frac{-x^3}{2y^5z^2}\right)^{-3}$

$\left(\frac{2y^5z^2}{-x^3}\right)^3=\boxed{\frac{-8y^{15}z^6}{x^9}}$

⑫ $(4n^{4x}-3m^{x-5})^2$

$\boxed{16n^{8x}-24n^{4x}m^{x-5}+9m^{2x-10}}$

⑬ $a^7-a^3b^4+8b^7-8a^4b^3$
$a^3(a^4-b^4)+8b^3(b^4-a^4)$
$a^3(a^4-b^4)-8b^3(a^4-b^4)$
$(a^4-b^4)(a^3-8b^3)$
$\boxed{(a^2+b^2)(a+b)(a-b)(a-2b)(a^2+2ab+4b^2)}$

⑭ $2x^3+6xy^2-2y^3-6x^2y$
$2(x^3+3xy^2-y^3-3x^2y)$
$2(x^3-y^3+3xy^2-3x^2y)$
$2[(x-y)(x^2+xy+y^2)+3xy(y-x)]$
$2[(x-y)(x^2+xy+y^2)-3xy(x-y)]$
$2(x-y)(x^2+xy+y^2-3xy)$
$2(x-y)(x^2-2xy+y^2)$
$2(x-y)(x-y)^2=\boxed{2(x-y)^3}$

⑮ $x^2-1-8xy+16y^2$
$(x^2-8xy+16y^2)-1$
$(x-4y)^2-1$
$\boxed{(x-4y+1)(x-4y-1)}$

Polynomials

ANSWER KEY: UNIT 1 REVIEW & PRACTICE

① $(5a)(6a^2b)(3ab^3)+(2a)^2(3b^3)(2a^2b)$

$(90a^4b^4)+(24a^4b^4) = \boxed{114a^4b^4}$

② $\dfrac{-9x^{-2}y^3z^{-1}}{3x^{-4}yz^2} = \boxed{\dfrac{-3x^2y^2}{z^3}}$

③ $\left(\dfrac{2y^{-2}}{-2}\right)^{-1}\left(\dfrac{m^2n}{y}\right)^{-2}$

$\left(\dfrac{-1}{y^2}\right)^{-1}\left(\dfrac{y}{m^2n}\right)^{2}$

$\left(\dfrac{y^2}{-1}\right)\left(\dfrac{y^2}{m^4n^2}\right) = \boxed{\dfrac{-y^4}{m^4n^2}}$

④ $\dfrac{6n^{2x+3}}{3n^{x-4}} = 2n^{(2x+3)-(x-4)}$

$\boxed{2n^{x+7}}$

⑤ $x^{-1}y^2(x^{-3}y - xy^{-3} + x^2y^{-4})$

$x^{-4}y^3 - y^{-1} + xy^{-2}$

$\boxed{\dfrac{y^3}{x^4} - \dfrac{1}{y} + \dfrac{x}{y^2}}$

⑥ $(4x^{a+2} - 3y^{3a})^2$

$\boxed{16x^{2a+4} - 24x^{a+2}y^{3a} + 9y^{6a}}$

⑦ $(x^{n+1} + y^{2n-1})(x^{n+1} - y^{2n-1})$

$\boxed{x^{2n+2} - y^{4n-2}}$

⑧ $12n^2 - 7n - 10$

$12n^2 - 15n + 8n - 10$

$3n(4n-5) + 2(4n-5) = \boxed{(4n-5)(3n+2)}$

⑨ $(3a-4b)^2 - (a+3b)^2$

$[(3a-4b)+(a+3b)]\,[(3a-4b)-(a+3b)]$

$\boxed{(4a-b)(2a-7b)}$

⑩ $5x^3y + 40y^4$

$5y(x^3 + 8y^3) = \boxed{5y(x+2y)(x^2-2xy+4y^2)}$

⑪ $x^5 - x^3y^2 + 27y^5 - 27x^2y^3$

$x^3(x^2-y^2) + 27y^3(y^2-x^2)$

$x^3(x^2-y^2) - 27y^3(x^2-y^2)$

$(x^2-y^2)(x^3-27y^3)$

$\boxed{(x+y)(x-y)(x-3y)(x^2+3xy+9y^2)}$

⑫ $x^2 - 9 - 6xy + 9y^2$

$(x^2 - 6xy + 9y^2) - 9$

$(x-3y)^2 - 9$

$\boxed{(x-3y+3)(x-3y-3)}$

⑬ $m^4n + m^2n - mn^4 - mn^2$

$mn(m^3 + m - n^3 - n)$

$mn(m^3 - n^3 + m - n)$

$mn\left[(m-n)(m^2+mn+n^2) + 1(m-n)\right]$

$\boxed{mn(m-n)(m^2+mn+n^2+1)}$

⑭ $3x^3 + 9x^2 + 9x + 3$

$3(x^3 + 3x^2 + 3x + 1)$

$3(x^3 + 1 + 3x^2 + 3x)$

$3\left[(x+1)(x^2-x+1) + 3x(x+1)\right]$

$3\left[(x+1)(x^2-x+1+3x)\right] = 3(x+1)(x^2+2x+1)$

$3(x+1)(x+1)(x+1) = \boxed{3(x+1)^3}$

Polynomials

ANSWER KEY: UNIT 1 REVIEW & PRACTICE

(15) $4n^3-8n^2+16n-32+\frac{61}{n+2}$

$n+2 \;|\; 4n^4 \qquad\qquad -3$

$4n^4+8n^3$

$-8n^3$

$-8n^3-16n^2$

$16n^2$

$16n^2+32n$

$-32n-3$

$-32n-64$

61

(16) $(x^6-9x^4+2x^3-18x+12)(x+3)^{-1}$

−3	1	0	−9	2	0	−18	12
		−3	9	0	−6	18	0
	1	−3	0	2	−6	0	12

$\boxed{x^5-3x^4+2x^2-6x+\frac{12}{x+3}}$

(17) $(4x^3+x-3)(2x-1)^{-1}$

$(2x^3+\frac{1}{2}x-\frac{3}{2})(x-\frac{1}{2})^{-1}$

1/2	2	0	1/2	−3/2
		1	1/2	1/2
	2	1	1	−1

$\boxed{2x^2+x+1-\frac{2}{2x-1}}$

(18) $x^3+2x^2-13x+10$

factors of 10: $\pm1, \pm2, \pm5, \pm10$

$(1)^3+2(1)^2-13(1)+10=0$

$(x-1)$

$x^2+3x-10$

$x-1 \;|\; x^3+2x^2-13x+10$

x^3-x^2

$3x^2-13x$

$3x^2-3x$

$-10x+10$

$-10x+10$

$(x-1)(x^2+3x-10) = \boxed{(x-1)(x+5)(x-2)}$

(19) $n^3-9n^2+26n-24$

factors of -24: $\pm1, \pm2, \pm3, \pm4, \pm6, \pm8, \pm12, \pm24$

$(2)^3-9(2)^2+26(2)-24=0$

$(n-2)$

$n^2-7n+12$

$n-2 \;|\; n^3-9n^2+26n-24$

n^3-2n^2

$-7n^2+26n$

$-7n^2+14n$

$12n-24$

$12n-24$

$(n-2)(n^2-7n+12) = \boxed{(n-2)(n-3)(n-4)}$

(20) $x^9+y^9 = \boxed{(x+y)(x^8-x^7y+x^6y^2-x^5y^3+x^4y^4-x^3y^5+x^2y^6-xy^7+y^8)}$

(21) $32x^{10}-y^{20} = [(2x^2)-(y^4)][(2x^2)^4+(2x^2)^3(y^4)+(2x^2)^2(y^4)^2+(2x^2)(y^4)^3+(y^4)^4]$

$\boxed{(2x^2-y^4)(16x^8+8x^6y^4+4x^4y^8+2x^2y^{12}+y^{16})}$

Second & Third Order Systems

ANSWER KEY 2.1

① $6x-4y=-6$
$3x+y=3 \rightarrow y=3-3x$

$6x-4(3-3x)=-6$
$6x-12+12x=-6$
$18x=6$
$x=1/3$

$y=3-3(1/3)$
$y=3-1$
$y=2$

$\boxed{(1/3, 2)}$

② $8x+3y=4$ mult. by 3
$4x-9y=-5$

$24x+9y=12$
$4x-9y=-5$
$28x=7$
$x=1/4$

$4(1/4)-9y=-5$
$1-9y=-5$
$-9y=-6$
$y=2/3$

$\boxed{(1/4, 2/3)}$

③ $x+y+z=-1$
$2x+y=2$
$-3x=-9 \rightarrow x=3$

$2(3)+y=2$
$6+y=2$
$y=-4$

$(3)+(-4)+z=-1$
$z=0$

$\boxed{(3,-4,0)}$

④ $2x+4y-z=-3$
$y+z=4$
$2y=-2 \rightarrow y=-1$

$(-1)+z=4$
$z=5$

$2x+4(-1)-(5)=-3$
$2x=6 \quad x=3$

$\boxed{(3,-1,5)}$

⑤ a) $x+y+z=-1$
b) $2x-y+z=19$
c) $3x-2y-4z=16$

a) $x+y+z=-1$
b) $2x-y+z=19$
$3x+2z=18$

a) $2x+2y+2z=-2$
c) $3x-2y-4z=16$
$5x-2z=14$

$3x+2z=18$
$5x-2z=14$
$8x=32$
$x=4$

$3(4)+2z=18$
$2z=6$
$z=3$

$\boxed{(4,-8,3)}$

a) $(4)+y+(3)=-1$
$y=-8$

⑥ a) $x-2y+z=3$
b) $2x+y-2z=31$
c) $-x+2y+3z=-23$

a) $x-2y+z=3$
c) $-x+2y+3z=-23$
$4z=-20$
$z=-5$

a) $x-2y+z=3$
b) $4x+2y-4z=62$
$5x-3z=65$

$5x-3(-5)=65$
$5x=50$
$x=10$

a) $(10)-2y+(-5)=3$
$-2y=-2$
$y=1$

$\boxed{(10, 1, -5)}$

Second & Third Order Systems

ANSWER KEY 2.1

⑦ a) $2x+3y+z=7$
b) $x+y+z=4$
c) $3x+4y-2z=6$

$\boxed{(5/2,\ 1/4,\ 5/4)}$

a) $2x+3y+z=7$
b) $-x-y-z=-4$
$x+2y=3$

b) $2x+2y+2z=8$
c) $3x+4y-2z=6$
$5x+6y=14$

$x+2y=3$ mult. by -3
$5x+6y=14$

$-3x-6y=-9$
$5x+6y=14$
$2x=5$
$x=5/2$

$(5/2)+2y=3$
$2y=1/2$
$y=1/4$

b) $(5/2)+(1/4)+z=4$
$11/4+z=4$
$z=5/4$

⑧ a) $x+8y+2z=-24$
b) $3x+y+7z=-3$
c) $4x-3y+6z=9$

$\boxed{(0,-3,0)}$

a) $-3x-24y-6z=72$
b) $3x+y+7z=-3$
$-23y+z=69$

a) $-4x-32y-8z=96$
c) $4x-3y+6z=9$
$-35y-2z=105$

$-23y+z=69$ mult. by 2
$-35y-2z=105$

$-46y+2z=138$
$-35y-2z=105$
$-81y=243$
$y=-3$

$-23(-3)+z=69$
$z=0$

a) $x+8(-3)+2(0)=-24$
$x=0$

Determinants & Cramer's Rule

ANSWER KEY 2.2

① $\begin{vmatrix} 7 & 8 \\ -8 & 0 \end{vmatrix}$ $0 - (-64) =$ **64**

② $\begin{vmatrix} 5 & 5 \\ 5 & 5 \end{vmatrix}$ $25 - 25 =$ **0**

③ $\begin{vmatrix} -6 & -2 \\ 2 & 6 \end{vmatrix}$ $-36 - (-4) =$ **-32**

④ $\begin{vmatrix} 8 & -1 \\ 13 & 0 \end{vmatrix}$ $0 - (-13) =$ **13**

⑤ $\begin{vmatrix} 24 & 6 \\ -13 & -4 \end{vmatrix}$ $-96 - (-78) =$ **-18**

⑥ $\begin{vmatrix} 18 & -5 \\ -9 & 11 \end{vmatrix}$ $198 - (45) =$ **153**

⑦ $\begin{vmatrix} -13 & -11 \\ -17 & -12 \end{vmatrix}$ $156 - 187 =$ **-31**

⑧ $\begin{vmatrix} -6 & 7 \\ -9 & 10 \end{vmatrix}$ $-60 - (-63) =$ **3**

⑨ $5x + 4y = -1$
$2x - y = 10$

$x = \dfrac{\begin{vmatrix} -1 & 4 \\ 10 & -1 \end{vmatrix}}{\begin{vmatrix} 5 & 4 \\ 2 & -1 \end{vmatrix}} = \dfrac{-39}{-13}$ $(1) - (40) = -39$ $(-5) - (8) = -13$

$y = \dfrac{\begin{vmatrix} 5 & -1 \\ 2 & 10 \end{vmatrix}}{-13} = \dfrac{52}{-13}$ $(50) - (-2) = 52$

(3, -4)

⑩ $3a + 8 = -b$ $\quad 3a + b = -8$
$4a - 2b = -14$ $\quad 4a - 2b = -14$

$a = \dfrac{\begin{vmatrix} -8 & 1 \\ -14 & -2 \end{vmatrix}}{\begin{vmatrix} 3 & 1 \\ 4 & -2 \end{vmatrix}} = \dfrac{30}{-10}$ $(16) - (-14)$ $(-6) - (4)$

$b = \dfrac{\begin{vmatrix} 3 & -8 \\ 4 & -14 \end{vmatrix}}{-10} = \dfrac{-10}{-10}$ $(-42) - (-32)$

(-3, 1)

⑪ $2x - 3y = 19$ $\quad 2x - 3y = 19$
$6x + 3 = -6y$ $\quad 6x + 6y = -3$

$x = \dfrac{\begin{vmatrix} 19 & -3 \\ -3 & 6 \end{vmatrix}}{\begin{vmatrix} 2 & -3 \\ 6 & 6 \end{vmatrix}} = \dfrac{105}{30}$ $(114) - (9)$ $(12) - (-18)$

$y = \dfrac{\begin{vmatrix} 2 & 19 \\ 6 & -3 \end{vmatrix}}{30} = \dfrac{-120}{30}$ $(-6) - (114)$

(7/2, -4)

⑫ $6a + 7 = 5b$ $\quad 6a - 5b = -7$
$2a - 3b = 7$ $\quad 2a - 3b = 7$

$a = \dfrac{\begin{vmatrix} -7 & -5 \\ 7 & -3 \end{vmatrix}}{\begin{vmatrix} 6 & -5 \\ 2 & -3 \end{vmatrix}} = \dfrac{56}{-8}$ $(21) - (-35)$ $(-18) - (-10)$

$b = \dfrac{\begin{vmatrix} 6 & -7 \\ 2 & 7 \end{vmatrix}}{-8} = \dfrac{56}{-8}$ $(42) - (-14)$

(-7, -7)

continued

Determinants & Cramer's Rule

ANSWER KEY 2.2

(13) a) $2x - y + 4z = 7$
b) $x - 3y + z = -2$
c) $3x - 2y + 2z = -2$

$\boxed{(-2, 1, 3)}$

a) $2x - y + 4z = 7$
b) $-2x + 6y - 2z = 4$
$5y + 2z = 11$

b) $-3x + 9y - 3z = 6$
c) $3x - 2y + 2z = -2$
$7y - z = 4$

$5y + 2z = 11$
$7y - z = 4$ mult. by 2

$5y + 2z = 11$
$14y - 2z = 8$
$19y = 19$
$y = 1$

$7(1) - z = 4$
$-z = -3$
$z = 3$

b) $x - 3(1) + (3) = -2$
$x = -2$

(14) independent — different lines
consistent — at least one solution (exactly one)

(15) independent — different lines
inconsistent — no solutions

(16) dependent — same line
consistent — at least one solution (infinitely many)

Expansion of Minors & Diagonals

ANSWER KEY 2.3

① $\begin{vmatrix} 1 & 3 & -2 \\ 2 & -1 & 1 \\ -1 & 2 & 3 \end{vmatrix} = \boxed{-32}$

(1) $\begin{vmatrix} -1 & 1 \\ 2 & 3 \end{vmatrix}$ $-(3)\begin{vmatrix} 2 & 1 \\ -1 & 3 \end{vmatrix}$ $+(-2)\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix}$

$(-3)-(2)$ $\quad (6)-(-1)$ $\quad (4)-(1)$

$(1)(-5) - (3)(7) + (-2)(3) = -32$

② $\begin{vmatrix} -1 & 1 & 2 \\ 2 & 1 & 0 \\ 3 & 6 & -2 \end{vmatrix} = \boxed{24}$

$(-1)\begin{vmatrix} 1 & 0 \\ 6 & -2 \end{vmatrix} - (1)\begin{vmatrix} 2 & 0 \\ 3 & -2 \end{vmatrix} + (2)\begin{vmatrix} 2 & 1 \\ 3 & 6 \end{vmatrix}$

$(-2)-(0)$ $\quad (-4)-(0)$ $\quad (12)-(3)$

$(-1)(-2) - (1)(-4) + (2)(9) = 24$

③ $\begin{vmatrix} 1 & -1 & 1 \\ 4 & 3 & 1 \\ 0 & 5 & 2 \end{vmatrix} = \boxed{29}$

$(1)\begin{vmatrix} 3 & 1 \\ 5 & 2 \end{vmatrix} - (-1)\begin{vmatrix} 4 & 1 \\ 0 & 2 \end{vmatrix} + (1)\begin{vmatrix} 4 & 3 \\ 0 & 5 \end{vmatrix}$

$(6)-(5)$ $\quad (8)-(0)$ $\quad (20)-(0)$

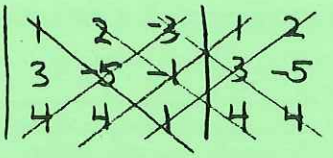
$(1)(1) - (-1)(8) + (1)(20) = 29$

④ $\begin{vmatrix} 3 & -1 & 2 \\ 0 & 4 & 1 \\ 5 & -2 & -3 \end{vmatrix} = \boxed{-75}$

$(3)\begin{vmatrix} 4 & 1 \\ -2 & -3 \end{vmatrix} - (-1)\begin{vmatrix} 0 & 1 \\ 5 & -3 \end{vmatrix} + (2)\begin{vmatrix} 0 & 4 \\ 5 & -2 \end{vmatrix}$

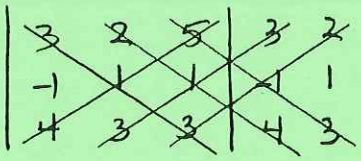
$(-12)-(-2)$ $\quad (0)-(5)$ $\quad (0)-(20)$

$(3)(-10) - (-1)(-5) + (2)(-20) = -75$

⑤  $= \boxed{-111}$

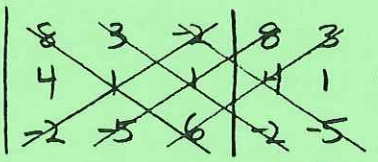
$(-5) + (-8) + (-36) - (60) - (-4) - (6)$

$(-5) + (-8) + (-36) + (-60) + (4) + (-6)$

⑥  $= \boxed{-21}$

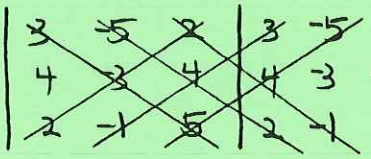
$(9) + (8) + (-15) - (20) - (9) - (-6)$

$(9) + (8) + (-15) + (-20) + (-9) + (6)$

⑦  $= \boxed{46}$

$(48) + (-6) + (40) - (4) - (-40) - (72)$

$(48) + (-6) + (40) + (-4) + (40) + (-72)$

⑧  $= \boxed{31}$

$(-45) + (-40) + (-8) - (-12) - (-12) - (-100)$

$(-45) + (-40) + (-8) + (12) + (12) + (100)$

continued

Expansion of Minors & Diagonals

ANSWER KEY 2.3

⑨ $2x - 3y = 0$
$6x + 5y = 7$

$$x = \frac{(0)-(-21)}{(10)-(-18)} = \frac{\begin{vmatrix} 0 & 3 \\ 7 & 5 \end{vmatrix}}{\begin{vmatrix} 2 & 3 \\ 6 & 5 \end{vmatrix}} = \frac{21}{28}$$

$$y = \frac{(14)-(0)}{28} = \frac{\begin{vmatrix} 2 & 0 \\ 6 & 7 \end{vmatrix}}{28} = \frac{14}{28}$$

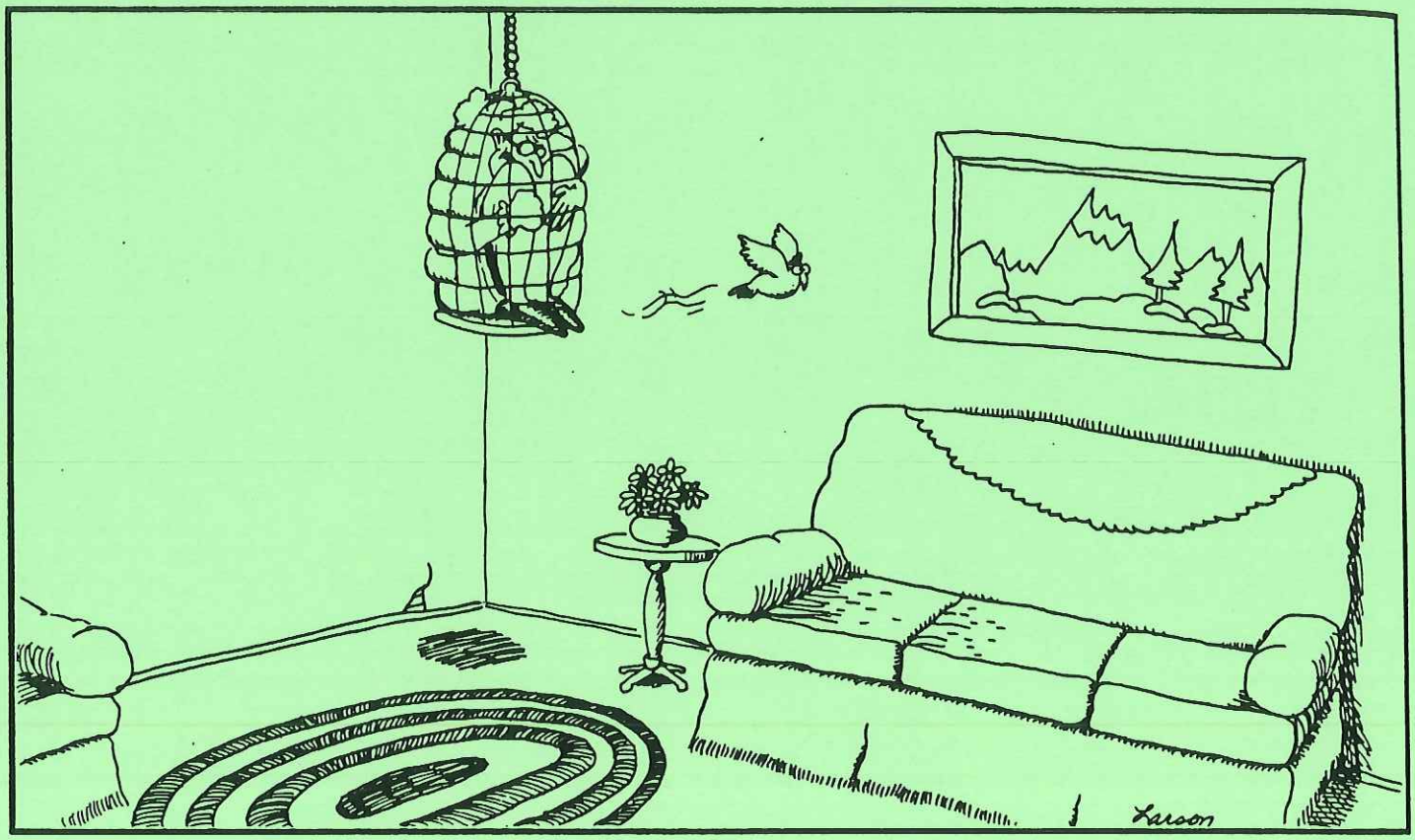
$\boxed{(3/4, 1/2)}$

⑩ $3x + 4y = 6$
$2x + 5y = 11$

$$x = \frac{(30)-(44)}{(15)-(8)} = \frac{\begin{vmatrix} 6 & 4 \\ 11 & 5 \end{vmatrix}}{\begin{vmatrix} 3 & 4 \\ 2 & 5 \end{vmatrix}} = \frac{-14}{7}$$

$$y = \frac{(33)-(12)}{7} = \frac{\begin{vmatrix} 3 & 6 \\ 2 & 11 \end{vmatrix}}{7} = \frac{21}{7}$$

$\boxed{(-2, 3)}$



"You'll never get away with this!"

Cramer's Rule: Third Order

ANSWER KEY 2.4

① $\begin{vmatrix} 2 & -1 & 1 \\ 1 & 2 & 6 \\ 3 & -1 & 2 \end{vmatrix}$ Denominator "-3"

$(2)\begin{vmatrix} 2 & 6 \\ -1 & 2 \end{vmatrix} - (-1)\begin{vmatrix} 1 & 6 \\ 3 & 2 \end{vmatrix} + (1)\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}$

$(4)-(-6) \quad (2)-(18) \quad (-1)-(-6)$

$(2)(10) - (-1)(-16) + (1)(-7) = -3$

$x = \dfrac{-9}{-3} \quad y = \dfrac{-18}{-3} \quad z = \dfrac{6}{-3}$

$\boxed{(3, 6, -2)}$

$x = \dfrac{\begin{vmatrix} -2 & -1 & 1 \\ 3 & 2 & 6 \\ -1 & -1 & 2 \end{vmatrix}}{-3}$ $(-2)\begin{vmatrix} 2 & 6 \\ -1 & 2 \end{vmatrix} - (-1)\begin{vmatrix} 3 & 6 \\ -1 & 2 \end{vmatrix} + (1)\begin{vmatrix} 3 & 2 \\ -1 & -1 \end{vmatrix}$

$(4)-(-6) \quad (6)-(-6) \quad (-3)-(-2)$

$(-2)(10) - (-1)(12) + (1)(-1) = -9$

$y = \dfrac{\begin{vmatrix} 2 & -2 & 1 \\ 1 & 3 & 6 \\ 3 & -1 & 2 \end{vmatrix}}{-3}$ $(2)\begin{vmatrix} 3 & 6 \\ -1 & 2 \end{vmatrix} - (-2)\begin{vmatrix} 1 & 6 \\ 3 & 2 \end{vmatrix} + (1)\begin{vmatrix} 1 & 3 \\ 3 & -1 \end{vmatrix}$

$(6)-(-6) \quad (2)-(18) \quad (-1)-(9)$

$(2)(12) - (-2)(-16) + (1)(-10) = -18$

$z = \dfrac{\begin{vmatrix} 2 & -1 & -2 \\ 1 & 2 & 3 \\ 3 & -1 & -1 \end{vmatrix}}{-3}$ $(2)\begin{vmatrix} 2 & 3 \\ -1 & -1 \end{vmatrix} - (-1)\begin{vmatrix} 1 & 3 \\ 3 & -1 \end{vmatrix} + (-2)\begin{vmatrix} 1 & 2 \\ 3 & -1 \end{vmatrix}$

$(-2)-(-3) \quad (-1)-(9) \quad (-1)-(6)$

$(2)(1) - (-1)(-10) + (-2)(-7) = 6$

② $\begin{vmatrix} 3 & -1 & 2 \\ 6 & -3 & 1 \\ -3 & -2 & 2 \end{vmatrix}$ Denominator "-39"

$(3)\begin{vmatrix} -3 & 1 \\ -2 & 2 \end{vmatrix} - (-1)\begin{vmatrix} 6 & 1 \\ -3 & 2 \end{vmatrix} + (2)\begin{vmatrix} 6 & -3 \\ -3 & -2 \end{vmatrix}$

$(-6)-(-2) \quad (12)-(-3) \quad (-12)-(9)$

$(3)(-4) - (-1)(15) + (2)(-21) = -39$

$x = \dfrac{13}{-39} \quad y = \dfrac{-78}{-39} \quad z = \dfrac{-273}{-39}$

$\boxed{(-1/3, 2, 7)}$

$x = \dfrac{\begin{vmatrix} 11 & -1 & 2 \\ -1 & -3 & 1 \\ 11 & -2 & 2 \end{vmatrix}}{-39}$ $(11)\begin{vmatrix} -3 & 1 \\ -2 & 2 \end{vmatrix} - (-1)\begin{vmatrix} -1 & 1 \\ 11 & 2 \end{vmatrix} + (2)\begin{vmatrix} -1 & -3 \\ 11 & -2 \end{vmatrix}$

$(-6)-(-2) \quad (-2)-(11) \quad (2)-(-33)$

$(11)(-4) - (-1)(-13) + (2)(35) = 13$

$y = \dfrac{\begin{vmatrix} 3 & 11 & 2 \\ 6 & -1 & 1 \\ -3 & 11 & 2 \end{vmatrix}}{-39}$ $(3)\begin{vmatrix} -1 & 1 \\ 11 & 2 \end{vmatrix} - (11)\begin{vmatrix} 6 & 1 \\ -3 & 2 \end{vmatrix} + (2)\begin{vmatrix} 6 & -1 \\ -3 & 11 \end{vmatrix}$

$(-2)-(11) \quad (12)-(-3) \quad (66)-(3)$

$(3)(-13) - (11)(15) + (2)(63) = -78$

$z = \dfrac{\begin{vmatrix} 3 & -1 & 11 \\ 6 & -3 & -1 \\ -3 & -2 & 11 \end{vmatrix}}{-39}$ $(3)\begin{vmatrix} -3 & -1 \\ -2 & 11 \end{vmatrix} - (-1)\begin{vmatrix} 6 & -1 \\ -3 & 11 \end{vmatrix} + (11)\begin{vmatrix} 6 & -3 \\ -3 & -2 \end{vmatrix}$

$(-33)-(2) \quad (66)-(3) \quad (-12)-(9)$

$(3)(-35) - (-1)(63) + (11)(-21) = -273$

③ $\begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & 1 \\ 1 & 3 & -2 \end{vmatrix}$ Denominator "0"

$(2)\begin{vmatrix} 2 & 1 \\ 3 & -2 \end{vmatrix} - (-1)\begin{vmatrix} 3 & 1 \\ 1 & -2 \end{vmatrix} + (3)\begin{vmatrix} 3 & 2 \\ 1 & 3 \end{vmatrix}$

$(-4)-(3) \quad (-6)-(1) \quad (9)-(2)$

$(2)(-7) - (-1)(-7) + (3)(7) = 0$

No distinct solution

An undefined value results from "0" in the denominator

Cramer's Rule: Third Order

ANSWER KEY 2.4

④  Denominator "15"

(3) + (18) + (−6) − (9) − (3) − (−12)

(3) + (18) + (−6) + (−9) + (−3) + (12)

(3, 1, 6)

$x = \dfrac{45}{15} = 3$

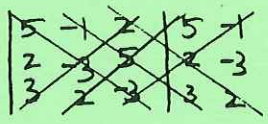
(−39) + (−48) + (−69) − (−24) − (−39) − (−138)

$y = \dfrac{15}{15} = 1$

(−69) + (−117) + (48) − (−207) − (−24) − (78)

$z = \dfrac{90}{15} = 6$

(−6) + (138) + (−26) − (39) − (23) − (−32)

⑤  Denominator "0"

(45) + (−15) + (8) − (−18) − (50) − (6)

No distinct solution

⑥  Denominator "50"

(6) + (4) + (12) − (−6) − (2) − (−24)

$(6, -\frac{1}{2}, 2)$

$x = \dfrac{300}{50} = 6$

(60) + (−4) + (90) − (6) − (20) − (−180)

$y = \dfrac{-25}{50} = \dfrac{-1}{2}$

(−45) + (10) + (−6) − (45) − (−1) − (−60)

$z = \dfrac{100}{50} = 2$

(2) + (60) + (40) − (−20) − (30) − (−8)

⑦ $x + 2y = 5$

$x - y = 6 \rightarrow x = 6 + y$

$(17/3, -1/3)$

$(6 + y) + 2y = 5$

$3y = -1$

$y = -1/3$

$x - (-1/3) = 6$

$x = 17/3$

⑧ $(3x - 5y = -13) \times (4) = 12x - 20y = -52$

$(4x + 3y = 2) \times (-3) = -12x - 9y = -6$

$-29y = -58$

$y = 2$

$3x - 5(2) = -13$

$3x = -3$

$x = -1$

(−1, 2)

⑨ $7x - 8y = 11$

$9x - 2y = 3$

$x = \dfrac{\begin{vmatrix} 11 & -8 \\ 3 & -2 \end{vmatrix}}{\begin{vmatrix} 7 & -8 \\ 9 & -2 \end{vmatrix}} = \dfrac{2}{58}$ (−22) − (−24) / (−14) − (−72)

$y = \dfrac{\begin{vmatrix} 7 & 11 \\ 9 & 3 \end{vmatrix}}{58} = \dfrac{-78}{58}$ (21) − (99)

$(1/29, -39/29)$

Systems

ANSWER KEY: UNIT 2 REVIEW & PRACTICE

① $x - 3y = -3 \rightarrow x = 3y - 3$
$4x + 9y = 2$

$4(3y-3)+9y=2$
$12y - 12 + 9y = 2$
$21y = 14$
$y = 2/3$

$x = 3(2/3) - 3$
$x = -1$

$\boxed{(-1, 2/3)}$

② $2x + 3y = 5$ mult. by -2
$-3x + 6y = 12$

$-4x - 6y = -10$
$-3x + 6y = 12$
$-7x = 2$
$x = -2/7$

$2(-2/7) + 3y = 5$
$3y = 5 + 4/7$
$3y = 39/7$
$y = 13/7$

$\boxed{(-2/7, 13/7)}$

③ a) $4x + 3y + z = -10$
b) $x - 12y + 2z = -5$
c) $x + 18y + z = 4$

$\boxed{(-3, 1/3, 1)}$

a) $4x + 3y + z = -10$
b) $-4x + 48y - 8z = 20$
$51y - 7z = 10$

b) $x - 12y + 2z = -5$
c) $-x - 18y - z = -4$
$-30y + z = -9$

mult. by 7

$51y - 7z = 10$
$-210y + 7z = -63$
$-159y = -53$
$y = 1/3$

$-30(1/3) + z = -9$
$z = 1$

$x - 12(1/3) + 2(1) = -5$
$x - 4 + 2 = -5$
$x = -3$

④ $\begin{vmatrix} -3 & 7 \\ 4 & 9 \end{vmatrix}$

$(-27) - (28) = \boxed{-55}$

⑤ $\begin{vmatrix} -2 & 0 \\ 6 & 5 \end{vmatrix}$

$(-10) - (0) = \boxed{-10}$

⑥ $3x + 2y = 40$
$x - 7y = -2$

$\boxed{(12, 2)}$

$x = \dfrac{\begin{vmatrix} 40 & 2 \\ -2 & -7 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 1 & -7 \end{vmatrix}} = \dfrac{-276}{-23}$ $\quad (-280)-(-4)$ $\quad (-21)-(2)$

$y = \dfrac{\begin{vmatrix} 3 & 40 \\ 1 & -2 \end{vmatrix}}{-23} = \dfrac{-46}{-23}$ $\quad (-6)-(40)$

⑦ $\begin{vmatrix} 4 & -5 & 3 \\ -3 & 2 & 0 \\ 2 & -1 & 4 \end{vmatrix} = \boxed{-31}$

$(4)\begin{vmatrix} 2 & 0 \\ -1 & 4 \end{vmatrix} - (-5)\begin{vmatrix} -3 & 0 \\ 2 & 4 \end{vmatrix} + (3)\begin{vmatrix} -3 & 2 \\ 2 & -1 \end{vmatrix}$

$(8)-(0) \quad (-12)-(0) \quad (3)-(4)$

$(4)(8) - (-5)(-12) + (3)(-1) = -31$

⑧ $\begin{vmatrix} 6 & 3 & 6 & 3 \\ -1 & 1 & -4 & 1 \\ -2 & 4 & -2 & 4 \end{vmatrix} = \boxed{136}$

$(12) + (24) + (-4) - (-2) - (-96) - (-6)$

$(12) + (24) + (-4) + (2) + (96) + (6)$

Systems

ANSWER KEY: UNIT 2 REVIEW & PRACTICE

(9) $2a - b + 3c = 5$
$3a + 2b - 5c = 7$
$a - 4b + 11c = 3$

Denominator

$$\begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & -5 \\ 1 & -4 & 11 \end{vmatrix} = 0$$

$$(2)\begin{vmatrix} 2 & -5 \\ -4 & 11 \end{vmatrix} - (-1)\begin{vmatrix} 3 & -5 \\ 1 & 11 \end{vmatrix} + (3)\begin{vmatrix} 3 & 2 \\ 1 & -4 \end{vmatrix}$$

$(22) - (20) \qquad (33) - (-5) \qquad (-12) - (2)$

$(2)(2) - (-1)(38) + (3)(-14) = 0$

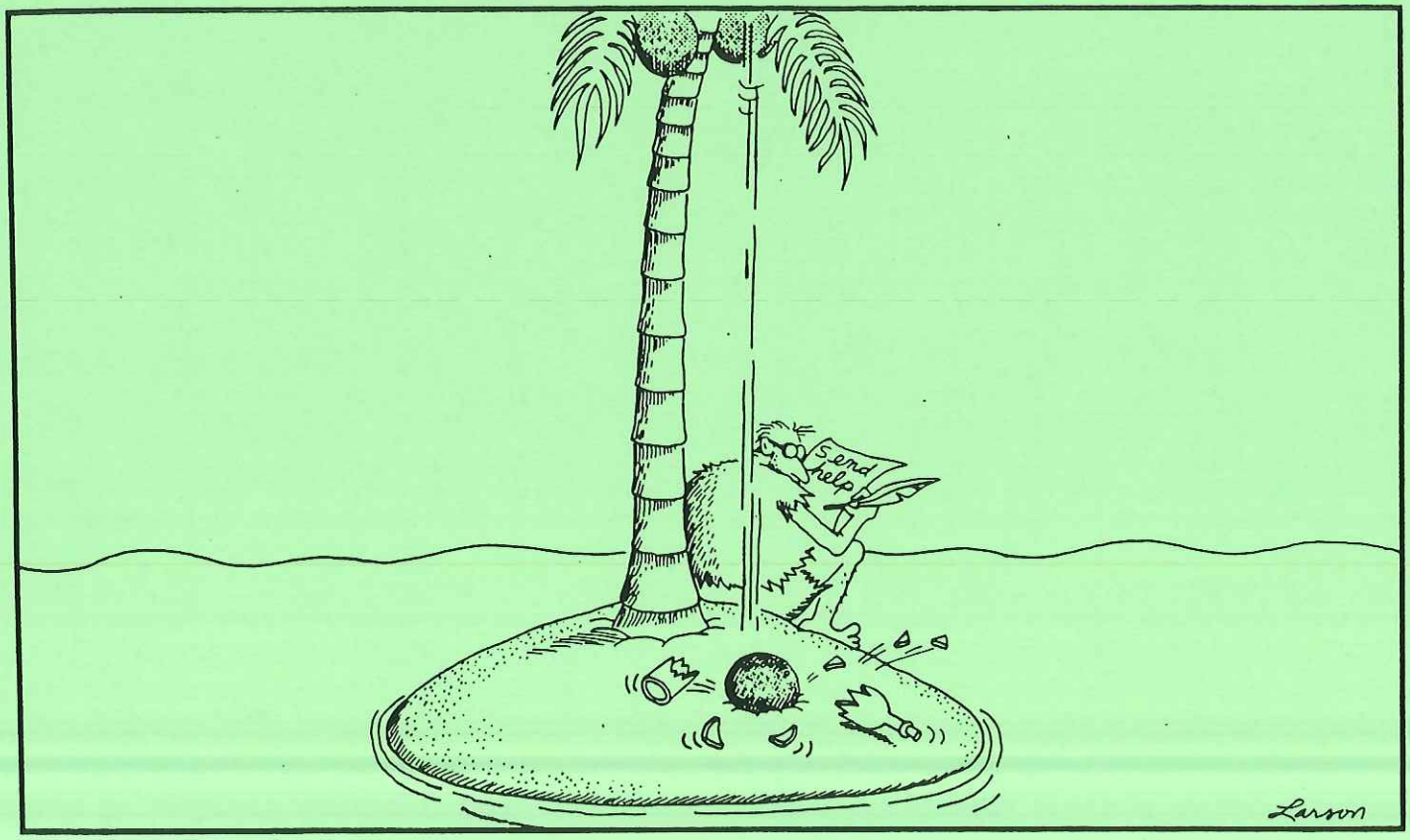
NO UNIQUE SOLUTION

(10) $3x + 4y + z = 10$
$6x - 2y - z = 6$
$3x + 6y - 2z = 2$

$$x = \frac{\begin{vmatrix} 10 & 4 & 1 \\ 6 & -2 & -1 \\ 2 & 6 & -2 \end{vmatrix}}{\begin{vmatrix} 3 & 4 & 1 \\ 6 & -2 & -1 \\ 3 & 6 & -2 \end{vmatrix}} = \frac{180}{108}$$

$$x = \frac{(40) + (-8) + (36) - (-4) - (-60) - (-48)}{(12) + (-12) + (36) - (-6) - (-18) - (-48)}$$

$x = \boxed{5/3}$



Problem Solving

ANSWER KEY 3.1

① $r \cdot t = d$

Truck $45 \cdot 12 = 540$

Car $r \cdot 10 = 10r$

$10r = 45 \cdot 12$

$10r = 540$

$r = 54$ **54 mph**

② $r \cdot t = d$

Car 1 $r \cdot 3 = 3r$

Car 2 $r+10 \cdot 3 = 3r+30$

$3r + (3r+30) = 240$

$6r = 210$

$r = 35$ **35 and 45 mph**

③ $r \cdot t = d$

Org. Speed $r \cdot 3 = 3r$

New Speed $r+4 \cdot 2 = 2r+8$

$3r = 2r + 8$

$r = 8$ **12 mph**

$r+4 = 12$

④ $(55)(5) = 275$ miles - 1st 5 hrs

$662 - 275 = 387$ miles to go

$387 \div 9 =$ **43 mph** - final 9 hrs

⑤ htu → 3 digit number

a) $h+t+u = 15$

b) $h - (t+u) = 1$

continued

new number: $100u + 10t + h$

c) $(100u+10t+h) = (100h+10t+u) - 495$

$99u - 99h = -495$

a) $h+t+u = 15$

b) $h-t-u = 1$

$2h = 16$

$h = 8$

c) $99u - 99(8) = -495$

$99u - 792 = -495$

$99u = 297$

$u = 3$

a) $(8) + t + (3) = 15$

$t = 4$

843

⑥ htu → 3 digit number

a) $h+t+u = 17$

b) $t = 3h$

c) $h+t = u-1$

a) $h+t+u = 17$

c) $-h-t+u = 1$

$2u = 18$

$u = 9$

a) $h + (3h) + (9) = 17$

$4h = 8$

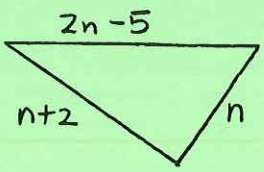
$h = 2$

b) $t = 3(2)$

$t = 6$

269

⑦



$(n) + (n+2) + (2n-5) = 45$

$4n - 3 = 45$

$4n = 48$

$n = 12$

$n+2 = 14$

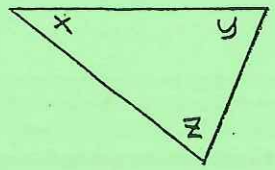
$2n-5 = 19$

12, 14, 19 cm

Problem Solving

ANSWER KEY 3.1

⑧



x smallest ∠
y middle ∠
z largest ∠

a) $x+y+z=180$
b) $z=x+15$ $\quad -x+z=15$
c) $(y+z)-2x=24$ $\quad -2x+y+z=24$

a) $-x-y-z=-180$
c) $-2x+y+z=24$
$-3x = -156$
$x = 52$

b) $-(52)+z=15$
$z=67$

a) $(52)+y+(67)=180$
$y=61$

52° $\boxed{61^\circ}$ 67°

⑨ n = amount (kg) of pure copper

$.40(20)+.50(10)+1.00(n)=.66(30+n)$
$8+5+n=19.8+.66n$
$.34n=6.8$
$n=20$ $\quad \boxed{20\text{ kg}}$

⑩ n = amount (ℓ) of 70% solution

$.90(30)+.70(n)=.82(30+n)$
$27+.7n=24.6+.82n$
$-.12n=-2.4$
$n=20$ $\quad \boxed{20\ \ell}$

⑪ n = sales for one week

$240+.08(n)=680$
$.08n=440$
$n=5500$ $\quad \boxed{\$5500}$

⑫ $\dfrac{\text{aft. tax}}{\text{org. pr.}}$ $\dfrac{23.52}{n}=\dfrac{105}{100}$

$105n=2352$
$n=22.4$
$23.52-22.40=$ $\boxed{\$1.12}$

⑬ $.08(n)=(2)(.06)(8000-n)-40$
$.08n=960-.12n-40$
$.2n=920$
$n=4600$
$8000-n=3400$

$\boxed{\$4600 \text{ at } 8\%}$
$\boxed{\$3400 \text{ at } 6\%}$

⑭ $\dfrac{\text{purch. pr.}}{\text{org. pr.}}$ $\dfrac{27.88}{n}=\dfrac{85}{100}$

$85n=2788$
$n=32.8$

$32.80-27.88=4.92$

She saved $\boxed{\$4.92}$

Problem Solving

ANSWER KEY 3.1

(15)

	r	· t	= d
with wind	r+w	· 1/20	= 1
against wind	r-w	· 1/4	= 1

$(\frac{1}{20}r + \frac{1}{20}w = 1) \times 20$

$(\frac{1}{4}r - \frac{1}{4}w = 1) \times 4$

$r + w = 20$

$r - w = 4$

$2r = 24$

$r = 12$

$w = 8$

8 mph

(16)

	r	· t	= d
upstream	r-c	· 1/3	= 3
downstream	r+c	· 1/5	= 3

$(\frac{1}{3}r - \frac{1}{3}c = 3) \times 3$

$(\frac{1}{5}r + \frac{1}{5}c = 3) \times 5$

$r - c = 9$

$r + c = 15$

$2r = 24$

$r = 12$

12 mph

(17)

	r	· t	= d
bike	15	· t	= 15t
walk	4	· 5-t	= 20-4t

$15t + (20 - 4t) = 53$

$11t = 33$

$t = 3$

distance walked:

$20 - 4t$

$20 - 4(3) = 8$ **8 miles**

(18)

a) $h + t + u = 14$

b) $t - 2h = 1$

c) $u + 3 = h$

a) $h + t + u = 14$

c) $-h \quad + u = 3$

$t + 2u = 11$

a) $2h + 2t + 2u = 28$

b) $-2h + t \quad = 1$

$3t + 2u = 29$

$t + 2u = 11$

$-3t - 2u = -29$

$-2t = -18$

$t = 9$

b) $(9) - 2h = 1$

$-2h = -8$

$h = 4$

c) $u + 3 = (4)$

$u = 1$

491

(19)

a) $2h - (t+u) = 3$

b) $(100u + 10t + h) = (100h + 10t + u) + 198$

$99u - 99h = 198$

$u - h = 2$

c) $h + t + u = 12$

507

a) $2h - t - u = 3$

c) $h + t + u = 12$

$3h = 15$

$h = 5$

b) $u - (5) = 2$

$u = 7$

c) $(5) + t + (7) = 12$

$t = 0$

(20)

$.25(32) - 0(n) = .40(32 - n)$

$8 = 12.8 - .4n$

$.4n = 4.8$

$n = 12$

12 l

Problem Solving

ANSWER KEY 3.1

(21) n = weekly salary
$100n$ = sales

$4n + .075(100n) = 1380$
$4n + 7.5n = 1380$
$11.5n = 1380$
$n = 120$ **$120**

(22) aft. tax / Sale price $\frac{11.34}{n} = \frac{105}{100}$

$105n = 1134$
$n = 10.80$

Sale price / org price $\frac{10.80}{n} = \frac{90}{100}$

$90n = 1080$
$n = 12$ **$12.00**

(23) $.06n = .08(4000-n) + 100$
$.06n = 320 - .08n + 100$
$.14n = 420$
$n = 3000$

\$3000 at 6% = $(3000)(.06) = 180$
\$1000 at 8% = $(1000)(.08) = 80$
260

$260 interest

$260 \div 4000 = .065$ **$6\frac{1}{2}\%$**

(24)

	r	$\cdot$	t	$=$	d
minute hand	60	$\cdot$	t	$=$	$60t$
hour hand	5	$\cdot$	t	$=$	$5t$

The hour hand has a 25 minute head start at 5:00.

$60t = 5t + 25$
$55t = 25$
$t = 5/11$ hours

$(5/11)(60) = 300/11 = 27\frac{3}{11}$ minutes

$(3/11)(60) = 180/11 \approx 16$ seconds

5:27:16

(25)

	r	$\cdot$	t	$=$	d
minute hand	60	$\cdot$	t	$=$	$60t$
hour hand	5	$\cdot$	t	$=$	$5t$

The hour hand has a 15 minute head start at 3:00.

$60t = 5t + 15$
$55t = 15$
$t = 3/11$ hours

$(3/11)(60) = 180/11 = 16\frac{4}{11}$ minutes

$(4/11)(60) = 240/11 \approx 22$ seconds

3:16:22

Problem Solving

ANSWER KEY 3.1

(26)

	r	·	t	=	d
Conrad	6	·	t	=	6t
Chris	10	·	t	=	10t

$6t + 10t = 1$

$16t = 1$

$t = 1/16$ hours

$\left(\frac{1}{16}\right)(60) = \frac{60}{16} = \boxed{3\frac{3}{4} \text{ minutes}}$

(27)

	r	·	t	=	d
Boy	8	·	5/4	=	10
Father	r	·	1/4	=	10

If the father drives $\frac{1}{4}$ hour and leaves 1 hour later, the boy rides $1\frac{1}{4}$ hours

$\frac{1}{4}r = 10 \quad r = \boxed{40 \text{ mph}}$

(28)

	r	·	t	=	d
uphill	8	·	t	=	8t
downhill	12	·	t-1/4	=	12t-3

$8t = 12t - 3$

$-4t = -3$

$t = 3/4$

$8t = 8(3/4) = 6$

$\boxed{6 \text{ miles}}$

(29)

	r	·	t	=	d
out	25	·	t	=	25t
back	40	·	$3\frac{1}{4} - t$	=	130-40t

$25t = 130 - 40t$

$65t = 130$

$t = 2$

$25t = 25(2) = \boxed{50 \text{ miles}}$

(30)

	r	·	t	=	d
with wind	r+w	·	3	=	1200
against wind	r-w	·	5	=	1200

$(3r + 3w = 1200) \div 3 \quad r + w = 400$

$(5r - 5w = 1200) \div 5 \quad r - w = 240$

$2r = 640$

$r = 320$

$w = 80$

rate in still air = 320 mph

wind = 80 mph

4 hours:

2 hours with wind $2(320+80)$

2 hours no wind $2(320)$

$2(320+80) + 2(320) = 1440$

$\boxed{1440 \text{ miles}}$

Binomial Theorem

ANSWER KEY 17.1

① $1 \quad 5\left(\frac{4}{2}\right) \quad 10 \quad 10 \quad 5 \quad 1$

② $1 \quad 4\left(\frac{3}{2}\right) \quad 6 \quad 4 \quad 1$

③ $1 \quad 8\left(\frac{7}{2}\right) \quad 28\left(\frac{6}{3}\right) \quad 56\left(\frac{5}{4}\right) \quad 70 \quad 56 \quad 28 \quad 8 \quad 1$

④ $1 \quad 13\left(\frac{12}{2}\right) \quad 78\left(\frac{11}{3}\right) \quad 286\left(\frac{10}{4}\right) \quad 715\left(\frac{9}{5}\right) \quad 1287\left(\frac{8}{6}\right) \quad 1716 \quad 1716 \quad 1287 \quad 715 \quad 286 \quad 78 \quad 13 \quad 1$

⑤ $(x+m)^4 \rightarrow 1 \quad 4\left(\frac{3}{2}\right) \quad 6 \quad 4 \quad 1$

$x^4 + 4x^3m + 6x^2m^2 + 4xm^3 + m^4$

⑥ $(r+s)^6 \rightarrow 1 \quad 6\left(\frac{5}{2}\right) \quad 15\left(\frac{4}{3}\right) \quad 20 \quad 15 \quad 6 \quad 1$

$r^6 + 6r^5s + 15r^4s^2 + 20r^3s^3 + 15r^2s^4 + 6rs^5 + s^6$

⑦ $(b-z)^5 \rightarrow 1 \quad 5\left(\frac{4}{2}\right) \quad 10 \quad 10 \quad 5 \quad 1$

$b^5 - 5b^4z + 10b^3z^2 - 10b^2z^3 + 5bz^4 - z^5$

⑧ $(r-m)^8 \rightarrow 1 \quad 8\left(\frac{7}{2}\right) \quad 28\left(\frac{6}{3}\right) \quad 56\left(\frac{5}{4}\right) \quad 70 \quad 56 \quad 28 \quad 8 \quad 1$

$r^8 - 8r^7m + 28r^6m^2 - 56r^5m^3 + 70r^4m^4 - 56r^3m^5 + 28r^2m^6 - 8rm^7 + m^8$

⑨ $(3r+y)^4 \rightarrow 1 \quad 4\left(\frac{3}{2}\right) \quad 6 \quad 4 \quad 1$

$(3r)^4 + 4(3r)^3y + 6(3r)^2y^2 + 4(3r)y^3 + y^4$

$81r^4 + 108r^3y + 54r^2y^2 + 12ry^3 + y^4$

⑩ $(2b+x)^6 \rightarrow 1 \quad 6\left(\frac{5}{2}\right) \quad 15\left(\frac{4}{3}\right) \quad 20 \quad 15 \quad 6 \quad 1$

$(2b)^6 + 6(2b)^5x + 15(2b)^4x^2 + 20(2b)^3x^3 + 15(2b)^2x^4 + 6(2b)x^5 + x^6$

$64b^6 + 192b^5x + 240b^4x^2 + 160b^3x^3 + 60b^2x^4 + 12bx^5 + x^6$

⑪ $(3x-2y)^5 \rightarrow 1 \quad 5\left(\frac{4}{2}\right) \quad 10 \quad 10 \quad 5 \quad 1$

$(3x)^5 - 5(3x)^4(2y) + 10(3x)^3(2y)^2 - 10(3x)^2(2y)^3 + 5(3x)(2y)^4 - (2y)^5$

$243x^5 - 810x^4y + 1080x^3y^2 - 720x^2y^3 + 240xy^4 - 32y^5$

(continued)

Binomial Theorem

ANSWER KEY 17.1

(12) $(2m-3)^6 \rightarrow 1 \quad 6\left(\frac{5}{2}\right) \quad 15\left(\frac{4}{3}\right) \quad 20 \quad 15 \quad 6 \quad 1$

$(2m)^6 - 6(2m)^5(3) + 15(2m)^4(3)^2 - 20(2m)^3(3)^3 + 15(2m)^2(3)^4 - 6(2m)(3)^5 + (3)^6$

$64m^6 - 576m^5 + 2160m^4 - 4320m^3 + 4860m^2 - 2916m + 729$

(13) $(2n+3m)^8 \rightarrow 1 \quad 8\left(\frac{7}{2}\right) \quad 28\left(\frac{6}{3}\right) \quad 56\left(\frac{5}{4}\right) \quad 70 \quad 56 \quad 28 \quad 8 \quad 1$

$(2n)^8 + 8(2n)^7(3m) + 28(2n)^6(3m)^2 + 56(2n)^5(3m)^3 + 70(2n)^4(3m)^4 + 56(2n)^3(3m)^5 + 28(2n)^2(3m)^6 + 8(2n)(3m)^7 + (3m)^8$

$256n^8 + 3072n^7m + 16{,}128n^6m^2 + 48{,}384n^5m^3 + 90{,}720n^4m^4 + 108{,}864n^3m^5 + 81{,}648n^2m^6 + 34{,}992nm^7 + 6561m^8$

(14) $(x-4y)^9 \rightarrow 1 \quad 9\left(\frac{8}{2}\right) \quad 36\left(\frac{7}{3}\right) \quad 84\left(\frac{6}{4}\right) \quad 126 \quad 126 \quad 84 \quad 36 \quad 9 \quad 1$

$x^9 - 9x^8(4y) + 36x^7(4y)^2 - 84x^6(4y)^3 + 126x^5(4y)^4 - 126x^4(4y)^5 + 84x^3(4y)^6 - 36x^2(4y)^7 + 9x(4y)^8 - (4y)^9$

$x^9 - 36x^8y + 576x^7y^2 - 5376x^6y^3 + 32{,}256x^5y^4 - 129{,}024x^4y^5 + 344{,}064x^3y^6 - 589{,}824x^2y^7 + 589{,}824xy^8 - 262{,}144y^9$

(15) $a^5 - b^5 = (a-b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$

(16) $32x^5 + y^{10}$

$(2x)^5 + (y^2)^5$

$(2x+y^2)\left[(2x)^4 - (2x)^3(y^2) + (2x)^2(y^2)^2 - (2x)(y^2)^3 + (y^2)^4\right]$

$(2x+y^2)(16x^4 - 8x^3y^2 + 4x^2y^4 - 2xy^6 + y^8)$

(17) $128a^{14} - b^{21}$

$(2a^2)^7 - (b^3)^7$

$(2a^2 - b^3)\left[(2a^2)^6 + (2a^2)^5(b^3) + (2a^2)^4(b^3) + (2a^2)^3(b^3)^3 + (2a^2)^2(b^3)^4 + (2a^2)(b^3)^5 + (b^3)^6\right]$

$(2a^2 - b^3)(64a^{12} + 32a^{10}b^3 + 16a^8b^6 + 8a^6b^9 + 4a^4b^{12} + 2a^2b^{15} + b^{18})$

(18) $243x^{15} - 1$

$(3x^3)^5 - 1$

$(3x^3 - 1)\left[(3x^3)^4 + (3x^3)^3(1) + (3x^3)^2(1)^2 + (3x^3)(1)^3 + (1)^4\right]$

$(3x^3 - 1)(81x^{12} + 27x^9 + 9x^6 + 3x^3 + 1)$

Sigma Notation & Arithmetic Series

ANSWER KEY 17.2

① $\sum_{K=2}^{6} (3+2K)$ $n=5$ $d=2$ $a_1=7$

$S=\frac{n}{2}[2a_1+(n-1)d]$

$S=\frac{5}{2}[2(7)+4(2)]$

$S=\frac{5}{2}(22)=\boxed{55}$

$\boxed{7+9+11}$

② $\sum_{m=3}^{9} (5-3m)$ $n=7$ $d=-3$ $a_1=-4$

$S=\frac{n}{2}[2a_1+(n-1)d]$

$S=\frac{7}{2}[2(-4)+6(-3)]$

$S=\frac{7}{2}(-26)=\boxed{-91}$

$\boxed{(-4)+(-7)+(-10)}$

③ $\sum_{x=2}^{10} (8-3x)$ $n=9$ $d=-3$ $a_1=2$

$S=\frac{n}{2}[2a_1+(n-1)d]$

$S=\frac{9}{2}[2(2)+8(-3)]$

$S=\frac{9}{2}(-20)=\boxed{-90}$

$\boxed{2+(-1)+(-4)}$

④ $\sum_{n=1}^{30} (2n-1)$ $n=30$ $d=2$ $a_1=1$

$S=\frac{n}{2}[2a_1+(n-1)d]$

$S=\frac{30}{2}[2(1)+29(2)]$

$S=15(60)=\boxed{900}$

$\boxed{1+3+5}$

⑤ $\sum_{n=1}^{40} (3n+2)$ $n=40$ $d=3$ $a_1=5$

$S=\frac{n}{2}[2a_1+(n-1)d]$

$S=\frac{40}{2}[2(5)+39(3)]$

$S=20(127)=\boxed{2540}$

$\boxed{5+8+11}$

⑥ $\sum_{x=10}^{50} (3x-1)$ $n=41$ $d=3$ $a_1=29$

$S=\frac{n}{2}[2a_1+(n-1)d]$

$S=\frac{41}{2}[2(29)+40(3)]$

$S=\frac{41}{2}(178)=\boxed{3649}$

$\boxed{29+32+35}$

⑦ $\sum_{m=21}^{75} (2m+5)$ $n=55$ $d=2$ $a_1=47$

$S=\frac{n}{2}[2a_1+(n-1)d]$

$S=\frac{55}{2}[2(47)+54(2)]$

$S=\frac{55}{2}(202)=\boxed{5555}$

$\boxed{47+49+51}$

⑧ $n=50$ $a_1=1$ $d=2$

$S=\frac{n}{2}[2a_1+(n-1)d]$

$S=\frac{50}{2}[2(1)+49(2)]$

$S=25(100)=\boxed{2500}$

⑨ $100\div 6=16\frac{2}{3}$

$n=16$ $d=6$ $a_1=6$

$S=\frac{n}{2}[2a_1+(n-1)d]$

$S=\frac{16}{2}[2(6)+15(6)]$

$S=8(102)=\boxed{816}$

⑩ $n=30$ $a_1=21$ $d=1$

$S=\frac{n}{2}[2a_1+(n-1)d]$

$S=\frac{30}{2}[2(21)+29(1)]$

$S=15(71)=\boxed{1065}$

Sigma Notation & Arithmetic Series

ANSWER KEY 17.2

(11) $(m+3n)^7 \rightarrow 1 \quad 7\left(\frac{6}{1}\right) \quad 21\left(\frac{5}{2}\right) \quad 35 \quad 35 \quad 21 \quad 7 \quad 1$

$m^7 + 7m^6(3n) + 21m^5(3n)^2 + 35m^4(3n)^3 + 35m^3(3n)^4 + 21m^2(3n)^5 + 7m(3n)^6 + (3n)^7$

$m^7 + 21m^6n + 189m^5n^2 + 945m^4n^3 + 2835m^3n^4 + 5103m^2n^5 + 5103mn^6 + 2187n^7$

(12) $(2x-5y)^6 \rightarrow 1 \quad 6\left(\frac{5}{1}\right) \quad 15\left(\frac{4}{2}\right) \quad 20 \quad 15 \quad 6 \quad 1$

$(2x)^6 - 6(2x)^5(5y) + 15(2x)^4(5y)^2 - 20(2x)^3(5y)^3 + 15(2x)^2(5y)^4 - 6(2x)(5y)^5 + (5y)^6$

$64x^6 - 960x^5y + 6000x^4y^2 - 20{,}000x^3y^3 + 37{,}500x^2y^4 - 37{,}500xy^5 + 15{,}625y^6$

(13) $256x^{14} - 2y^{21}$

$2(128x^{14} - y^{21})$

$2(2x^2)^7 - (y^3)^7$

$2(2x^2 - y^3)\left[(2x^2)^6 + (2x^2)^5(y^3) + (2x^2)^4(y^3)^2 + (2x^2)^3(y^3)^3 + (2x^2)^2(y^3)^4 + (2x^2)(y^3)^5 + (y^3)^6\right]$

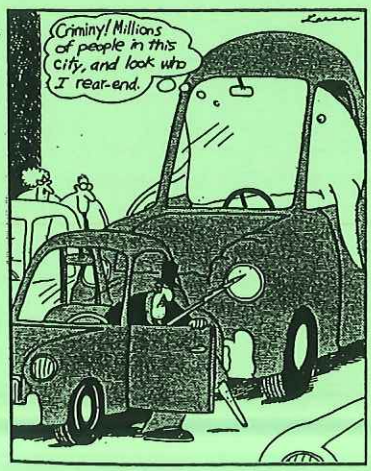
$2(2x^2 - y^3)(64x^{12} + 32x^{10}y^3 + 16x^8y^6 + 8x^6y^9 + 4x^4y^{12} + 2x^2y^{15} + y^{18})$

(14) $1024a^5b^{10} + c^{15}$

$(4ab^2)^5 + (c^3)^5$

$(4ab^2 + c^3)\left[(4ab^2)^4 - (4ab^2)^3(c^3) + (4ab^2)^2(c^3)^2 - (4ab^2)(c^3)^3 + (c^3)^4\right]$

$(4ab^2 + c^3)(256a^4b^8 - 64a^3b^6c^3 + 16a^2b^4c^6 - 4ab^2c^9 + c^{12})$



Geometric Series

ANSWER KEY 17.3

① $\sum_{c=1}^{8} 3^c$ geometric series $n=8$ $r=3$ $a_1=3$

$S = \frac{a_1 - a_1 r^n}{1-r} = \frac{3-3(3)^8}{1-3}$

9840 | 3 + 9 + 27

② $\sum_{t=-2}^{6} 3(2)^{t+3}$ geometric series $n=9$ $r=2$ $a_1=6$

$S = \frac{a_1 - a_1 r^n}{1-r} = \frac{6-6(2)^9}{1-2}$

3066 | 6 + 12 + 24

③ $\sum_{x=-3}^{3} (2x+2)$ arithmetic series $n=7$ $d=2$ $a_1=-4$

$S = \frac{n}{2}[2a_1 + (n-1)d] = \frac{7}{2}(-8+12)$

14 | (-4) + (-2) + (0)

④ $\sum_{e=3}^{12} 3(-2)^e$ geometric series $n=10$ $r=-2$ $a_1=-24$

$S = \frac{a_1 - a_1 r^n}{1-r} = \frac{-24-(-24)(-2)^{10}}{1-(-2)}$

8184 | (-24) + (48) + (-96)

⑤ $\sum_{n=0}^{6} (24-9n)$ arithmetic series $n=7$ $d=-9$ $a_1=24$

$S = \frac{n}{2}[2a_1 + (n-1)d] = \frac{7}{2}(48+(-54))$

-21 | 24 + 15 + 6

⑥ $\sum_{k=1}^{7} 2^{k-2}$ geometric series $n=7$ $r=2$ $a_1=\frac{1}{2}$

$S = \frac{a_1 - a_1 r^n}{1-r} = \frac{\frac{1}{2} - (\frac{1}{2})(2)^7}{1-2}$

$63\frac{1}{2}$ | $\frac{1}{2} + 1 + 2$

⑦ $\sum_{n=2}^{11} 4(-3)^{n-1}$ geometric series $n=10$ $r=-3$ $a_1=-12$

$S = \frac{a_1 - a_1 r^n}{1-r} = \frac{(-12)-(-12)(-3)^{10}}{1-(-3)}$

177,144 | (-12) + (36) + (-108)

⑧ $(x-y)^7$

1 7 (6/2) 21 (5/3) 35 35 21 7 1

$x^7 - 7x^6y + 21x^5y^2 - 35x^4y^3 + 35x^3y^4 - 21x^2y^5 + 7xy^6 - y^7$

(continued)

Geometric Series

ANSWER KEY 17.3

⑨ $(2x+3y)^5 \rightarrow 1 \quad 5 \quad (\frac{4}{2}) \quad 10 \quad 10 \quad 5 \quad 1$

$(2x)^5 + 5(2x)^4(3y) + 10(2x)^3(3y)^2 + 10(2x)^2(3y)^3 + 5(2x)(3y)^4 + (3y)^5$

$32x^5 + 240x^4y + 720x^3y^2 + 1080x^2y^3 + 810xy^4 + 243y^5$

⑩ $(3n-4m)^6 \rightarrow 1 \quad 6(\frac{5}{2}) \quad 15(\frac{4}{3}) \quad 20 \quad 15 \quad 6 \quad 1$

$(3n)^6 - 6(3n)^5(4m) + 15(3n)^4(4m)^2 - 20(3n)^3(4m)^3 + 15(3n)^2(4m)^4 - 6(3n)(4m)^5 + (4m)^6$

$729n^6 - 5832n^5m + 19440n^4m^2 - 34,560n^3m^3 + 34560n^2m^4 - 18432nm^5 + 4096m^6$

⑪ $64x^5 - 2y^{10}$

$2(32x^5 - y^{10})$

$2[(2x)^5 - (y^2)^5]$

$2(2x-y^2)[(2x)^4 + (2x)^3(y^2) + (2x)^2(y^2)^2 + (2x)(y^2)^3 + (y^2)^4]$

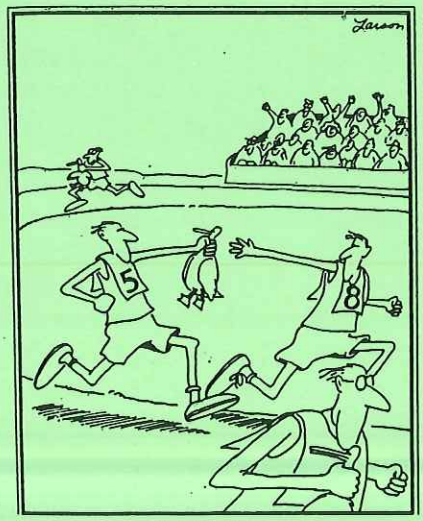
$2(2x-y^2)(16x^4 + 8x^3y^2 + 4x^2y^4 + 2xy^6 + y^8)$

⑫ $a^7b^7 + 128c^7$

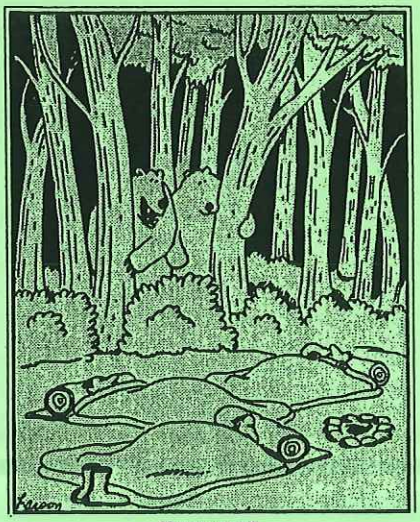
$(ab)^7 + (2c)^7$

$(ab+2c)[(ab)^6 - (ab)^5(2c) + (ab)^4(2c)^2 - (ab)^3(2c)^3 + (ab)^2(2c)^4 - (ab)(2c)^5 + (2c)^6]$

$(ab+2c)(a^6b^6 - 2a^5b^5c + 4a^4b^4c^2 - 8a^3b^3c^3 + 16a^2b^2c^4 - 32abc^5 + 64c^6)$



The duck relays



"Sandwiches!"

Infinite Geometric Series

ANSWER KEY 17.4

① $a_1 = \frac{1}{2}$ $r = \frac{2}{3}$

$S = \frac{a_1}{1-r} = \frac{\frac{1}{2}}{1-\frac{2}{3}} = \frac{\frac{1}{2}}{\frac{1}{3}} = \boxed{\frac{3}{2}}$

② $a_1 = 12$ $r = \frac{1}{4}$

$S = \frac{a_1}{1-r} = \frac{12}{1-\frac{1}{4}} = \frac{12}{\frac{3}{4}} = \boxed{16}$

③ $a_1 = 1$ $r = \frac{-1}{3}$

$S = \frac{a_1}{1-r} = \frac{1}{1-(\frac{-1}{3})} = \frac{1}{\frac{4}{3}} = \boxed{\frac{3}{4}}$

④ $a_1 = 1$ $r = -3$ **no sum**

r is not between -1 and 1

⑤ $n = 5$ $d = 4$

$\boxed{\sum_{x=1}^{5} (4x-2)}$

⑥ $n = 5$ $d = -6$

$\boxed{\sum_{x=1}^{5} (-6x+20)}$

⑦ $n = 6$ $r = 1/2$

$\boxed{\sum_{x=1}^{6} \left(\frac{1}{2}\right)^{x-4}}$

⑧ $n = 6$ $r = -\frac{1}{3}$

$\boxed{\sum_{x=1}^{6} \left(\frac{-1}{3}\right)^{x-3}}$

⑨ $n = 5$ $r = \frac{-1}{2}$

$\boxed{\sum_{x=1}^{5} \frac{-1}{3}\left(\frac{-1}{2}\right)^{x}}$

⑩ $n = 5$ $r = 1/4$

$\boxed{\sum_{x=1}^{5} 160\left(\frac{1}{4}\right)^{x}}$

⑪ $a_1 = 80$ $r = .9$

$S = \frac{a_1}{1-r} = \frac{80}{1-.9} = \frac{80}{.1} = \boxed{800 \text{ feet}}$

⑫

downward	upward
$a_1 = 12$ $r = .7$	$a_1 = 8.4$ $r = .7$
$S = \frac{a_1}{1-r} = \frac{12}{.3}$	$S = \frac{a_1}{1-r} = \frac{8.4}{.3}$
$S = 40$	$S = 28$

$40 + 28 = \boxed{68 \text{ feet}}$

⑬ $(2a-3b)^8 \rightarrow$ 1 8($\frac{7}{2}$) 28($\frac{6}{3}$) 56($\frac{5}{4}$)

70 56 28 8 1

$(2a)^8 - 8(2a)^7(3b) + 28(2a)^6(3b)^2 - 56(2a)^5(3b)^3$
$+ 70(2a)^4(3b)^4 - 56(2a)^3(3b)^5 + 28(2a)^2(3b)^6$
$- 8(2a)(3b)^7 + (3b)^8$

$\boxed{256a^8 - 3072a^7b + 16128a^6b^2 - 48{,}384a^5b^3 + 90{,}720a^4b^4 - 108{,}864a^3b^5 + 81{,}648a^2b^6 - 34{,}992ab^7 + 6561b^8}$

⑭ $\sum_{x=-2}^{11} (4x-3)$ $n = 14$ $d = 4$ $a_1 = -11$

$\frac{14}{2}[2(-11) + 13(4)]$

$7(-22+52) = \boxed{210}$

$\boxed{(-11) + (-7) + (-3)}$

⑮ $\sum_{a=4}^{12} 2(4)^{a-5}$ $n = 9$ $r = 4$ $a_1 = 1/2$

$\frac{\frac{1}{2} - \frac{1}{2}(4)^9}{1-4}$

$\boxed{43{,}690.5}$

$\boxed{\left(\frac{1}{2}\right) + (2) + (8)}$

Infinite Geometric Series

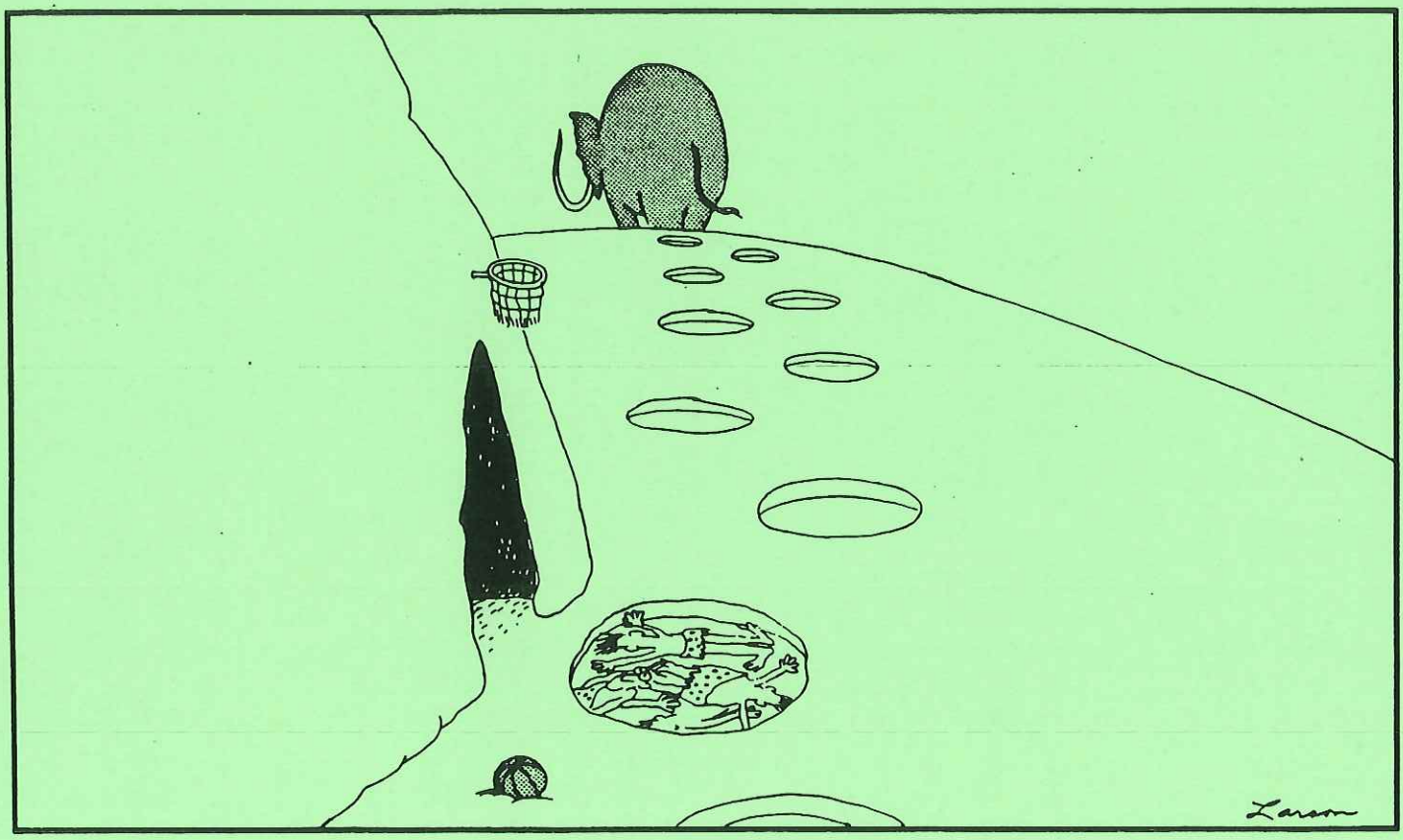
ANSWER KEY 17.4

(16) $x^5y^{10} + 32z^{10}$

$(xy^2)^5 + (2z^2)^5$

$(xy^2+2z^2)\left[(xy^2)^4 - (xy^2)^3(2z^2) + (xy^2)^2(2z^2)^2 - (xy^2)(2z)^3 + (2z^2)^4\right]$

$(xy^2+2z^2)(x^4y^8 - 2x^3y^6z^2 + 4x^2y^4z^4 - 8xy^2z^6 + 16z^8)$



Sequence & Series

ANSWER KEY: UNIT 17 REVIEW & PRACTICE

① $(x+y)^7 \rightarrow 1 \quad 7\left(\frac{6}{2}\right) \quad 21\left(\frac{5}{3}\right) \quad 35 \quad 35 \quad 21 \quad 7 \quad 1$

$x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$

② $(3a+4b)^4 \rightarrow 1 \quad 4\left(\frac{3}{2}\right) \quad 6 \quad 4 \quad 1$

$(3a)^4 + 4(3a)^3(4b) + 6(3a)^2(4b)^2 + 4(3a)(4b)^3 + (4b)^4$

$81a^4 + 432a^3b + 864a^2b^2 + 768ab^3 + 256b^4$

③ $(n-3m)^5 \rightarrow 1 \quad 5\left(\frac{4}{2}\right) \quad 10 \quad 10 \quad 5 \quad 1$

$n^5 - 5n^4(3m) + 10n^3(3m)^2 + 10n^2(3m)^3 + 5n(3m)^4 + (3m)^5$

$n^5 - 15n^4m + 90n^3m^2 - 270n^2m^3 + 405nm^4 - 243m^5$

④ $(2x-3y)^6 \rightarrow 1 \quad 6\left(\frac{5}{2}\right) \quad 15\left(\frac{4}{3}\right) \quad 20 \quad 15 \quad 6 \quad 1$

$(2x)^6 - 6(2x)^5(3y) + 15(2x)^4(3y)^2 - 20(2x)^3(3y)^3 + 15(2x)^2(3y)^4 - 6(2x)(3y)^5 + (3y)^6$

$64x^6 - 576x^5y + 2160x^4y^2 - 4320x^3y^3 + 4860x^2y^4 - 2916xy^5 + 729y^6$

⑤ $\sum_{x=1}^{7} (4x-5)$ $n=7$, $d=4$, $a_1=-1$

arithmetic series

$S = \frac{n}{2}[2a_1 + (n-1)d]$

$\frac{7}{2}(-2+24) =$ **77**

(-1) + (3) + (7)

⑥ $\sum_{c=3}^{10} (8-5c)$ $n=8$, $d=-5$, $a_1=-7$

arithmetic series

$S = \frac{n}{2}[2a_1 + (n-1)d]$

$\frac{8}{2}(-14 + (-35)) =$ **-196**

(-7) + (-12) + (-17)

⑦ $\sum_{m=-2}^{9} 2^m$ $n=12$, $r=2$, $a_1=1/4$

geometric series

$S = \frac{a_1 - a_1 r^n}{1-r} = \frac{\frac{1}{4} - \frac{1}{4}(2)^{12}}{1-2}$

1023.75

(1/4) + (1/2) + (1)

⑧ $\sum_{e=2}^{11} \frac{1}{2}(4)^{e-2}$ $n=10$, $r=4$, $a_1=1/2$

geometric series

$S = \frac{a_1 - a_1 r^n}{1-r}$

$\frac{\frac{1}{2} - \frac{1}{2}(4)^{10}}{1-4} =$ **174,762.5**

1/2 + 2 + 8

Sequence & Series

ANSWER KEY: UNIT 17 REVIEW & PRACTICE

(9) $3+1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots$

$a_1 = 3 \quad r = \frac{1}{3} \quad S = \frac{a_1}{1-r} = \frac{3}{1-\frac{1}{3}} = \frac{3}{\frac{2}{3}} = \boxed{\frac{9}{2}}$

(10) $8-6+\frac{9}{2}-\frac{27}{8}+\ldots$

$a_1 = 8 \quad r = -\frac{3}{4} \quad S = \frac{a_1}{1-r} = \frac{8}{1-(-\frac{3}{4})} = \frac{8}{\frac{7}{4}} = \boxed{\frac{32}{7}}$

(11) arithmetic series

$n = 51 \quad d = 2 \quad a_1 = 250$

$S = \frac{51}{2}[2(250)+50(2)] = \boxed{15{,}300}$

(12) infinite geometric series

$r = 9/10 \quad a_1 = 50$

$S = \frac{50}{1-9/10} = \frac{50}{.1} = \boxed{500 \text{ cm}}$

(13) $6-2-10-18-26$

arithmetic series

$n = 5 \quad d = -8 \quad a_1 = 6$

$\boxed{\sum_{x=1}^{5} (-8x+14)}$

(14) $20+11+2-7-16-25$

arithmetic series

$n = 6 \quad d = -9 \quad a_1 = 20$

$\boxed{\sum_{x=1}^{6} (-9x+29)}$

(15) $3+9+27+81+243$

geometric series

$n = 5 \quad r = 3 \quad a_1 = 3$

$\boxed{\sum_{x=1}^{5} 3^x}$

(16) $10-5+5/2-5/4+5/8$

$n = 5 \quad r = -1/2 \quad a_1 = 10$

$\boxed{\sum_{x=1}^{5} -20(-1/2)^x}$

(17) $4x^5+972y^{10}$

$4(x^5+243y^{10})$

$4[x^5+(3y^2)^5]$

$4(x+3y^2)[x^4-x^3(3y^2)+x^2(3y^2)^2-x(3y^2)^3+(3y^2)^4]$

$4(x+3y^2)(x^4-3x^3y^2+9x^2y^4-27xy^6+81y^8)$

(18) $a^7b^{14}-128c^7$

$(ab^2)^7-(2c)^7$

$(ab^2-2c)[(ab^2)^6+(ab^2)^5(2c)+(ab^2)^4(2c)^2+(ab^2)^3(2c)^3+(ab^2)^2(2c)^4+(ab^2)(2c)^5+(2c)^6]$

$(ab^2-2c)(a^6b^{12}+2a^5b^{10}c+4a^4b^8c^2+8a^3b^6c^3+16a^2b^4c^4+32ab^2c^5+64c^6)$

Linear Permutations

ANSWER KEY 18.1

① $5 \cdot 5 \cdot 5 \cdot 5 \cdot 5 =$ **3125** ind.

② $24 \cdot 24 \cdot 10 \cdot 10 \cdot 9 =$ **518,400** ind.

③ $15 \cdot 12 \cdot 10 =$ **1800** ind.

④ $6 \cdot 18 \cdot 7 =$ **756** ind.

⑤ $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 =$ **720** dep.

⑥ $7 \cdot 6 \cdot 5 \cdot 4 =$ **840** dep.

⑦ $$\frac{P(6,4)}{P(5,3)} = \frac{\left(\frac{6!}{2!}\right)}{\left(\frac{5!}{2!}\right)} = \frac{360}{60} = \boxed{6}$$

⑧ $$\frac{P(10,3)}{P(5,3)} = \frac{\left(\frac{10!}{7!}\right)}{\left(\frac{5!}{2!}\right)} = \frac{720}{60} = \boxed{12}$$

⑨ $$\frac{P(6,3) \cdot P(4,2)}{P(5,2)} = \frac{\left(\frac{6!}{3!}\right)\left(\frac{4!}{2!}\right)}{\left(\frac{5!}{3!}\right)} = \frac{1440}{20}$$

72

⑩ $$\frac{P(5,3)}{P(8,5) \cdot P(5,5)} = \frac{\left(\frac{5!}{2!}\right)}{\left(\frac{8!}{3!}\right)\left(\frac{5!}{0!}\right)} =$$

$$\frac{60}{806,400} = \boxed{1/13,440}$$

⑪ $$\frac{12!}{5!\,3!\,4!} = \boxed{27,720}$$

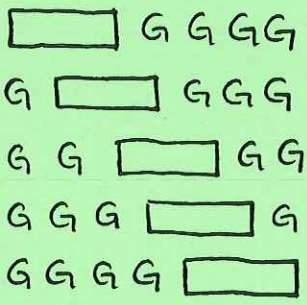
⑫ $$\frac{17!}{8!\,5!\,3!} = \boxed{12,252,240}$$

⑬ $10! = \boxed{3,628,800}$

⑭ $$\frac{6!}{2!2!} = \boxed{180}$$

⑮ $$\frac{8!}{3!5!} = \boxed{56}$$

⑯ algebra books → 5!
geometry books → 4!
algebra books together can go in 5 possible places:



$5!\,4! \cdot 5 =$ **14,400**

Circular Permutations

ANSWER KEY 18.2

① circular, reflective

② circular, not reflective

③ linear, not reflective

④ circular, not reflective

⑤ linear, not reflective

⑥ linear, reflective

⑦ $\frac{(6-1)!}{2} = \boxed{60}$

⑧ $\frac{(8-1)!}{2} = \boxed{2520}$

⑨ $(5-1)! = \boxed{24}$

⑩ $(4-1)!(2) = \boxed{12}$

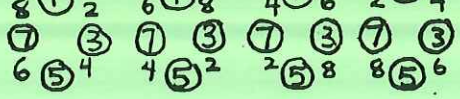
the 2 people seated next to each other can be counted as "1" person - but there are (2) ways they can be seated next to each other.

⑪ $(6-1)! = \boxed{120}$

⑫ $(4-1)!(4-1)!(4) = \boxed{144}$

men cir (4-1)!
women cir (4-1)!

4 permutations (example):



⑬ $6! = \boxed{720}$ linear

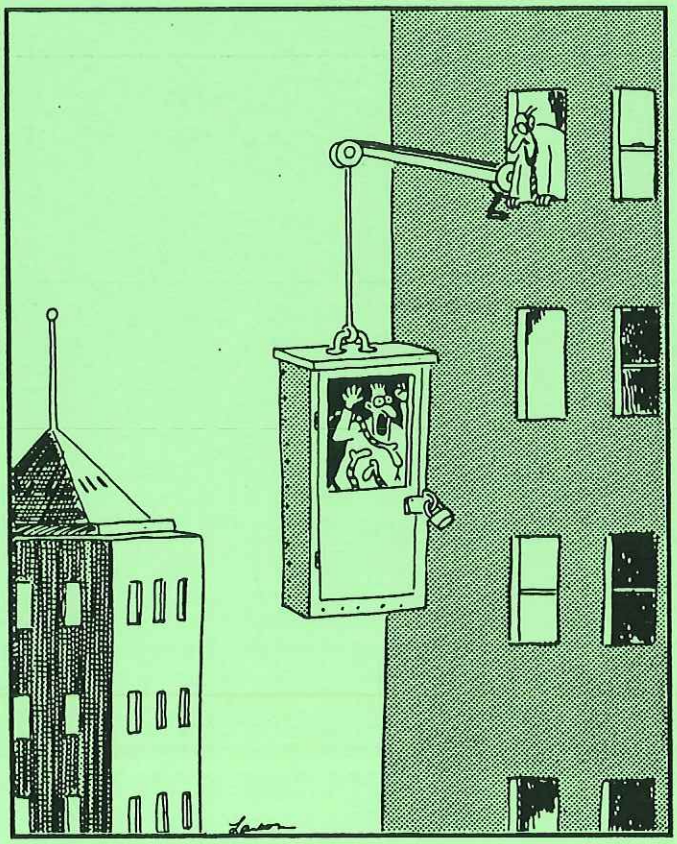
⑭ $7! = \boxed{5040}$

President has (1) possible seat.
Advisors are linear (7!)

⑮ $1 \cdot 4 \cdot 8 \cdot 7 \cdot 6 = \boxed{1344}$

↓ ↓
6 5 7 8 9

⑯ $\frac{9!}{5!\,4!} = \boxed{126}$



Professor Gallagher and his controversial technique of simultaneously confronting the fear of heights, snakes, and the dark.

Combinations

ANSWER KEY 18.3

① $\frac{8!}{3!5!} \cdot \frac{7!}{4!3!} = \boxed{1960}$

② $\frac{24!}{3!21!} = \boxed{2024}$

③ $C(12,5) = \frac{12!}{7!5!} = \boxed{792}$

④ $C(14,9) = \frac{14!}{5!9!} = \boxed{2002}$

⑤ $C(9,4) = \frac{9!}{5!4!} = \boxed{126}$

⑥ $C(85,2) = \frac{85!}{83!2!} = \boxed{3570}$

⑦ $C(27,25) = \frac{27!}{25!2!} = \boxed{351}$

⑧ $C(8,3) = \frac{8!}{5!3!} = \boxed{56}$

⑨ $C(13,5) \cdot 4 = \frac{13!}{8!5!} \cdot 4 = \boxed{5148}$

⑩ $C(13,1) \cdot C(13,1) \cdot C(13,1) \cdot C(13,1) \cdot C(4,4)$
$13 \cdot 13 \cdot 13 \cdot 13 \cdot 1 = \boxed{28,561}$

⑪ $C(9,5) = \frac{9!}{4!5!} = \boxed{126}$

⑫ $\boxed{0}$ only 4 red marbles

⑬ $C(4,2) \cdot C(6,2) \cdot C(9,1)$
$\frac{4!}{2!2!} \cdot \frac{6!}{4!2!} \cdot \frac{9!}{8!1!} = \boxed{810}$

⑭

2 red, 3 white	$C(4,2) \cdot C(6,3) =$	120
2 red, 3 blue	$C(4,2) \cdot C(9,3) =$	504
2 white, 3 red	$C(6,2) \cdot C(4,3) =$	60
2 white, 3 blue	$C(6,2) \cdot C(9,3) =$	1260
2 blue, 3 red	$C(9,2) \cdot C(4,3) =$	144
2 blue, 3 white	$C(9,2) \cdot C(6,3) =$	720
		$\boxed{2808}$

⑮ $C(8,5) = \frac{8!}{5!3!} = \boxed{56}$

⑯ $C(8,3) \cdot C(10,2) = \frac{8!}{3!5!} \cdot \frac{10!}{8!2!} = \boxed{2520}$

⑰ $C(8,1) \cdot C(10,4) = \frac{8!}{7!1!} \cdot \frac{10!}{6!4!} = \boxed{1680}$

⑱ $C(10,5) = \frac{10!}{5!5!} = \boxed{252}$

⑲ $1 \cdot 6 \cdot 8 \cdot 7 \cdot 6 = \boxed{2016}$
↑ ↑
8 digits 2 thru 7

⑳ $\frac{8!}{2!3!} = \boxed{3360}$

㉑ $\frac{(7-1)!}{2} = \frac{6!}{2} = \boxed{360}$

Combinations & Permutations

ANSWER KEY: UNIT 18 REVIEW & PRACTICE

① $P(11,4) = \frac{11!}{7!} = \boxed{7920}$

② $P(5,5) = \frac{5!}{0!} = 5! = \boxed{120}$

③ $C(9,3) = \frac{9!}{6!3!} = \boxed{84}$

④ $C(13,5) = \frac{13!}{8!5!} = \boxed{1287}$

⑤ $7 \cdot 4 = \boxed{28}$

⑥ $8! = \boxed{40,320}$

⑦ $P(12,8) = \frac{12!}{4!} = \boxed{19,958,400}$

⑧ $\frac{10!}{2!2!} = \boxed{907,200}$

⑨ $(7-1)! = \boxed{720}$

⑩ $(9-1)! = \boxed{40,320}$

⑪ $\frac{(8-1)!}{2} = \boxed{2520}$

⑫ $\frac{6!}{2} = \boxed{360}$

⑬ $C(12,5) = \frac{12!}{7!5!} = \boxed{792}$

⑭ $C(10,2) \cdot C(8,2)$

$\frac{10!}{8!2!} \cdot \frac{8!}{6!2!} = \boxed{1260}$

⑮ $C(5,4) \cdot C(7,3)$

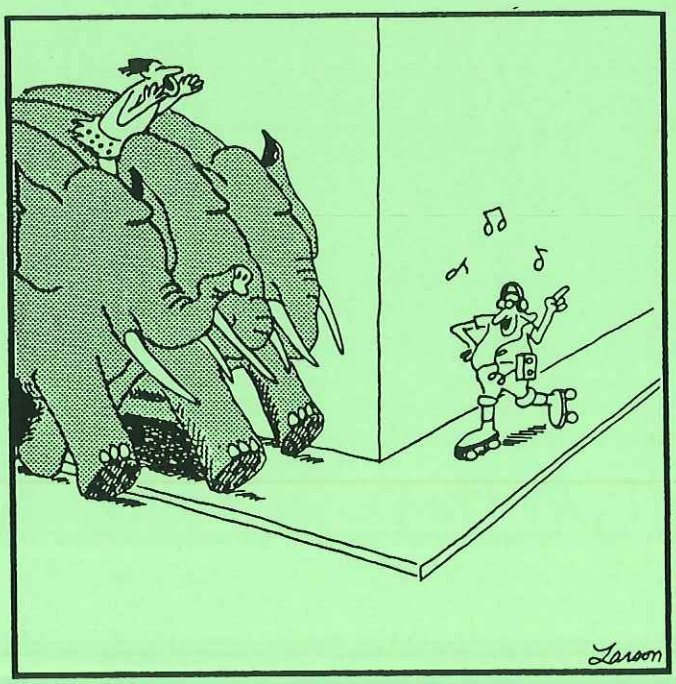
$\frac{5!}{1!4!} \cdot \frac{7!}{4!3!} = \boxed{175}$

⑯ $C(13,2) \cdot C(13,2) \cdot C(13,2) \cdot C(13,2)$

$\left(\frac{13!}{11!2!}\right)^4 = \boxed{37,015,056}$

⑰ $C(13,3) \cdot C(13,2) \cdot C(13,1) \cdot P(4,3)$

$\frac{13!}{10!3!} \cdot \frac{13!}{11!2!} \cdot \frac{13!}{12!1!} \cdot \frac{4!}{1!}$ $\boxed{6,960,096}$



Brian has a rendezvous with destiny.

Probability

ANSWER KEY 19.1

① $\frac{C(7,3)}{C(16,3)} = \frac{\frac{7!}{4!3!}}{\frac{16!}{13!3!}} = \frac{35}{560} = \boxed{\frac{1}{16}}$

② $\frac{C(5,3)}{C(16,3)} = \frac{\frac{5!}{2!3!}}{\frac{16!}{13!3!}} = \frac{10}{560} = \boxed{\frac{1}{56}}$

③ $\frac{C(7,2)\cdot C(5,1)}{C(16,3)} = \frac{\frac{7!}{5!2!}\cdot\frac{5!}{4!1!}}{\frac{16!}{13!3!}} = \frac{105}{560} = \boxed{\frac{3}{16}}$

④ $\frac{C(7,1)\cdot C(4,1)\cdot C(5,1)}{C(16,3)} = \frac{\frac{7!}{6!1!}\cdot\frac{4!}{3!1!}\cdot\frac{5!}{4!1!}}{560}$

$\frac{140}{560} = \boxed{\frac{1}{4}}$

⑤ $\frac{C(3,1)\cdot C(4,1)}{C(7,2)} = \frac{\frac{3!}{2!1!}\cdot\frac{4!}{3!1!}}{\frac{7!}{5!2!}} = \frac{12}{21}$

$\boxed{4/7}$

⑥ $\frac{C(3,2)}{C(7,2)} = \frac{\frac{3!}{1!2!}}{21} = \frac{3}{21} = \boxed{\frac{1}{7}}$

⑦ $\frac{C(4,2)}{C(7,2)} = \frac{\frac{4!}{2!2!}}{21} = \frac{6}{21} = \boxed{\frac{2}{7}}$

⑧ Probability: $\frac{C(8,4)}{C(17,4)} = \frac{\frac{8!}{4!4!}}{\frac{17!}{13!4!}} = \frac{70}{2380} = \frac{1}{34}$ odds: $\boxed{1 \text{ to } 33}$

⑨ Probability: $\frac{C(9,2)\cdot C(8,2)}{C(17,4)} = \frac{\frac{9!}{7!2!}\cdot\frac{8!}{6!2!}}{2380} = \frac{1008}{2380}$

$\frac{1008}{2380} = \frac{252}{595} = \frac{36}{85}$ odds: $\boxed{36 \text{ to } 49}$

⑩ $\frac{C(9,4)}{C(17,4)} = \frac{\frac{9!}{5!4!}}{2380} = \frac{126}{2380} = \boxed{\frac{9}{170}}$

⑪ $\frac{C(8,3)\cdot C(9,1)}{C(17,4)} = \frac{\frac{8!}{5!3!}\cdot\frac{9!}{8!1!}}{2380}$

$\frac{504}{2380} = \boxed{\frac{18}{85}}$

⑫ Probability: $\frac{C(4,5)}{C(52,5)} \rightarrow \boxed{0}$ There are only 4 aces in the deck

⑬ Probability: $\frac{C(12,5)}{C(52,5)} = \frac{\frac{12!}{7!5!}}{\frac{52!}{47!5!}} = \frac{792}{2,598,960}$

$\frac{33}{108,290} \rightarrow$ odds: $\boxed{33 \text{ to } 108,257}$

⑭ Probability: $\frac{C(13,3)\cdot C(13,2)\cdot P(4,2)}{C(52,5)}$

(continued)

Probability

ANSWER KEY 19.1

$$\frac{\frac{13!}{10!3!}\cdot\frac{13!}{11!2!}\cdot\frac{4!}{2!}}{2{,}598{,}960} = \frac{267{,}696}{2{,}598{,}960}$$

$$\frac{23308}{216{,}580} = \frac{5{,}577}{54{,}145} = \frac{429}{4165}$$

odds:

429 to 3736

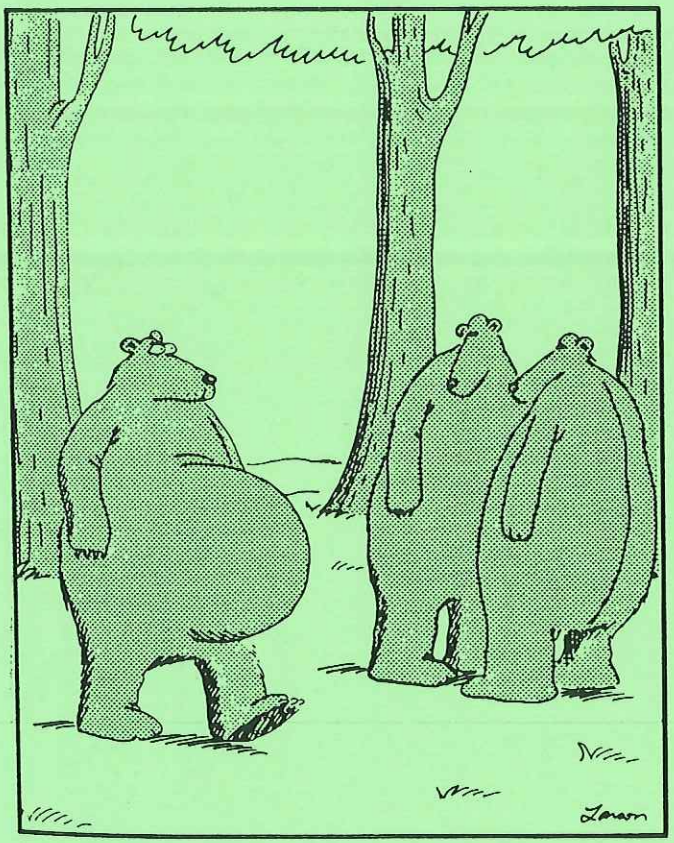
(15) Probability:

$$\frac{C(13,5)\cdot P(4,1)}{C(52,5)} = \frac{\frac{13!}{8!5!}\cdot\frac{4!}{3!}}{2{,}598{,}960}$$

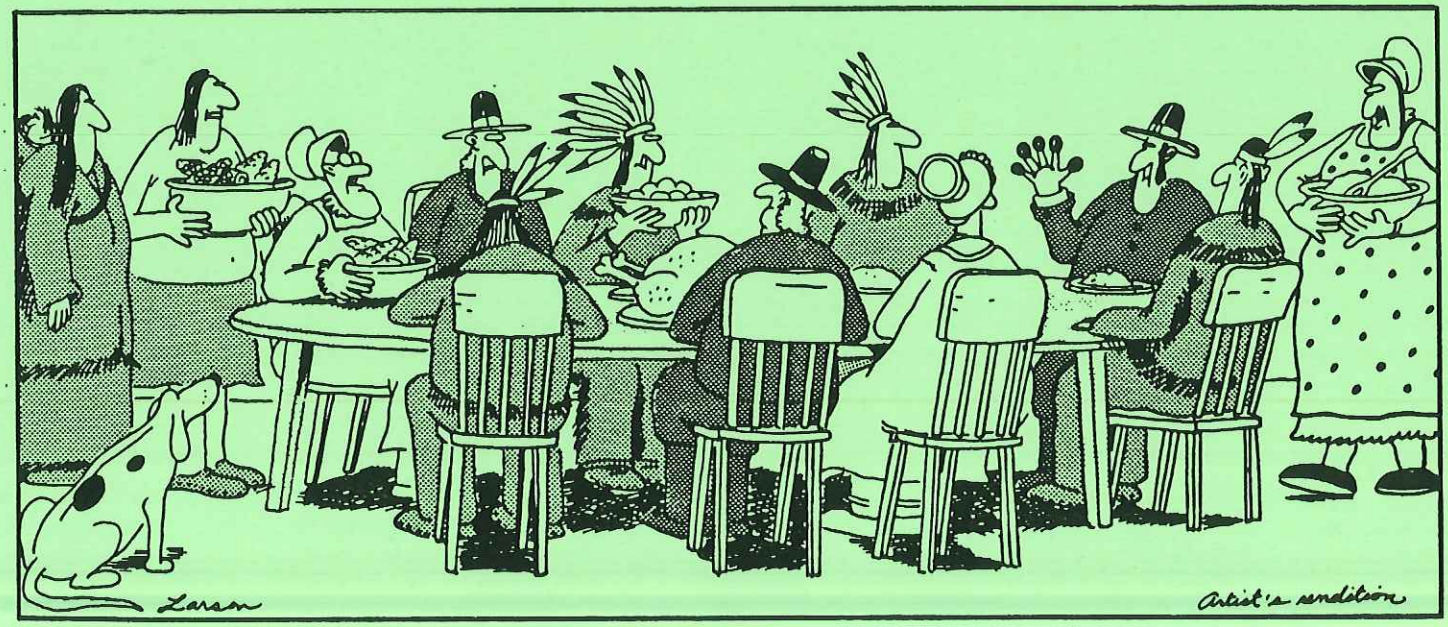
$$\frac{5148}{2{,}598{,}960} = \frac{33}{16{,}660}$$

odds:

33 to 16,627



Impolite as they were, the other bears could never help staring at Larry's enormous deer gut.



Thomas Sullivan, a blacksmith who attended the original Thanksgiving dinner, is generally credited as being the first person to stick olives on all his fingers.

Independence & Exclusivity

ANSWER KEY 19.2

① $\frac{3}{7} \cdot \frac{2}{6} = \boxed{\frac{1}{7}} \approx .143$

② $\frac{3}{7} \cdot \frac{3}{7} = \boxed{\frac{9}{49}} \approx .184$

③ $\frac{4}{7} \cdot \frac{3}{6} = \boxed{\frac{2}{7}} \approx .286$

④ $\frac{C(6,2)}{C(16,2)} = \frac{\frac{6!}{4!2!}}{\frac{16!}{14!2!}} = \boxed{\frac{1}{8}} = .125$

⑤ $\frac{5}{16} \cdot \frac{5}{15} = \boxed{\frac{5}{48}} \approx .104$

⑥ $\frac{C(5,1) \cdot C(5,1)}{C(16,2)} = \boxed{\frac{5}{24}} \approx .208$

⑦ $\frac{5}{6} \cdot \frac{5}{6} = \boxed{\frac{25}{36}} \approx .694$

⑧ $\frac{1}{6} \cdot \frac{1}{6} = \boxed{\frac{1}{36}} \approx .028$

⑨ $\frac{1}{6} \cdot \frac{5}{6} = \boxed{\frac{5}{36}} \approx .139$

⑩ $\frac{6}{6} \cdot \frac{5}{6} = \boxed{\frac{5}{6}} \approx .833$

⑪ $\frac{C(5,1) \cdot C(6,2) + C(6,3)}{C(11,3)} = \boxed{\frac{19}{33}} \approx .576$

⑫ $\frac{C(5,2) \cdot C(6,1) + C(5,3)}{C(11,3)} = \boxed{\frac{14}{33}} \approx .424$

⑬ $\frac{4}{52} \cdot \frac{3}{51} + \frac{12}{52} \cdot \frac{11}{51} = \boxed{\frac{12}{221}} \approx .054$

⑭ $\frac{4}{52} \cdot \frac{3}{51} + \frac{26}{52} \cdot \frac{25}{51} - \frac{2}{52} \cdot \frac{1}{51}$

$\boxed{\frac{55}{221}} \approx .249$

⑮ $\frac{26}{52} \cdot \frac{25}{51} + \frac{12}{52} \cdot \frac{11}{51} - \frac{6}{52} \cdot \frac{5}{51} = \boxed{\frac{188}{663}} \approx .284$

⑯ $\frac{4}{52} \cdot \frac{3}{51} + \frac{4}{52} \cdot \frac{3}{51} + \frac{26}{52} \cdot \frac{25}{51} - \frac{2}{52} \cdot \frac{1}{51} - \frac{2}{52} \cdot \frac{1}{51}$

$\boxed{\frac{331}{1326}} \approx .253$

⑰ $\frac{C(6,6) + C(8,6)}{C(14,6)} = \boxed{\frac{29}{3003}} \approx .010$

⑱ $\frac{C(8,5) \cdot C(6,1) + C(8,6)}{C(14,6)} = \boxed{\frac{4}{33}} \approx .121$

⑲ $\frac{C(8,5) \cdot C(6,1) + C(8,6) + C(8,1) \cdot C(6,5) + C(6,6}{C(14,6)}$

$\boxed{\frac{59}{429}} \approx .138$

Binomial Trials

ANSWER KEY 19.3

① $C(4,0)T^4$

$1\left(\frac{1}{3}\right)^4 = \boxed{\frac{1}{81}} \approx .012$

② $C(4,4)H^4 + C(4,3)H^3T$

$1\left(\frac{2}{3}\right)^4 + 4\left(\frac{2}{3}\right)^3\left(\frac{1}{3}\right)$

$\frac{16}{81} + \frac{32}{81} = \frac{48}{81} = \boxed{\frac{16}{27}} \approx .593$

③ $C(4,2)H^2T^2 + C(4,1)HT^3 + C(4,0)T^4$

$6\left(\frac{2}{3}\right)^2\left(\frac{1}{3}\right)^2 + 4\left(\frac{2}{3}\right)\left(\frac{1}{3}\right)^3 + 1\left(\frac{1}{3}\right)^4$

$\frac{24}{81} + \frac{8}{81} + \frac{1}{81} = \frac{33}{81} = \boxed{\frac{11}{27}} \approx .407$

④ $C(5,3)H^3O^2$

$\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^2$

$10\left(\frac{1}{125}\right)\left(\frac{16}{25}\right) = \frac{160}{3125} = \boxed{\frac{32}{625}} \approx .051$

⑤ $C(5,5)H^5 + C(5,4)H^4O$

$1\left(\frac{1}{5}\right)^5 + 5\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right)$

$\frac{1}{3125} + \frac{20}{3125} = \boxed{\frac{21}{3125}} \approx .007$

⑥ $C(5,5)H^5 + C(5,4)H^4O + C(5,3)H^3O^2$
$+ C(5,2)H^2O^3$

$1\left(\frac{1}{5}\right)^5 + 5\left(\frac{1}{5}\right)^4\left(\frac{4}{5}\right) + 10\left(\frac{1}{5}\right)^3\left(\frac{4}{5}\right)^2$
$+ 10\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^3 = \frac{1+20+160+640}{3125}$

$\boxed{\frac{821}{3125}} \approx .263$

⑦ $C(10,10)U^{10}$

$1\left(\frac{2}{5}\right)^{10} = \boxed{\frac{1024}{9,765,625}} \approx .000105$

⑧ $C(10,5)U^5D^5$

$252\left(\frac{2}{5}\right)^5\left(\frac{3}{5}\right)^5 = \boxed{\frac{1,959,552}{9,765,625}} \approx .201$

⑨ $C(10,10)U^{10} + C(10,9)U^9D + C(10,8)U^8D^2$
$+ C(10,7)U^7D^3 + C(10,6)U^6D^4$

$1\left(\frac{2}{5}\right)^{10} + 10\left(\frac{2}{5}\right)^9\left(\frac{3}{5}\right) + 45\left(\frac{2}{5}\right)^8\left(\frac{3}{5}\right)^2$
$+ 120\left(\frac{2}{5}\right)^7\left(\frac{3}{5}\right)^3 + 210\left(\frac{2}{5}\right)^6\left(\frac{3}{5}\right)^4$

$\frac{1024 + 15360 + 103680 + 414720 + 1088640}{9,765,625}$

$\boxed{\frac{1,623,424}{9,765,625}} \approx .166$

⑩ $\frac{C(13,2) \cdot C(13,1) \cdot P(4,2)}{C(52,3)}$

$\frac{78 \cdot 13 \cdot 12}{22,100} = \frac{12,168}{22,100} = \frac{234}{425}$

Probability:

$\frac{234}{425}$

odds: $\boxed{234 \text{ to } 191}$

Binomial Trials

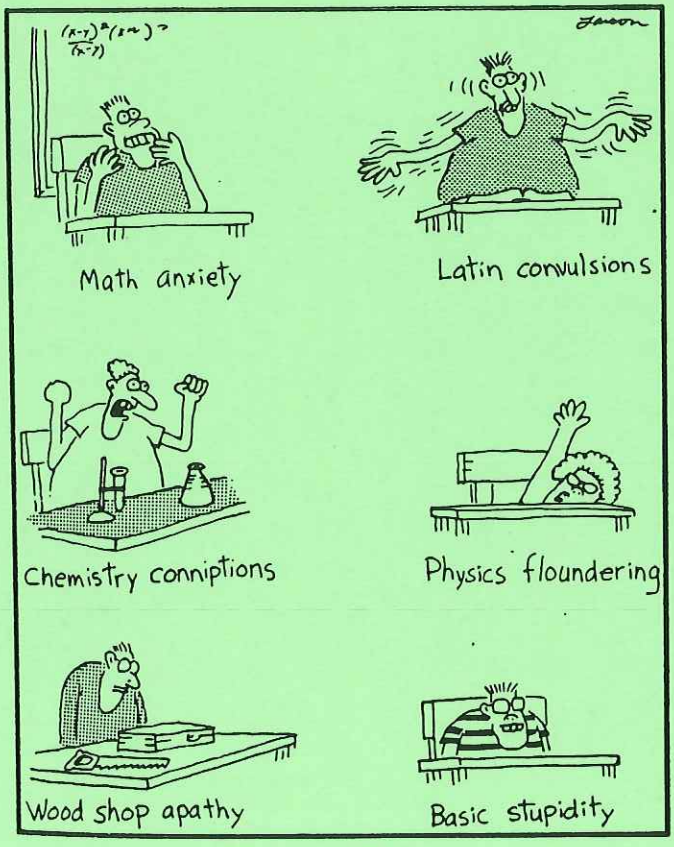
ANSWER KEY 19.3

(11) $$\frac{C(6,1) \cdot C(4,3)}{C(10,4)}$$

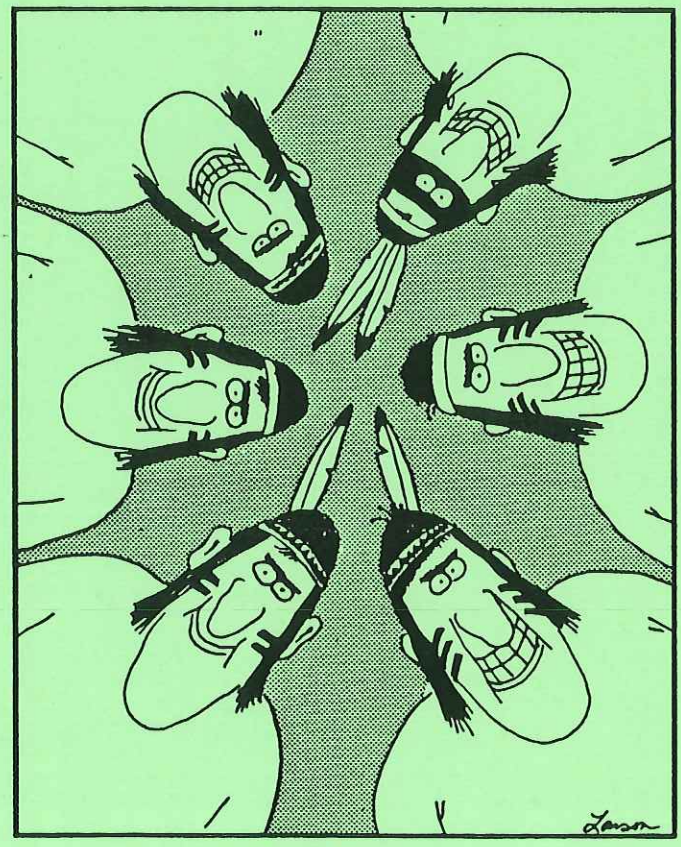
$$\frac{6 \cdot 4}{210} = \frac{24}{210} = \boxed{\frac{4}{35}}$$

(12) $$\frac{C(5,2) + C(4,2) - C(2,2)}{C(9,2)}$$

$$\frac{10+6-1}{36} = \frac{15}{36} = \boxed{\frac{5}{12}}$$



Classroom afflictions



Custer's last view

Mathematics of Chance

ANSWER KEY: UNIT 19 REVIEW & PRACTICE

① $\frac{C(3,3)}{C(10,3)} = \boxed{\frac{1}{120}} \approx .008$

② $\frac{C(4,3)+C(3,3)}{C(10,3)} = \frac{4+1}{120} = \boxed{\frac{1}{24}} \approx .042$

③ $\frac{C(4,3)\cdot C(6,1) + C(4,4)\cdot C(6,0)}{C(10,4)}$

$\frac{(4)(6)+(1)(1)}{210} = \boxed{\frac{5}{42}} \approx .119$

④ $\frac{C(12,4)}{C(52,4)} = \frac{495}{270,725} = \boxed{\frac{99}{54,145}}$

$\approx .002$

⑤ $\frac{26}{52} + \frac{12}{52} - \frac{6}{52} = \frac{32}{52} = \boxed{\frac{8}{13}} \approx .615$

⑥ $\frac{C(13,3)\cdot C(13,1)\cdot P(4,2)}{C(52,4)}$

$\frac{(286)(13)(12)}{270,725} = \boxed{\frac{3432}{20,825}} \approx .165$

⑦ $C(7,5)\,H^5T^2 = 21\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^2$

$\boxed{\frac{21}{128}} \approx .164$

⑧ $C(7,3)\,H^3T^4 + C(7,2)\,H^2T^5 + C(7,1)\,HT^6 + C(7,0)\,T^7$

$35\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^4 + 21\left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^5 + 7\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^6 + \left(\frac{1}{2}\right)^7$

$\frac{35+21+7+1}{128} = \frac{64}{128} = \boxed{\frac{1}{2}}$

⑨ $C(7,3)\,H^3T^4 + C(7,4)\,H^4T^3$

$35\left(\frac{1}{2}\right)^3\left(\frac{1}{2}\right)^4 + 35\left(\frac{1}{2}\right)^4\left(\frac{1}{2}\right)^3$

$\frac{35+35}{128} = \frac{70}{128} = \boxed{\frac{35}{64}} \approx .547$

⑩ $C(8,6)\,S^6F^2$ $\approx .209$

$28\left(\frac{3}{5}\right)^6\left(\frac{2}{5}\right)^2 = \boxed{\frac{81,648}{390,625}}$

⑪ $C(8,8)S^8 + C(8,7)S^7F^1$

$\left(\frac{3}{5}\right)^8 + 8\left(\frac{3}{5}\right)^7\left(\frac{2}{5}\right)$

$\frac{6561+34,992}{390,625} = \boxed{\frac{41553}{390,625}} \approx .106$

⑫ $C(8,8)S^8 + C(8,1)S^1F^7 + C(8,0)F^8$

$\left(\frac{3}{5}\right)^8 + 8\left(\frac{3}{5}\right)\left(\frac{2}{5}\right)^7 + \left(\frac{2}{5}\right)^8$

$\frac{6561+3072+256}{390,625} = \boxed{\frac{9889}{390,625}}$

$\approx .025$

Mathematics of Chance

ANSWER KEY: UNIT 19 REVIEW & PRACTICE

(13) $\frac{4}{10} \cdot \frac{3}{9} = \frac{12}{90} = \boxed{\frac{2}{15}} \approx .133$

(14) $\frac{20}{52} \cdot \frac{19}{51} + \frac{26}{52} \cdot \frac{25}{51} - \frac{10}{52} \cdot \frac{9}{51}$

$$\frac{380 + 650 - 90}{2652} = \boxed{\frac{235}{663}}$$

$\approx .354$

(15) $\frac{3}{6} \cdot \frac{3}{6} = \frac{1}{4}$ probability

odds are $\boxed{1 \text{ to } 3}$