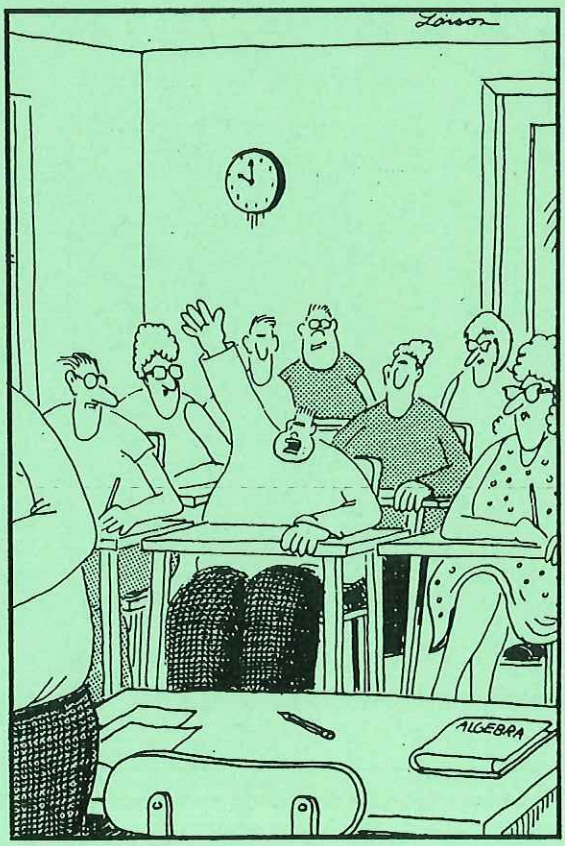
Friendship Junior High School
Accelerated Math Program
Mr. Lavine (Room 102A)

A.T.I.M.

Advanced Topics In Mathematics

ATIM
End Of The Year Review

1. *Polynomials*
2. *Systems*
Problem Solving
4. *Inequalities & Coordinate Graphing*
Linear Programming
6. *Roots & Radicals*
7. *Rat. Expon. & Complex Numbers*
8. *Quadratics*
9. *Rational Expressions*
Problem Solving
Plane Geometry
Solid Geometry
Introduction to Two-Column Proofs
Triangle Trigonometry
Law of Sines & Cosines
Trigonometric Functions
17. *Sequence & Series*
Combinations & Permutations
Mathematics of Chance
20. *Matrices*
21. *Logarithms*
22. *Conics*



"Mr. Lavine, may I be excused?
My brain is full."

Polynomials

ANSWER KEY: UNIT 1 DEMONSTRATION

① $ab^{-2}(a^2b + a^{-1}b^3 - a^3b^{-1})$

$a^3b^{-1} + b - a^4b^{-3}$

$\boxed{\frac{a^3}{b} + b - \frac{a^4}{b^3}}$

② $\frac{-18a^{-2}b^3c^{-4}}{12ab^{-2}c^{-3}} = \boxed{\frac{-3b^5}{2a^3c}}$

③ $(5n^{3x} - 2m^{x+2})^2$

$\boxed{25n^{6x} - 20n^{3x}m^{x+2} + 4m^{2x+4}}$

④ $54a^3b - 2b^4$

$2b(27a^3 - b^3)$

$\boxed{2b(3a-b)(9a^2+3ab+b^2)}$

⑤ $5ab + b^2 + 5ac - c^2$

$5ab + 5ac + b^2 - c^2$

$5a(b+c) + (b+c)(b-c)$

$\boxed{(b+c)(5a+b-c)}$

⑥ $m^3 - 3m^2a + 3ma^2 - a^3$

$m^3 - a^3 - 3m^2a + 3ma^2$

$(m-a)(m^2+ma+a^2) - 3ma(m-a)$

$(m-a)(m^2 - 2ma + a^2)$

$(m-a)(m-a)^2$

$\boxed{(m-a)^3}$

⑦

$$\begin{array}{r|l} x+2 & x^3 - 2x^2 + 4x - 8 + {}^{20}/_{x+2} \\ & x^4 \qquad\qquad\qquad +4 \\ & x^4 + 2x^3 \\ \hline & -2x^3 \\ & -2x^3 - 4x^2 \\ \hline & 4x^2 \\ & 4x^2 + 8x \\ \hline & -8x + 4 \\ & -8x - 16 \\ \hline & 20 \end{array}$$

⑧ $(2a^4 - 5a^3 + 2a - 3)(a-1)^{-1}$

1	2	-5	0	2	-3
		2	-3	-3	-1
	2	-3	-3	-1	-4

$\boxed{2a^3 - 3a^2 - 3a - 1 - {}^4/_{a-1}}$

⑨ $(12x^4 + 7x^2 - 9x + 11.)(2x-3)^{-1}$

$(6x^4 + \frac{7}{2}x^2 - \frac{9}{2}x + \frac{11}{2})(x - 3/2)^{-1}$

3/2	6	0	7/2	-9/2	11/2
		9	27/2	51/2	63/2
	6	9	17	21	37

$\boxed{6x^3 + 9x^2 + 17x + 21 + {}^{74}/_{2x-3}}$

⑩ $\left(\frac{-a^2b^3c^{-1}}{3a^{-4}b^{-1}c^3}\right)^{-2} = \left(\frac{-a^2b^4}{3c^4}\right)^{-2} = \left(\frac{-3c^4}{a^2b^4}\right)^2 = \boxed{\frac{9c^8}{a^4b^8}}$

Polynomials

ANSWER KEY: UNIT 1 REVIEW

① $(5a)(6a^2b)(3ab^3) + (2a)^2(3b^3)(2a^2b)$

$(90a^4b^4) + (24a^4b^4) = \boxed{114a^4b^4}$

② $\dfrac{-9x^{-2}y^3z^{-1}}{3x^{-4}yz^2} = \boxed{\dfrac{-3x^2y^2}{z^3}}$

③ $\left(\dfrac{2y^{-2}}{-2}\right)^{-1}\left(\dfrac{m^2n}{y}\right)^{-2}$

$\left(\dfrac{-1}{y^2}\right)^{-1}\left(\dfrac{y}{m^2n}\right)^{2}$

$\left(\dfrac{y^2}{-1}\right)\left(\dfrac{y^2}{m^4n^2}\right) = \boxed{\dfrac{-y^4}{m^4n^2}}$

④ $\dfrac{6n^{2x+3}}{3n^{x-4}} = 2n^{(2x+3)-(x-4)}$

$\boxed{2n^{x+7}}$

⑤ $x^{-1}y^2(x^{-3}y - xy^{-3} + x^2y^{-4})$

$x^{-4}y^3 - y^{-1} + xy^{-2}$

$\boxed{\dfrac{y^3}{x^4} - \dfrac{1}{y} + \dfrac{x}{y^2}}$

⑥ $(4x^{a+2} - 3y^{3a})^2$

$\boxed{16x^{2a+4} - 24x^{a+2}y^{3a} + 9y^{6a}}$

⑦ $(x^{n+1} + y^{2n-1})(x^{n+1} - y^{2n-1})$

$\boxed{x^{2n+2} - y^{4n-2}}$

⑧ $12n^2 - 7n - 10$

$12n^2 - 15n + 8n - 10$

$3n(4n-5) + 2(4n-5) = \boxed{(4n-5)(3n+2)}$

⑨ $(3a-4b)^2 - (a+3b)^2$

$[(3a-4b)+(a+3b)]\,[(3a-4b)-(a+3b)]$

$\boxed{(4a-b)(2a-7b)}$

⑩ $5x^3y + 40y^4$

$5y(x^3 + 8y^3) = \boxed{5y(x+2y)(x^2-2xy+4y^2)}$

⑪ $x^5 - x^3y^2 + 27y^5 - 27x^2y^3$

$x^3(x^2-y^2) + 27y^3(y^2-x^2)$

$x^3(x^2-y^2) - 27y^3(x^2-y^2)$

$(x^2-y^2)(x^3-27y^3)$

$\boxed{(x+y)(x-y)(x-3y)(x^2+3xy+9y^2)}$

⑫ $x^2 - 9 - 6xy + 9y^2$

$(x^2 - 6xy + 9y^2) - 9$

$(x-3y)^2 - 9$

$\boxed{(x-3y+3)(x-3y-3)}$

⑬ $m^4n + m^2n - mn^4 - mn^2$

$mn(m^3 + m - n^3 - n)$

$mn(m^3 - n^3 + m - n)$

$mn[(m-n)(m^2+mn+n^2) + 1(m-n)]$

$\boxed{mn(m-n)(m^2+mn+n^2+1)}$

⑭ $3x^3 + 9x^2 + 9x + 3$

$3(x^3 + 3x^2 + 3x + 1)$

$3(x^3 + 1 + 3x^2 + 3x)$

$3[(x+1)(x^2-x+1) + 3x(x+1)]$

$3[(x+1)(x^2-x+1+3x)] = 3(x+1)(x^2+2x+1)$

$3(x+1)(x+1)(x+1) = \boxed{3(x+1)^3}$

Polynomials

ANSWER KEY: UNIT 1 REVIEW

(15)

$$n+2 \;\overline{)\; 4n^4 \qquad\qquad -3}$$

$4n^3 - 8n^2 + 16n - 32 + \frac{61}{n+2}$

$4n^4 + 8n^3$

$-8n^3$

$-8n^3 - 16n^2$

$16n^2$

$16n^2 + 32n$

$-32n - 3$

$-32n - 64$

61

(16) $(x^6 - 9x^4 + 2x^3 - 18x + 12)(x+3)^{-1}$

-3	1	0	-9	2	0	-18	12
		-3	9	0	-6	18	0
	1	-3	0	2	-6	0	12

#16 continued:

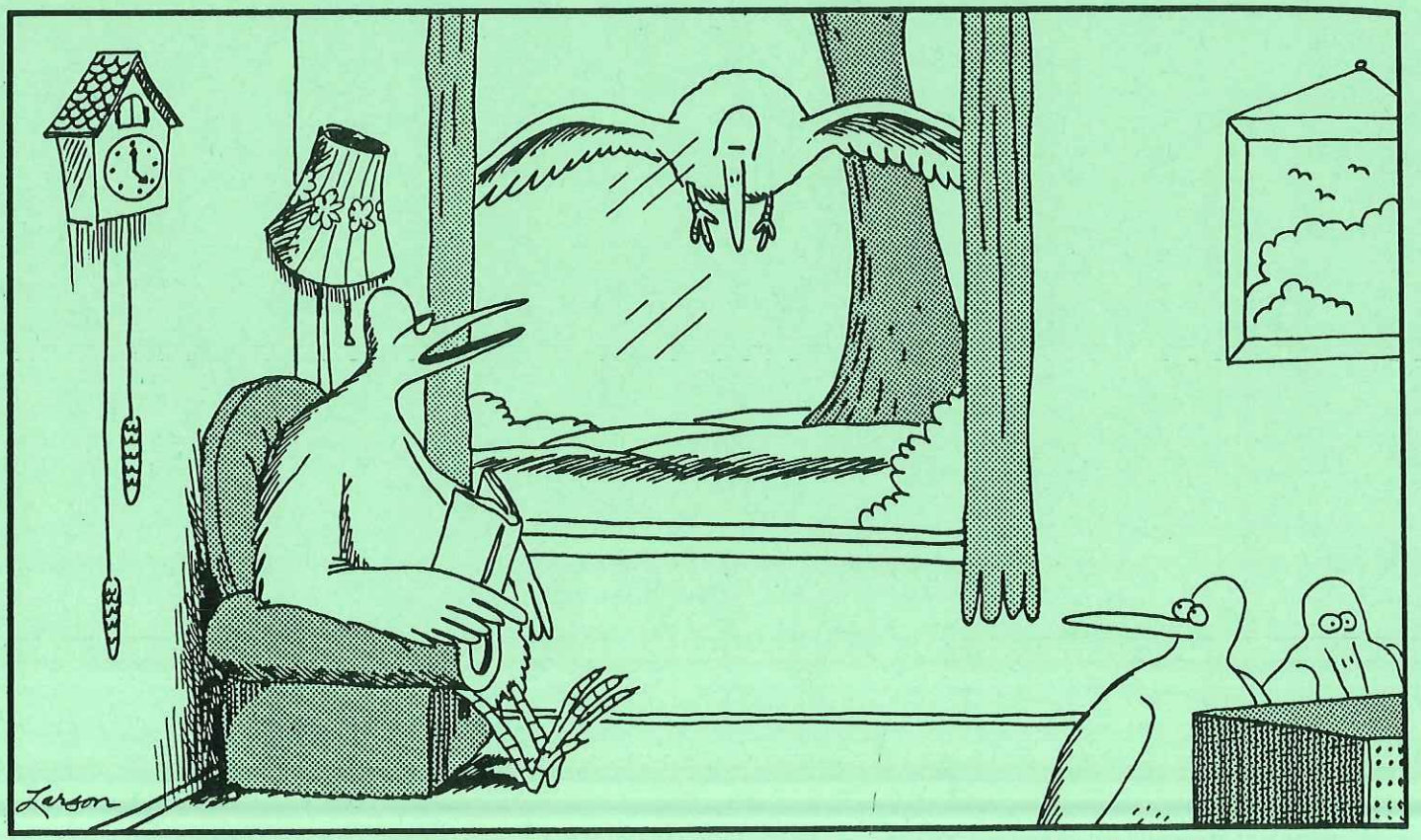
$$\boxed{x^5 - 3x^4 + 2x^2 - 6x + \frac{12}{x+3}}$$

(17) $(4x^3 + x - 3)(2x-1)^{-1}$

$(2x^3 + \frac{1}{2}x - \frac{3}{2})(x - \frac{1}{2})^{-1}$

$\frac{1}{2}$	2	0	$\frac{1}{2}$	$-\frac{3}{2}$
		1	$\frac{1}{2}$	$\frac{1}{2}$
	2	1	1	-1

$$\boxed{2x^2 + x + 1 - \frac{2}{2x-1}}$$



"Oh, this should be interesting. ... Looks as if your father has forgotten about the front window again."

Systems

ANSWER KEY: UNIT 2 DEMONSTRATION

① $x - 3y = -3 \rightarrow x = 3y - 3$
$4x + 9y = 2$

$4(3y-3)+9y=2$
$12y-12+9y=2$
$21y=14$
$y=2/3$

$x=3(2/3)-3$
$x=-1$

$\boxed{(-1, 2/3)}$

② $2x+3y=5$ mult. by -2
$-3x+6y=12$

$-4x-6y=-10$
$-3x+6y=12$
$-7x=2$
$x=-2/7$

$2(-2/7)+3y=5$
$3y=5+4/7$
$3y=39/7$
$y=13/7$

$\boxed{(-2/7, 13/7)}$

③ a) $4x+3y+z=-10$
b) $x-12y+2z=-5$
c) $x+18y+z=4$

$\boxed{(-3, 1/3, 1)}$

a) $4x+3y+z=-10$
b) $-4x+48y-8z=20$
$51y-7z=10$

b) $x-12y+2z=-5$
c) $-x-18y-z=-4$
$-30y+z=-9$

mult. by 7

$51y-7z=10$
$-210y+7z=-63$
$-159y=-53$
$y=1/3$

$-30(1/3)+z=-9$
$z=1$

$x-12(1/3)+2(1)=-5$
$x-4+2=-5$
$x=-3$

④ $\begin{vmatrix} -3 & 7 \\ 4 & 9 \end{vmatrix}$

$(-27)-(28) = \boxed{-55}$

⑤ $\begin{vmatrix} -2 & 0 \\ 6 & 5 \end{vmatrix}$

$(-10)-(0) = \boxed{-10}$

⑥ $3x+2y=40$
$x-7y=-2$

$\boxed{(12, 2)}$

$x = \dfrac{\begin{vmatrix} 40 & 2 \\ -2 & -7 \end{vmatrix}}{\begin{vmatrix} 3 & 2 \\ 1 & -7 \end{vmatrix}} = \dfrac{-276}{-23}$

$(-280)-(-4)$
$(-21)-(2)$

$y = \dfrac{\begin{vmatrix} 3 & 40 \\ 1 & -2 \end{vmatrix}}{-23} = \dfrac{-46}{-23}$

$(-6)-(40)$

⑦ $\begin{vmatrix} 4 & -5 & 3 \\ -3 & 2 & 0 \\ 2 & -1 & 4 \end{vmatrix} = \boxed{-31}$

$(4)\begin{vmatrix} 2 & 0 \\ -1 & 4 \end{vmatrix} - (-5)\begin{vmatrix} -3 & 0 \\ 2 & 4 \end{vmatrix} + (3)\begin{vmatrix} -3 & 2 \\ 2 & -1 \end{vmatrix}$

$(8)-(0)$ $\quad (-12)-(0)$ $\quad (3)-(4)$

$(4)(8) - (-5)(-12) + (3)(-1) = -31$

⑧ $\begin{vmatrix} 6 & 3 & 6 & 3 \\ -1 & 1 & -4 & 1 \\ -2 & 4 & -2 & 4 \end{vmatrix} = \boxed{136}$

$(12)+(24)+(-4)-(-2)-(-96)-(-6)$
$(12)+(24)+(-4)+(2)+(96)+(6)$

Systems

ANSWER KEY: UNIT 2 DEMONSTRATION

(9) $2a - b + 3c = 5$
$3a + 2b - 5c = 7$
$a - 4b + 11c = 3$

Denominator

$$\begin{vmatrix} 2 & -1 & 3 \\ 3 & 2 & -5 \\ 1 & -4 & 11 \end{vmatrix} = 0$$

$$(2)\begin{vmatrix} 2 & -5 \\ -4 & 11 \end{vmatrix} - (-1)\begin{vmatrix} 3 & -5 \\ 1 & 11 \end{vmatrix} + (3)\begin{vmatrix} 3 & 2 \\ 1 & -4 \end{vmatrix}$$

$(22) - (20) \qquad (33) - (-5) \qquad (-12) - (2)$

$(2)(2) - (-1)(38) + (3)(-14) = 0$

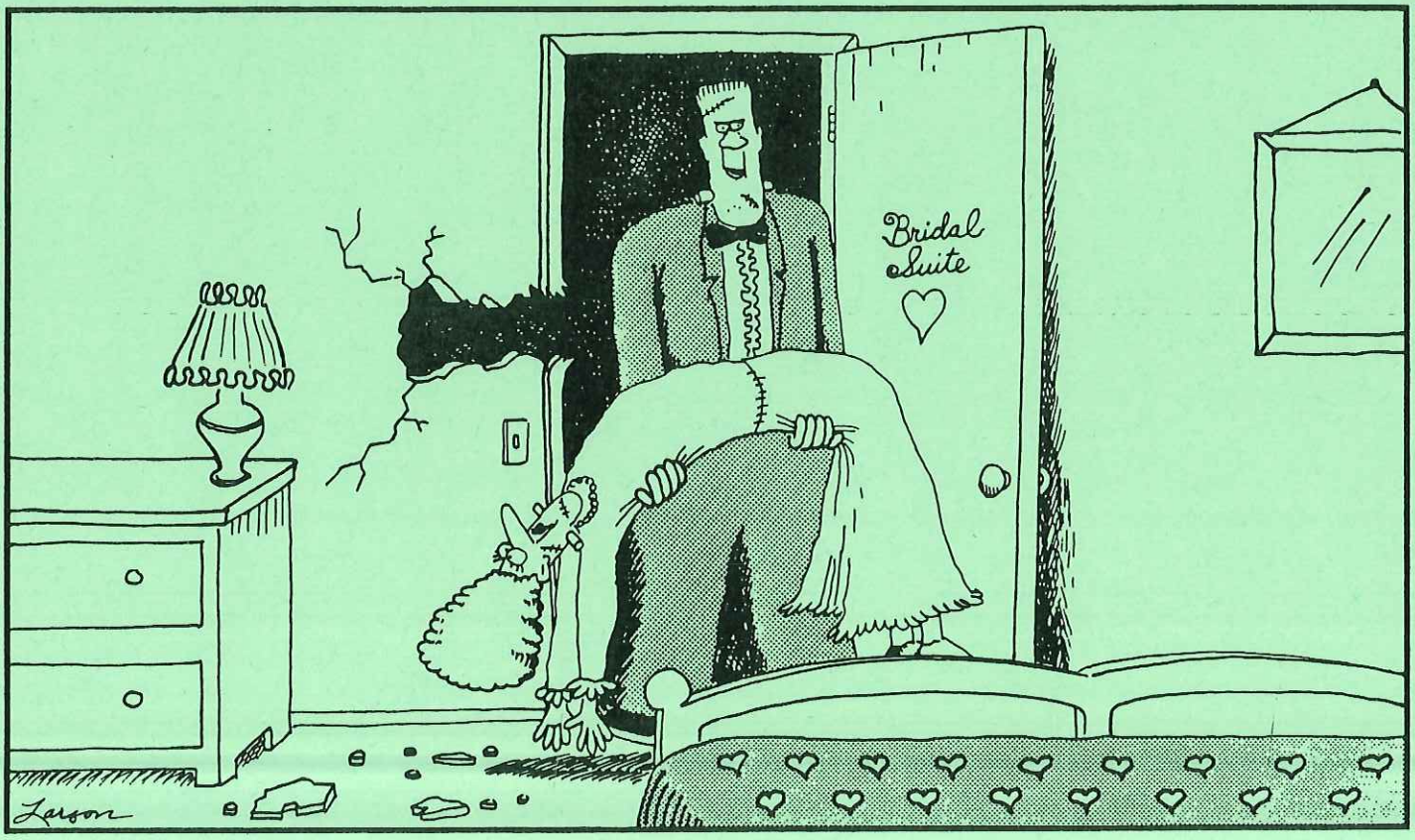
NO UNIQUE SOLUTION

(10) $3x + 4y + z = 10$
$6x - 2y - z = 6$
$3x + 6y - 2z = 2$

$$x = \frac{\begin{vmatrix} 10 & 4 & 1 \\ 6 & -2 & -1 \\ 2 & 6 & -2 \end{vmatrix}}{\begin{vmatrix} 3 & 4 & 1 \\ 6 & -2 & -1 \\ 3 & 6 & -2 \end{vmatrix}} = \frac{180}{108}$$

$$x = \frac{(40) + (-8) + (36) - (-4) - (-60) - (-48)}{(12) + (-12) + (36) - (-6) - (-18) - (-48)}$$

$x = \boxed{5/3}$



Systems

ANSWER KEY: UNIT 2 REVIEW

① $4x+y=9 \rightarrow y=9-4x$
$3x-2y=4$

$3x-2(9-4x)=4$
$3x-18+8x=4$
$11x=22$
$x=2$

$y=9-4(2)$
$y=1$
$\boxed{(2,1)}$

② $3x-4y=23$
$9x+2y=-15$ mult. by 2

$3x-4y=23$
$18x+4y=-30$
$21x=-7$
$x=-1/3$

$3(-\frac{1}{3})-4y=23$
$-4y=24$
$y=-6$
$\boxed{(-1/3,-6)}$

③ $\begin{vmatrix} -8 & -1 \\ 2 & -3 \end{vmatrix}$ $(24)-(-2)=\boxed{26}$

④ $2x-3y=8$
$5x+6y=11$ $\boxed{(3,-2/3)}$

$x=\dfrac{\begin{vmatrix} 8 & -3 \\ 11 & 6 \end{vmatrix}}{\begin{vmatrix} 2 & -3 \\ 5 & 6 \end{vmatrix}}$ $y=\dfrac{\begin{vmatrix} 2 & 8 \\ 5 & 11 \end{vmatrix}}{\begin{vmatrix} 2 & -3 \\ 5 & 6 \end{vmatrix}}$

⑤ a) $a+b+c=0$
b) $3a-2b+5c=1$
c) $2a+b+2c=-1$ $\boxed{(-4,1,3)}$

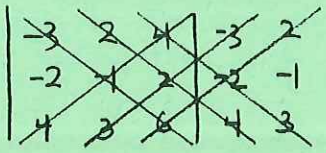
a) $-3a-3b-3c=0$
b) $3a-2b+5c=1$
$-5b+2c=1$

a) $-2a-2b-2c=0$
c) $2a+b+2c=-1$
$-b=-1$
$b=1$

$-5(1)+2c=1$
$2c=6$
$c=3$

$a+(1)+(3)=0$
$a=-4$

⑥ $\begin{vmatrix} -4 & 0 & 1 \\ 2 & -3 & 4 \\ 5 & 2 & 3 \end{vmatrix} = (-4)\begin{vmatrix} -3 & 4 \\ 2 & 3 \end{vmatrix} - (0)\begin{vmatrix} 2 & 4 \\ 5 & 3 \end{vmatrix} + (1)\begin{vmatrix} 2 & -3 \\ 5 & 2 \end{vmatrix}$
$(-9)-(8)$ $(6)-(20)$ $(4)-(-15)$
$(-4)(-17)-(0)(-14)+(1)(19)=\boxed{87}$

⑦ 

$(18)+(16)+(-24)-(-16)-(-18)-(-24)=\boxed{68}$

⑧ $x+5y+3z=0$
$2x-5y+3z=11$
$x+10y-6z=-7$

$z=\dfrac{\begin{vmatrix} 1 & 5 & 3 & 1 & 5 \\ 2 & -5 & 3 & 2 & -5 \\ 1 & 10 & -6 & 1 & 10 \end{vmatrix}}{\begin{vmatrix} 1 & 5 & 3 & 1 & 5 \\ 2 & -5 & 3 & 2 & -5 \\ 1 & 10 & -6 & 1 & 10 \end{vmatrix}}=\boxed{\frac{1}{3}}$

$z=\dfrac{(35)+(55)+(0)-(0)-(110)-(-70)}{(30)+(15)+(60)-(-15)-(30)-(-60)}=\dfrac{50}{150}$

Inequalities & Coordinate Graphing

ANSWER KEY: UNIT 4 DEMONSTRATION

① $4n+3 \geq 2(2n-2)-3$

$4n+3 \geq 4n-4-3$

$+3 \geq -7$ **all solutions**

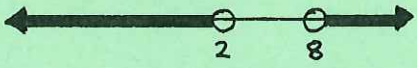
② $-4 \leq 2n+4 \leq 12$

$\boxed{-4 \leq n \leq 4}$

③ $|x-5| > 3$

$x-5 > 3$ or $x-5 < -3$

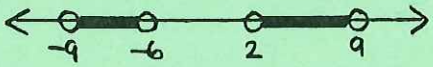
$\boxed{x > 8 \text{ or } x < 2}$



④ $|n+2| > 4$ and $|n| < 9$

$(n+2 > 4$ or $n+2 < -4)$ and $(n < 9$ and $n > -9)$

$\boxed{(n > 2 \text{ or } n < -6) \text{ and } (-9 < n < 9)}$



⑤ $3x-2y=5$ parallel slope $= \frac{3}{2}$

$y = \frac{3}{2}x+b$ $(6,2)$

$(2) = \frac{3}{2}(6)+b$

$b = -7$

$\boxed{y = \frac{3}{2}x-7}$

y-int $\boxed{(0,-7)}$

x-int $\boxed{(14/3, 0)}$

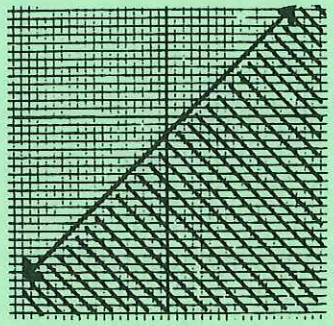
⑥ $(-2,5)$ $(-6,8)$ $\frac{(5)-(8)}{(-2)-(-6)} = \frac{-3}{4}$

$\boxed{y-5 = -\frac{3}{4}(x+2)}$

or

$\boxed{y-8 = -\frac{3}{4}(x+6)}$

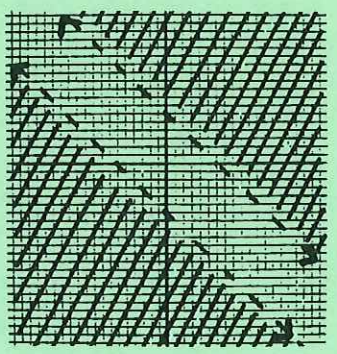
⑦ $y \leq x-2$



⑧ $|x+y| > 6$

$x+y > 6$ or $x+y < -6$

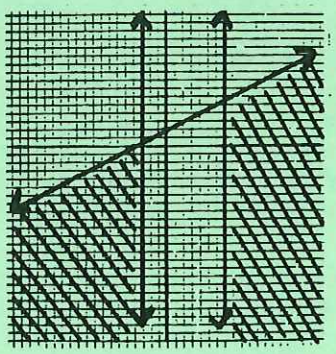
$y > -x+6$ or $y < -x-6$



⑨ $|x-2| \geq 5$ and $y \leq \frac{1}{2}x+2$

$(x-2 \geq 5$ or $x-2 \leq -5)$ and $(y \leq \frac{1}{2}x+2)$

$(x \geq 7$ or $x \leq -3)$ and $(y \leq \frac{1}{2}x+2)$



Inequalities & Coordinate Graphing

ANSWER KEY: UNIT 4 REVIEW

① $2(x+4)+\frac{x}{3} < 4x-1$

$2x+8+\frac{x}{3} < 4x-1$

$6x+24+x < 12x-3$

$-5x < -27$ $\boxed{x > 27/5}$

② $\frac{x+3}{4} \geq \frac{2(x-3)}{3}$

$3x+9 \geq 8x-24$

$-5x \geq -33$ $\boxed{x \leq 33/5}$

③ $3(2x-3) \geq 2(3x+2)$

$6x-9 \geq 6x+4$

$-9 \geq 4$ $\boxed{\text{no solutions}}$

④ $4(x-1) < 4x+5$

$4x-4 < 4x+5$

$-4 < 5$ $\boxed{\text{all solutions}}$

⑤ $-1 < n+4 < 9$

$\boxed{-5 < n < 5}$

⑥ $|3x+6| > 12$

$3x+6 > 12$ or $3x+6 < -12$

$3x > 6$ or $3x < -18$

$\boxed{x > 2 \text{ or } x < -6}$

⑦ $3 \leq |n-4| \leq 10$

$|n-4| \geq 3$ and $|n-4| \leq 10$

$(n-4 \geq 3$ or $n-4 \leq -3)$ and $(n-4 \leq 10$ and $n-4 \geq -10)$

$(n \geq 7$ or $n \leq 1)$ and $(n \leq 14$ and $n \geq -6)$

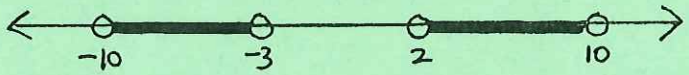
$\boxed{(n \geq 7 \text{ or } n \leq 1) \text{ and } (-6 \leq n \leq 14)}$

⑧ $|2x+1| > 5$ and $|x| < 10$

$(2x+1 > 5$ or $2x+1 < -5)$ and $(x < 10$ and $x > -10)$

$(2x > 4$ or $2x < -6)$ and $(x < 10$ and $x > -10)$

$\boxed{(x > 2 \text{ or } x < -3) \text{ and } (-10 < x < 10)}$



⑨ $5x-3y=4$ parallel slope $= 5/3$

$y = \frac{5}{3}x+b$ $(6,-2)$ $\boxed{y = \frac{5}{3}x-12}$

$(-2) = \frac{5}{3}(6)+b$

$b = -12$

y-int $\boxed{(0,-12)}$

x-int $\boxed{(36/5,0)}$

⑩ $y = -3/4x+7$ perpendicular slope $= 4/3$

$\frac{4}{3} = \frac{A}{-B}$ $A=4$ $B=-3$ $Ax+By=C$ $(2,5)$

$4x-3y=C$

$4(2)-3(5) = -7$

y-int $\boxed{(0,7/3)}$

x-int $\boxed{(-7/4,0)}$

$\boxed{4x-3y=-7}$

⑪ $(-2,6)$ $(10,-1)$

$\frac{(6)-(-1)}{(-2)-(10)} = \frac{7}{-12}$

$\boxed{y-6 = \frac{-7}{12}(x+2)}$

-or-

$\boxed{y+1 = -7/12(x-10)}$

⑫ $2x+3y=-4$

$3y = -2x-4$

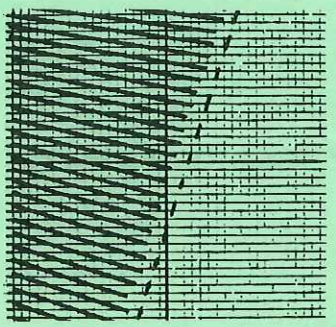
$\boxed{y = -2/3x-4/3}$

Inequalities & Coordinate Graphing

ANSWER KEY: UNIT 4 REVIEW

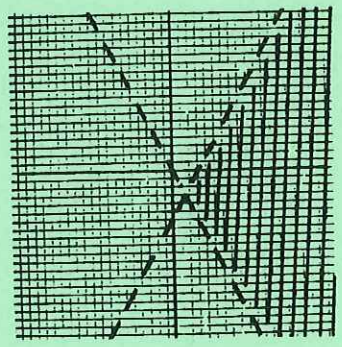
(13) $3x - y < 8$

$-y < -3x + 8 \rightarrow y > 3x - 8$



(14) $2x - y > 6 \rightarrow y < 2x - 6$

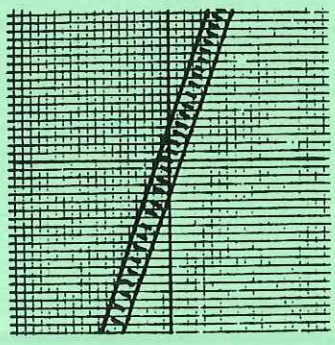
$y > -2x$



(15) $|3x - y| \leq 4$

$3x - y \leq 4$ and $3x - y \geq -4$

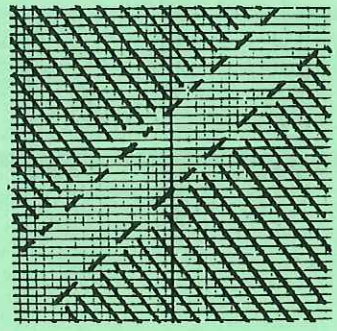
$y \geq 3x - 4$ and $y \leq 3x + 4$



(16) $|x - y| > 5$

$x - y > 5$ or $x - y < -5$

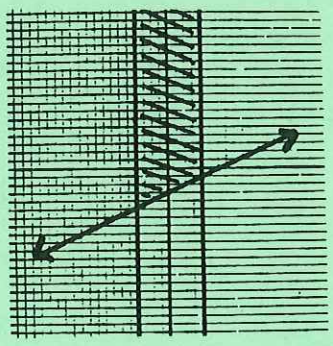
$y < x - 5$ or $y > x + 5$



(17) $|x| \leq 4$ and $x - 2y \leq 6$

$(x \leq 4$ and $x \geq -4)$ and $(-2y \leq -x + 6)$

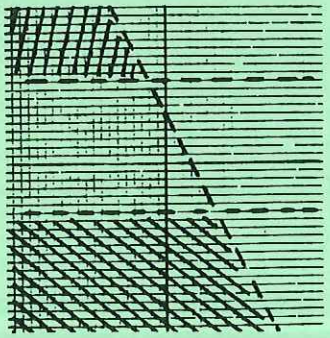
$(-4 \leq x \leq 4)$ and $(y \geq \frac{1}{2}x - 3)$



(18) $|y - 2| > 8$ and $2x + y < 6$

$(y - 2 > 8$ or $y - 2 < 8)$ and $(y < -2x + 6)$

$(y > 10$ or $y < -6)$ and $(y < -2x + 6)$



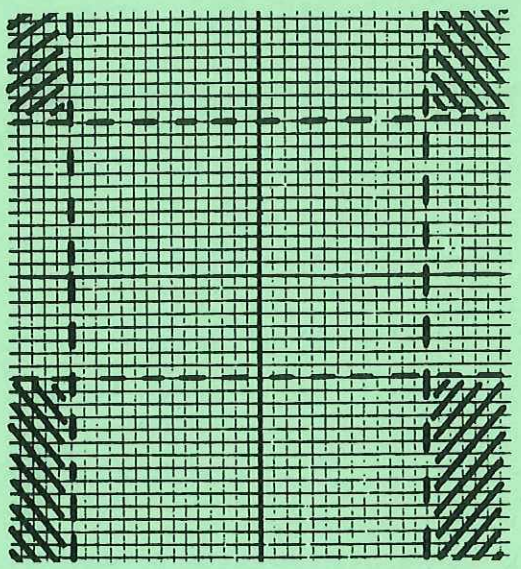
Inequalities & Coordinate Graphing

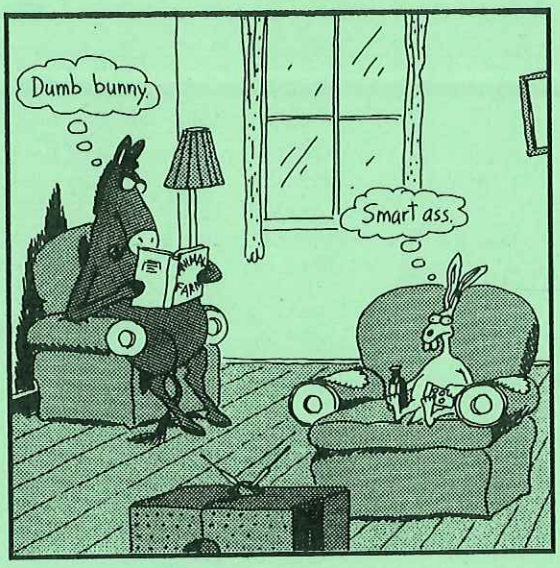
ANSWER KEY: UNIT 4 REVIEW

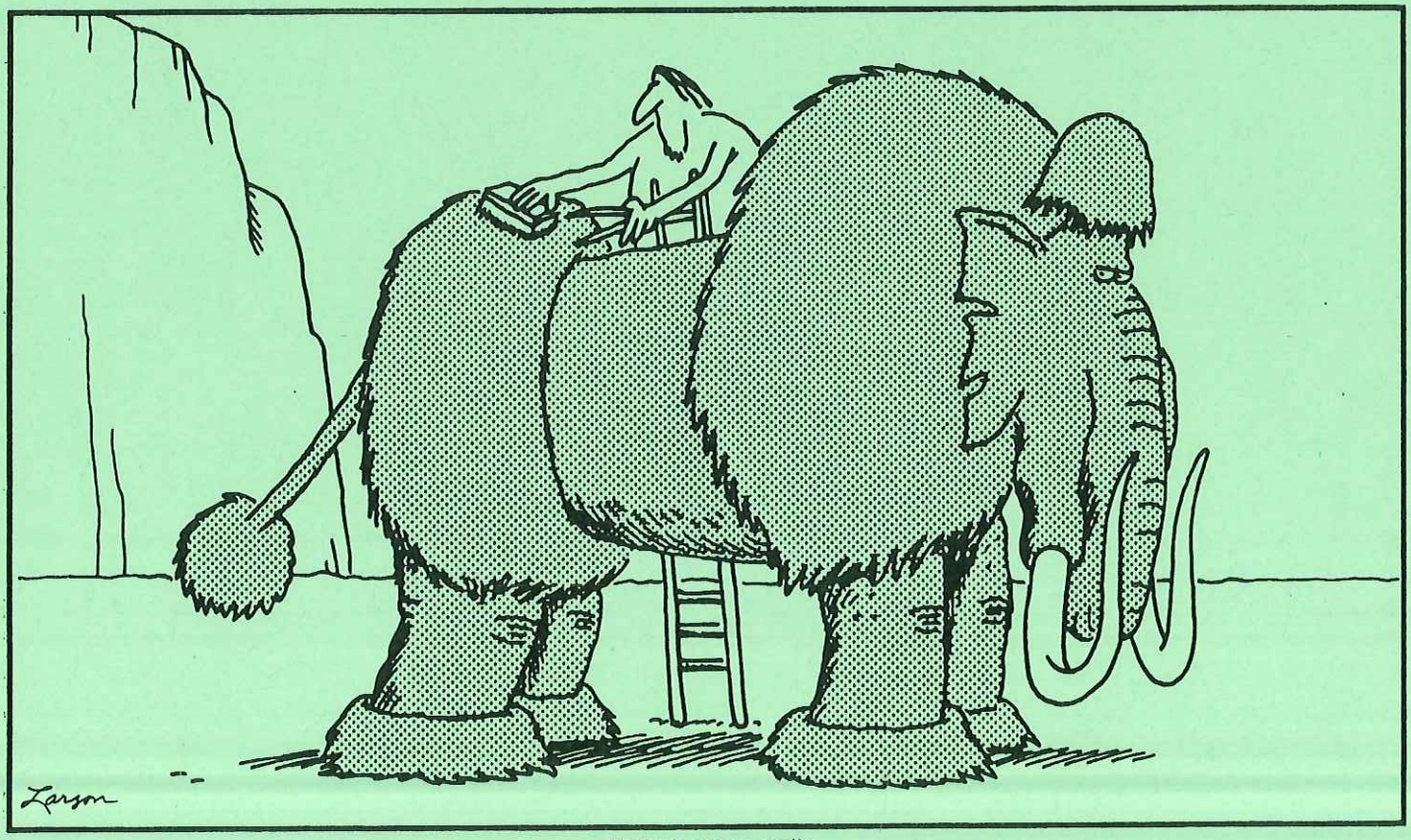
(19) $|x+1| > 14$ and $|y-2| > 10$

$(x+1 > 14$ or $x+1 < -14)$ and $(y-2 > 10$ or $y-2 < -10)$

$(x > 13$ or $x < -15)$ and $(y > 12$ or $y < -8)$







French mammoth

Roots & Radicals

ANSWER KEY: UNIT 6 DEMONSTRATION

① $\sqrt[4]{81x^6y^5z^{10}} = \boxed{3|x|yz^2\sqrt[4]{x^2yz^2}}$

② $\sqrt[3]{(3n-2)^3} = \boxed{3n-2}$

③ $\sqrt[4]{18a^3bc^5} \cdot \sqrt[4]{27a^2b^6c}$

$\sqrt[4]{2\cdot 3^5a^5b^7c^6} = \boxed{3abc\sqrt[4]{6ab^3c^2}}$

④ $(4-2\sqrt{2})(\sqrt{2}-4)$

$4\sqrt{2}-16-4+8\sqrt{2} = \boxed{12\sqrt{2}-20}$

⑤ $\frac{3-2\sqrt{3}}{2\sqrt{3}+3} \cdot \frac{2\sqrt{3}-3}{2\sqrt{3}-3} = \frac{6\sqrt{3}-9-12+6\sqrt{3}}{12-9}$

$\frac{12\sqrt{3}-21}{3} = \boxed{4\sqrt{3}-7}$

⑥ $\sqrt{\frac{1}{5}} - 2\sqrt{20} + 3\sqrt{5}$

$\frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} - 4\sqrt{5} + 3\sqrt{5}$

$\frac{\sqrt{5}}{5} - \sqrt{5} = \frac{\sqrt{5}}{5} - \frac{5\sqrt{5}}{5} = \boxed{\frac{-4\sqrt{5}}{5}}$

⑦ $\sqrt{3x+1} + 2 = 7$

$\sqrt{3x+1} = 5$ sq. both sides

$3x+1 = 25$

$3x = 24$

$\boxed{x=8}$

⑧ $\sqrt{2x+1} - 2 = \sqrt{2x-7}$ sq. both sides

$(2x+1) - 4\sqrt{2x+1} + 4 = 2x-7$

$-4\sqrt{2x+1} = -12$

$\sqrt{2x+1} = 3$ sq. both sides

$2x+1 = 9$

$2x = 8$ $\boxed{x=4}$

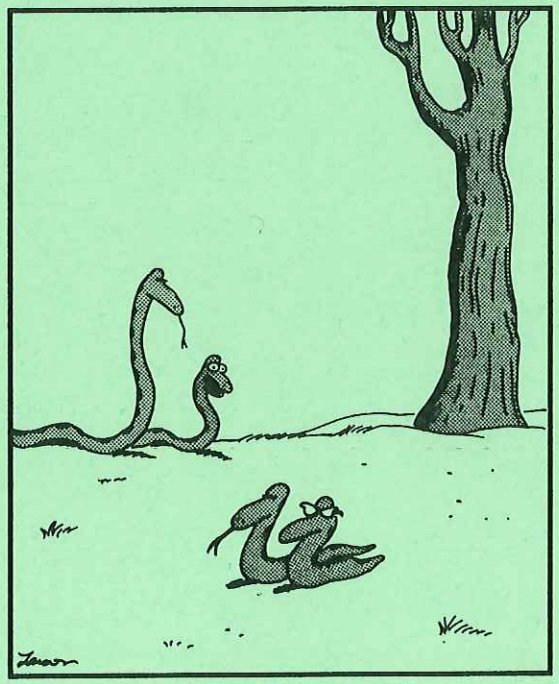
⑨ $\sqrt{4x+29} = (x+6)$ sq. both sides

$4x+29 = x^2+12x+36$

$x^2+8x+7 = 0$

$(x+7)(x+1) = 0$

$\boxed{x=-1}$ or ~~7~~ -7 does not check



"Look, Dad! ... Snidgets!"

Roots & Radicals

ANSWER KEY: UNIT 6 REVIEW

① $\sqrt[4]{64x^5y^6z^9} = \boxed{2|xy|z^2\sqrt[4]{4xy^2z}}$

② $\sqrt{72a^5b^7c^6}$ $\boxed{6a^2|b^3c^3|\sqrt{2ab}}$

③ $\sqrt[3]{-40n^7m^4}$ $\boxed{-2n^2m\sqrt[3]{5nm}}$

④ $\sqrt{n^2-10n+25} = \boxed{|n-5|}$

⑤ $\sqrt[3]{(3x-y)^5} = \boxed{(3x-y)\sqrt[3]{(3x-y)^2}}$

must use ↑ parenthesis

⑥ $\sqrt[4]{24a^2b^3c^5} \cdot \sqrt[4]{48a^3b^3c^6}$

$\sqrt[4]{2^7\cdot3^2\cdot a^5b^6c^{11}} = \boxed{2abc^2\sqrt[4]{72ab^2c^3}}$

no ab. value for a or b. both are + or both −

⑦ $\sqrt{3mn} + \sqrt{27m^3n}$

$\sqrt{3mn} + 3|m|\sqrt{3mn} = \boxed{(1+3|m|)\sqrt{3mn}}$

must have parenthesis ↑

⑧ $(4-\sqrt{3})(6-\sqrt{3})$

$24-4\sqrt{3}-6\sqrt{3}+3 = \boxed{27-10\sqrt{3}}$

⑨ $\sqrt[3]{\frac{2}{5n}} = \frac{\sqrt[3]{2}}{\sqrt[3]{5n}} \cdot \frac{\sqrt[3]{25n^2}}{\sqrt[3]{25n^2}} = \boxed{\frac{\sqrt[3]{50n^2}}{5n}}$

⑩ $\frac{2-\sqrt{2}}{4+2\sqrt{2}} \cdot \frac{4-2\sqrt{2}}{4-2\sqrt{2}} = \frac{8-4\sqrt{2}-4\sqrt{2}+4}{16-8}$

$\frac{12-8\sqrt{2}}{8} = \boxed{\frac{3-2\sqrt{2}}{2}}$

⑪ $\sqrt{\frac{1}{3}} + \sqrt{75} - 2\sqrt{3}$

$\frac{\sqrt{1}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} + 5\sqrt{3} - 2\sqrt{3}$

continued

$\frac{\sqrt{3}}{3} + 3\sqrt{3} = \frac{\sqrt{3}}{3} + \frac{9\sqrt{3}}{3} = \boxed{\frac{10\sqrt{3}}{3}}$

⑫ $\sqrt[3]{y+1} = 3$ cube both sides

$y+1 = 27$ $\boxed{y=26}$

⑬ $\sqrt{y+12}+1 = \sqrt{y+21}$ sq. both sides

$(y+12)+2\sqrt{y+12}+1 = y+21$

$2\sqrt{y+12} = 8$

$\sqrt{y+12} = 4$ sq. both sides

$y+12 = 16$ $\boxed{y=4}$

⑭ $\sqrt{5+2x} = x-5$ sq. both sides

$5+2x = x^2-10x+25$

$x^2-12x+20=0$

$(x-10)(x-2)=0$

$\boxed{x=10}$ or ~~x~~ 2 does not check

⑮ $\sqrt{x+11}-x=-9$

$\sqrt{x+11} = x-9$ sq. both sides

$x+11 = x^2-18x+81$

$x^2-19x+70=0$

$(x-14)(x-5)=0$

$\boxed{x=14}$ or ~~5~~ 5 does not check

⑯ $r = \sqrt[3]{\frac{2mM}{c}}$ cube both sides

$r^3 = \frac{2mM}{c}$

$r^3c = 2mM$

$\boxed{c = \frac{2mM}{r^3} \text{ for } r \neq 0}$

Rational Exponents & Complex Numbers

ANSWER KEY: UNIT 7 DEMONSTRATION

① $\sqrt{8x^3y^4} = \sqrt{2^3x^3y^4} = \boxed{2^{3/2}x^{3/2}y^2}$

② $x^{1/3}y^{1/2}z^{3/2}$

$x^{2/6}y^{3/6}z^{9/6} = \sqrt[6]{x^2y^3z^9}$

$\boxed{z\sqrt[6]{x^2y^3z^3}}$

③ $\sqrt[6]{8n^9} = \sqrt[6]{2^3n^9}$

$2^{3/6}n^{9/6} = 2^{1/2}n^{3/2}$

$\sqrt{2n^3} = \boxed{n\sqrt{2n}}$

④ $\dfrac{x^{5/2}}{x^{1/2}+y^{1/2}} \cdot \dfrac{x^{1/2}-y^{1/2}}{x^{1/2}-y^{1/2}} = \boxed{\dfrac{x^3-x^{5/2}y^{1/2}}{x-y}}$

⑤ $(\sqrt{-8})(\sqrt{-6}) = (2i\sqrt{2})(i\sqrt{6})$

$2i^2\sqrt{12} = 4i^2\sqrt{3} = \boxed{-4\sqrt{3}}$

⑥ $(3i^3)(-4i)^2 = (3i^3)(16i^2)$

$48i^5 = \boxed{48i}$

⑦ $2n^2+18=0$

$2n^2 = -18$

$n^2 = -9$

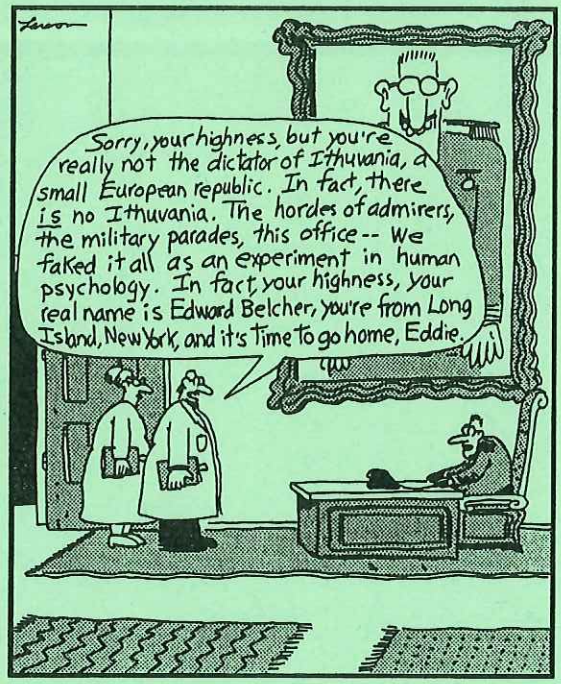
$n = \pm\sqrt{-9}$

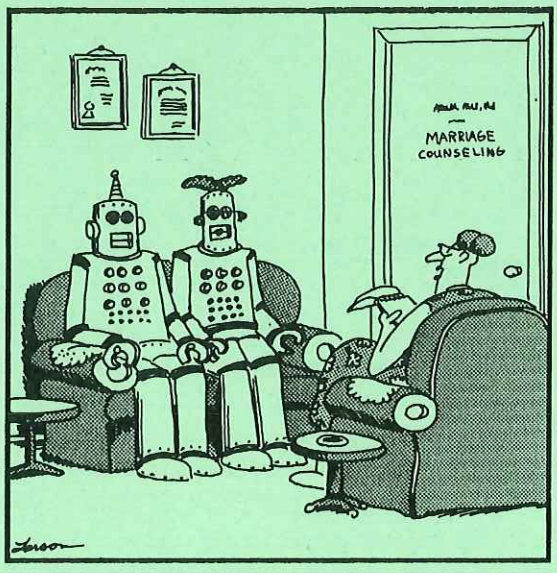
$\boxed{n = \pm 3i}$

⑧ $\dfrac{3+3i}{1-2i} \cdot \dfrac{1+2i}{1+2i} = \dfrac{3+9i+6i^2}{1-4i^2} = \boxed{\dfrac{-3+9i}{5}}$

⑨ $\left(\dfrac{a^{-3/2}}{3^6b^{-2/3}}\right)^{-1/2} = \dfrac{a^{3/4}}{3^{-3}b^{1/3}}$

$\dfrac{27a^{3/4}}{b^{1/3}} \cdot \dfrac{b^{2/3}}{b^{2/3}} = \boxed{\dfrac{27a^{3/4}b^{2/3}}{b}}$





"The problem, as I see it, is that you both are extremely adept at pushing each other's buttons."

Rational Exponents & Complex Numbers

ANSWER KEY: UNIT 7 REVIEW

① $\sqrt[3]{16x^5y^6z^8} = \sqrt[3]{2^4x^5y^6z^8}$

$\boxed{2^{4/3}x^{5/3}y^2z^{8/3}}$

② $\sqrt[4]{96a^5b^7c^8} = \sqrt[4]{2^5\cdot 3\cdot a^5\cdot b^7c^8}$

$\boxed{2^{5/4}\cdot 3^{1/4}a^{5/4}b^{7/4}c^2}$

③ $r^2s^{1/3}y^{1/2} = r^2s^{2/6}y^{3/6} = \boxed{r^2\sqrt[6]{s^2y^3}}$

④ $(3x)^{1/2}x^{1/4} = 3^{1/2}x^{1/2}x^{1/4}$

$3^{1/2}x^{3/4} = 3^{2/4}x^{3/4} = \boxed{\sqrt[4]{9x^3}}$

⑤ $\sqrt[6]{27} = \sqrt[6]{3^3} = 3^{3/6} = 3^{1/2} = \boxed{\sqrt{3}}$

⑥ $\sqrt[4]{64x^{10}} = \sqrt[4]{2^6x^{10}} = 2^{6/4}x^{10/4}$

$2^{3/2}x^{5/2} = \sqrt{2^3x^5} = \boxed{2x^2\sqrt{2x}}$

⑦ $(25^{3/4})^{2/3} = 25^{1/2} = (5^2)^{1/2} = \boxed{5}$

⑧ $(8^{2/3})(16^{-3/4}) = (2^3)^{2/3}(2^4)^{-3/4}$

$(2^2)(2^{-3}) = 2^{-1} = \boxed{1/2}$

⑨ $\dfrac{x^{2/3}}{x^{2/3}-x^{-1/3}}\cdot\dfrac{x^{1/3}}{x^{1/3}} = \boxed{\dfrac{x}{x-1}}$

⑩ $\dfrac{x^{1/2}-y^{1/2}}{x^{1/2}+y^{1/2}}\cdot\dfrac{x^{1/2}-y^{1/2}}{x^{1/2}-y^{1/2}} = \boxed{\dfrac{x-2x^{1/2}y^{1/2}+y}{x-y}}$

⑪ $\left(\dfrac{a^{-2/3}}{2^{1/2}a^2}\right)^{-1/2} = \dfrac{a^{1/3}}{2^{-1/4}a^{-1}} = \boxed{2^{1/4}a^{4/3}}$

⑫ $\left(\dfrac{2^4a}{a^2b^{-1}}\right)^{-1/2} = \dfrac{2^{-2}a^{-1/2}}{a^{-1}b^{1/2}} = \dfrac{a^{1/2}}{4b^{1/2}}$

$\dfrac{a^{1/2}}{4b^{1/2}}\cdot\dfrac{b^{1/2}}{b^{1/2}} = \boxed{\dfrac{a^{1/2}b^{1/2}}{4b}}$

⑬ $(\sqrt{-8})(\sqrt{-12}) = (2i\sqrt{2})(2i\sqrt{3})$

$4i^2\sqrt{6} = \boxed{-4\sqrt{6}}$

⑭ $(\sqrt{-6})(\sqrt{-4})(\sqrt{-3}) = (i\sqrt{6})(2i)(i\sqrt{3})$

$2i^3\sqrt{18} = 6i^3\sqrt{2} = \boxed{-6i\sqrt{2}}$

⑮ $(3i^3)(2i)^2 = (3i^3)(4i^2)$

$12i^5 = \boxed{12i}$

⑯ $(-2i)^4(-4i^3) = (16i^4)(-4i^3)$

$-64i^7 = \boxed{64i}$

⑰ $2n^2 = -27/8$

$n^2 = -27/16$

$n = \pm\dfrac{\sqrt{-27}}{4}$

$\boxed{n = \pm\dfrac{3i\sqrt{3}}{4}}$

⑱ $4x^2+75=0$

$x^2 = -75/4$

$x = \pm\dfrac{\sqrt{-75}}{2}$

$\boxed{x = \pm\dfrac{5i\sqrt{3}}{2}}$

⑲ $(7+2i)(5-3i)$

$35-21i+10i-6i^2$

$35-11i-6i^2 = \boxed{41-11i}$

Rational Exponents & Complex Numbers

ANSWER KEY: UNIT 7 REVIEW

(20) $(3+8i)(3-8i)$ conjugates

$(3)^2 - (8i)^2$

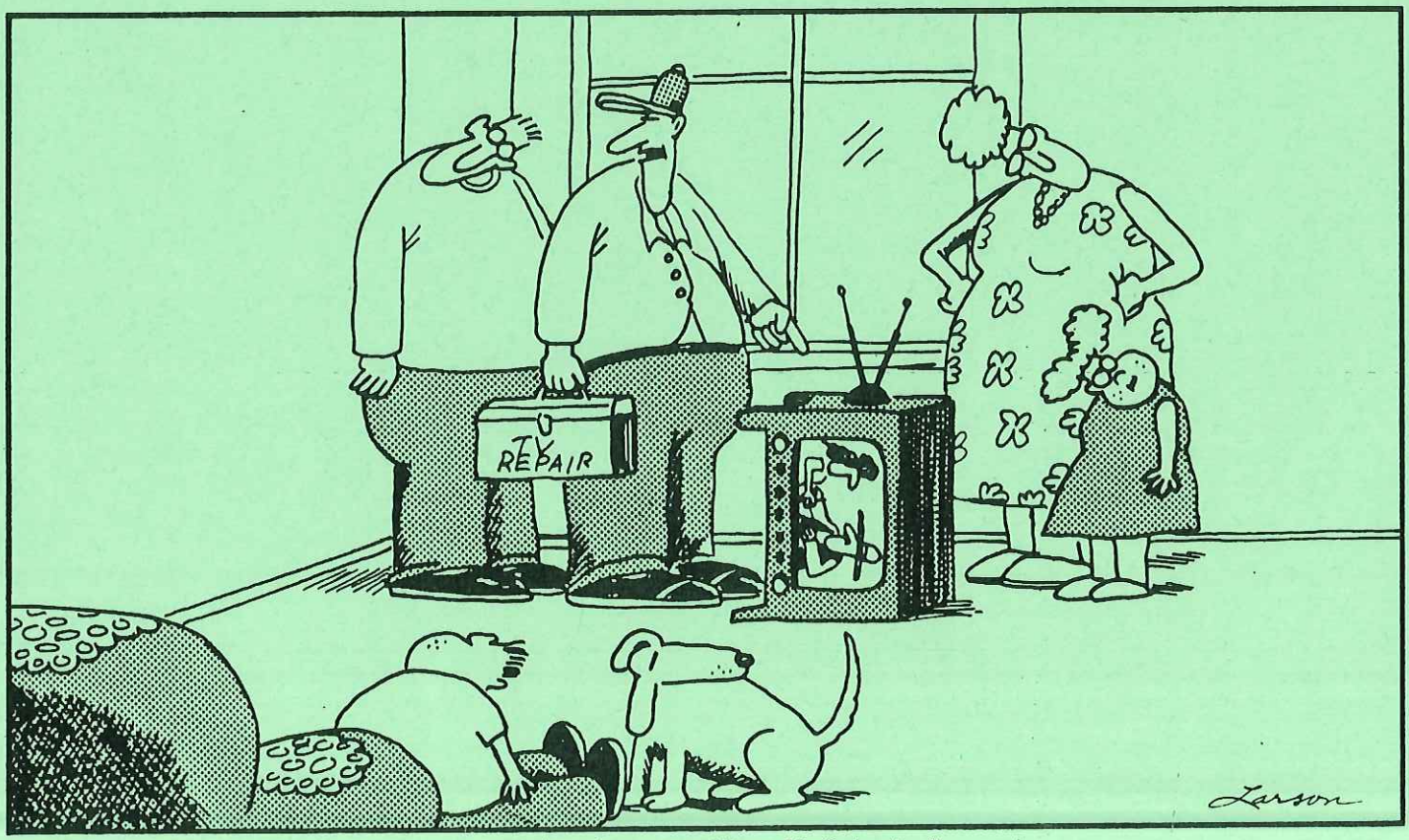
$9 - 64i^2$

$9 - (-64) = \boxed{73}$

(21) $\frac{4+3i}{1-2i} \cdot \frac{1+2i}{1+2i} = \frac{4+8i+3i+6i^2}{1-4i^2}$

$\frac{4+11i+6i^2}{1-(-4)} = \boxed{\frac{-2+11i}{5}}$

(22) $\frac{2-2i}{2+2i} \cdot \frac{2-2i}{2-2i} = \frac{4-8i+4i^2}{4-4i^2} = \frac{-8i}{8} = \boxed{-i}$



"Well, here's your problem, Mr. Schueler."

Quadratics

ANSWER KEY: UNIT 8 DEMONSTRATION

① $2x^2+3x+2=0$

$x^2+\frac{3}{2}x = -1$

$x^2+\frac{3}{2}x+\frac{9}{16} = -1+\frac{9}{16}$

$(x+\frac{3}{4})^2 = \frac{-7}{16}$

$x+\frac{3}{4} = \frac{\pm\sqrt{-7}}{4}$ $\boxed{x=\frac{-3\pm i\sqrt{7}}{4}}$

② $2x^2+2x+3=0$

$a=2$ $b=2$ $c=3$

$\frac{-(2)\pm\sqrt{(2)^2-4(2)(3)}}{2(2)}$

$\frac{-2\pm\sqrt{-20}}{4} = \frac{-2\pm 2i\sqrt{5}}{4}$

$\boxed{x=\frac{-1\pm i\sqrt{5}}{2}}$

③ $4x^2-8x+13=0$

$b^2-4ac = (-8)^2-4(4)(13) = \boxed{-144}$

$\boxed{\text{2 imaginary roots}}$

④ sum: $(-\frac{3}{5})+(\frac{1}{2}) = -\frac{1}{10}$

prod: $(-\frac{3}{5})(\frac{1}{2}) = \frac{-3}{10}$

$x^2+\frac{1}{10}x-\frac{3}{10}=0$ $\boxed{10x^2+x-3=0}$

⑤ sum: $(4+\sqrt{3})+(4-\sqrt{3}) = 8$

prod: $(4+\sqrt{3})(4-\sqrt{3}) = 16-3 = 13$

$\boxed{x^2-8x+13=0}$

⑥ $x^4-9=0$

$(x^2-3)(x^2+3)$

$x^2=3$ $x^2=-3$

$x=\pm\sqrt{3}$ $x=\pm i\sqrt{3}$ $\boxed{x=\pm\sqrt{3},\pm i\sqrt{3}}$

⑦ $x-4\sqrt{x}-45=0$

$(\sqrt{x}-9)(\sqrt{x}+5)=0$

$\sqrt{x}=9$ $\sqrt{x}=-5$

$x=81$ $x=\cancel{25}$ $\boxed{x=81}$ 25 does not check out

⑧ $y \geq -x^2+10x-21$

$x=\frac{-b}{2a}$ $\boxed{x=5}$ axis

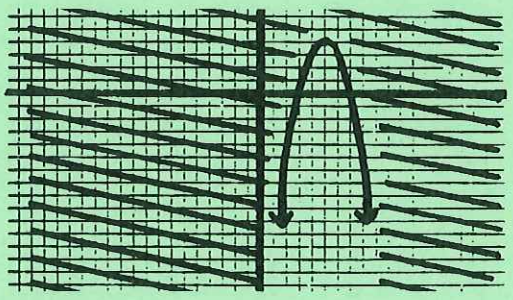
$-(5)^2+10(5)-21=4$ $\boxed{(5,4)}$ max. pt.

x	y	
4	3	$-(4)^2+10(4)-21=3$
3	0	$-(3)^2+10(3)-21=0$
2	-5	$-(2)^2+10(2)-21=-5$

$-x^2+10x-21=0$

$x^2-10x+21=0$

$(x-7)(x-3)=0$ $\boxed{x\leq 3 \text{ or } x\geq 7}$



⑨ $bx^2+cx+a=0$

$x^2+\frac{c}{b}x = \frac{-a}{b}$

$x^2+\frac{c}{b}x+\frac{c^2}{4b^2} = \frac{-a}{b}+\frac{c^2}{4b^2}$

$(x+\frac{c}{2b})^2 = \frac{c^2-4ab}{4b^2}$

$x+\frac{c}{2b} = \frac{\pm\sqrt{c^2-4ab}}{2b}$

$\boxed{x=\frac{-c\pm\sqrt{c^2-4ab}}{2b}}$

Quadratics

ANSWER KEY: UNIT 8 REVIEW

① $6x^2+7x-3=0$

$6x^2+9x-2x-3=0$

$3x(2x+3)-1(2x+3)=0$

$(2x+3)(3x-1)=0$ $\boxed{x=-3/2,\ 1/3}$

② $2n^2+n-21=0$

$n^2+\frac{1}{2}n = \frac{21}{2}$

$n^2+\frac{1}{2}n+\frac{1}{16} = \frac{21}{2}+\frac{1}{16}$

$(n+\frac{1}{4})^2 = \frac{169}{16}$

$n+\frac{1}{4} = \pm\frac{13}{4}$

$n=\frac{-1\pm13}{4}$ $\boxed{n=-\frac{7}{2},\ 3}$

③ $3x^2+4x+2=0$

$x^2+\frac{4}{3}x = -\frac{2}{3}$

$x^2+\frac{4}{3}x+\frac{4}{9} = -\frac{2}{3}+\frac{4}{9}$

$(x+\frac{2}{3})^2 = -\frac{2}{9}$

$x+\frac{2}{3} = \frac{\pm\sqrt{-2}}{3}$

$\boxed{x=\frac{-2\pm i\sqrt{2}}{3}}$

④ $2a^2-5a+4=0$

$a=2$, $b=-5$, $c=4$

$\frac{-(-5)\pm\sqrt{(-5)^2-4(2)(4)}}{2(2)}$

$\frac{5\pm\sqrt{-7}}{4} = \boxed{a=\frac{5\pm i\sqrt{7}}{4}}$

⑤ $ax^2+bx+3b=0$

$a=a$, $b=b$, $c=3b$

$\frac{-b\pm\sqrt{b^2-4a(3b)}}{2a}$

$\boxed{x=\frac{-b\pm\sqrt{b^2-12ab}}{2a}}$

⑥ $bx^2+acx+c=0$

$a=b$, $b=ac$, $c=c$

$\frac{-ac\pm\sqrt{(ac)^2-4(b)(c)}}{2(b)}$

$\boxed{x=\frac{-ac\pm\sqrt{a^2c^2-4bc}}{2b}}$

⑦ $4x^2-40x+25=0$

$b^2-4ac = (-40)^2-4(4)(25) = \boxed{1200}$

$\boxed{\text{2 irrational roots}}$

⑧ $2y^2+6y+5=0$

$b^2-4ac = (6)^2-4(2)(5) = \boxed{-4}$

$\boxed{\text{2 imaginary roots}}$

⑨ sum: $(\frac{3}{4})+(\frac{1}{3}) = \frac{13}{12}$ $x^2-\frac{13}{12}x+\frac{1}{4}=0$

prod: $(\frac{3}{4})(\frac{1}{3}) = \frac{1}{4}$ $\boxed{12x^2-13x+3=0}$

⑩ sum: $(2+\sqrt{3})+(2-\sqrt{3}) = 4$

prod: $(2+\sqrt{3})(2-\sqrt{3}) = 4-3 = 1$

$\boxed{x^2-4x+1=0}$

⑪ sum: $(5+3i)+(5-3i) = 10$

prod: $(5+3i)(5-3i) = 25-9i^2 = 34$

$\boxed{x^2-10x+34=0}$

⑫ $x-4\sqrt{x}-32=0$

$(\sqrt{x}-8)(\sqrt{x}+4)=0$

$\sqrt{x}=8$ $\sqrt{x}=-4$

$x=64$ $x=16$

$\boxed{x=64}$

16 does not check

⑬ $x^4-12x^2+27=0$

$(x^2-9)(x^2-3)=0$

$x^2=3$

$x=\pm\sqrt{3}$

$(x+3)(x-3)$

$x=\pm3$

$\boxed{x=\pm3,\pm\sqrt{3}}$

Quadratics

ANSWER KEY: UNIT 8 REVIEW

(14) $x^{2/3} - 9x^{1/3} + 20 = 0$

$(x^{1/3})^2 - 9x^{1/3} + 20 = 0$

$(x^{1/3} - 5)(x^{1/3} - 4) = 0$

$x^{1/3} = 5 \quad x^{1/3} = 4$

$x = 125 \quad x = 64$

$\boxed{x = 125, 64}$

(15) $n^6 + 9n^3 + 8 = 0$

$(n^3)^2 + 9n^3 + 8 = 0$

$(n^3 + 8)(n^3 + 1) = 0$

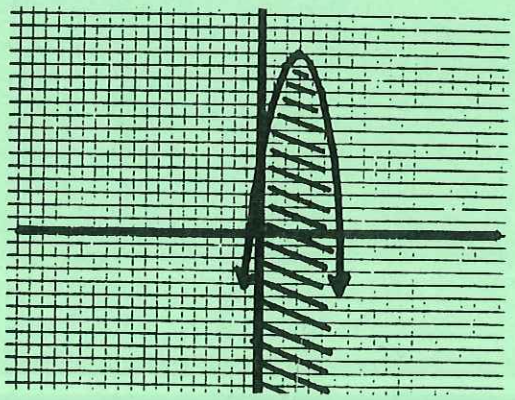
$(n+2)(n^2 - 2n + 4)(n+1)(n^2 - n + 1) = 0$

$n = -2$ $\quad \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)}$ $\quad n = -1$ $\quad \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$

$\frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm 2i\sqrt{3}}{2}$ $\quad \frac{1 \pm \sqrt{-3}}{2}$

$1 \pm i\sqrt{3}$ $\quad \frac{1 \pm i\sqrt{3}}{2}$

$\boxed{n = -2, -1, 1 \pm i\sqrt{3}, \frac{1 \pm i\sqrt{3}}{2}}$



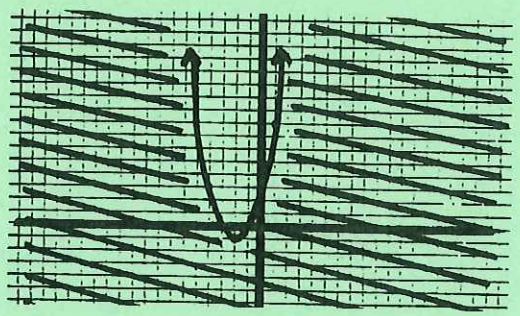
(16) $y \leq x^2 + 4x + 3 \quad x = \frac{-b}{2a}$ $\boxed{x = -2}$ axis

$(-2)^2 + 4(-2) + 3 = -1$ $\boxed{(-2, -1)}$ min. pt.

x	y	
-1	0	$(-1)^2 + 4(-1) + 3 = 0$
0	3	$(0)^2 + 4(0) + 3 = 3$
1	8	$(1)^2 + 4(1) + 3 = 8$

$x^2 + 4x + 3 = 0$

$(x+3)(x+1) = 0$ $\quad \boxed{x \leq -3 \text{ or } x \geq -1}$



(17) $y \leq -x^2 + 6x + 5 \quad x = \frac{-b}{2a}$ $\boxed{x = 3}$ axis

$-(3)^2 + 6(3) + 5 = 14$ $\boxed{(3, 14)}$ max. pt.

x	y	
2	13	$-(2)^2 + 6(2) + 5 = 13$
1	10	$-(1)^2 + 6(1) + 5 = 10$
0	5	$-(0)^2 + 6(0) + 5 = 5$

$-x^2 + 6x + 5 = 0$

$x^2 - 6x - 5 = 0$

$\frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-5)}}{2(1)}$

$\frac{6 \pm \sqrt{56}}{2} = \frac{6 \pm 2\sqrt{14}}{2} \quad x = 3 \pm \sqrt{14}$

$\boxed{3 - \sqrt{14} \leq x \leq 3 + \sqrt{14}}$

← graph

Rational Expressions

ANSWER KEY: UNIT 9 DEMONSTRATION

① $\dfrac{x^2+7x+10}{x+2} \div \dfrac{x^2+2x-15}{x^2-5x+6}$

$\dfrac{(x+5)(x+2)}{(x+2)} \cdot \dfrac{(x-3)(x-2)}{(x-3)(x+5)} = \boxed{x-2}$

② $3n+1+\dfrac{1}{3n-1} = \dfrac{(3n+1)(3n-1)+1}{(3n-1)}$

$\dfrac{(9n^2-1)+1}{3n-1} = \boxed{\dfrac{9n^2}{3n-1}}$

③ $\dfrac{\dfrac{x+y}{x}}{\dfrac{1}{x}+\dfrac{1}{y}} = \dfrac{\dfrac{x+y}{x}}{\dfrac{y+x}{xy}}$

$\dfrac{x+y}{x} \cdot \dfrac{xy}{x+y} = \boxed{y}$

④ $\left[\dfrac{y}{y-3}+\dfrac{6}{y+3}=1\right](y-3)(y+3)$

$y(y+3)+6(y-3) = 1(y-3)(y+3)$

$y^2+3y+6y-18 = y^2-9$

$9y = 9 \quad \boxed{y=1}$

⑤ $\dfrac{n+5+\dfrac{3}{n+1}}{n-1-\dfrac{3}{n+1}} = \dfrac{\dfrac{(n+5)(n+1)+3}{n+1}}{\dfrac{(n-1)(n+1)-3}{n+1}}$

$\dfrac{n^2+6n+8}{n+1} \cdot \dfrac{n+1}{n^2-4} = \dfrac{(n+4)(n+2)(n+1)}{(n+1)(n+2)(n-2)} = \boxed{\dfrac{n+4}{n-2}}$

⑥ $\dfrac{x_1}{x_2} = \dfrac{y_1}{y_2} \quad \dfrac{7}{x} = \dfrac{21}{-5} \quad 21x = -35 \quad \boxed{x = -5/3}$

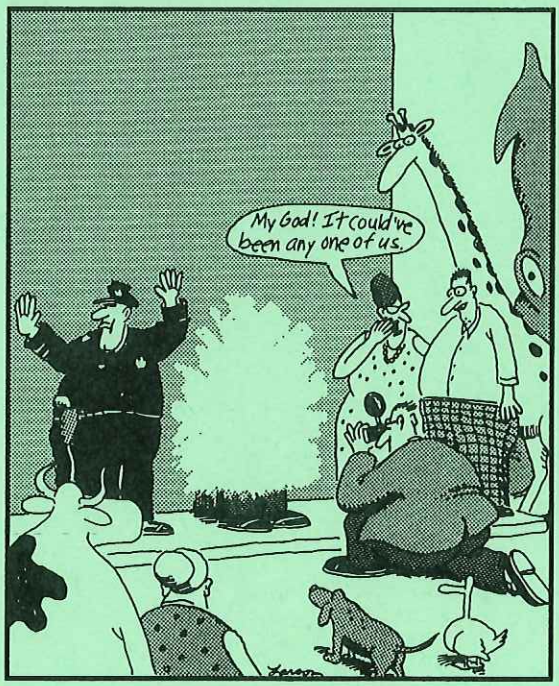
⑦ $\dfrac{x_1}{x_2} = \dfrac{y_2}{y_1} \quad \dfrac{60}{5} = \dfrac{y}{40} \quad 5y = 2400 \quad y = \boxed{480 \text{ cm}}$

⑧ $\left[\dfrac{1}{n-2} = \dfrac{2n+1}{n^2+2n-8} + \dfrac{2}{n+4}\right](n+4)(n-2)$

$1(n+4) = 2n+1+2(n-2)$

$n+4 = 2n+1+2n-4$

$-3n = -7 \quad \boxed{n = 7/3}$



Drive-by erasings

Rational Expressions

ANSWER KEY: UNIT 9 REVIEW

① $\frac{a^3-b^3}{b^2-a^2} \cdot \frac{a+b}{a^2+ab+b^2}$

$\frac{(a-b)(a^2+ab+b^2)(a+b)}{(b-a)(b+a)(a^2+ab+b^2)} = \frac{(a-b)}{-(a-b)} = \boxed{-1}$

② $\frac{x^2-11x+24}{x^2-18x+80} \div \frac{x^2-9x+20}{x^2-15x+50}$

$\frac{(x-8)(x-3)}{(x-8)(x-10)} \cdot \frac{(x-5)(x-10)}{(x-5)(x-4)} = \boxed{\frac{x-3}{x-4}}$

③ $\frac{x^2-2x+1}{y-5} \div \frac{(x-1)^2}{y^2-25}$

$\frac{(x-1)(x-1)}{(y-5)} \cdot \frac{(y+5)(y-5)}{(x-1)^2} = \boxed{y+5}$

④ $\frac{3}{m-2} + \frac{2}{2-m} = \frac{3}{m-2} - \frac{2}{m-2} = \boxed{\frac{1}{m-2}}$

⑤ $\frac{3x+2}{3x-6} - \frac{x+2}{x^2-4} = \frac{3x+2}{3(x-2)} - \frac{x+2}{x^2-4}$

$\frac{(3x+2)(x+2) - (x+2)(3)}{3(x^2-4)}$

$\frac{(3x^2+8x+4) - (3x+6)}{3(x^2-4)}$

$\frac{3x^2+5x-2}{3(x^2-4)} = \frac{(3x-1)(x+2)}{3(x-2)(x+2)} = \boxed{\frac{3x-1}{3x-6}}$

⑥ $\frac{2n}{n^2-5n} - \frac{-3n}{n-5}$

$\frac{2n-(-3n)(n)}{n(n-5)} = \frac{2n+3n^2}{n(n-5)} = \frac{n(2+3n)}{n(n-5)} = \boxed{\frac{(2+3n)}{(n-5)}}$

⑦ $\frac{\frac{x}{y} - \frac{y}{x}}{\frac{1}{x} + \frac{1}{y}} = \frac{\frac{x^2-y^2}{xy}}{\frac{y+x}{xy}}$

$\frac{(x-y)(x+y)}{xy} \cdot \frac{xy}{(x+y)} = \boxed{x-y}$

⑧ $\frac{\frac{m+4}{m} - \frac{3}{m+5}}{\frac{m-1}{m^2+5m} + \frac{3}{m+5}} = \frac{\frac{(m+4)(m+5)-3(m)}{m(m+5)}}{\frac{(m-1)+3(m)}{m^2+5m}}$

$\frac{\frac{(m^2+9m+20)-3m}{m^2+5m}}{\frac{4m-1}{m^2+5m}}$

$\frac{m^2+6m+20}{(m^2+5m)} \cdot \frac{(m^2+5m)}{4m-1} = \boxed{\frac{m^2+6m+20}{4m-1}}$

⑨ $\frac{n+5+\frac{4}{n+1}}{n+3} = \frac{\frac{(n+5)(n+1)+4}{n+1}}{n+3}$

$\frac{\left(\frac{n^2+6n+9}{n+1}\right)}{(n+3)} = \frac{(n+3)^2}{n+1} \cdot \frac{1}{(n+3)} = \boxed{\frac{n+3}{n+1}}$

Rational Expressions

ANSWER KEY: UNIT 9 REVIEW

(10) $\frac{3x}{2x-5} + \frac{2x}{5-2x} = \frac{x-1}{2x+5}$

$\left[\frac{3x}{2x-5} - \frac{2x}{2x-5} = \frac{x-1}{2x+5}\right](2x-5)(2x+5)$

$3x(2x+5) - 2x(2x+5) = (x-1)(2x-5)$

$6x^2 + 15x - 4x^2 - 10x = 2x^2 - 7x + 5$

$12x = 5$

$\boxed{x = 5/12}$

(11) $\left[\frac{3}{x+2} + \frac{12}{x^2-4} = \frac{-1}{x-2}\right](x^2-4)$

$3(x-2) + 12 = -1(x+2)$

$3x - 6 + 12 = -x - 2$

$4x = -8$

$x = -2$ $\boxed{\text{no solutions}}$

-2 is an excluded value

(12) $\left[\frac{x+3}{x+2} = 2 - \frac{3}{x^2+5x+6}\right](x+2)(x+3)$

$(x+3)(x+3) = 2(x+3)(x+2) - 3$

$x^2 + 6x + 9 = 2x^2 + 10x + 12 - 3$

$x^2 + 4x = 0$

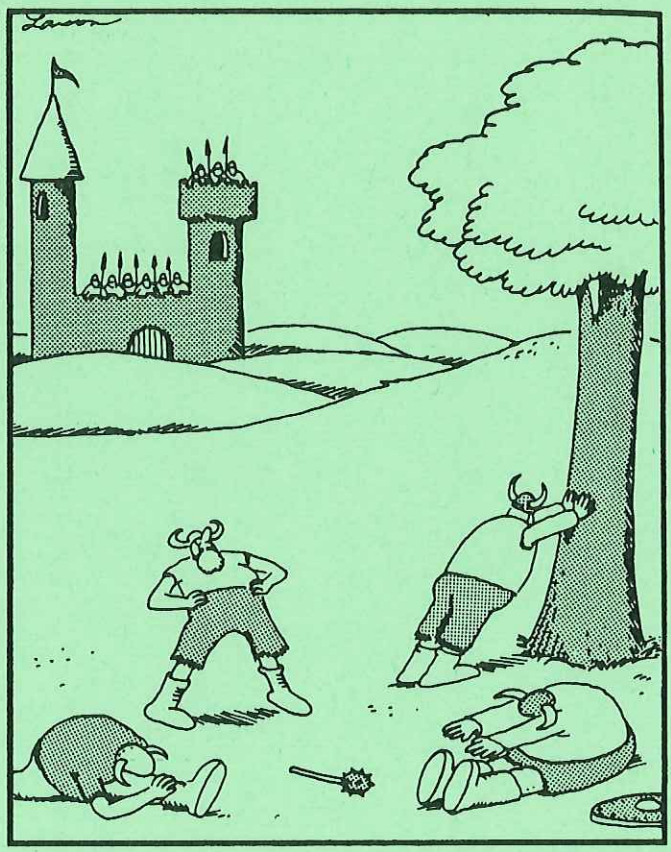
$x(x+4) = 0$ $\boxed{x = 0, -4}$

(13) $\frac{x_1}{x_2} = \frac{y_2}{y_1}$ $\frac{5/2}{-3/5} = \frac{y}{9}$

$-\frac{3}{5}y = \frac{45}{2}$ $\boxed{y = -75/2}$

(14) $\frac{x_1}{x_2} = \frac{y_1}{y_2}$ $\frac{-3}{x} = \frac{-8}{6}$ $-8x = -18$ $\boxed{x = 9/4}$

(15) $\frac{x_1}{x_2} = \frac{y_2}{y_1}$ $\frac{6}{x} = \frac{12}{10}$ $12x = 60$ $\boxed{x = 5}$



The Vikings, of course, knew the importance of stretching before an attack.

Sequence & Series

ANSWER KEY: UNIT 17 DEMONSTRATION

① $(x-y)^7 \rightarrow 1 \quad 7\left(\frac{6}{2}\right) \quad 21\left(\frac{5}{3}\right) \quad 35 \quad 35 \quad 21 \quad 7 \quad 1$

$x^7 - 7x^6y + 21x^5y^2 - 35x^4y^3 + 35x^3y^4 - 21x^2y^5 + 7xy^6 - y^7$

② $(a+3b)^5 \rightarrow 1 \quad 5\left(\frac{4}{1}\right) \quad 10 \quad 10 \quad 5 \quad 1$

$a^5 + 5a^4(3b) + 10a^3(3b)^2 + 10a^2(3b)^3 + 5a(3b)^4 + (3b)^5$

$a^5 + 15a^4b + 90a^3b^2 + 270a^2b^3 + 405ab^4 + 243b^5$

③ $\sum_{x=2}^{10} (3x+5)$ $\quad n=9,\ d=3,\ a_1=11$

$s = \frac{9}{2}[2(11) + 8(3)]$

207 $\quad$ 11 + 14 + 17

④ $\sum_{x=1}^{7} (3)^{x+1}$ $\quad n=7,\ r=3,\ a_1=9$

$s = \frac{9-9(3)^7}{1-(3)} =$ 9837

9 + 27 + 81

⑤ $(-20) + (-10) + (-5) + (-5/2) + \ldots$

$r = 1/2 \quad a_1 = -20$

$\frac{-20}{1-1/2} = \frac{-20}{1/2} =$ −40

⑥ $n = 30 \quad d = 2 \quad a_1 = 2$

$\frac{30}{2}[2(2) + 29(2)] =$ 930

⑦ $n=6,\ d=-5,\ a_1=16$ $\quad \sum_{x=1}^{6} (-5x+21)$

As witnesses later recalled,
two small dogs just waltzed into the place,
grabbed the cat, and waltzed out.

Sequence & Series

ANSWER KEY: UNIT 17 REVIEW

① $(x+y)^7 \rightarrow 1 \quad 7\left(\frac{6}{2}\right) \quad 21\left(\frac{5}{3}\right) \quad 35 \quad 35 \quad 21 \quad 7 \quad 1$

$x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7$

② $(3a+4b)^4 \rightarrow 1 \quad 4\left(\frac{3}{2}\right) \quad 6 \quad 4 \quad 1$

$(3a)^4 + 4(3a)^3(4b) + 6(3a)^2(4b)^2 + 4(3a)(4b)^3 + (4b)^4$

$81a^4 + 432a^3b + 864a^2b^2 + 768ab^3 + 256b^4$

③ $(n-3m)^5 \rightarrow 1 \quad 5\left(\frac{4}{2}\right) \quad 10 \quad 10 \quad 5 \quad 1$

$n^5 - 5n^4(3m) + 10n^3(3m)^2 - 10n^2(3m)^3 + 5n(3m)^4 - (3m)^5$

$n^5 - 15n^4m + 90n^3m^2 - 270n^2m^3 + 405nm^4 - 243m^5$

④ $(2x-3y)^6 \rightarrow 1 \quad 6\left(\frac{5}{2}\right) \quad 15\left(\frac{4}{3}\right) \quad 20 \quad 15 \quad 6 \quad 1$

$(2x)^6 - 6(2x)^5(3y) + 15(2x)^4(3y)^2 - 20(2x)^3(3y)^3 + 15(2x)^2(3y)^4 - 6(2x)(3y)^5 + (3y)^6$

$64x^6 - 576x^5y + 2160x^4y^2 - 4320x^3y^3 + 4860x^2y^4 - 2916xy^5 + 729y^6$

⑤ $\sum_{x=1}^{7} (4x-5)$ $n=7$, $d=4$, $a_1=-1$

arithmetic series

$S = \frac{n}{2}[2a_1 + (n-1)d]$

$\frac{7}{2}(-2+24) =$ **77**

(-1) + (3) + (7)

⑥ $\sum_{c=3}^{10} (8-5c)$ $n=8$, $d=-5$, $a_1=-7$

arithmetic series

$S = \frac{n}{2}[2a_1 + (n-1)d]$

$\frac{8}{2}(-14 + (-35)) =$ **-196**

(-7) + (-12) + (-17)

⑦ $\sum_{m=-2}^{9} 2^m$ $n=12$, $r=2$, $a_1=1/4$

geometric series

$S = \frac{a_1 - a_1 r^n}{1-r} = \frac{\frac{1}{4} - \frac{1}{4}(2)^{12}}{1-2}$

1023.75

(1/4) + (1/2) + (1)

⑧ $\sum_{e=2}^{11} \frac{1}{2}(4)^{e-2}$ $n=10$, $r=4$, $a_1=1/2$

geometric series

$S = \frac{a_1 - a_1 r^n}{1-r}$

$\frac{\frac{1}{2} - \frac{1}{2}(4)^{10}}{1-4} =$ **174,762.5**

1/2 + 2 + 8

Sequence & Series

ANSWER KEY: UNIT 17 REVIEW

(9) $3+1+\frac{1}{3}+\frac{1}{9}+\frac{1}{27}+\ldots$

$a_1 = 3$ $\quad r = \frac{1}{3}$ $\quad S = \frac{a_1}{1-r} = \frac{3}{1-\frac{1}{3}} = \frac{3}{\frac{2}{3}} = \boxed{\frac{9}{2}}$

(10) $8-6+\frac{9}{2}-\frac{27}{8}+\ldots$

$a_1 = 8$ $\quad r = -\frac{3}{4}$ $\quad S = \frac{a_1}{1-r} = \frac{8}{1-(-\frac{3}{4})} = \frac{8}{\frac{7}{4}} = \boxed{\frac{32}{7}}$

(11) arithmetic series

$n = 51 \quad d = 2 \quad a_1 = 250$

$S = \frac{51}{2}[2(250)+50(2)] = \boxed{15{,}300}$

(12) infinite geometric series

$r = 9/10 \quad a_1 = 50$

$S = \frac{50}{1-9/10} = \frac{50}{.1} = \boxed{500 \text{ cm}}$

(13) $6-2-10-18-26$

arithmetic series

$n = 5$
$d = -8$
$a_1 = 6$

$\boxed{\sum_{x=1}^{5} (-8x+14)}$

(14) $20+11+2-7-16-25$

arithmetic series

$n = 6$
$d = -9$
$a_1 = 20$

$\boxed{\sum_{x=1}^{6} (-9x+29)}$

(15) $3+9+27+81+243$

geometric series

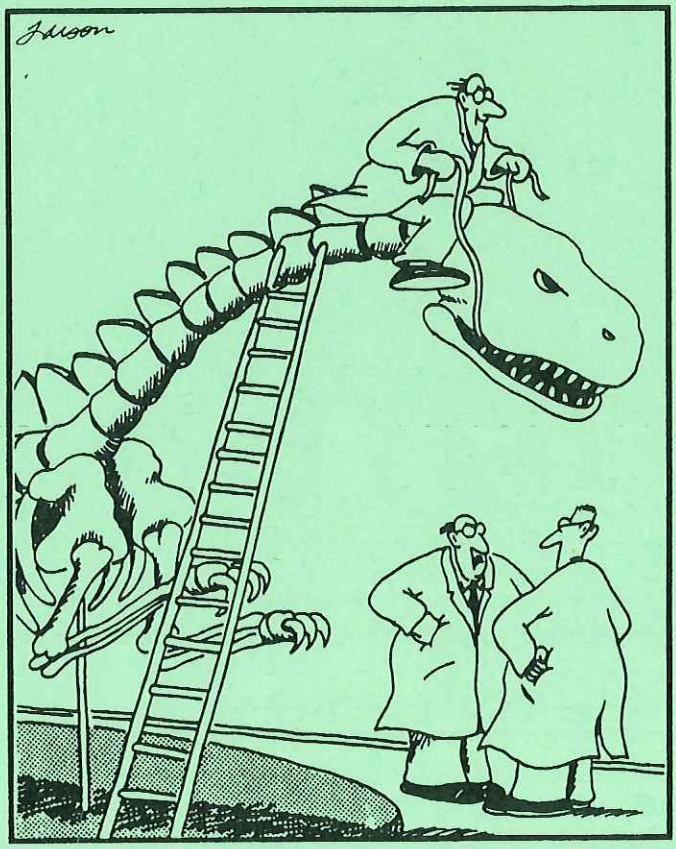
$n = 5$
$r = 3$
$a_1 = 3$

$\boxed{\sum_{x=1}^{5} 3^x}$

(16) $10-5+5/2-5/4+5/8$

$n = 5$
$r = -1/2$
$a_1 = 10$

$\boxed{\sum_{x=1}^{5} -20(-1/2)^x}$



"I assume you're being facetious, Andrews ... I distinctly yelled 'second' before you did."

Matrices

ANSWER KEY: UNIT 20 DEMONSTRATION

① $\begin{bmatrix} 2 & 3 & 9 \\ 1 & 2 & 7 \end{bmatrix}$ $\times -2$

$\times 3$ (add to row 1)

$\begin{bmatrix} -1 & 0 & 3 \\ 1 & 2 & 7 \end{bmatrix}$ (add to row 2)

$\begin{bmatrix} -1 & 0 & 3 \\ 0 & 2 & 10 \end{bmatrix}$ $\div -1$

$\div 2$

$\begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 5 \end{bmatrix}$ $x = -3$

$y = 5$

$\boxed{(-3, 5)}$

② $\begin{bmatrix} 1 & 2 & 2 & 1 \\ 3 & -1 & 2 & 8 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ $\times -1$ (add to row 2)

$\begin{bmatrix} 1 & 2 & 2 & 1 \\ 2 & -3 & 0 & 7 \\ 1 & 1 & 1 & 0 \end{bmatrix}$ $\times (-2)$ (add to row 1)

$\begin{bmatrix} -1 & 0 & 0 & 1 \\ 2 & -3 & 0 & 7 \\ 1 & 1 & 1 & 0 \end{bmatrix}$

$\boxed{(-1, -3, 4)}$

$-x = 1$ $\quad x = -1$

$2(-1) - 3y = 7$ $\quad y = -3$

$(-1) + (-3) + z = 0$ $\quad z = 4$

③ $\begin{matrix} -2(-6) + 3(9) + 0(3) \\ 4(-6) + (-1)(9) + 6(3) \end{matrix} \rightarrow -\frac{1}{3} \begin{bmatrix} 39 \\ -15 \end{bmatrix} = \begin{bmatrix} -13 \\ 5 \end{bmatrix}$

④ $\begin{matrix} -4(8) + (-5)(0) & -4(2) + (-5)(-1) \\ 2(8) + 6(0) & 2(2) + 6(-1) \end{matrix}$ $\begin{bmatrix} -32 & -3 \\ 16 & -2 \end{bmatrix}$

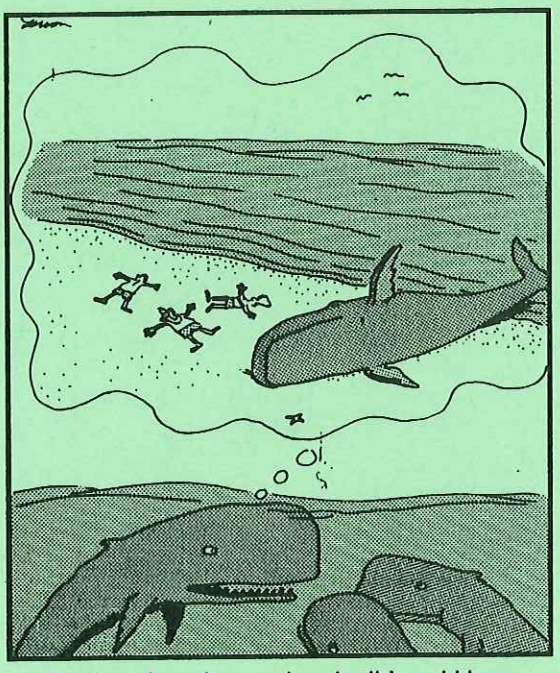
⑤ $-\begin{vmatrix} 5 & 8 \\ 6 & -2 \end{vmatrix} = 58$ $\quad +\begin{vmatrix} 4 & 8 \\ -3 & -2 \end{vmatrix} = 16$ $\quad -\begin{vmatrix} 4 & 5 \\ -3 & 6 \end{vmatrix} = -39$

$(-10) - (48)$ $\quad (-8) - (-24)$ $\quad (24) - (-15)$

$\boxed{58 \quad 16 \quad -39}$

⑥ $\begin{vmatrix} 2 & 4 \\ -7 & 6 \end{vmatrix} = (12) - (-28) = 40$ Reciprocal $1/40$

Inverse $\rightarrow \frac{1}{40} \begin{bmatrix} 6 & -4 \\ 7 & 2 \end{bmatrix}$



"And so there I was—beached! I could hear voices all around me, but I couldn't go forward or back. And then it hit me: I could *roll!*"

Matrices

ANSWER KEY: UNIT 20 DEMONSTRATION

⑦ $\begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$-\begin{bmatrix} -2 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = -\begin{bmatrix} -2 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = -\begin{bmatrix} -2 & 3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ -4 \end{bmatrix}$ **(-5, -4)**

$\begin{vmatrix} 2 & -3 \\ 1 & -2 \end{vmatrix} = (-4)-(-3) = -1$ Reciprocal -1

Inverse → $-\begin{bmatrix} -2 & 3 \\ -1 & 2 \end{bmatrix}$

Multiplying Right Side:

$\begin{matrix} -2(2)+3(3) \\ -1(2)+2(3) \end{matrix} \rightarrow -\begin{bmatrix} 5 \\ 4 \end{bmatrix} = \begin{bmatrix} -5 \\ -4 \end{bmatrix}$

⑧

1	3	0	1	3
1	4	-2	1	4
2	1	0	2	1

$(8)+(-12)+(0)-(0)$
$-(-2)-(6) = -8$
Reciprocal = $-1/8$

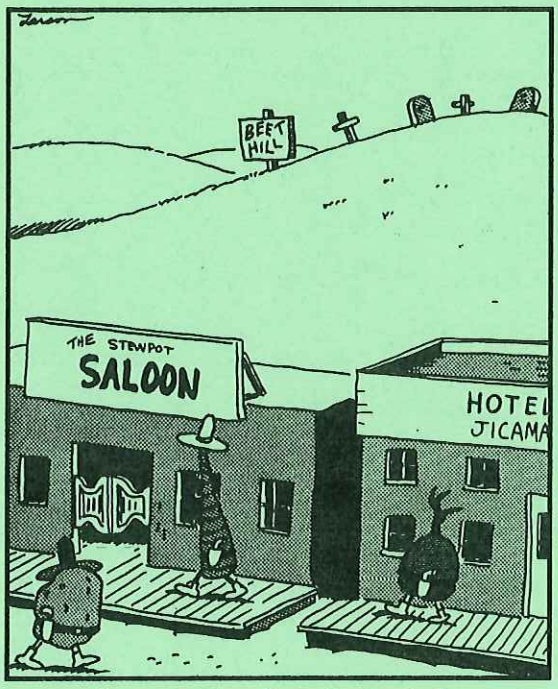
$+\begin{vmatrix} 4 & -2 \\ 1 & 2 \end{vmatrix} = 10$ $(8)-(-2)$ $\quad -\begin{vmatrix} 1 & -2 \\ 2 & 2 \end{vmatrix} = -6$ $(2)-(-4)$ $\quad +\begin{vmatrix} 1 & 4 \\ 2 & 1 \end{vmatrix} = -7$ $(1)-(8)$

$-\begin{vmatrix} 3 & 0 \\ 1 & 2 \end{vmatrix} = -6$ $(6)-0$ $\quad +\begin{vmatrix} 1 & 0 \\ 2 & 2 \end{vmatrix} = 2$ $(2)-0$ $\quad -\begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = 5$ $(1)-(6)$

$+\begin{vmatrix} 3 & 0 \\ 4 & -2 \end{vmatrix} = -6$ $(-6)-0$ $\quad -\begin{vmatrix} 1 & 0 \\ 1 & -2 \end{vmatrix} = 2$ $(-2)-0$ $\quad +\begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} = 1$ $(4)-(3)$

$\begin{bmatrix} 10 & -6 & -7 \\ -6 & 2 & 5 \\ -6 & 2 & 1 \end{bmatrix}$ Transpose → $\begin{bmatrix} 10 & -6 & -6 \\ -6 & 2 & 2 \\ -7 & 5 & 1 \end{bmatrix}$

Inverse → $\frac{-1}{8}\begin{bmatrix} 10 & -6 & -6 \\ -6 & 2 & 2 \\ -7 & 5 & 1 \end{bmatrix}$



It was no place for yellow squash.

Matrices

ANSWER KEY: UNIT 20 REVIEW

① $\begin{bmatrix} 3 & 4 & 12 \\ 5 & 2 & -8 \end{bmatrix}$ $\times -2$ (add to row 1)

$\begin{bmatrix} -7 & 0 & 28 \\ 5 & 2 & -8 \end{bmatrix}$ $\times 5$ (add to row 2)
$\times 7$

$\begin{bmatrix} -7 & 0 & 28 \\ 0 & 14 & 84 \end{bmatrix}$ $\div -7$
$\div 14$

$\begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 6 \end{bmatrix}$ $x = -4$
$y = 6$

$\boxed{(-4, 6)}$

② $\begin{bmatrix} 2 & 3 & -1 & -7 \\ 4 & -1 & -6 & 10 \\ 1 & 2 & 3 & -2 \end{bmatrix}$ $\times -6$ (add to row 2)

$\begin{bmatrix} 2 & 3 & -1 & -7 \\ -8 & -19 & 0 & 52 \\ 1 & 2 & 3 & -2 \end{bmatrix}$ $\times 3$
(add to row 1)

$\begin{bmatrix} 7 & 11 & 0 & -23 \\ -8 & -19 & 0 & 52 \\ 1 & 2 & 3 & -2 \end{bmatrix}$ $\times 19$
$\times 11$ (add to row 1)

$\begin{bmatrix} 45 & 0 & 0 & 135 \\ -8 & -19 & 0 & 52 \\ 1 & 2 & 3 & -2 \end{bmatrix}$

$\boxed{(3, -4, 1)}$

$45x = 135$ $\quad x = 3$
$-8(3) - 19y = 52$ $\quad y = -4$
$(3) + 2(-4) + 3z = -2$ $\quad z = 1$

③ $\begin{matrix} 2(-3) + 3(-5) & 2(1) + 3(-1) \\ -4(-3) + 0(-5) & -4(1) + 0(-1) \end{matrix}$ $\begin{bmatrix} -21 & -1 \\ 12 & -4 \end{bmatrix}$

④ $\begin{matrix} -2(6) + 3(-4) + 4(10) \\ 0(6) + 2(-4) + 5(10) \end{matrix} \rightarrow \frac{-1}{2} \begin{bmatrix} 16 \\ 42 \end{bmatrix} = \begin{bmatrix} -8 \\ -21 \end{bmatrix}$

⑤ $\begin{matrix} -2(-2) + (-1)(5) + 0(3) & -2(3) + (-1)(-1) + 0(0) \\ 3(-2) + (0)(5) + 4(3) & 3(3) + 0(-1) + 4(0) \\ 4(-2) + (2)(5) + (-3)(3) & 4(3) + 2(-1) + (-3)(0) \end{matrix}$

$\begin{bmatrix} -1 & -5 \\ 6 & 9 \\ -7 & 10 \end{bmatrix}$

⑥ $+\begin{vmatrix} 4 & -5 \\ -3 & 7 \end{vmatrix} = 13$ $\quad -\begin{vmatrix} 3 & -5 \\ 2 & 7 \end{vmatrix} = -31$ $\quad +\begin{vmatrix} 3 & 4 \\ 2 & -3 \end{vmatrix} = -17$

$(28) - (15)$ $\quad (21) - (-10)$ $\quad (-9) - (8)$

$\boxed{13 \quad -31 \quad -17}$

⑦ $-\begin{vmatrix} -1 & 0 \\ -3 & 7 \end{vmatrix} = 7$ $\quad +\begin{vmatrix} 6 & 0 \\ 2 & 7 \end{vmatrix} = 42$ $\quad -\begin{vmatrix} 6 & -1 \\ 2 & -3 \end{vmatrix} = 16$

$(-7) - (0)$ $\quad (42) - (0)$ $\quad (-18) - (-2)$

$\boxed{7 \quad 42 \quad 16}$

⑧ $\begin{vmatrix} 4 & 3 \\ -6 & 8 \end{vmatrix} = 50$

$(32) - (-18)$

Reciprocal
$1/50$

$\frac{1}{50} \begin{bmatrix} 8 & -3 \\ 6 & 4 \end{bmatrix}$

continued

Matrices

ANSWER KEY: UNIT 20 REVIEW

(9)

$$\begin{vmatrix} 2 & 0 & 6 & 2 & 0 \\ -4 & 3 & 8 & 4 & 3 \\ 5 & -1 & 7 & 5 & -1 \end{vmatrix}$$

$(42) + (0) + (24) - (90) - (-16) - (0) = -8$

Reciprocal $= -1/8$

$$+\begin{vmatrix} 3 & 8 \\ -1 & 7 \end{vmatrix} = 29 \quad (21) - (-8)$$

$$-\begin{vmatrix} -4 & 8 \\ 5 & 7 \end{vmatrix} = 68 \quad (-28) - (40)$$

$$+\begin{vmatrix} -4 & 3 \\ 5 & -1 \end{vmatrix} = -11 \quad (4) - (15)$$

$$-\begin{vmatrix} 0 & 6 \\ -1 & 7 \end{vmatrix} = -6 \quad 0 - (-6)$$

$$+\begin{vmatrix} 2 & 6 \\ 5 & 7 \end{vmatrix} = -16 \quad (14) - (30)$$

$$-\begin{vmatrix} 2 & 0 \\ 5 & -1 \end{vmatrix} = 2 \quad (-2) - 0$$

$$+\begin{vmatrix} 0 & 6 \\ 3 & 8 \end{vmatrix} = -18 \quad 0 - (18)$$

$$-\begin{vmatrix} 2 & 6 \\ -4 & 8 \end{vmatrix} = -40 \quad (16) - (-24)$$

$$+\begin{vmatrix} 2 & 0 \\ -4 & 3 \end{vmatrix} = 6 \quad (6) - 0$$

$$\begin{bmatrix} 29 & 68 & -11 \\ -6 & -16 & 2 \\ -18 & -40 & 6 \end{bmatrix} \xrightarrow{\text{Transpose}} \begin{bmatrix} 29 & -6 & -18 \\ 68 & -16 & -40 \\ -11 & 2 & 6 \end{bmatrix}$$

$$\text{Inverse} \rightarrow \frac{-1}{8}\begin{bmatrix} 29 & -6 & -18 \\ 68 & -16 & -40 \\ -11 & 2 & 6 \end{bmatrix}$$

(10)

$$\begin{bmatrix} 3 & 5 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 15 \end{bmatrix}$$

$$\frac{-1}{14}\begin{bmatrix} -3 & -5 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \frac{-1}{14}\begin{bmatrix} -3 & -5 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{-1}{14}\begin{bmatrix} -3 & -5 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 15 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \end{bmatrix} \qquad \boxed{(6, -3)}$$

Reciprocal of Determinant:

$$\begin{vmatrix} 3 & 5 \\ 1 & -3 \end{vmatrix} = (-9) - (5) = -14$$

Reciprocal $-1/14$

Inverse of Coefficient Matrix:

$$\frac{-1}{14}\begin{bmatrix} -3 & -5 \\ -1 & 3 \end{bmatrix}$$

Multiplying Right side of Equation:

$$\begin{matrix} -3(3) + (-5)(15) \\ -1(3) + (3)(15) \end{matrix} \rightarrow \frac{-1}{14}\begin{bmatrix} -84 \\ 42 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \end{bmatrix}$$

Logarithms

ANSWER KEY: UNIT 21 DEMONSTRATION

① $5^2=25 \rightarrow \boxed{\log_5 25 = 2}$

② $\log_9 3 = \frac{1}{2} \rightarrow \boxed{9^{1/2} = 3}$

③ $\log_2 (3x-2) = \log_2 (2x+6)$

$3x-2 = 2x+6 \rightarrow \boxed{x=8}$

④ $\log_9 (x+4) + \log_9 (x-4) = 1$

$\log_9 (x+4)(x-4) = 1$

$9^1 = x^2 - 16$

$x^2 - 25 = 0$

$(x+5)(x-5) = 0$

$x = 5, -5$ $\boxed{x=5}$

⑤ $\log 51.2$ $\quad 51.2 = 5.12 \times 10^1$

mantissa $5.12 = .7093$

characteristic $= 1$ $\boxed{1.7093}$

⑥ $\log_3 12 = x$

$3^x = 12$

$\log 3^x = \log 12$

$x \log 3 = \log 12$

$x = \dfrac{\log 12}{\log 3}$

$x = \boxed{2.2619}$

⑦ $27^{2x+1} = 2^{4x-1}$

$\log 27^{2x+1} = \log 2^{4x-1}$

$(2x+1)\log 27 = (4x-1)\log 2$

$(2x)\log 27 + \log 27 = (4x)\log 2 - \log 2$

$(2x)\log 27 - (4x)\log 2 = -\log 2 - \log 27$

$x(2\log 27 - 4\log 2) = -\log 2 - \log 27$

$x = \dfrac{-\log 2 - \log 27}{\log 27^2 - \log 2^4}$

$x = \dfrac{-\log 2 - \log 27}{\log 729 - \log 16} \approx \boxed{-1.044}$

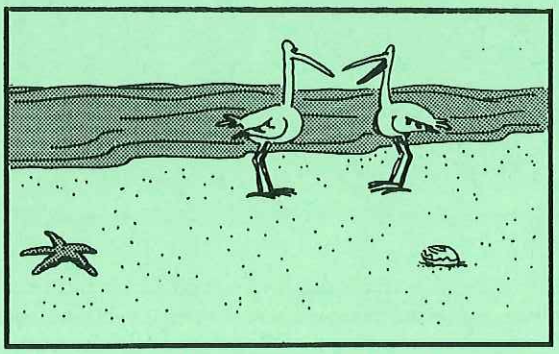
⑧ $\log_2 [\log_9 (\log_3 x)] = -1$

$2^{-1} = \log_9 (\log_3 x)$

$\log_9 (\log_3 x) = \frac{1}{2}$

$9^{1/2} = \log_3 x \qquad 3^3 = x$

$\log_3 x = 3 \qquad \boxed{x = 27}$



"You're not fooling me, Ned. ... Taking a long walk on the beach sounds romantic, but I know you're just looking for crustaceans."

Logarithms

ANSWER KEY: UNIT 21 REVIEW

① $3^{-2} = 1/9 \rightarrow \boxed{\log_3 1/9 = -2}$

② $\log_4 2 = 1/2 \rightarrow \boxed{4^{1/2} = 2}$

③ $\log_4(1-2x) = \log_4(x+10)$

$1-2x = x+10$

$-3x = 9 \rightarrow \boxed{x = -3}$

④ $\log_6(n-3) + \log_6(n+2) = \log_3 3$

$\log_6(n-3)(n+2) = 1$

$6^1 = n^2 - n - 6$

$n^2 - n - 12 = 0$

$(n-4)(n+3) = 0$

$n = 4$ or -3 $\boxed{n=4}$

⑤ $2\log_2 x - \frac{1}{2}\log_2 16 = 4$

$\log_2 x^2 - \log_2 16^{1/2} = 4$

$\log_2\left(\frac{x^2}{4}\right) = 4$

$2^4 = \frac{x^2}{4}$

$16 = \frac{x^2}{4}$ $\boxed{x = 8}$

⑥ $\log 437 \rightarrow 437 = 4.37 \times 10^2$

mantissa $4.37 = .6405$

characteristic $= 2$ $\boxed{2.6405}$

⑦ $\log x = .5011 - 2$

mantissa $.5011 = 3.17$

characteristic -2 $\boxed{.0317}$

⑧ $\log_3 8 = x$

$3^x = 8$

$\log 3^x = \log 8$

$x \log 3 = \log 8$

$x = \frac{\log 8}{\log 3} = \boxed{1.8928}$

⑨ $4^{3x-2} = 6^{2x+1}$

$\log 4^{3x-2} = \log 6^{2x+1}$

$(3x-2)\log 4 = (2x+1)\log 6$

$(3x)\log 4 - (2)\log 4 = (2x)\log 6 + \log 6$

$(3x)\log 4 - (2x)\log 6 = 2\log 4 + \log 6$

$x(3\log 4 - 2\log 6) = 2\log 4 + \log 6$

$x = \frac{\log 4^2 + \log 6}{\log 4^3 - \log 6^2} = \frac{\log 16 + \log 6}{\log 64 - \log 36}$

$\boxed{x = 7.9330}$

⑩ $\log_3[\log_8(\log_3 x)] = -1$

$3^{-1} = \log_8(\log_3 x)$

$\log_8(\log_3 x) = 1/3$

$8^{1/3} = \log_3 x$

$\log_3 x = 2$

$3^2 = x \rightarrow \boxed{x = 9}$

Conics

ANSWER KEY: UNIT 22 DEMONSTRATION

① $y^2-8y-8x+56=0$

$8x = y^2-8y+56$

$8x = y^2-8y+16+56-16$

$8x = (y-4)^2+40$

$\boxed{x = \frac{1}{8}(y-4)^2+5}$

a) (5,4)

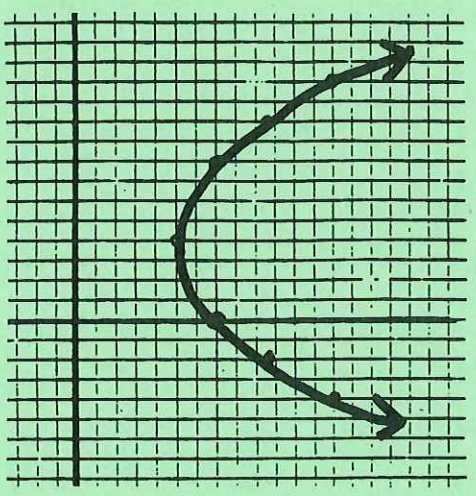
b) right

c) $y=4$

d) 8

e) $x=3$

f) (7,4)



② $x^2+y^2+6x-4y-7=0$

$x^2+6x \quad +y^2-4y = 7$

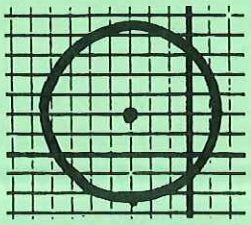
$(x^2+6x+9)+(y^2-4y+4)=7+9+4$

$\boxed{(x+3)^2+(y-2)^2=20}$

a) (-3, 2)

b) $r = 2\sqrt{5}$

$r \approx 4.47$



③ $16x^2+9y^2-96x-72y+144=0$

$16x^2-96x \quad +9y^2-72y \quad = -144$

$16(x^2-6x+9)+9(y^2-8y+16)=-144+144+144$

$16(x-3)^2+9(y-4)^2=144$

$\boxed{\frac{(x-3)^2}{9}+\frac{(y-4)^2}{16}=1}$

$a = 4$

$b = 3$

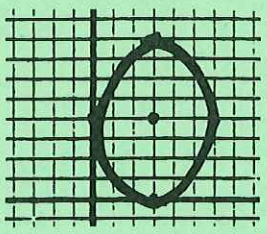
$c = \sqrt{7}$

a) (3,4)

b) 8

c) 6

d) $(3, 4\pm\sqrt{7})$



④ $3y^2-x^2-6y-10x-34=0$

$3y^2-6y \quad -x^2-10x = 34$

$3(y^2-2y+1)-(x^2+10x+25)=34+3-25$

$3(y-1)^2-(x+5)^2=12$

$\boxed{\frac{(y-1)^2}{4}-\frac{(x+5)^2}{12}=1}$

$a = 2$

$b = 2\sqrt{3}$

$c = 4$

a) (-5,1)

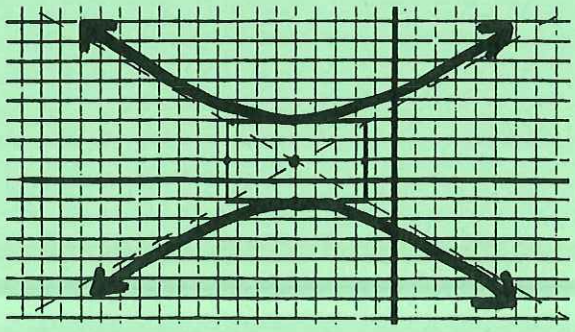
b) 4

c) $4\sqrt{3}$

d) (-5,5)(-5,-3)

e) $\pm\sqrt{3}/3$

f) $y-1 = \pm\frac{\sqrt{3}}{3}(x+5)$



Conics

ANSWER KEY: UNIT 22 REVIEW

① $x^2+6x+6y+3=0$

$-6y = x^2+6x+3$

$-6y = x^2+6x+9+3-9$

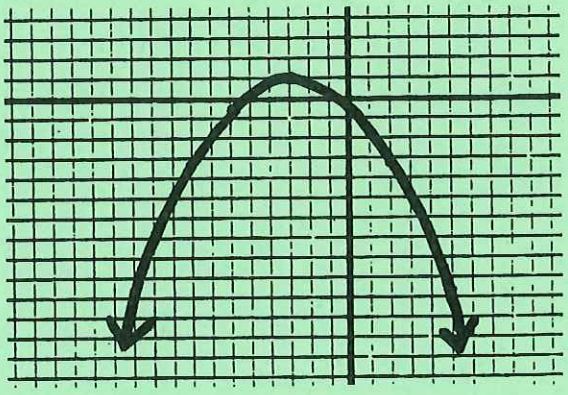
$-6y = (x+3)^2-6$

$$\boxed{y = \frac{-1}{6}(x+3)^2+1}$$

a) $(-3,1)$ d) 6

b) down e) $y = 2\frac{1}{2}$

c) $x=-3$ f) $(-3,-\frac{1}{2})$



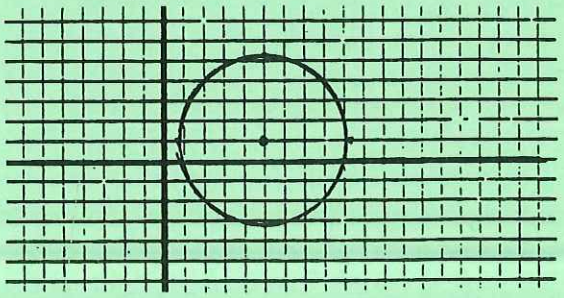
② $x^2+y^2-10x-2y+8=0$

$x^2-10x \quad +y^2-2y = -8$

$(x^2-10x+25)+(y^2-2y+1) = -8+25+1$

$$\boxed{(x-5)^2+(y-1)^2 = 18}$$

a) $(5,1)$ b) $r = 3\sqrt{2} \approx 4.24$



③ $4x^2+25y^2-16x+350y+1141=0$

$4x^2-16x \quad +25y^2+350y = -1141$

$4(x^2-4x+4)+25(y^2+14y+49) = -1141+16+1225$

$4(x-2)^2+25(y+7)^2 = 100$

$$\boxed{\frac{(x-2)^2}{25}+\frac{(y+7)^2}{4}=1}$$

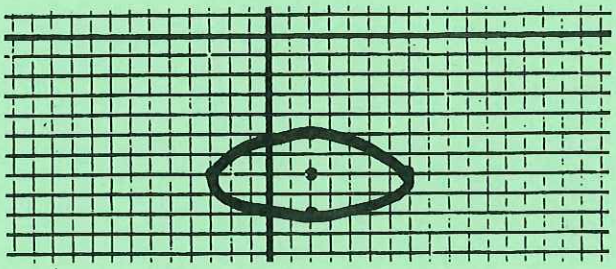
$a=5$

$b=2$

$c=\sqrt{21}$

a) $(2,-7)$ c) 4

b) 10 d) $(2\pm\sqrt{21},-7)$



④ $4x^2-9y^2+24x+108y-324=0$

$4x^2+24x \quad -9y^2+108y = 324$

$4(x^2+6x+9)-9(y^2-12y+36) = 324+36-324$

$4(x+3)^2-9(y-6)^2 = 36$

$$\boxed{\frac{(x+3)^2}{9}-\frac{(y-6)^2}{4}=1}$$

$a=3$

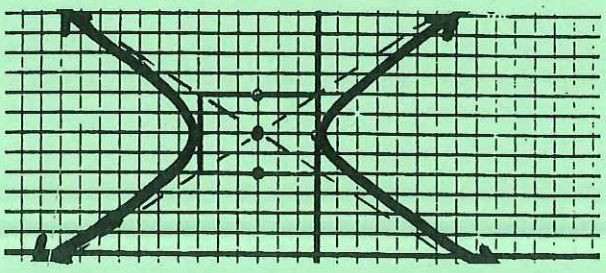
$b=2$

$c=\sqrt{13}$

a) $(-3,6)$ d) $(-3\pm\sqrt{13},6)$

b) 6 e) $\pm 2/3$

c) 4 f) $y-6 = \pm\frac{2}{3}(x+3)$



Conics

ANSWER KEY: UNIT 22 REVIEW

⑤ Parabola with focus (1,5) and directrix $x=3$

$(h,k) = (2,5)$
latus is 4 units
opens left

$x = -\frac{1}{4}(y-5)^2 + 2$

⑥ Ellipse: foci (3,1) (3,9) minor axis of length 6

$(h,k) = (3,5)$
$c = 4 \quad b = 3 \quad c^2 = a^2 - b^2$
$a = 5 \qquad 16 = a^2 - 9$
vertical

$\frac{(x-3)^2}{9} + \frac{(y-5)^2}{25} = 1$

⑦ Circle: center (4,3) passing through (6,8)

distance formula:
$r = \sqrt{(4-6)^2 + (3-8)^2}$
$r = \sqrt{29}$

$(x-4)^2 + (y-3)^2 = 29$

⑧ Hyperbola: foci $(6, -2 \pm \sqrt{34})$ trans. axis = 6 units

vertical hyperbola
$c = \sqrt{34} \quad a = 3 \quad c^2 = a^2 + b^2$
$b = 5 \qquad 34 = 9 + b^2$

$(h,k) = (6,-2)$

$\frac{(y+2)^2}{9} - \frac{(x-6)^2}{25} = 1$

⑨ $(h,k) = (-3,-1) \quad r = 6$
$(x+3)^2 + (y+1)^2 = 36$

⑩ $(h,k) = (6,2) \quad$ latus $= 12$
$x = \frac{1}{12}(y-2)^2 + 6$

⑪ $(h,k) = (-2,4) \quad a = 5 \quad b = 2$
$\frac{(x+2)^2}{25} + \frac{(y-4)^2}{4} = 1$

⑫ $(h,k) = (-3,2) \quad a = 3 \quad b = 4$
$\frac{(y-2)^2}{9} - \frac{(x+3)^2}{16} = 1$