

Friendship Junior High School
Accelerated Math Program
Mr. Lavine (Room 102A)

A.T.I.M.

Advanced Topics In Mathematics

ATIM
End Of The Year Review

- ① *Polynomials*
- ② *Systems*
Problem Solving
- ④ *Inequalities & Coordinate Graphing*
Linear Programming
- ⑥ *Roots & Radicals*
- ⑦ *Rat. Expon. & Complex Numbers*
- ⑧ *Quadratics*
- ⑨ *Rational Expressions*
Problem Solving
Plane Geometry
Solid Geometry
Introduction to Two-Column Proofs
Triangle Trigonometry
Law of Sines & Cosines
Trigonometric Functions
- ⑰ *Sequence & Series*
Combinations & Permutations
Mathematics of Chance
- ⑳ *Matrices*
- ㉑ *Logarithms*
- ㉒ *Conics*



"Mr. Lavine, may I be excused?
My brain is full."

Polynomials

UNIT 1 DEMONSTRATION

Simplify:

$$\textcircled{1} ab^{-2}(a^2b+a^{-1}b^3-a^3b^{-1})$$

$$\textcircled{2} \frac{-18a^{-2}b^3c^{-4}}{12ab^{-2}c^{-3}}$$

$$\textcircled{3} (5n^{3x} - 2m^{x+2})^2$$

Factor completely:

$$\textcircled{4} 54a^3b - 2b^4$$

$$\textcircled{5} 5ab + b^2 + 5ac - c^2$$

$$\textcircled{6} m^3 - 3m^2a + 3ma^2 - a^3$$

Polynomial division:

$$\textcircled{7} (x^4 + 4)(x + 2)^{-1}$$

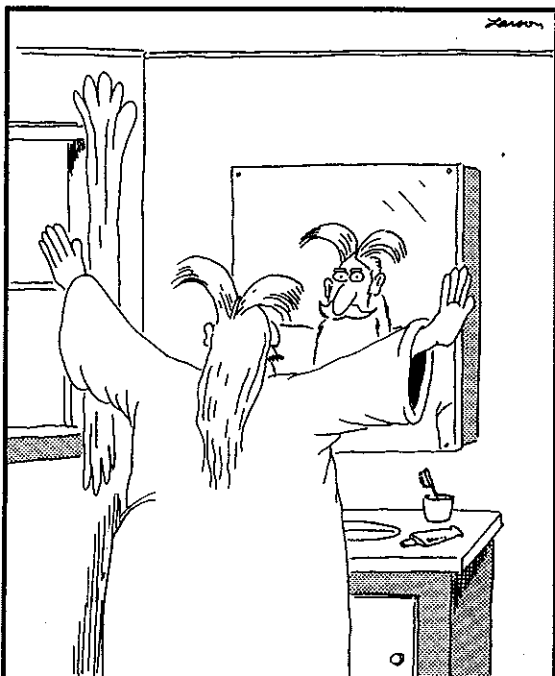
Synthetic division:

$$\textcircled{8} (2a^4 - 5a^3 + 2a - 3)(a - 1)^{-1}$$

$$\textcircled{9} (12x^4 + 7x^2 - 9x + 11)(2x - 3)^{-1}$$

Simplify:

$$\textcircled{10} \left(\frac{-a^{-2}b^3c^{-1}}{3a^{-4}b^{-1}c^3} \right)^{-2}$$



Moses parting his hair



Eskimo rescue units

Polynomials

UNIT 1 REVIEW

Simplify:

$$\textcircled{1} (5a)(6a^2b)(3ab^3) + (2a)^2(3b^3)(2a^2b)$$

$$\textcircled{2} \frac{-9x^{-2}y^3z^{-1}}{3x^{-4}yz^2}$$

$$\textcircled{3} \left(\frac{2y^{-2}}{-2}\right)^{-1} \left(\frac{m^2n}{y}\right)^{-2}$$

$$\textcircled{4} \frac{6n^{2x+3}}{3n^{x-4}}$$

$$\textcircled{5} x^{-1}y^2(x^{-3}y - xy^{-3} + x^2y^{-4})$$

$$\textcircled{6} (4x^{a+2} - 3y^{3a})^2$$

$$\textcircled{7} (x^{n+1} + y^{2n-1})(x^{n+1} - y^{2n-1})$$

Factor completely:

$$\textcircled{8} 12n^2 - 7n - 10$$

$$\textcircled{9} (3a - 4b)^2 - (a + 3b)^2$$

$$\textcircled{10} 5x^3y + 40y^4$$

$$\textcircled{11} x^5 - x^3y^2 + 27y^5 - 27x^2y^3$$

$$\textcircled{12} x^2 - 9 - 6xy + 9y^2$$

$$\textcircled{13} m^4n + m^2n - mn^4 - mn^2$$

$$\textcircled{14} 3x^3 + 9x^2 + 9x + 3$$

Polynomial division:

$$\textcircled{15} (4n^4 - 3)(n + 2)^{-1}$$

Synthetic division:

$$\textcircled{16} (x^6 - 9x^4 + 2x^3 - 18x + 12)(x + 3)^{-1}$$

$$\textcircled{17} (4x^3 + x - 3)(2x - 1)^{-1}$$



When cliff divers belly flop

Systems

UNIT 2 DEMONSTRATION

Solve using substitution:

$$\begin{aligned} \textcircled{1} \quad x - 3y &= -3 \\ 4x + 9y &= 2 \end{aligned}$$

Solve using elimination:

$$\begin{aligned} \textcircled{2} \quad 2x + 3y &= 5 \\ -3x + 6y &= 12 \end{aligned}$$

Solve using elimination and substitution:

$$\begin{aligned} \textcircled{3} \quad 4x + 3y + z &= -10 \\ x - 12y + 2z &= -5 \\ x + 18y + z &= 4 \end{aligned}$$

Determine the value:

$$\textcircled{4} \begin{vmatrix} -3 & 7 \\ 4 & 9 \end{vmatrix} \quad \textcircled{5} \begin{vmatrix} -2 & 0 \\ 6 & 5 \end{vmatrix}$$

Solve using Cramer's Rule:

$$\begin{aligned} \textcircled{6} \quad 3x + 2y &= 40 \\ x - 7y &= -2 \end{aligned}$$

Determine the value of a third order determinant

using expansion of minors:

$$\textcircled{7} \begin{vmatrix} 4 & -5 & 3 \\ -3 & 2 & 0 \\ 2 & -1 & 4 \end{vmatrix}$$

Determine the value using diagonals:

$$\textcircled{8} \begin{vmatrix} 6 & 3 & 1 \\ -1 & 1 & -4 \\ -2 & 4 & 2 \end{vmatrix}$$

Use expansion of minors to determine if a unique solution exists for this system:

$$\begin{aligned} \textcircled{9} \quad 2a - b + 3c &= 5 \\ 3a + 2b - 5c &= 7 \\ a - 4b + 11c &= 3 \end{aligned}$$

Use diagonals to solve for x in this system:

$$\begin{aligned} \textcircled{10} \quad 3x + 4y + z &= 10 \\ 6x - 2y - z &= 6 \\ 3x + 6y - 2z &= 2 \end{aligned}$$

Systems

UNIT 2 REVIEW

Solve using substitution:

$$\begin{aligned} \textcircled{1} \quad & 4x + y = 9 \\ & 3x - 2y = 4 \end{aligned}$$

Solve using elimination:

$$\begin{aligned} \textcircled{2} \quad & 3x - 4y = 23 \\ & 9x + 2y = -15 \end{aligned}$$

Determine the value:

$$\textcircled{3} \quad \begin{vmatrix} -8 & -1 \\ 2 & -3 \end{vmatrix}$$

Solve using Cramer's Rule:

$$\begin{aligned} \textcircled{4} \quad & 2x - 3y = 8 \\ & 5x + 6y = 11 \end{aligned}$$

Solve using elimination and substitution:

$$\begin{aligned} \textcircled{5} \quad & a + b + c = 0 \\ & 3a - 2b + 5c = 1 \\ & 2a + b + 2c = -1 \end{aligned}$$

Determine the value using expansion of minors:

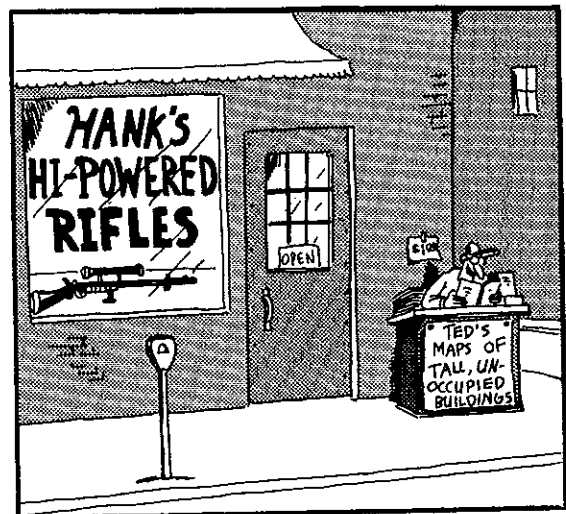
$$\textcircled{6} \quad \begin{vmatrix} -4 & 0 & 1 \\ 2 & -3 & 4 \\ 5 & 2 & 3 \end{vmatrix}$$

Determine the value using diagonals:

$$\textcircled{7} \quad \begin{vmatrix} -3 & 2 & 4 \\ -2 & -1 & 2 \\ 4 & 3 & 6 \end{vmatrix}$$

Determine z using diagonals or expansion:

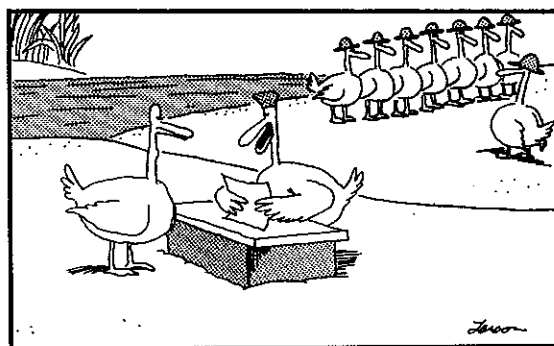
$$\begin{aligned} \textcircled{8} \quad & x + 5y + 3z = 0 \\ & 2x - 5y + 3z = 11 \\ & x + 10y - 6z = -7 \end{aligned}$$



Problem Solving

UNIT 3 DEMONSTRATION

- ① Mr. Davis made a trip to the lake in 3 hours. The return trip took $3\frac{1}{2}$ hours. He drove 5 mph faster on his way out to the lake. How many miles was the round trip?
- ② The sum of the digits of a three digit number is 10. When the digits are reversed, the new number is 99 less than the original. The hundreds digit equals the sum of the tens and units digits. Find the number.
- ③ How many liters of a 70% salt solution should be mixed with 60 liters of a 20% solution to produce a 40% solution?
- ④ Bud invested \$7000, part at 8% and part at 10%. His total interest for the year was \$650. How much was invested at 8%?
- ⑤ Maria rode her bike 6 miles into town with the wind at her back in 18 min. On her way home against the wind she traveled only 4 miles in 30 min. What is the speed of the wind?
- ⑥ Stacy bought a clock on sale and paid \$11.55. It was on sale for 25% off. After the discount was deducted, she had a coupon for an additional \$2.50 off. After the coupon, she had to pay a 5% sales tax. Determine the original price.
- ⑦ At what time between 8:00 and 9:00 will the hands on a clock coincide? (Answer to the nearest sec.)



"Can't use you, son ... says your feet aren't flat."

Problem Solving

UNIT 3 REVIEW

① Two cars start driving toward each other at 1:30 PM from a distance of 162 miles apart. If one car goes 5mph faster than the other and they meet at 3:30 PM, what are their rates?

② The sum of the digits of a three digit number is 11. The hundreds digit is twice as large as the units digit. If the digits are reversed, the new number is 198 less than the original. Find the original number.

③ In a triangle, the largest angle is 10° less than 3 times the smallest. The middle angle is 10° greater than twice the smallest. Find the measures of all three angles.

④ Lisa rowed her boat 3 miles downstream in 10 minutes. Rowing upstream, the same distance took 3 times as long. Find the rate of the current.

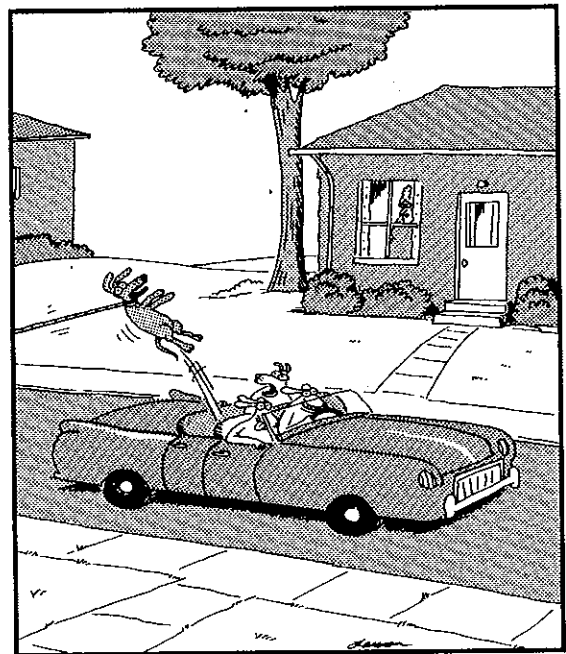
⑤ Kevin paid \$4.77 for a book on sale for 25% off

after a 6% sales tax was added on. How much was the original price?

⑥ How many liters of water must be added to 15 liters of a 90% salt solution to bring the concentration of salt down to 27%?

"Clock" Problem

⑦ At what time between 6:00 and 7:00 will the hands of a clock coincide? (Answer to the nearest second)



Careening through the neighborhood with reckless abandon, none of them suspected that Tuffy was still tied up.

Inequalities & Coordinate Graphing

UNIT 4 DEMONSTRATION

Solve and graph on a number line:

- ① $4n + 3 \geq 2(2n - 2) - 3$
- ② $-4 \leq 2n + 4 \leq 12$
- ③ $|x - 5| > 3$
- ④ $|n + 2| > 4$ and $|n| < 9$

Understanding linear equations:

- ⑤ Write an equation in slope-intercept form for a line parallel to $3x - 2y = 5$ passing through the point $(6, 2)$. Then identify the intercepts.
- ⑥ Write an equation in point-slope form (not slope-intercept form) for a line passing through $(-2, 5)$ and $(-6, 8)$.

Graph the inequality:

⑦ $y \leq x - 2$

Graph the system:

⑧ $|x + y| > 6$

Graph the system:

⑨ $|x - 2| \geq 5$
 $y \leq \frac{1}{2}x + 2$



"That's right, the forty-ninth floor. ... And you better hurry—she's hanging by a thread."

Inequalities & Coordinate Graphing

UNIT 4 REVIEW

Solve and graph on a number line:

$$\textcircled{1} 2(x+4) + \frac{x}{3} < 4x-1$$

$$\textcircled{2} \frac{x+3}{4} \geq \frac{2(x-3)}{3}$$

$$\textcircled{3} 3(2x-3) \geq 2(3x+2)$$

$$\textcircled{4} 4(x-1) < 4x+5$$

$$\textcircled{5} -1 < n+4 < 9$$

$$\textcircled{6} |3x+6| > 12$$

$$\textcircled{7} 3 \leq |n-4| \leq 10$$

$$\textcircled{8} |2x+1| > 5 \text{ and } |x| < 10$$

Linear equations:

$\textcircled{9}$ Write an equation in slope-intercept form for a line parallel to $5x-3y=4$ through $(6, -2)$. Identify the intercepts.

$\textcircled{10}$ Write an equation in standard form for a line perpendicular to $y=\frac{3}{4}x+7$ through $(2, 5)$. Identify the intercepts.

$\textcircled{11}$ Write an equation in point-slope form for a line through $(-2, 6)$ and $(10, -1)$.

$\textcircled{12}$ Change $2x+3y=-4$ to slope-intercept form

Graph the inequality:

$$\textcircled{13} 3x-y < 8$$

Graph the system:

$$\textcircled{14} \begin{aligned} 2x-y &> 6 \\ y &> -2x \end{aligned}$$

$$\textcircled{15} |3x-y| \leq 4$$

$$\textcircled{16} |x-y| > 5$$

$$\textcircled{17} \begin{aligned} |x| &\leq 4 \\ x-2y &\leq 6 \end{aligned}$$

$$\textcircled{18} \begin{aligned} |y-2| &> 8 \\ 2x+y &< 6 \end{aligned}$$

$$\textcircled{19} \begin{aligned} |x+1| &> 14 \\ |y-2| &> 10 \end{aligned}$$

Linear Programming

UNIT 5 DEMONSTRATION

Graph the system and determine the maximum and minimum values:

① Function:
 $f(x,y) = -2x + 2y$

Conditions:

$$y \leq 2$$

$$2x - y \leq 4$$

$$-2x - y \leq 4$$

② Function:
 $f(x,y) = 3x - y$

Conditions:

$$y \leq x$$

$$y \leq -5x + 18$$

$$y \geq -2$$

Define variables and graph a system of inequalities based on stated conditions:

- ③ A manufacturer makes 2 kinds of televisions - portables and consoles. He has equipment to

manufacture up to 500 portables and up to 300 consoles. Time permits him to manufacture no more than 600 television sets in all. Customer advance orders require him to make at least 100 portables. If he realizes a profit of \$60 on each portable and \$70 on each console, how many of each should he manufacture to maximize his profit?



Scene from *Insurance Salesman of the Opera*

Linear Programming

UNIT 5 REVIEW

Graph each system of inequalities. Identify the vertices. Find the max. and min. values:

① Function: $f(x,y) = 5x - 3y$

Conditions:

$$\begin{aligned} y &\geq 0 \\ 0 &\leq x \leq 5 \\ -x + y &\leq 2 \\ x + y &\leq 6 \end{aligned}$$

② Function: $f(x,y) = 2x + 4y$

Conditions:

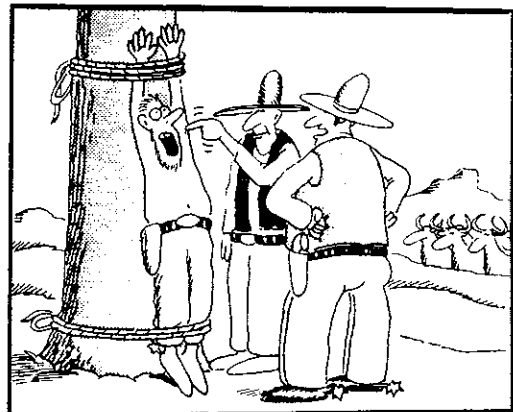
$$\begin{aligned} x &\geq 0 \\ y &\geq 0 \\ x + y &\leq 3 \\ 3x + y &\leq 6 \end{aligned}$$

Solve each problem by establishing a graph based on a system of inequalities subject to conditions. Use a function to evaluate:

- ③ A carpenter makes bookcases in 2 sizes: large and small. It takes 4 hrs. to make a large one and 2 hrs. for a small one. The carpenter can spend up to 24 hours per week working on them. He makes a profit of \$50 on a large bookcase and \$20 on a small one. To satisfy customers, he must make at least 2 of each size

per week. How many of each size per week should be made to maximize profit?

- ④ A tire manufacturer has 800 units of rubber to use in producing radial tires and tractor tires. Each radial tire requires 5 units of rubber while each tractor tire requires 20 units. Labor costs are \$12 for a radial and \$12 for a tractor tire. The company budgets a maximum of \$1200 for labor. Customer needs require the company to make at least 40 radial tires. If each radial results in a \$10 profit and each tractor tire results in a \$25 profit, how many of each should the company make to maximize profit?



Sorry, Mister...

But this is how we handle cattle rustlers in these parts.

Roots & Radicals

UNIT 6 DEMONSTRATION

Simplify:

$$\textcircled{1} \sqrt[4]{81x^6y^5z^{10}}$$

$$\textcircled{2} \sqrt[3]{(3n-2)^3}$$

Compute and simplify:

$$\textcircled{3} \sqrt[4]{18a^3bc^5} \cdot \sqrt[4]{27a^2b^6c}$$

$$\textcircled{4} (4-2\sqrt{2})(\sqrt{2}-4)$$

Rationalize and simplify:

$$\textcircled{5} \frac{3-2\sqrt{3}}{2\sqrt{3}+3}$$

$$\textcircled{6} \sqrt{\frac{1}{5}} - 2\sqrt{20} + 3\sqrt{5}$$

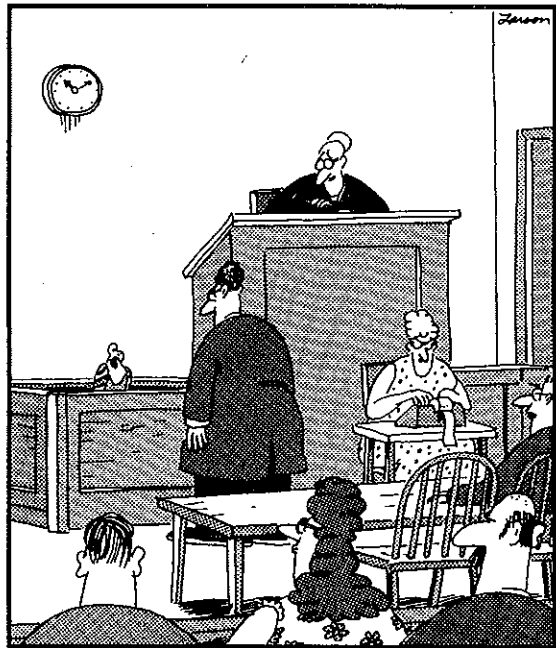
Solve:

$$\textcircled{7} \sqrt{3x+1} + 2 = 7$$

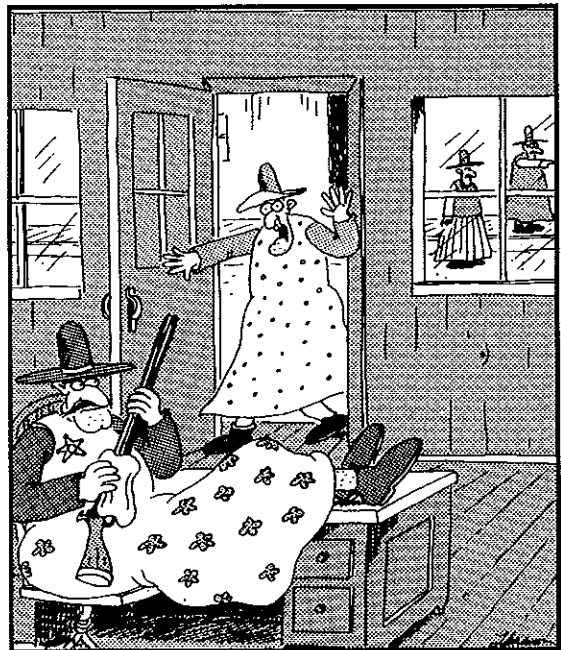
$$\textcircled{8} \sqrt{2x+1} - 2 = \sqrt{2x-7}$$

Solve:

$$\textcircled{9} \sqrt{4x+29} = (x+6)$$



"No, I never said that. ... Well, I actually did say it, but after he said it. He said it, then I said it. I'm a mimic—that's what I do."



"Sheriff! Ben Wiggins is ridin' into town, and he's wearin' that same little chiffon number that he wore when he shot Jake Sutton!"

Roots & Radicals

UNIT 6 REVIEW

Simplify:

$$\textcircled{1} \sqrt[4]{64x^5y^6z^9}$$

$$\textcircled{2} \sqrt{72a^5b^7c^6}$$

$$\textcircled{3} \sqrt[3]{-40n^7m^4}$$

$$\textcircled{4} \sqrt{n^2-10n+25}$$

$$\textcircled{5} \sqrt[3]{(3x-y)^5}$$

Compute and simplify:

$$\textcircled{6} \sqrt[4]{24a^2b^3c^5} \cdot \sqrt[4]{48a^3b^3c^6}$$

$$\textcircled{7} \sqrt{3mn} + \sqrt{27m^3n}$$

$$\textcircled{8} (4-\sqrt{3})(6-\sqrt{3})$$

Rationalize and simplify:

$$\textcircled{9} \sqrt[3]{\frac{2}{5n}}$$

$$\textcircled{10} \frac{2-\sqrt{2}}{4+2\sqrt{2}}$$

$$\textcircled{11} \sqrt{\frac{1}{3}} + \sqrt{75} - 2\sqrt{3}$$

Solve each equation:

$$\textcircled{12} \sqrt[3]{y+1} = 3$$

$$\textcircled{13} \sqrt{y+12} + 1 = \sqrt{y+21}$$

$$\textcircled{14} \sqrt{5+2x} = x-5$$

$$\textcircled{15} \sqrt{x+11} - x = -9$$

Solve for c:

$$\textcircled{16} r = \sqrt[3]{\frac{2mM}{c}}$$



Origin of the expression, "Putting on the dog."

Rational Exponents & Complex Numbers

UNIT 7 REVIEW

Express in rational form:

① $\sqrt[3]{16x^5y^6z^8}$

② $\sqrt[4]{96a^5b^7c^8}$

Express in radical form:

③ $r^2s^{\frac{1}{3}}y^{\frac{1}{2}}$

⑤ $\sqrt[6]{27}$

④ $(3x)^{\frac{1}{2}}x^{\frac{1}{4}}$

⑥ $\sqrt[4]{64x^{10}}$

Evaluate:

⑦ $(25^{\frac{3}{4}})^{\frac{2}{3}}$

⑧ $(8^{\frac{2}{3}})(16^{-\frac{3}{4}})$

Simplify in rational form:

⑨ $\frac{x^{\frac{2}{3}}}{x^{\frac{2}{3}} - x^{-\frac{1}{3}}}$

⑪ $\left(\frac{a^{-\frac{2}{3}}}{2^{\frac{1}{2}}a^2}\right)^{-\frac{1}{2}}$

⑩ $\frac{x^{\frac{1}{2}} - y^{\frac{1}{2}}}{x^{\frac{1}{2}} + y^{\frac{1}{2}}}$

⑫ $\left(\frac{2^4a}{a^2b^{-1}}\right)^{-\frac{1}{2}}$

Simplify:

⑬ $(\sqrt{-8})(\sqrt{-12})$

⑭ $(\sqrt{-6})(\sqrt{-4})(\sqrt{-3})$

⑮ $(3i^3)(2i)^2$

⑯ $(-2i)^4(-4i^3)$

Solve:

⑰ $2n^2 = -\frac{27}{8}$

⑱ $4x^2 + 75 = 0$

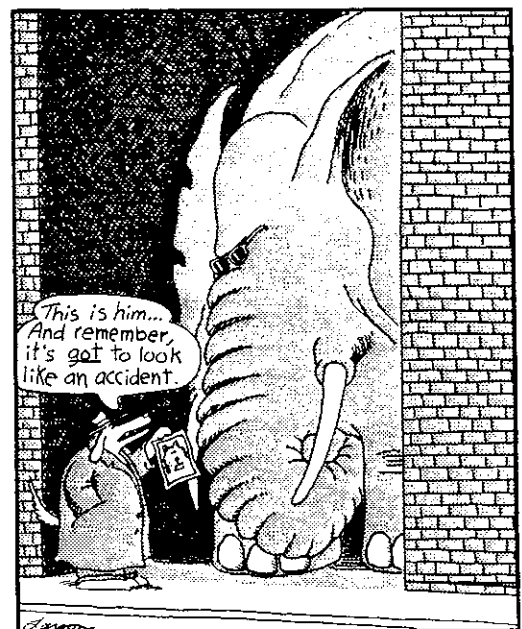
Simplify:

⑲ $(7+2i)(5-3i)$

⑳ $\frac{4+3i}{1-2i}$

㉑ $(3+8i)(3-8i)$

㉒ $\frac{2-2i}{2+2i}$



Hit elephants

Quadratics

UNIT 8 DEMONSTRATION

Complete the square:

① $2x^2 + 3x + 2 = 0$

Quadratic formula:

② $2x^2 + 2x + 3 = 0$

Use the discriminant to determine the nature of the roots:

③ $4x^2 - 8x + 13 = 0$

Write an equation with the given roots:

④ $-3/5, 1/2$

⑤ $4 \pm \sqrt{3}$

Solve equations in the quadratic form:

⑥ $x^4 - 9 = 0$

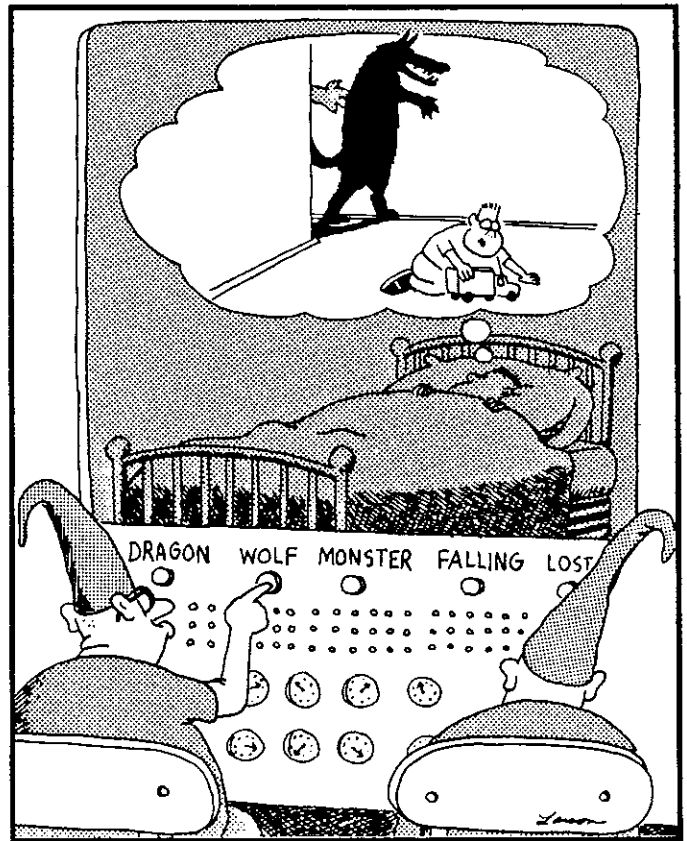
⑦ $x - 4\sqrt{x} - 45 = 0$

turning point (vertex), draw the graph, and indicate the algebraic solution:

⑧ $y \geq -x^2 + 10x - 21$

Solve (any method):

⑨ $bx^2 + cx + a = 0$



The nightmare makers

Indicate the axis, the

Quadratics

UNIT 8 REVIEW

Use factoring:

① $6x^2 + 7x - 3 = 0$

Complete the square:

② $2n^2 + n - 21 = 0$

③ $3x^2 + 4x + 2 = 0$

Quadratic formula:

④ $2a^2 - 5a + 4 = 0$

Any method:

⑤ $ax^2 + bx + 3b = 0$

⑥ $bx^2 + acx + c = 0$

Use the discriminant to determine the nature of the roots:

⑦ $4x^2 - 40x + 25 = 0$

⑧ $2y^2 + 6y + 5 = 0$

Determine an equation with the given roots:

⑨ $\frac{3}{4}, \frac{1}{3}$ ⑩ $2 \pm \sqrt{3}$ ⑪ $5 \pm 3i$

Quadratic form:

⑫ $x - 4\sqrt{x} - 32 = 0$

⑬ $x^4 - 12x^2 + 27 = 0$

⑭ $x^{2/3} - 9x^{1/3} + 20 = 0$

⑮ $n^6 + 9n^3 + 8 = 0$

Indicate the axis and vertex, draw the graph, and determine the solution (as a union or intersection):

⑯ $y \leq x^2 + 4x + 3$

⑰ $y \leq -x^2 + 6x + 5$



"Just a minute, young man! ... What are you taking from the jungle?"

Rational Expressions

UNIT 9 DEMONSTRATION

Divide and simplify:

$$\textcircled{1} \frac{x^2+7x+10}{x+2} \div \frac{x^2+2x-15}{x^2-5x+6}$$

Add and simplify:

$$\textcircled{2} 3n+1 + \frac{1}{3n-1}$$

Simplify:

$$\textcircled{3} \frac{\frac{x+y}{x}}{\frac{1}{x} + \frac{1}{y}}$$

Solve:

$$\textcircled{4} \frac{y}{y-3} + \frac{6}{y+3} = 1$$

Simplify:

$$\textcircled{5} \frac{n+5 + \frac{3}{n+1}}{n-1 - \frac{3}{n+1}}$$

Solve:

$$\textcircled{6} \text{ If } y \text{ varies directly as } x, \text{ and } x=7 \text{ when } y=21, \text{ find } x \text{ when } y=-5$$

$$\textcircled{7} \text{ If two boxes have the same depth and capacity, the length is inversely proportional to the width. One box is 60cm long and 40cm wide. A second box (same depth and capacity) is 5cm long. How wide is it?}$$

$$\textcircled{8} \frac{1}{n-2} = \frac{2n+1}{n^2+2n-8} + \frac{2}{n+4}$$



"Don't touch it, honey... it's just a face in the crowd."

Rational Expressions

UNIT 9 REVIEW

Multiply/Divide and simplify:

$$\textcircled{1} \frac{a^3 - b^3}{b^2 - a^2} \cdot \frac{a + b}{a^2 + ab + b^2}$$

$$\textcircled{2} \frac{x^2 - 11x + 24}{x^2 - 18x + 80} \div \frac{x^2 - 9x + 20}{x^2 - 15x + 50}$$

$$\textcircled{3} \frac{x^2 - 2x + 1}{y - 5} \div \frac{(x - 1)^2}{y^2 - 25}$$

Add/Subtract and simplify:

$$\textcircled{4} \frac{3}{m - 2} + \frac{2}{2 - m}$$

$$\textcircled{5} \frac{3x + 2}{3x - 6} - \frac{x + 2}{x^2 - 4}$$

$$\textcircled{6} \frac{2n}{n^2 - 5n} - \frac{-3n}{n - 5}$$

Simplify complex fractions:

$$\textcircled{7} \frac{\frac{x}{y} - \frac{y}{x}}{\frac{1}{x} + \frac{1}{y}}$$

$$\textcircled{8} \frac{\frac{m + 4}{m} - \frac{3}{m + 5}}{\frac{m - 1}{m^2 + 5m} + \frac{3}{m + 5}}$$

$$\textcircled{9} \frac{n + 5 + \frac{4}{n + 1}}{n + 3}$$

Solve:

$$\textcircled{10} \frac{3x}{2x - 5} + \frac{2x}{5 - 2x} = \frac{x - 1}{2x + 5}$$

$$\textcircled{11} \frac{3}{x + 2} + \frac{12}{x^2 - 4} = \frac{-1}{x - 2}$$

$$\textcircled{12} \frac{x + 3}{x + 2} = 2 - \frac{3}{x^2 + 5x + 6}$$

Variation:

$\textcircled{13}$ If y varies inversely as x , and $y = 9$ when $x = \frac{5}{2}$, find y when $x = -\frac{3}{5}$.

$\textcircled{14}$ If y varies directly as x , and $y = -8$ when $x = -3$, find x when $y = 6$.

$\textcircled{15}$ If area remains constant, a triangle's base and height vary inversely. If the height is 6 and the base is 10, find the height when the base is 12.

Problem Solving

UNIT 10 DEMONSTRATION

- ① Two numbers are in the ratio 6:11. If the first number is decreased by 4 and the second increased by 6, the resulting numbers are in the ratio 4:9. Find the original numbers.
- ② The sum of a number's reciprocal and twice the square of the reciprocal is 15. Find the original number.
- ③ Jean has a rug that is 9 by 12 ft. Along the outside of the rug is a uniform strip of uncovered floor with an area of 270 ft^2 . How wide is the strip?
- ④ A trip of 500 miles would take $2\frac{1}{2}$ hours less if the speed is increased by 10 mph. Find the original speed.
- ⑤ Andrea drives to the campground at 48 mph. She drove 60 mph on the return trip. What was her average speed?
- ⑥ If 8 adults can set up the campsite in 6 hours and 6 children can do the same job in 10 hours, how long would it take a crew of 2 adults and 3 children?
- ⑦ The fulcrum of a 16-foot seesaw is placed in the middle. Jason, who weighs 108 pounds, is seated 8 feet from the fulcrum. How far from the fulcrum should Laura sit if she weighs 132 pounds?

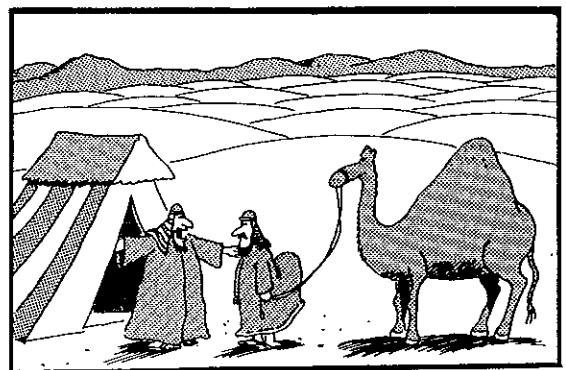


"You're darn lucky, Saunders... If that rhino had really respected you as an enemy, he'd have done a heck of a lot more than just slap your face."

Problem Solving

UNIT 10 REVIEW

- ① Two numbers are in a ratio of 3 to 5. If the first is increased by 3 and the second decreased by 2, the resulting numbers are in a ratio of 5 to 6. Find the original numbers.
- ② The sum of twice the reciprocal and twice the square of the reciprocal is $\frac{5}{8}$. Find two solutions.
- ③ An 8 by 12 ft. garden is surrounded by a sidewalk of uniform width. The sidewalk has an area of 125 ft^2 . What is the uniform width?
- ④ Increasing the speed by 9 mph allowed Robyn to cut 1 hour off her 180-mile trip. What was her original speed?
- ⑤ A carpenter can build a cabinet in 6 hours. His assistant can do the job in 10 hours. How long would it take for them to build a cabinet together if the assistant works on it twice as long as the carpenter? Indicate the time spent by each.
- ⑥ If 6 volunteers can do the job in 12 hours, and 4 hired workers can do the job in 6 hours, how long would it take for 3 volunteers and 2 hired workers to do the job together?
- ⑦ Allisyn drives to her aunt's house at 42 mph. She makes the return trip at 54 mph. What was her average speed for the round trip?
- ⑧ An 8-ounce weight is placed at one end of a yardstick and a 10-ounce weight is placed at the other end. If the yardstick is balanced, how far is the fulcrum from the 8-ounce weight?



"Abdul, my old friend! Come in, come in!
Have you traveled far!!"

Plane Geometry

REFERENCE PAGE

① Parallelogram

$$A = (\text{base})(ht)$$

② Rhombus, Kite

$$A = \frac{1}{2} (\text{prod. of diagonals})$$

③ Triangle

$$A = \frac{1}{2} (\text{base})(ht)$$

④ Trapezoid

$$A = \frac{1}{2} (\text{sum of bases})(ht)$$

$$A = (\text{median})(ht)$$

⑤ Circle

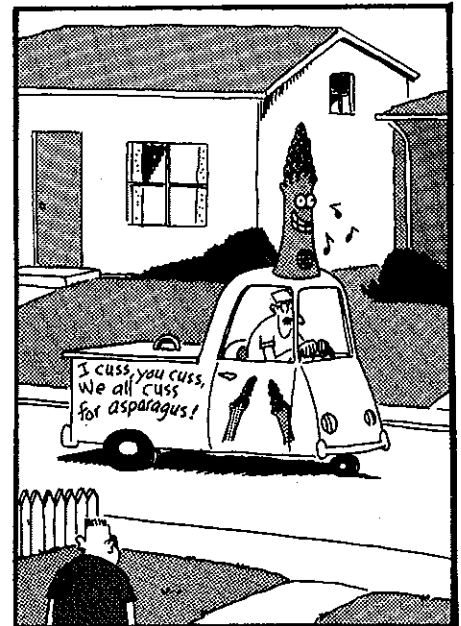
$$A = \pi r^2 \quad C = 2\pi r$$

⑥ Pythagorean Theorem & Triples

$$a^2 + b^2 = c^2$$

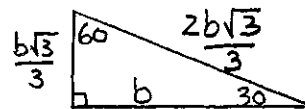
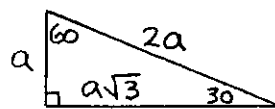
$$3-4-5 \quad 7-24-25$$

$$5-12-13 \quad 8-15-17$$

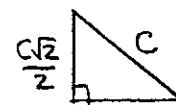
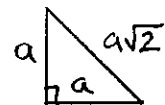


Failed marketing ploys

⑦ 30-60-90 Triangle



⑧ 45-45-90 Triangle

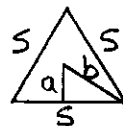


⑨ Equilateral Triangle



$$A = \frac{s^2\sqrt{3}}{4}$$

⑩ Regular Polygon

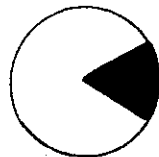


$a \rightarrow$ apothem

$b \rightarrow$ radius

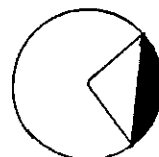
$$A = \frac{1}{2} (\text{apoth})(\text{per})$$

⑪ Sector of a Circle



$$A = \frac{\text{central angle}}{360} (\pi r^2)$$

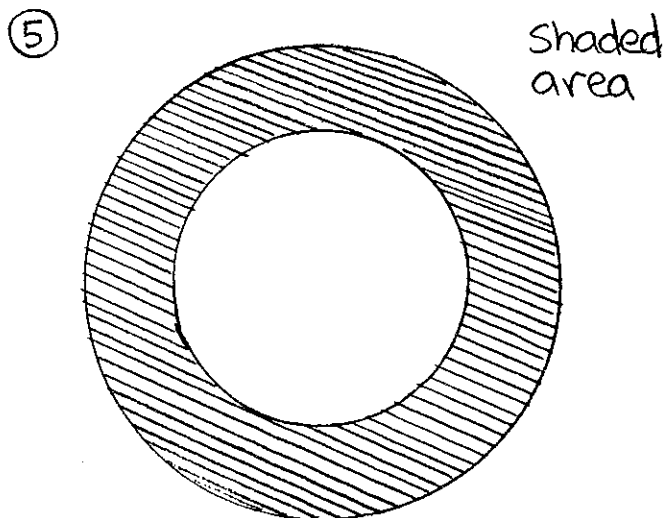
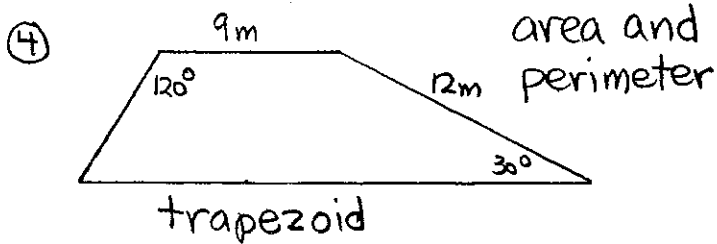
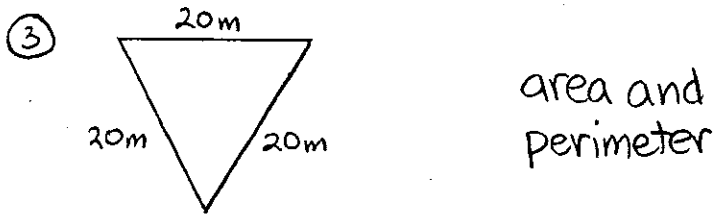
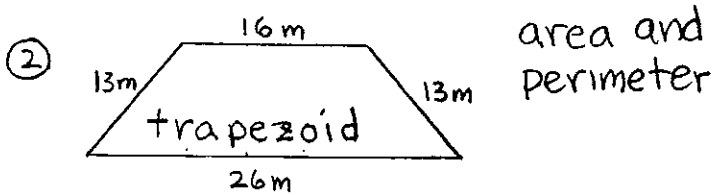
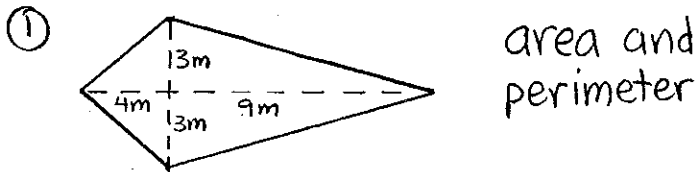
⑫ Segment of a Circle



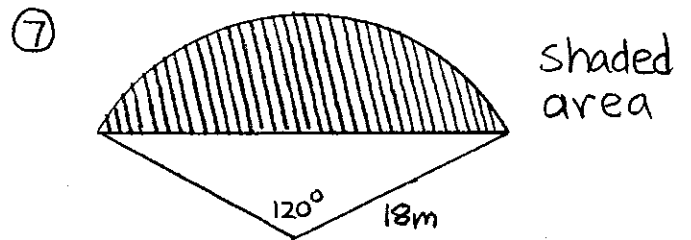
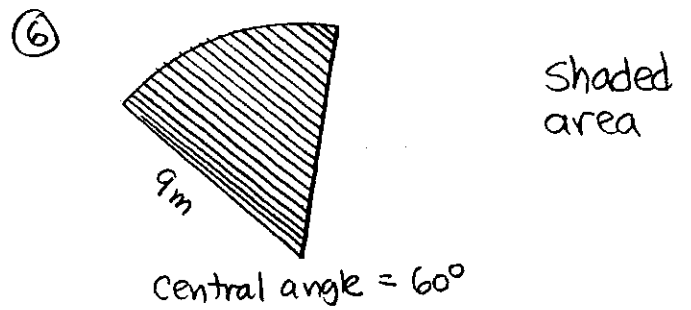
$$A = (\text{area of sector}) - (\text{area of triangle})$$

Plane Geometry

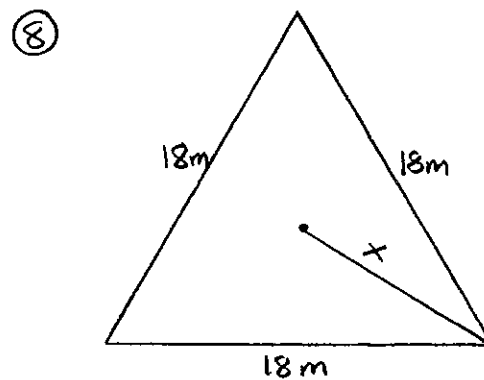
UNIT 11 DEMONSTRATION



inner $r = 10\text{m}$ outer $r = 16\text{m}$



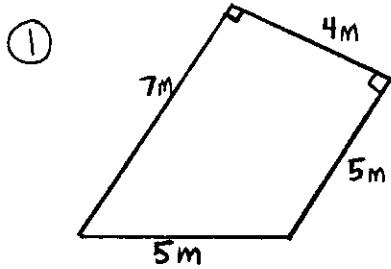
Solve for x :



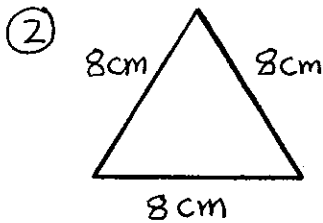
Plane Geometry

UNIT 11 REVIEW

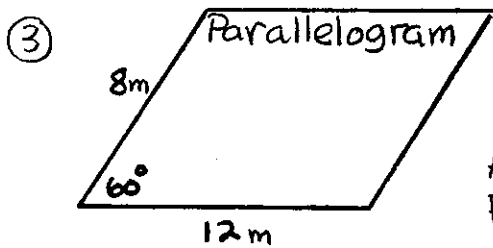
Determine the area (and the perimeter/circumference for selected problems):



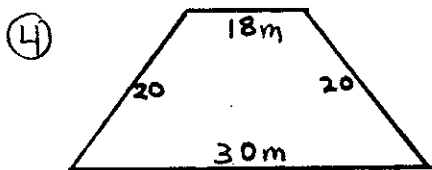
Area
Perimeter



Area
Perimeter



Area
Perimeter



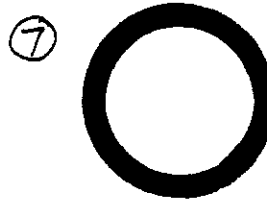
Isosceles Trapezoid

Area
Perimeter

⑤ Find the area of a circle if the circumference is 16π cm

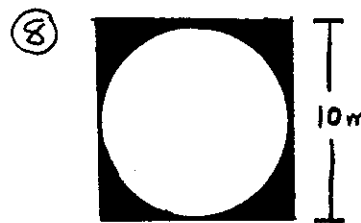


Area
Circumference

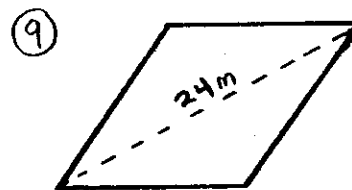


Area (only)
of the
shaded region

inside $r = 6m$ outside $r = 8m$



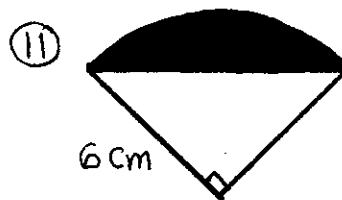
Area (only)
of the
shaded region



Area (only)

perimeter = 52 m (rhombus)

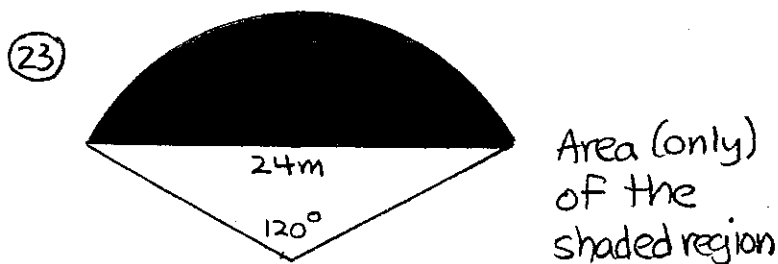
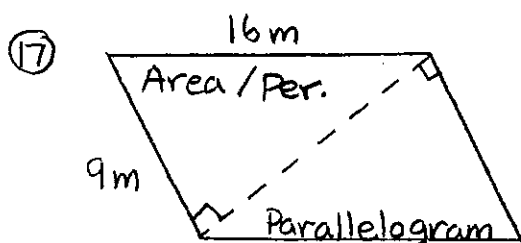
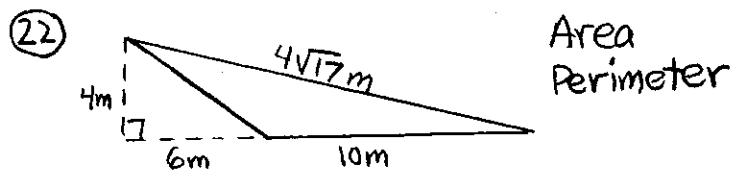
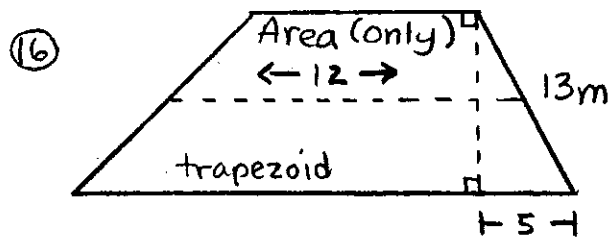
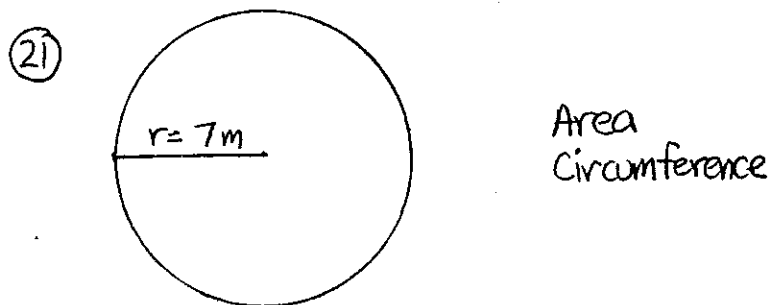
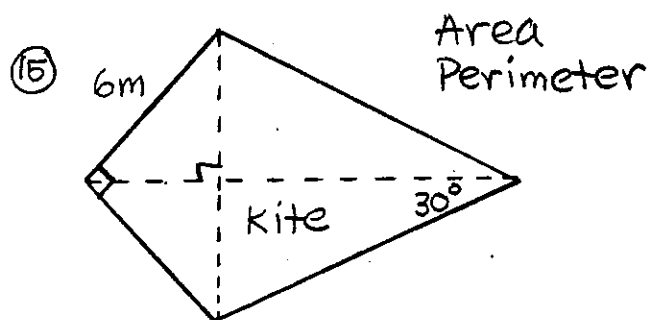
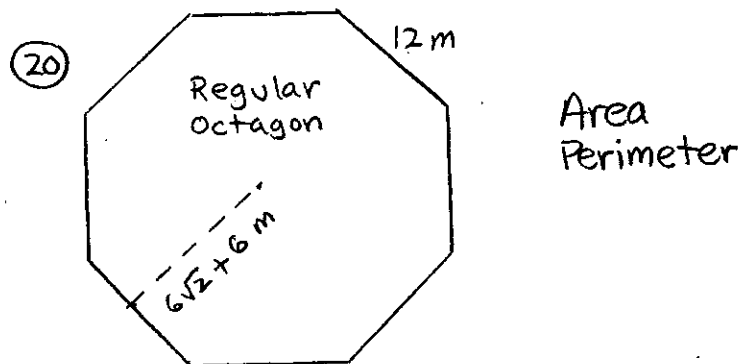
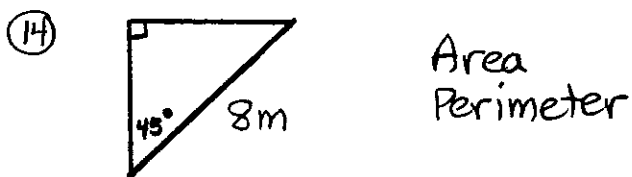
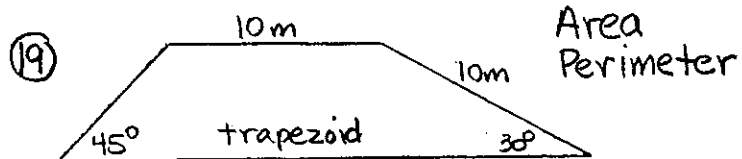
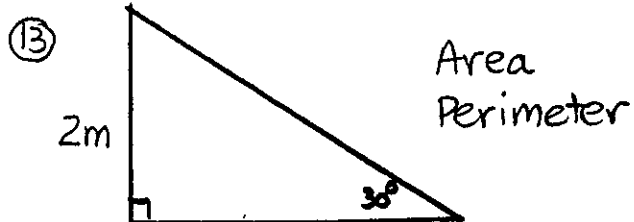
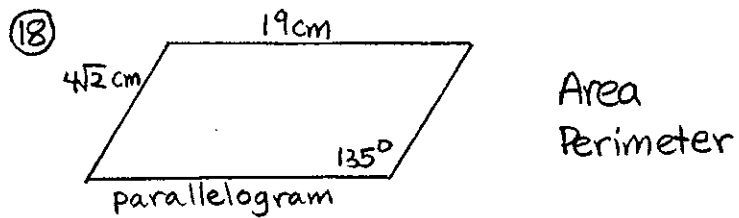
⑩ Find the area of an equilateral triangle whose perimeter is 21 cm.



Area (only)

Plane Geometry

UNIT 11 REVIEW



Solid Geometry

REFERENCE PAGE

① Prism

$$V = (\text{base area})(\text{height})$$

$$SA = 2(\text{base area}) + (\text{perimeter})(\text{height})$$

② Cylinder

$$V = (\text{base area})(\text{height})$$

$$V = (\pi r^2)(ht)$$

$$SA = 2(\text{base area}) + (\text{cir.})(ht)$$

$$SA = 2(\pi r^2) + (2\pi r)(ht)$$

③ Pyramid

$$V = \frac{1}{3}(\text{base area})(\text{height})$$

$$SA = (\text{base area}) + \frac{1}{2}(\text{perimeter})(\text{slant height})$$

④ Cone

$$V = \frac{1}{3}(\text{base area})(\text{height})$$

$$V = \frac{1}{3}(\pi r^2)(ht)$$

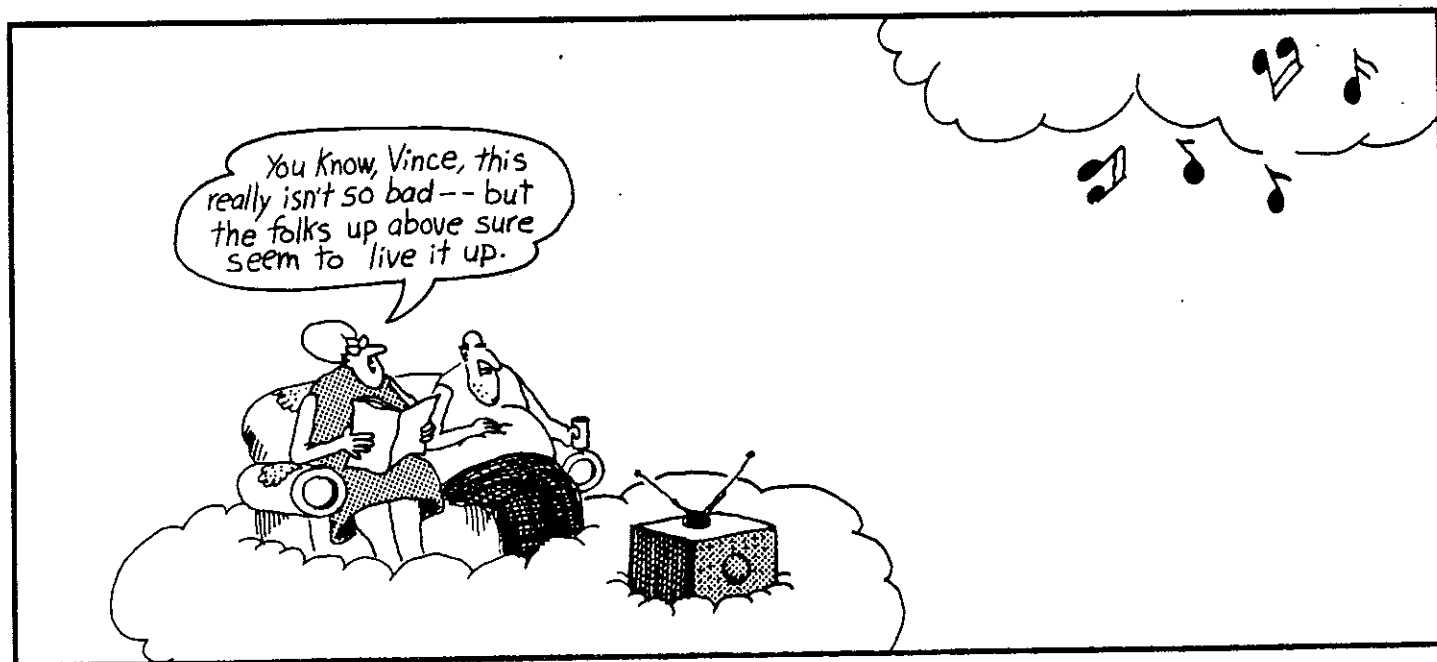
$$SA = (\text{base area}) + \frac{1}{2}(\text{cir.})(\text{slant ht})$$

$$SA = (\pi r^2) + \frac{1}{2}(2\pi r)(\text{slant ht})$$

⑤ Sphere

$$V = \frac{4}{3}\pi r^3$$

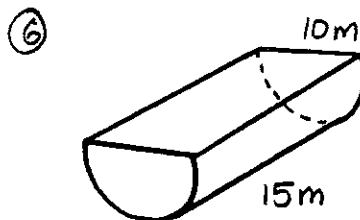
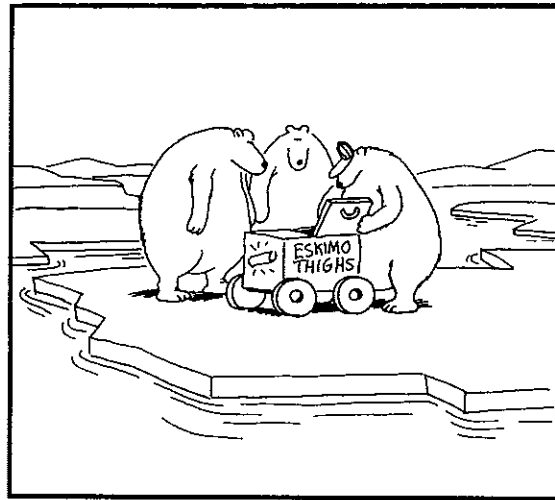
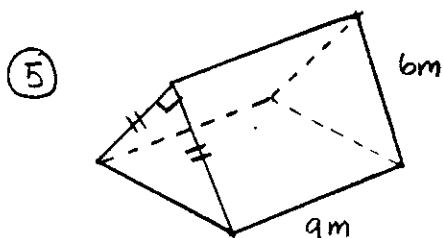
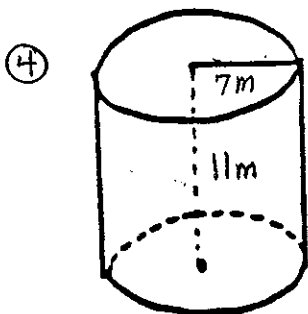
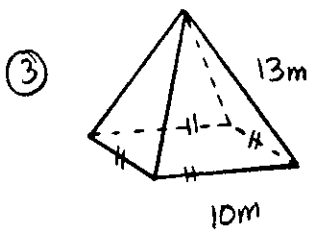
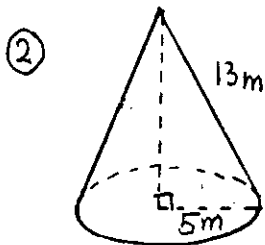
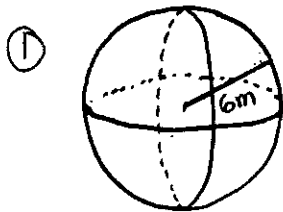
$$SA = 4\pi r^2$$



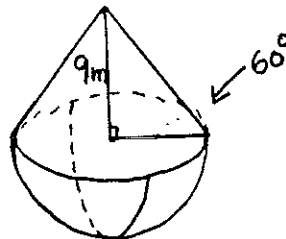
Solid Geometry

UNIT 12 DEMONSTRATION

Volume and surface area:



⑦ Volume (below)



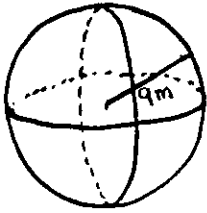
⑧ Surface area (above)

Solid Geometry

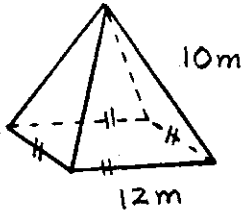
UNIT 12 REVIEW

Determine volume and surface area:

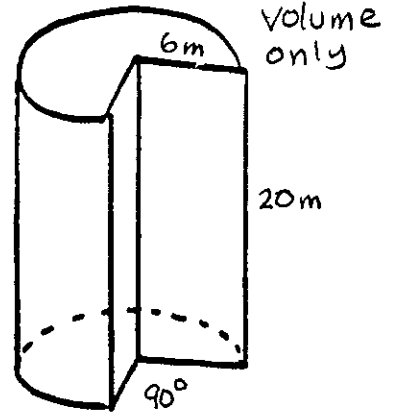
①



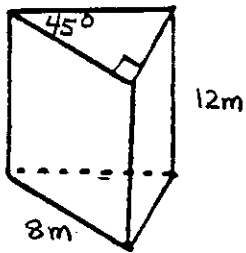
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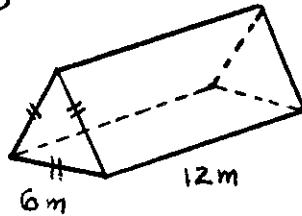
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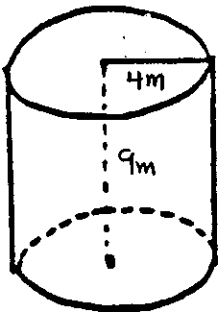
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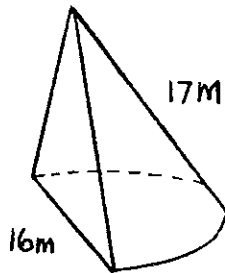
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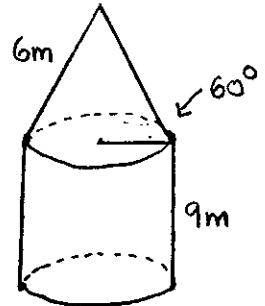
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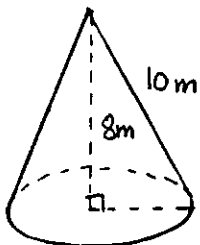
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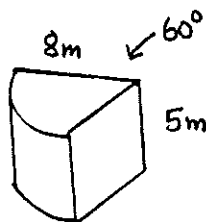
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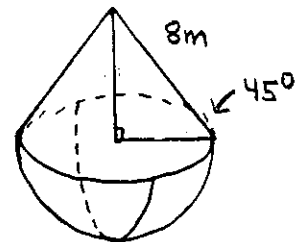
④



⑧



⑪



Volume only

Introduction to Two-Column Proofs

REFERENCE PAGE

DEFINITIONS

- | | |
|-------------------------|--|
| 1. SEGMENT MIDPOINT | Midpoint separates a segment into two congruent parts |
| 2. SEGMENT BISECTOR | Segment, line, or plane intersecting at segment midpoint |
| 3. RIGHT ANGLE | 90 degree angle |
| 4. LINEAR PAIR | Adjacent angles whose noncommon sides form a straight angle |
| 5. CONGRUENT ANGLES | Angles are congruent iff their measures are exactly equal |
| 6. CONGRUENT SEGMENTS | Segments are congruent iff their lengths are exactly equal |
| 7. COMPLEMENTARY ANGLES | Angles are complementary iff their measures sum to 90° |
| 8. SUPPLEMENTARY ANGLES | Angles are supplementary iff their measures sum to 180° |
| 9. VERTICAL ANGLES | Angles formed on opposite sides of intersecting lines |
| 10. PERPENDICULAR LINES | Lines intersecting at right angles |
| 11. ISOSCELES TRIANGLE | Triangle with two sides congruent |
| 12. RIGHT TRIANGLE | Triangle with one right angle |
| 13. CPCTC | Corresponding Parts of Congruent Triangles are Congruent |

POSTULATES

1. IDENTITY (Reflexive Property)
2. Two points form a line
3. Three noncollinear points form a plane
4. A line segment has exactly one midpoint
5. If two angles form a linear pair, then they are supplementary
6. SSS proves congruent triangles
7. SAS proves congruent triangles
8. ASA proves congruent triangles

Definition: Explanation of terms

Postulate: Statement accepted as true

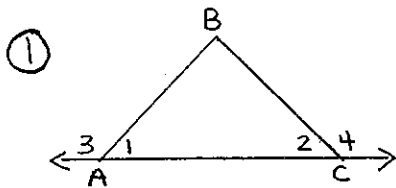
Theorem: Statement proven to be true

THEOREMS

1. Vertical angles are equal
2. Complements of equal angles are equal
3. Supplements of equal angles are equal
4. If two equal angles form a linear pair, then they are right angles
5. All right angles are equal
6. Perpendicular lines form congruent adjacent angles
7. The acute angles of a right triangle are complementary
8. AAS proves congruent triangles
9. In a triangle, sides opposite equal angles are equal
10. In a triangle, angles opposite equal sides are equal

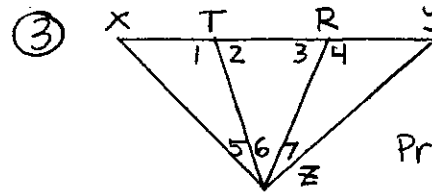
Introduction to Two-Column Proofs

UNIT 13 DEMONSTRATION



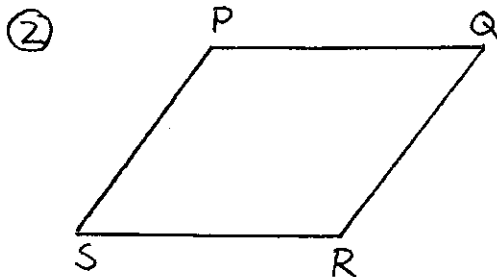
Given: $\overline{AB} \cong \overline{CB}$
 Prove: $\angle 3 \cong \angle 4$

Statements	
1. $\overline{AB} \cong \overline{CB}$	1.
2. $\angle 1 \cong \angle 2$	2.
3. $\angle 1$ and $\angle 3$, $\angle 2$ and $\angle 4$ are linear pairs	3.
4. $\angle 1$ and $\angle 3$, $\angle 2$ and $\angle 4$ are Supplementary	4.
5. $\angle 3 \cong \angle 4$	5.



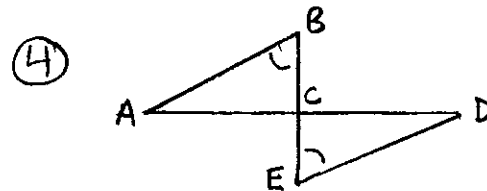
Given: $\overline{ZT} \cong \overline{ZR}$, $\overline{TX} \cong \overline{RY}$
 Prove: $\angle 5 \cong \angle 6$

Statements	
1. $\overline{ZT} \cong \overline{ZR}$, $\overline{TX} \cong \overline{RY}$	1.
2. $\angle 2 \cong \angle 3$	2.
3. $\angle 1$ and $\angle 2$, $\angle 3$ and $\angle 4$ are linear pairs	3.
4. $\angle 1$ and $\angle 2$, $\angle 3$ and $\angle 4$ are Supplementary	4.
5. $\angle 1 \cong \angle 4$	5.
6. $\triangle XTZ \cong \triangle YRZ$	6.
7. $\angle 5 \cong \angle 6$	7.



Given: $\overline{PQ} \cong \overline{RS}$, $\overline{PS} \cong \overline{RQ}$
 Prove: $\angle P \cong \angle R$

Statements	
1. $\overline{PQ} \cong \overline{RS}$, $\overline{PS} \cong \overline{RQ}$	1.
2. Draw \overline{SQ}	2.
3. $\overline{SQ} \cong \overline{QS}$	3.
4. $\triangle QSP \cong \triangle SQR$	4.
5. $\angle P \cong \angle R$	5.

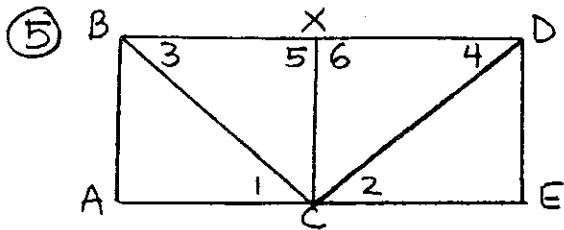


Given: $\overline{BE} \perp \overline{AD}$, C is midpoint of \overline{AD} , $\angle B \cong \angle E$
 Prove: $\overline{AB} \cong \overline{DE}$

Statements	
1. $\overline{BE} \perp \overline{AD}$, C is midpoint of \overline{AD} , $\angle B \cong \angle E$	1.
2. $\angle BCA$ and $\angle ECD$ are rt. \angle s	2.
3. $\angle BCA \cong \angle ECD$	3.
4. $\overline{AC} \cong \overline{DC}$	4.
5. $\triangle ABC \cong \triangle DEC$	5.
6. $\overline{AB} \cong \overline{DE}$	6.

Introduction to Two-Column Proofs

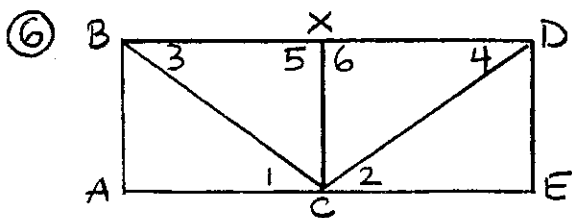
UNIT 13 DEMONSTRATION



Given: $\overline{BA} \perp \overline{AE}$, $\overline{DE} \perp \overline{AE}$, C is midpoint of \overline{AE} , $\angle ABC \cong \angle EDC$
 Prove: $\angle 1 \cong \angle 2$

Statements

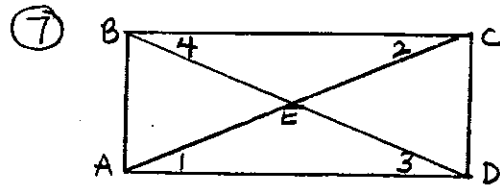
- | | |
|---|----|
| 1. $\overline{BA} \perp \overline{AE}$, $\overline{DE} \perp \overline{AE}$, C is midpoint of \overline{AE} , $\angle ABC \cong \angle EDC$ | 1. |
| 2. $\overline{AC} \cong \overline{EC}$ | 2. |
| 3. $\angle A$ and $\angle E$ are rt. \angle 's | 3. |
| 4. $\angle A \cong \angle E$ | 4. |
| 5. $\triangle BAC \cong \triangle DEC$ | 5. |
| 6. $\angle 1 \cong \angle 2$ | 6. |



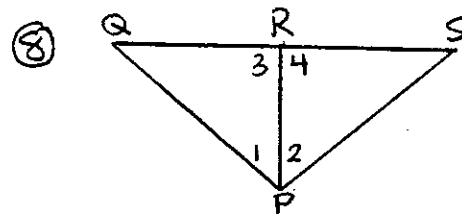
Given: $\overline{CB} \cong \overline{CD}$, $\angle 5 \cong \angle 6$
 Prove: $\triangle CBX \cong \triangle CDX$

Statements

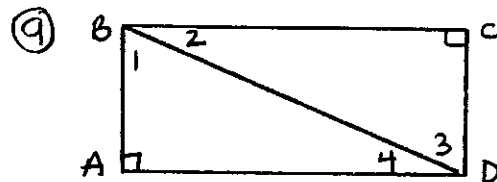
- | | |
|--|----|
| 1. $\overline{CB} \cong \overline{CD}$, $\angle 5 \cong \angle 6$ | 1. |
| 2. $\angle 3 \cong \angle 4$ | 2. |
| 3. $\triangle CBX \cong \triangle CDX$ | 3. |



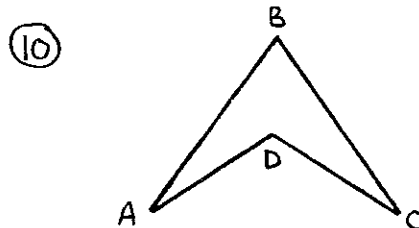
Given: E is midpoint of \overline{AC}
 $\angle 1 \cong \angle 2$
 Prove: $\angle 3 \cong \angle 4$



Given: $\overline{PR} \perp \overline{QS}$, $\angle Q \cong \angle S$
 Prove: R is the midpoint of \overline{QS}



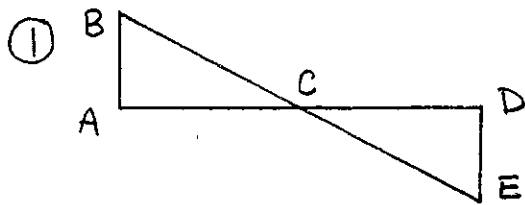
Given: $\angle A$ and $\angle C$ are rt. \angle 's
 $\angle 2 \cong \angle 4$
 Prove: $\overline{AB} \cong \overline{CD}$



Given: $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$
 Prove: $\angle A \cong \angle C$

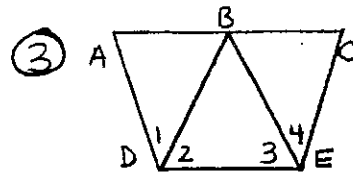
Introduction to Two-Column Proofs

UNIT 13 REVIEW



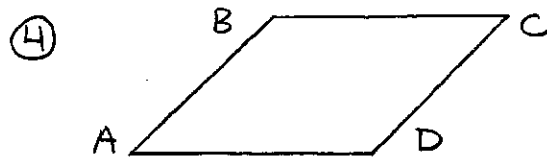
Given: $\angle A$ & $\angle D$ are rt. \angle 's,
 $\overline{AC} \cong \overline{DC}$
 Prove: $\angle B \cong \angle E$

Statements	
1. $\angle A$ & $\angle D$ are rt \angle 's, $\overline{AC} \cong \overline{DC}$	1.
2. $\angle A \cong \angle D$	2.
3. $\angle ACB$ and $\angle DCE$ are vertical angles	3.
4. $\angle ACB \cong \angle DCE$	4.
5. $\triangle ACB \cong \triangle DCE$	5.
6. $\angle B \cong \angle E$	6.

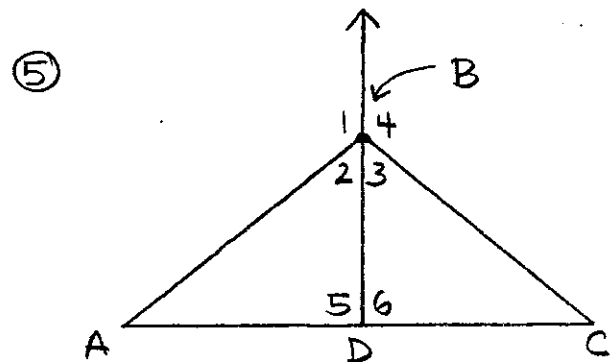


Given: $\angle A \cong \angle C$,
 $\overline{AD} \cong \overline{CE}$, B is mid-
 point of \overline{AC}
 Prove: $\angle 2 \cong \angle 3$

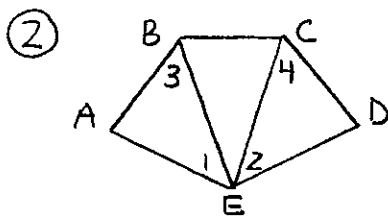
Statements	
1. $\angle A \cong \angle C$, $\overline{AD} \cong \overline{CE}$, B is midpoint of \overline{AC}	1.
2. $\overline{AB} \cong \overline{CB}$	2.
3. $\triangle ABD \cong \triangle CBE$	3.
4. $\overline{BD} \cong \overline{BE}$	4.
5. $\angle 2 \cong \angle 3$	5.



Given: $\overline{BC} \cong \overline{DA}$, $\overline{BA} \cong \overline{DC}$
 Prove: $\angle B \cong \angle D$



Given: $\angle 1 \cong \angle 4$, $\overline{BD} \perp \overline{AC}$
 Prove: D is the midpoint of \overline{AC}



Given: $\overline{BE} \perp \overline{AC}$,
 $\overline{CE} \perp \overline{DC}$, $\angle 1 \cong \angle 2$,
 $\overline{AE} \cong \overline{DE}$
 Prove: $\triangle BEC$ is isosceles

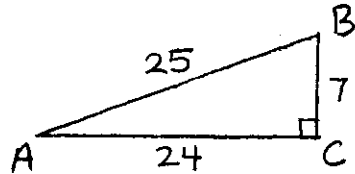
Statements	
1. $\overline{BE} \perp \overline{AC}$, $\overline{CE} \perp \overline{DC}$, $\angle 1 \cong \angle 2$, $\overline{AE} \cong \overline{DE}$	1.
2. $\angle 3$ and $\angle 4$ are rt. \angle 's	2.
3. $\angle 3 \cong \angle 4$	3.
4. $\triangle ABE \cong \triangle DCE$	4.
5. $\overline{BE} \cong \overline{CE}$	5.
6. $\triangle BEC$ is isosceles	6.

Triangle Trigonometry

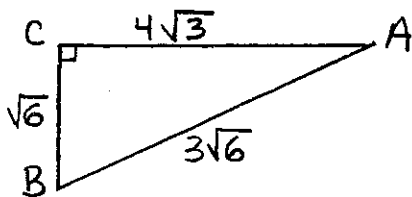
UNIT 14 DEMONSTRATION

- ① Find each value in fraction form:

- a) $\cos B$
b) $\sec A$



- ② Find each value in fraction form (rationalize as needed):



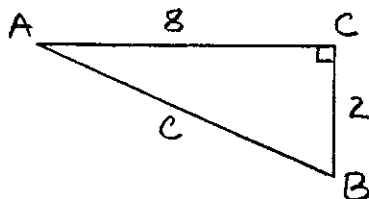
- a) $\tan A$
b) $\sin B$

- ③ Interpolate to the nearest minute:

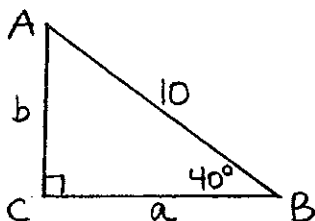
$$\tan A = 2.166$$

In problems 4 and 5, solve each triangle. Round sides to 2 decimal places and angles to the nearest min.

- ④ $\angle A =$
 $\angle B =$
 $c =$



- ⑤ $\angle A =$
 $a =$
 $b =$



Determine the value of each expression in fraction form. Rationalize if needed.

⑥ $(\sin^2 45^\circ)(\cot^2 30^\circ)$

⑦ $3 \sec 60^\circ - 2 \tan 45^\circ$

Determine the value:

⑧
$$\frac{4 \sin^2 60^\circ - \tan 45^\circ}{3 \sin 45^\circ}$$



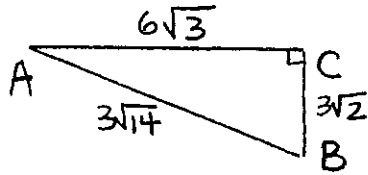
"Excuse me, sir, but could your entire family please step out of the car? ...Your faces are not in order."

Triangle Trigonometry

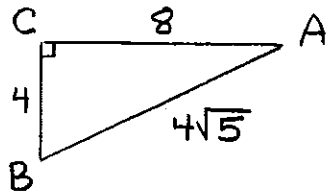
UNIT 14 REVIEW

Find each value in fraction form (rationalize as needed):

- ① a) $\tan A$
 b) $\csc B$
 c) $\sin A$
 d) $\cot A$



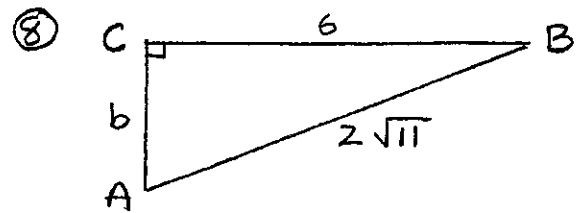
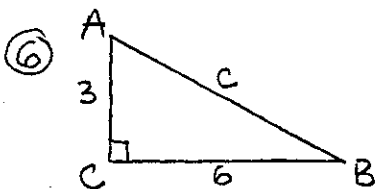
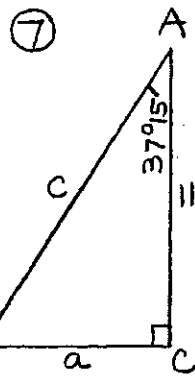
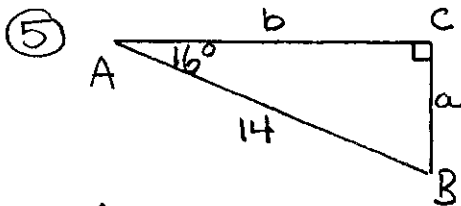
- ② a) $\cos B$
 b) $\tan A$
 c) $\sec B$
 d) $\sin A$



Interpolate to the nearest minute:

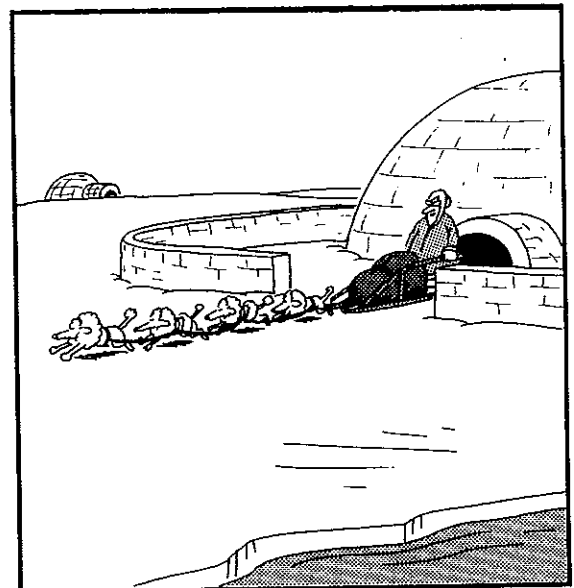
- ③ $\tan A = .4497$
 ④ $\sin A = .7478$

Solve each triangle. Round side measures to 2 decimal places and angles to the nearest minute:



Determine the value of each expression in fraction form:

- ⑨ $\cos^2 45^\circ - \sin 30^\circ$
 ⑩ $2 \tan^2 60^\circ + (\csc 30^\circ)(\sin^2 45^\circ)$
 ⑪ $\frac{2 \cos 60^\circ - \sin^2 45^\circ}{\csc 45^\circ}$
 ⑫ $\frac{(2 + \tan^2 60^\circ)(\sin 30^\circ)}{\sin 60^\circ}$



Beverly Hills of the North Pole

Law of Sines & Cosines

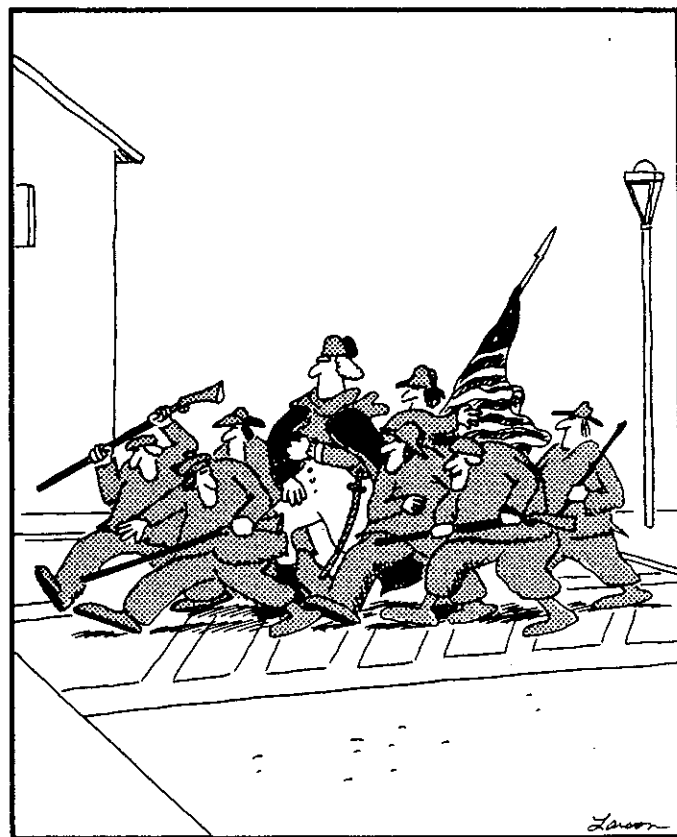
UNIT 15 DEMONSTRATION

Law of Sines and
Law of Cosines:

- ① From a point on the ground 50 m from the base of a flagpole, the angle of elevation to the top is 48° . How tall is the flagpole?
- ② The diagonal of a rectangle is 30 inches long and makes a 15° angle with one of the sides. Find the dimensions of the rectangle.
- ③ The base of a monument and two points on the ground are in a straight line. The two points are 50 m apart. The measurements of the angles of elevation to the top of the monument are 45° and 25° . Find the height of the monument.
- ④ An isosceles triangle has a vertex angle of 80° and a base of 20 cm. Determine the perimeter of the triangle.
- ⑤ A plane flew 1000 km north. It then changed direction by turning 20°

clockwise and flew for another 750 km. How far was the plane from its starting point?

- ⑥ Determine the area of a triangle with adjacent sides of 12 in. and 20 in. and an angle of $50^\circ 25'$ between them.
- ⑦ $\angle A = 20^\circ$, $b = 9$ cm, $c = 18$ cm
Determine $\angle C$ to the nearest minute.



Washington crossing the street

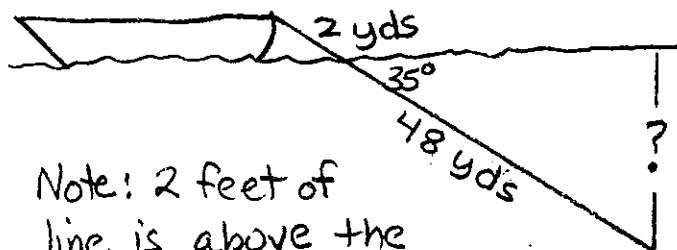
Law of Sines & Cosines

UNIT 15 REVIEW

Solve each problem:

- ① A 15-foot plank has one end on the ground and one end 6 feet off the ground. What is the measure of the angle formed by the plank and the ground (to the nearest minute)?

- ② An angler is trolling for salmon. Her line makes an angle of 35° with the water's surface. She has 50 yards of line out. How deep is her lure?



Note: 2 feet of line is above the water and 48 yards is below.

- ③ From Fred's position at 2nd base, the angle of elevation to the top of the scoreboard is 30° . Bryan, playing first base, is 25 feet closer. His angle is 45° . How high is the scoreboard?

- ④ Two planes leave an airport

at the same time. One flies 40° east of north and the other flies 20° east of south. If they both fly 200 mph, how far apart are they after 2 hours?

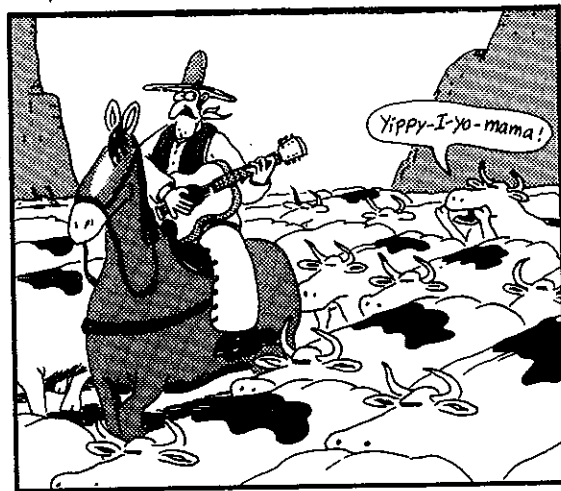
- ⑤ The sides of a triangle are 8 cm, 12 cm, and 15 cm. Determine the smallest angle to the nearest minute.

- ⑥ Determine the area:

$$a = 12 \text{ cm}, b = 15 \text{ cm}, \\ \angle C = 36^\circ$$

- ⑦ $\angle A = 25^\circ$, $b = 5 \text{ cm}$,
 $c = 13 \text{ cm}$

Determine $\angle C$ to the nearest minute. (Use a diagram!)



Trigonometric Functions

UNIT 16 DEMONSTRATION

① Determine the quadrant that contains the terminal side of an angle measuring $-\frac{7}{6}\pi$.

② Change a -240° angle to radians.

③ Change an angle measure of $\frac{7}{6}\pi$ to degrees.

④ Find the least positive coterminal angle to -1140° .

⑤ Determine a value for $(\sin 150^\circ)$ in radical form.

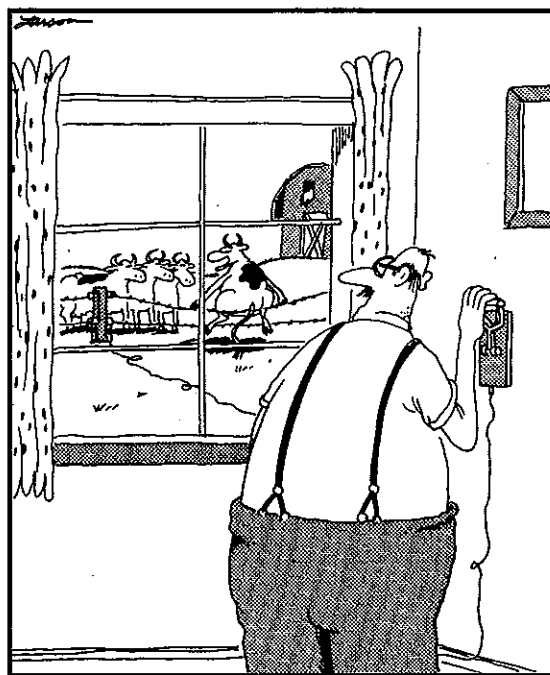
⑥ Determine a value for $(\cos -\frac{3}{4}\pi)$ in radical form.

⑦ Graph the curve:
 $y = -2 \sin \theta$

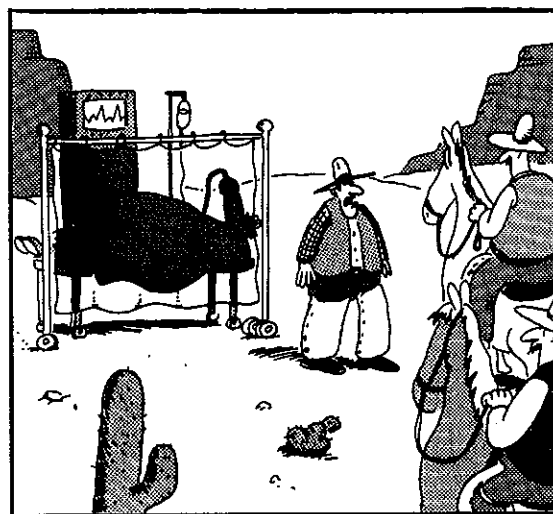
⑧ Graph: $y = \cos 2\theta$

⑨ Evaluate:

$$\frac{\frac{1}{2} (\cos 225^\circ) (\sin 120^\circ)}{(\cos 180^\circ)}$$



"Look, if it was electric, could I do this?"



"Am I glad you boys came along! ...
My horse seems to have come up lame."

Trigonometric Functions

UNIT 16 REVIEW

Determine the quadrant that contains the terminal side:

① -690° ② $\frac{7}{3}\pi$

Change to radians:

③ 900° ④ -240°

Change to degrees:

⑤ $-\frac{5}{6}\pi$ ⑥ $\frac{4}{3}\pi$

Find the least positive coterminal angle:

⑦ -1250° ⑧ $-\frac{5}{2}\pi$

Determine each value in radical form:

⑨ $\sin 600^\circ$ ⑩ $\cos \frac{5}{6}\pi$

⑪ $\cos -405^\circ$ ⑫ $\sin -\frac{17}{4}\pi$

Evaluate:

⑬ $\frac{4(\sin 135^\circ) - (\cos 390^\circ)}{\frac{1}{2}}$

⑭ $\frac{2(\cos 150^\circ)(\sin 300^\circ)}{(\cos 180^\circ)}$

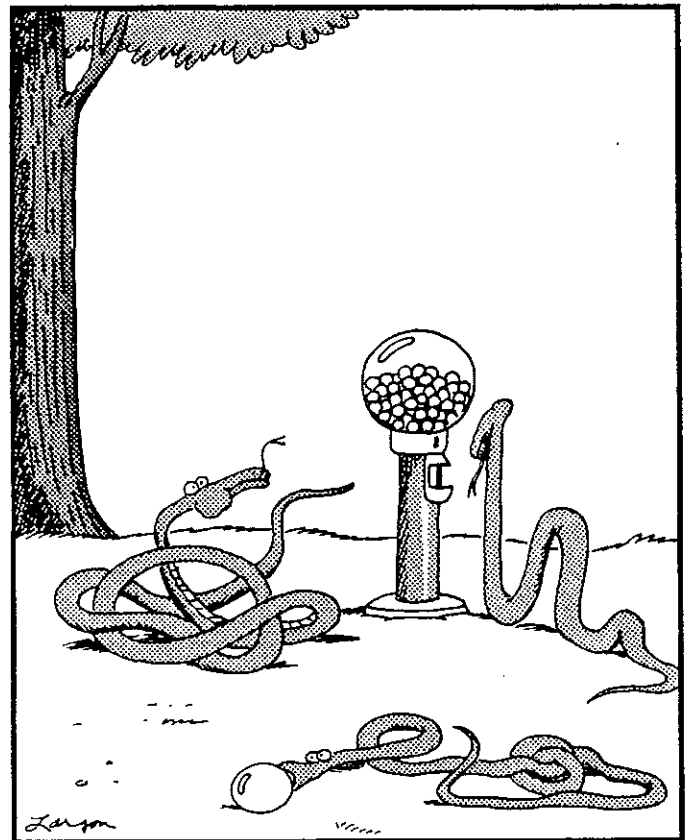
Graph each sine or cosine curve:

⑮ $y = 2 \sin \theta$

⑯ $y = -\frac{2}{3} \cos \frac{1}{2}\theta$

⑰ $-2y = 6 \sin 2\theta$

⑱ $\frac{1}{2}y = 2 \cos \frac{2}{3}\theta$



When snakes try to chew gum and crawl at the same time

Sequence & Series

UNIT 17 DEMONSTRATION

Expand each binomial:

① $(x - y)^7$

② $(a + 3b)^5$

Indicate the sum for each series and the first 3 terms:

③
$$\sum_{x=2}^{10} (3x+5)$$

④
$$\sum_{x=1}^7 (3)^{x+1}$$

Find the sum if possible:

⑤ $(-20) + (-10) + (-5) + (-\frac{5}{2}) + \dots$

Solve:

⑥ Find the sum of all even integers from 2 to 60

Solve:

⑦ Determine the summation notation:

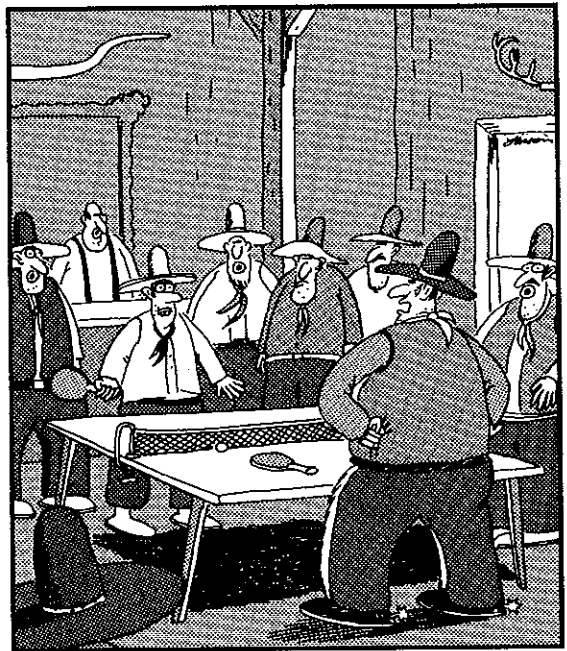
$(16) + (11) + (6) + (1) + (-4) + (-9)$

Reference:

$$S = \frac{n}{2} [2a_1 + (n-1)d]$$

$$S = \frac{a_1 - a_n r^n}{1-r}$$

$$S = \frac{a_1}{1-r} \text{ for } -1 < r < 1$$



"Well, kid, ya beat me—and now every punk packin' a paddle and tryin' to make a name for himself will come lookin' for you! ...
Welcome to hell, kid."

Sequence & Series

UNIT 17 REVIEW

Expand each binomial:

① $(x+y)^7$ ③ $(n-3m)^5$

② $(3a+4b)^4$ ④ $(2x-3y)^6$

Indicate the sum for each series and the first 3 terms:

⑤ $\sum_{x=1}^7 (4x-5)$

⑦ $\sum_{m=-2}^9 (2)^m$

⑥ $\sum_{c=3}^{10} (8-5c)$

⑧ $\sum_{e=2}^{11} \frac{1}{2}(4)^{e-2}$

Find the sum if possible:

⑨ $3 + 1 + \frac{1}{3} + \frac{1}{9} + \frac{1}{27} + \dots$

⑩ $8 - 6 + \frac{9}{2} - \frac{27}{8} + \dots$

Solve:

⑪ Find the sum of the even integers from 250 to 350.

⑫ A swinging pendulum moves 50 cm on its first swing and $\frac{9}{10}$ of the preceding distance

on each successive swing. How far will it move before stopping?

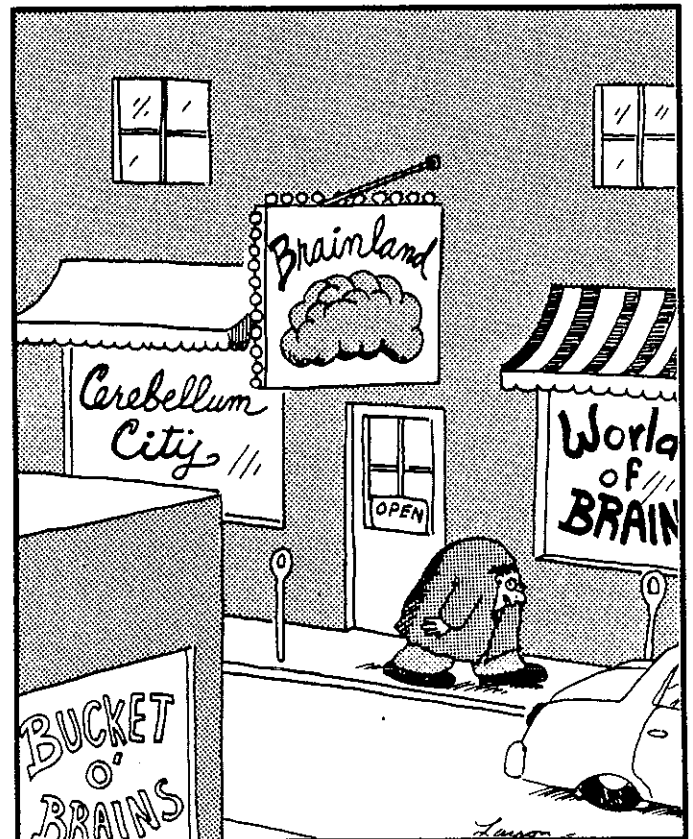
Determine the summation notation for each series:

⑬ $6 - 2 - 10 - 18 - 26$

⑭ $20 + 11 + 2 - 7 - 16 - 25$

⑮ $3 + 9 + 27 + 81 + 243$

⑯ $10 - 5 + \frac{5}{2} - \frac{5}{4} + \frac{5}{8}$



Igor goes shopping.

Combinations & Permutations

UNIT 18 DEMONSTRATION

Evaluate:

① $P(7, 5)$

② $C(6, 4)$

Solve:

③ A company makes crayons in 3 lengths, 2 widths, and 20 colors. How many different crayons do they make?

④ How many 3 boy - 3 girl subcommittees are possible if the Student Senate has 12 boys and 14 girls?

⑤ How many ways can the letters of "research" be arranged?

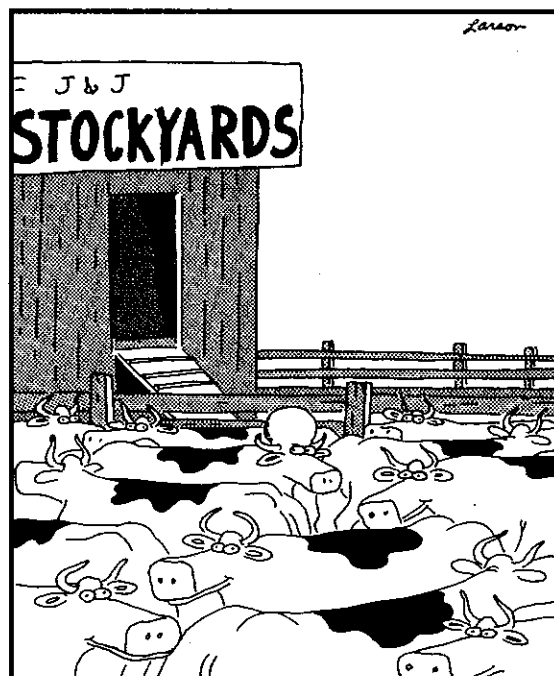
⑥ How many ways can nine beads be placed on a necklace with a clasp?

⑦ How many different teams of 9 softball players can be selected from 14 children?

⑧ How many ways can 11 children sit in a circle?

Solve:

⑨ How many ways can 3 cards be drawn from a 52-card deck so that 2 are from the same suit?



Only Claire, with her oversized brain, wore an expression of concern.

Combinations & Permutations

UNIT 18 REVIEW

Evaluate:

- ① $P(11, 4)$ ③ $C(9, 3)$
② $P(5, 5)$ ④ $C(13, 5)$

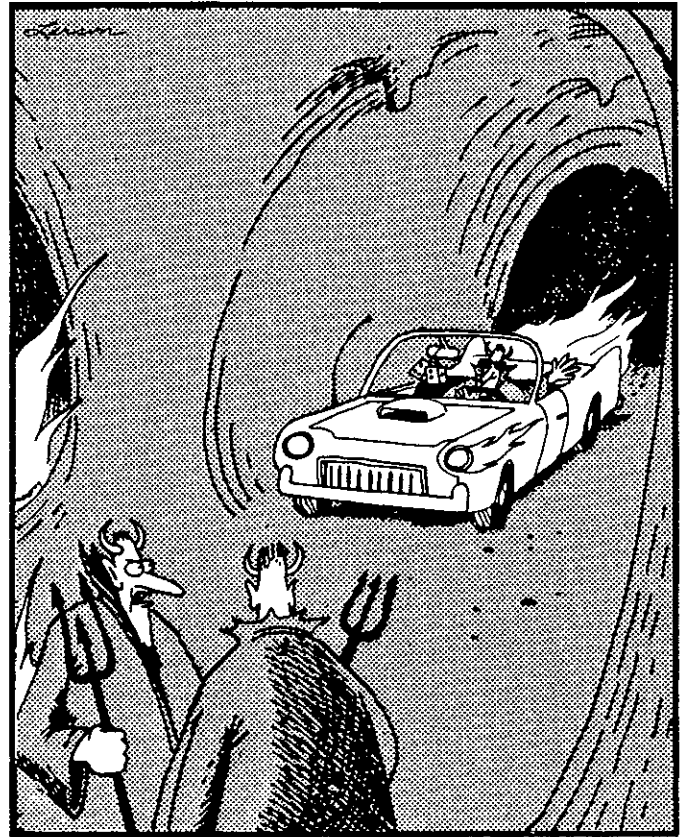
Solve:

- ⑤ A store sells seven different styles of gloves, each coming in a choice of four different colors. How many different gloves does the store sell?
- ⑥ How many ways can eight pictures be arranged in a straight line on the wall?
- ⑦ How many ways can eight students be assigned to a row of twelve lockers?
- ⑧ How many ways can the letters of the word "television" be arranged?
- ⑨ How many ways can seven people sit in a circle?
- ⑩ How many ways can nine different chairs be arranged around a circular table?
- ⑪ How many ways can eight keys be arranged on a key ring?
- ⑫ How many ways can six different color beads be arranged on a necklace with a clasp?
- ⑬ How many different starting units of five players can be selected from a team of twelve players?
- ⑭ How many math teams of 2 boys and 2 girls can be selected from an ATIM class with 10 boys and 8 girls?
- ⑮ There are five mystery books and seven romance novels on the reading list. How many lists of four mystery books and three romance novels can be made?
- ⑯ Eight cards are selected from a fifty-two card deck. How many different eight card combinations include two cards from each suit?
- ⑰ How many ways can 6 cards be drawn from a 52 card deck so that 3 are one suit and the remaining cards are from 2 other suits?

Mathematics of Chance

UNIT 19 DEMONSTRATION

- ① What is the probability of pulling out a penny followed by a dime from a coin purse that contains 5 dimes and 2 pennies?
- ② What are the odds of rolling a number higher than 1 on a single roll of a die?
- ③ In a bag, there are 6 white, 4 black, 2 brown, and 3 blue buttons. What is the probability of selecting 3 buttons - all of the same color?
- ④ Five cards are drawn from a standard deck. What is the probability of drawing 3 from one suit and 2 from another?
- ⑤ A coin is tossed 12 times. What is the probability of tossing 6 heads?
- ⑥ When Joe and Marilyn play ping pong, Marilyn wins 4 and Joe wins 1 out of every 5 games. In 6 games, what is the probability Marilyn will win more than 3 games?
- ⑦ From a standard deck, what is the probability of drawing 3 cards that are all red or all kings and queens?



"Well, here comes Roy again. He sure does think he's Hell on Wheels."

Mathematics of Chance

UNIT 19 REVIEW

In a bag are 4 black, 3 white, and 3 green marbles. with no replacement, what is the probability of selecting:

- ① 3 marbles - all white
- ② 3 marbles - all black or all green
- ③ 4 marbles - at least 3 black

Four cards are drawn from a standard deck. Determine the probability of drawing:

- ④ All four face cards
- ⑤ A red card or a face card on the first draw
- ⑥ Three from one suit

A coin is tossed 7 times. Determine the probability of:

- ⑦ 5 heads
- ⑧ At least 4 tails
- ⑨ 3 heads or 4 heads

A.J. makes 3 out of every 5 free throws he attempts during the basketball season. If he attempts 8, determine the probability he will:

- ⑩ Make 6 of them
- ⑪ Make at least 7 of them
- ⑫ Make more than 7 or less than 2

Determine the probability of drawing:

- ⑬ A black marble followed by a green marble from the bag in problems 1-3.
- ⑭ Two even number cards or two red cards from a standard deck.

Determine the odds of:

- ⑮ Rolling an odd number twice in a row with a standard die.

Matrices

UNIT 20 DEMONSTRATION

Solve using an augmented matrix:

$$\textcircled{1} \begin{cases} 2x + 3y = 9 \\ x + 2y = 7 \end{cases}$$

Solve using an augmented matrix and substitution:

$$\textcircled{2} \begin{cases} x + 2y + 2z = 1 \\ 3x - y + 2z = 8 \\ x + y + z = 0 \end{cases}$$

Find each product:

$$\textcircled{3} \frac{-1}{3} \begin{bmatrix} -2 & 3 & 0 \\ 4 & -1 & 6 \end{bmatrix} \begin{bmatrix} -6 \\ 9 \\ 3 \end{bmatrix}$$

$$\textcircled{4} \begin{bmatrix} -4 & -5 \\ 2 & 6 \end{bmatrix} \begin{bmatrix} 8 & 2 \\ 0 & -1 \end{bmatrix}$$

Find the signed minor for each element in the 2nd row:

$$\textcircled{5} \begin{bmatrix} 4 & 5 & 8 \\ 0 & -1 & 2 \\ -3 & 6 & -2 \end{bmatrix}$$

Find the inverse:

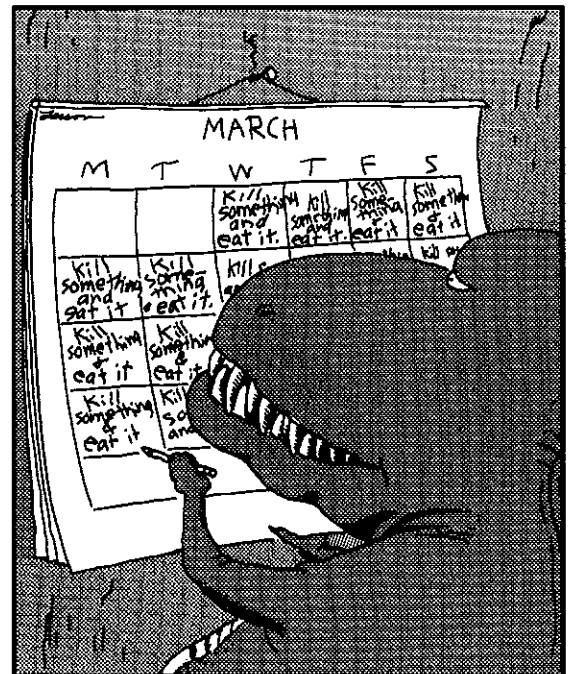
$$\textcircled{6} \begin{bmatrix} 2 & 4 \\ -7 & 6 \end{bmatrix}$$

Solve using an inverse matrix:

$$\textcircled{7} \begin{cases} 2x - 3y = 2 \\ x - 2y = 3 \end{cases}$$

Find the inverse:

$$\textcircled{8} \begin{bmatrix} 1 & 3 & 0 \\ 1 & 4 & -2 \\ 2 & 1 & 2 \end{bmatrix}$$



Jurassic calendars

Matrices

UNIT 20 REVIEW

Solve using an augmented matrix:

$$\begin{aligned} \textcircled{1} \quad & 3x + 4y = 12 \\ & 5x + 2y = -8 \end{aligned}$$

Solve using an augmented matrix and substitution:

$$\begin{aligned} \textcircled{2} \quad & 2x + 3y - z = -7 \\ & 4x - y - 6z = 10 \\ & x + 2y + 3z = -2 \end{aligned}$$

Find each product:

$$\textcircled{3} \quad \begin{bmatrix} 2 & 3 \\ -4 & 0 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ -5 & -1 \end{bmatrix}$$

$$\textcircled{4} \quad -\frac{1}{2} \begin{bmatrix} -2 & 3 & 4 \\ 0 & 2 & 5 \end{bmatrix} \begin{bmatrix} 6 \\ -4 \\ 10 \end{bmatrix}$$

$$\textcircled{5} \quad \begin{bmatrix} -2 & -1 & 0 \\ 3 & 0 & 4 \\ 4 & 2 & -3 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 5 & -1 \\ 3 & 0 \end{bmatrix}$$

Find the signed minor for each element:

$$\textcircled{6} \quad \text{In row 1} \quad \begin{bmatrix} 6 & -1 & 0 \\ 3 & 4 & -5 \\ 2 & -3 & 7 \end{bmatrix}$$

$$\textcircled{7} \quad \text{In row 2} \quad \begin{bmatrix} 6 & -1 & 0 \\ 3 & 4 & -5 \\ 2 & -3 & 7 \end{bmatrix}$$

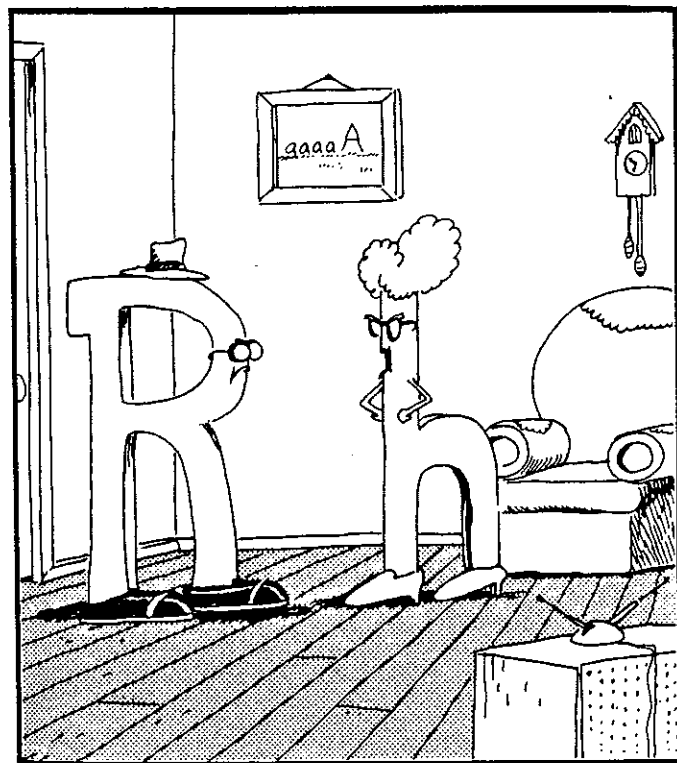
Find the inverse:

$$\textcircled{8} \quad \begin{bmatrix} 4 & 3 \\ -6 & 8 \end{bmatrix}$$

$$\textcircled{9} \quad \begin{bmatrix} 2 & 0 & 6 \\ -4 & 3 & 8 \\ 5 & -1 & 7 \end{bmatrix}$$

Solve using an inverse matrix:

$$\begin{aligned} \textcircled{10} \quad & 3x + 5y = 3 \\ & x - 3y = 15 \end{aligned}$$



"All right! All right! If you want the truth, off and on I've been seeing *all* the vowels—*a, e, i, o, u*... Oh, yes! And *sometimes y!*"

Logarithms

UNIT 21 DEMONSTRATION

Change to logarithmic form:

$$\textcircled{1} 5^2 = 25$$

Change to exponential form:

$$\textcircled{2} \log_9 3 = 1/2$$

Solve a basic logarithmic equation:

$$\textcircled{3} \log_2 (3x-2) = \log_2 (2x+6)$$

Use properties of logarithms to solve:

$$\textcircled{4} \log_9 (x+4) + \log_9 (x-4) = 1$$

Use the table of mantissas to solve:

$$\textcircled{5} \log 51.2$$

Approximate the value for this logarithm:

$$\textcircled{6} \log_3 12 \text{ (round to 4 places)}$$

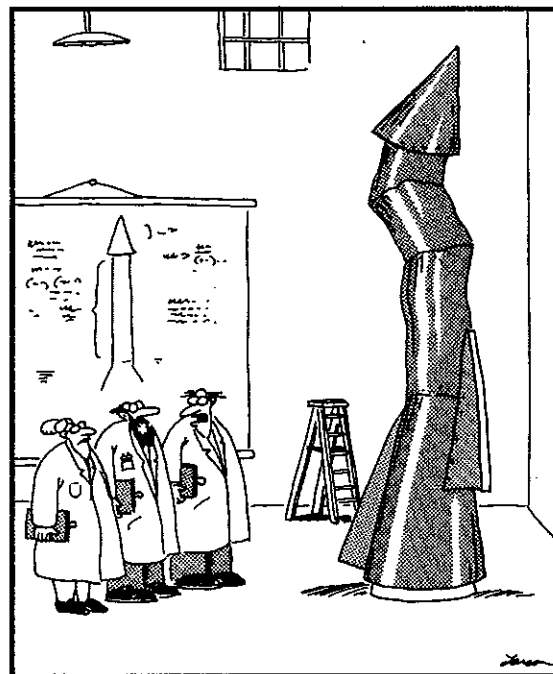
Solve an exponential equation:

$$\textcircled{7} 27^{2x+1} = 2^{4x-1}$$

Solve:

$$\textcircled{8} \log_2 [\log_9 (\log_3 x)] = -1$$

n	0	1	2	3	4
50	6990	6998	7007	7016	7024
51	7076	7084	7093	7101	7110
52	7160	7168	7177	7185	7193
53	7243	7251	7259	7267	7275
54	7324	7332	7340	7348	7356



"It's time we face reality, my friends...
We're not exactly rocket scientists."

Logarithms

UNIT 21 REVIEW

Change to logarithmic form:

$$\textcircled{1} 3^{-2} = \frac{1}{9}$$

Change to exponential form:

$$\textcircled{2} \log_4 2 = \frac{1}{2}$$

Solve a basic logarithmic equation:

$$\textcircled{3} \log_4 (1-2x) = \log_4 (x+10)$$

Use the properties of logarithms to solve:

$$\textcircled{4} \log_6 (n-3) + \log_6 (n+2) = \log_3 3$$

$$\textcircled{5} 2 \log_2 x - \frac{1}{2} \log_2 16 = 4$$

Use the table of mantissas to solve:

$$\textcircled{6} \log 437$$

$$\textcircled{7} \log x = .5011 - 2$$

Approximate the value

as a common logarithm:

$$\textcircled{8} \log_3 8 \text{ (round to 4 places)}$$

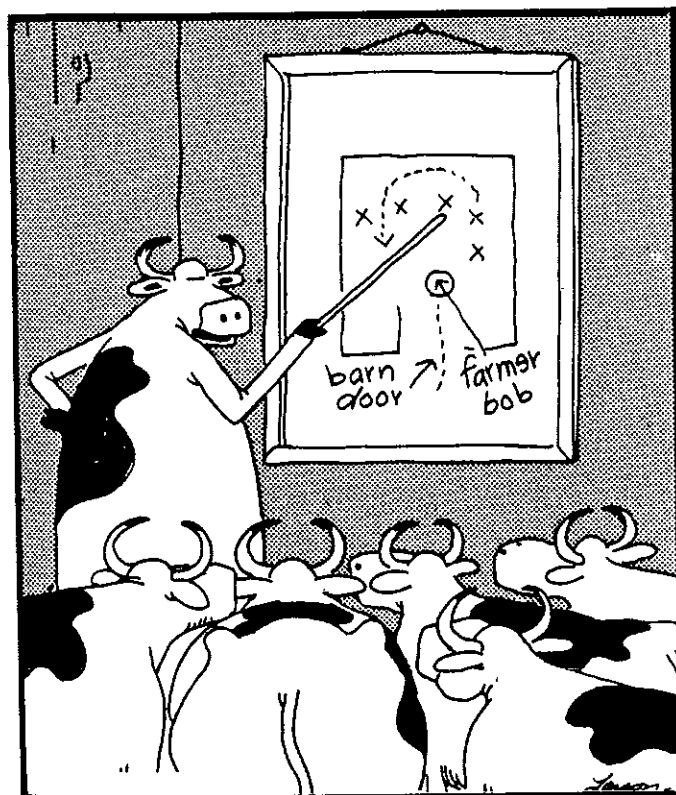
Solve an exponential equation:

$$\textcircled{9} 4^{3x-2} = 6^{2x+1}$$

(round to 4 places)

Solve:

$$\textcircled{10} \log_3 [\log_8 (\log_3 x)] = -1$$



"So when Farmer Bob comes through the door, three of us circle around and ... Muriel ... Are you chewing your cud while I'm talking?"

Common Logarithms of Numbers

n	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396

Common Logarithms of Numbers

n	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

Conics

UNIT 22 DEMONSTRATION

Put the equation in vertex form:

$$\textcircled{1} \quad y^2 - 8y - 8x + 56 = 0$$

Determine:

- vertex
- opening
- axis of symmetry
- length of latus
- directrix
- focus
- draw the graph

$$\textcircled{2} \quad x^2 + y^2 + 6x - 4y - 7 = 0$$

Determine:

- center
- radius
- draw the graph

$$\textcircled{3} \quad 16x^2 + 9y^2 - 96x - 72y + 144 = 0$$

Determine:

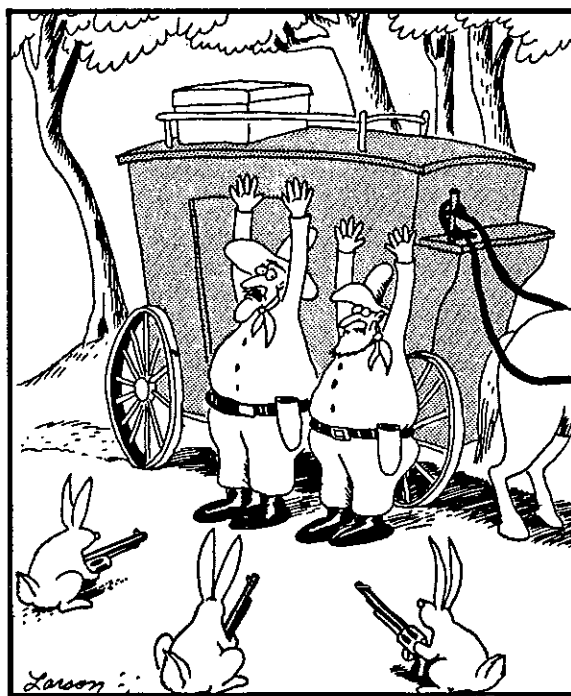
- center
- length of major axis
- length of minor axis
- foci
- draw the graph

$$\textcircled{4} \quad 3y^2 - x^2 - 6y - 10x - 34 = 0$$

Determine:

- center
- length of transverse axis
- length of conjugate axis
- foci
- asymptote slopes
- equations of asymptotes
- draw the graph

The review exercises will include additional related types of conics problems.



"This ain't gonna look good on our report, Leroy."

Conics

UNIT 22 REVIEW

Put each equation in vertex form and determine the indicated measures:

① $x^2 + 6x + 6y + 3 = 0$

- a) vertex
- b) opening
- c) axis of symmetry
- d) length of latus
- e) directrix
- f) focus
- g) draw the graph

② $x^2 + y^2 - 10x - 2y + 8 = 0$

- a) center
- b) radius
- c) draw the graph

③ $4x^2 + 25y^2 - 16x + 350y + 1141 = 0$

- a) center
- b) length of major axis
- c) length of minor axis
- d) foci
- e) draw the graph

④ $4x^2 - 9y^2 + 24x + 108y - 324 = 0$

- a) center
- b) length of transverse axis

- c) length of conjugate axis
- d) foci
- e) asymptote slopes
- f) equations of asymptotes
- g) draw the graph



"Remember, milk, eggs, loaf of bread ... and pick up one of those No-Penguin-Strips."

Determine the equation:

⑤ Parabola with focus (1,5) and directrix $x=3$

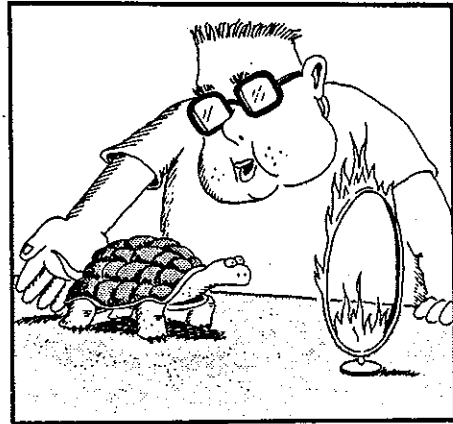
⑥ Ellipse with foci (3,1) and (3,9) and a minor axis of length 6

Conics

UNIT 22 REVIEW

- ⑦ Circle with a center at $(4, 3)$ passing through $(6, 8)$
- ⑧ Hyperbola with foci at $(6, -2 \pm \sqrt{34})$ and a transverse axis of length 6

Determine the equation for each graph:



"Through the hoop, Bob! Through the hoop!"

