Algebra II

REVIEW & PRACTICE

Polynomials

① Factor completely:
$6x^5y^2 + 48x^2y^5$

② Simplify:
$\left(\frac{ab^{-2}}{c^2}\right)^{-1}\left(\frac{a^{-2}c}{b}\right)^{-2}$

③ Synthetic division:
$(8x^3+6x-2)(2x+1)^{-1}$

Systems

④ Solve using Gramer's Rule:

$3x-2y=4$
$x-3y=13$

⑤ Determine the value using expansion of minors:

$$\begin{vmatrix} 3 & -1 & 3 \\ -2 & 0 & -2 \\ 1 & 4 & 3 \end{vmatrix}$$

⑥ Determine x using diagonals:

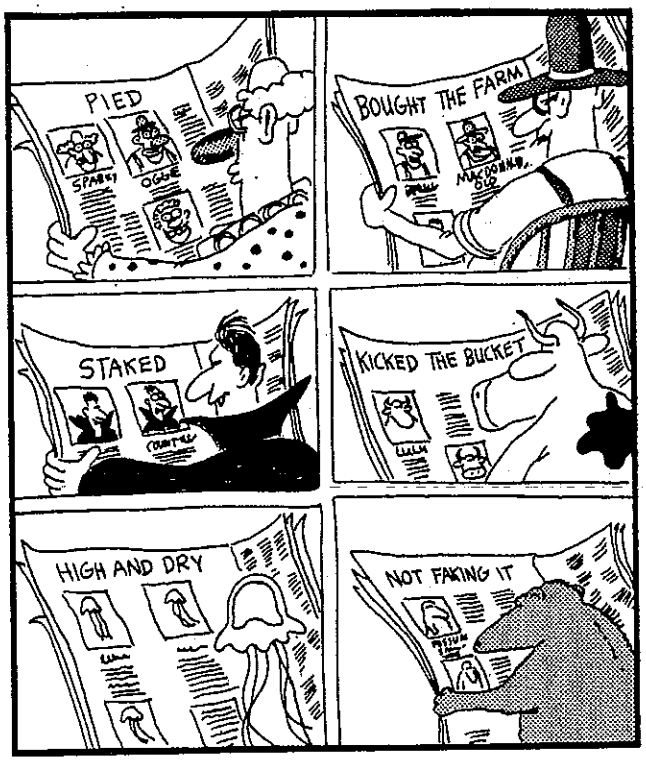
$$\begin{aligned} 2x-y+z &= -7 \\ x-2y+z &= -6 \\ 3x+y-2z &= -9 \end{aligned}$$

Inequalities & Coordinate Graphing

⑦ Solve and graph on number line:
$|x-4| < 6$

⑧ Write a linear equation in point-slope form for a line perpendicular to $2x-y=6$ through $(-4, 5)$

⑨ Graph the system:
$3x-4y \leq 12$
$y \geq -2x-8$



Specialized obituaries

Roots and Radicals

⑩ Simplify $\sqrt[4]{32a^2b^6c^5d^9}$

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(11) Rationalize and simplify: $\frac{2+2\sqrt{5}}{\sqrt{5}-2}$

(12) Solve: $\sqrt{2n+10}+3=5$

Rational Exponents & Complex Numbers

(13) Express in radical form:
$a^{2/3}\, b^{3/2}\, c^{5/6}$

(14) Simplify: $(\sqrt{-3})(\sqrt{-2})(\sqrt{-8})$

(15) Simplify: $\frac{2-2i}{2i+2}$

Quadratics

(16) Solve by completing the square:
$2x^2+3x+10=0$

(17) a) Use the discriminant to determine the nature of the roots:
$3x^2-4x+2=0$

b) Write an equation with these roots:
$\frac{1}{2}, -2$

Rational Expressions

(18) Divide and simplify:
$\frac{4n-24}{n^2-36} \div \frac{4n}{n^2+6n}$

(19) Simplify the complex fraction:
$\dfrac{3+\frac{5}{n+2}}{3-\frac{10}{n+7}}$

(20) Solve the equation:
$\frac{n}{n-5}+\frac{17}{25-n^2}=\frac{1}{n+5}$

Sequence and Series

(21) Expand: $(2a-b)^7$

(22) Indicate the sum and first 3 terms:
$\sum_{x=3}^{12} -4x+7$

(23) Write the series in summation notation:
$\frac{1}{3}-1+3-9+27-81$

(24) Write the 9th term of $(x-2y)^{11}$

(25) Find the 21st term of:
$-64, -58, -52, \ldots$

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Matrices

(26) Solve using an augmented matrix and substitution:

$$2a - b + 3c = 9$$
$$a + b + 4c = 9$$
$$3a - 2b - c = -2$$

(27) Find the product:

$$\begin{bmatrix} 3 & 0 & -2 \\ 2 & 5 & 3 \\ -1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 0 & 4 \\ -1 & -2 \end{bmatrix}$$

(28) Solve using an inverse matrix:

$$3x - 2y = 4$$
$$x - 3y = 13$$

Logarithms

(29) Round to 4 decimal places: $\log_4 18$

(30) Solve:

$$\log_2(x+3) - \log_2(5x+7) = -2$$

(31) Solve: (round to 1/1000)

$$4^{3x+1} = 3^{x+5}$$

Conics

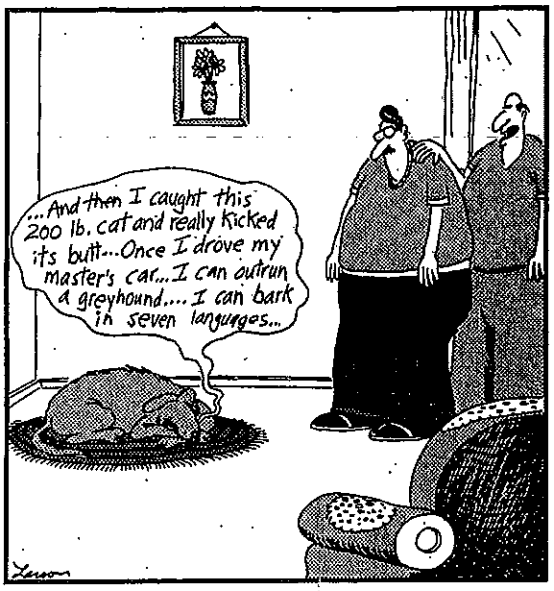
(32) Determine the equation of a parabola with a focus at (1,5) and a directrix of $x = 7$

(33) $9x^2 + 4y^2 + 54x - 48y + 189 = 0$

a) vertex form
b) center
c) length of major/minor axis
d) foci

(34) $3y^2 - 2x^2 + 6y + 4x - 17 = 0$

a) vertex form
b) length of transverse axis
c) length of conjugate axis
d) vertices
e) foci



"Edgar! Leave him be! ... Always best to let sleeping dogs lie."

Algebra II

REVIEW & PRACTICE - ANSWER KEY

① $6x^5y^2 + 48x^2y^5$

$6x^2y^2(x^3+8y^3)$

$6x^2y^2(x+2y)(x^2-2xy+4y^2)$

② $\left(\frac{ab^{-2}}{c^2}\right)^{-1}\left(\frac{a^{-2}c}{b}\right)^{-2}$

$\left(\frac{a^{-1}b^2}{c^{-2}}\right)\left(\frac{a^4c^{-2}}{b^{-2}}\right)$

$\left(\frac{c^2b^2}{a}\right)\left(\frac{a^4b^2}{c^2}\right)$

$\frac{a^4b^4c^2}{ac^2} = a^3b^4$

③ $(8x^3+6x-2)(2x+1)^{-1}$

$(4x^3+3x-1)(x+\frac{1}{2})^{-1}$

$-\frac{1}{2}$ | 4 0 3 −1

	−2	1	−2
4	−2	4	−3

$(-3)\times 2$

$4x^2-2x+4-\frac{6}{2x+1}$

④ $3x-2y=4$

$x-3y=13$

$$x=\frac{\begin{vmatrix}4&-2\\13&-3\end{vmatrix}}{\begin{vmatrix}3&-2\\1&-3\end{vmatrix}}=\frac{(-12)-(-26)}{(-9)-(-2)}=\frac{14}{-7}=-2$$

$$y=\frac{\begin{vmatrix}3&4\\1&13\end{vmatrix}}{-7}=\frac{(39)-(4)}{-7}=\frac{35}{-7}=-5$$

$(-2,-5)$

⑤ $\begin{vmatrix}3&-1&3\\-2&0&-2\\1&4&3\end{vmatrix}$

$3\begin{vmatrix}0&-2\\4&3\end{vmatrix}-(-1)\begin{vmatrix}-2&-2\\1&3\end{vmatrix}+3\begin{vmatrix}-2&0\\1&4\end{vmatrix}$

$(0)-(-8)\quad (-6)-(-2)\quad (-8)-(0)$

$(3)(8)-(-1)(-4)+(3)(-8)$

$24-(4)+(-24)$

-4

⑥ $2x-y+z=-7$

$x-2y+z=-6$

$3x+y-2z=-9$

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REVIEW & PRACTICE - ANSWER KEY

$$x = \frac{\begin{vmatrix} -7 & -1 & 1 & -7 & -1 \\ -6 & -2 & 1 & -6 & -2 \\ -9 & 1 & -2 & -9 & 1 \end{vmatrix}}{\begin{vmatrix} 2 & -1 & 1 & 2 & -1 \\ 1 & -2 & 1 & 1 & -2 \\ 3 & 1 & -2 & 3 & 1 \end{vmatrix}}$$

$$\frac{(-28)+(9)+(-6)-(18)-(-7)-(-12)}{(8)+(-3)+(1)-(-6)-(2)-(2)}$$

$$\frac{-24}{8} = -3 \qquad x = -3$$

⑦ $|x-4|<6$

$x-4<6$ and $x-4>-6$

$x<10$ and $x>-2$

$\boxed{-2<x<10}$

⑧ $2x-y=6$

Slope $-\frac{A}{B} = \frac{-2}{-1} = 2$

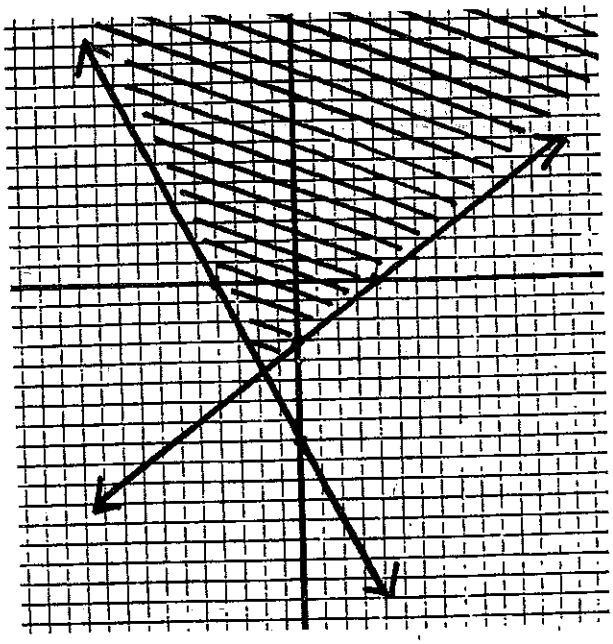
perpendicular slope $= -\frac{1}{2}$

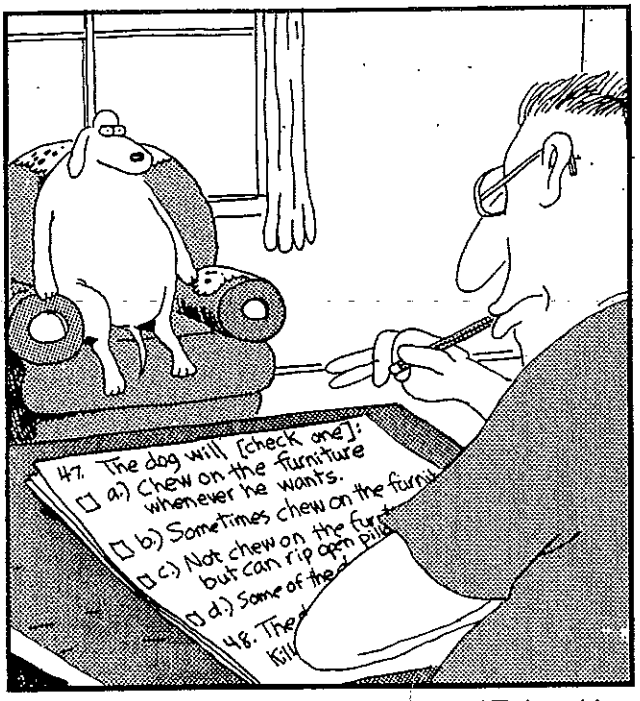
point-slope form:

$\boxed{y-5=-\frac{1}{2}(x+4)}$

⑨ $3x-4y \leq 12 \rightarrow y \geq \frac{3}{4}x-3$

$y \geq -2x-8$





The questions were getting harder, and Ted could feel Lucky's watchful glare from across the room. He had been warned, he recalled, that this was a breed that would sometimes test him.

Algebra II

REVIEW & PRACTICE - ANSWER KEY

⑩ $\sqrt[4]{32a^2b^6c^5d^9}$

$2|b|c|d^2\sqrt[4]{2a^2b^2cd}$

⑪ $\frac{2+2\sqrt{5}}{\sqrt{5}-2} \cdot \frac{\sqrt{5}+2}{\sqrt{5}+2}$

$\frac{2\sqrt{5}+4+10+4\sqrt{5}}{5-4} = 6\sqrt{5}+14$

⑫ $\sqrt{2n+10}+3=5$

$\sqrt{2n+10}=2$

$2n+10=4$

$2n=-6$

$n=-3$

⑬ $a^{2/3}b^{3/2}c^{5/6}$

$a^{4/6}b^{9/6}c^{5/6}$

$b\sqrt[6]{a^4b^3c^5}$

⑭ $(\sqrt{-3})(\sqrt{-2})(\sqrt{-8})$

$(i\sqrt{3})(i\sqrt{2})(2i\sqrt{2})$

$(2i^3\sqrt{12})$

$(4i^3\sqrt{3}) = -4i\sqrt{3}$

⑮ $\frac{2-2i}{2i+2}$

$\frac{2-2i}{2i+2} \cdot \frac{2i-2}{2i-2}$

$\frac{4i-4-4i^2+4i}{4i^2-4}$

$\frac{-4i^2+8i-4}{4i^2-4}$

$\frac{4+8i-4}{-4-4} = \frac{8i}{-8} = -i$

⑯ $2x^2+3x+10=0$

$x^2+\frac{3}{2}x+5=0$

$x^2+\frac{3}{2}x=-5$

$x^2+\frac{3}{2}x+\frac{9}{16}=-5+\frac{9}{16}$

$(x+\frac{3}{4})^2=\frac{-80+9}{16}$

$(x+\frac{3}{4})^2=\frac{-71}{16}$

$x+\frac{3}{4}=\pm\frac{i\sqrt{71}}{4}$

$\boxed{x=\frac{-3\pm i\sqrt{71}}{4}}$

Algebra II

REVIEW & PRACTICE - ANSWER KEY

(17) a) $3x^2-4x+2=0$

$b^2-4ac=(-4)^2-4(3)(2)$

discriminant $=16-24=-8$

no real roots (or two imaginary roots)

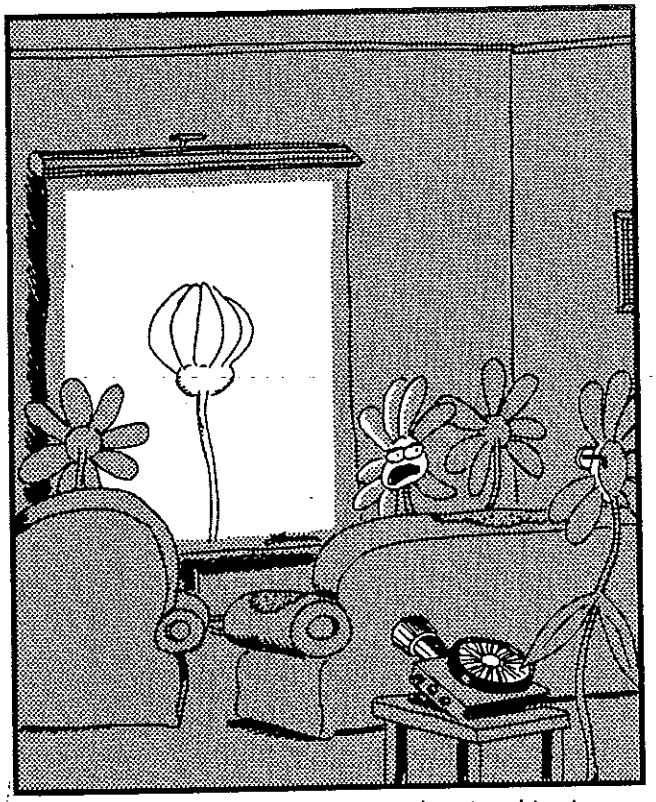
b) $\frac{1}{2}, -2$

sum $-\frac{3}{2}$

prod -1

$x^2+\frac{3}{2}x-1=0$

$\boxed{2x^2+3x-2=0}$



"Dang it, Morty! ... You're always showing this picture of me you took at 7 o'clock in the morning!"

(18) $\frac{4n-24}{n^2-36} \div \frac{4n}{n^2+6n}$

$\frac{4(n-6)}{(n+6)(n-6)} \cdot \frac{n(n+6)}{4n} = \boxed{1}$

(19) $\dfrac{3+\frac{5}{n+2}}{3-\frac{10}{n+7}}$

$\dfrac{\frac{3(n+2)+5}{n+2}}{\frac{3(n+7)-10}{n+7}} = \dfrac{\frac{3n+6+5}{n+2}}{\frac{3n+21-10}{n+7}}$

$\dfrac{\frac{3n+11}{n+2}}{\frac{3n+11}{n+7}} = \frac{3n+11}{n+2} \cdot \frac{n+7}{3n+11}$

$\boxed{\frac{n+7}{n+2}}$

Continued

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REVIEW & PRACTICE - ANSWER KEY

(20) $\frac{n}{n-5} + \frac{17}{25-n^2} = \frac{1}{n+5}$

$\frac{n}{(n-5)} + \frac{17}{(5+n)(5-n)} = \frac{1}{(n+5)}$

$\left[\frac{n}{(n-5)} - \frac{17}{(n+5)(n-5)} = \frac{1}{(n+5)}\right](n+5)(n-5)$

$n(n+5) - 17 = 1(n-5)$

$n^2+5n-17 = n-5$

$n^2+4n-12 = 0$

$(n+6)(n-2) = 0$

$\boxed{n = -6 \text{ or } 2}$

After many years of marital bliss, tension enters the Kent household.

(21) $(2a-b)^7$ 1 7 $(\frac{6}{2})$ 21 $(\frac{5}{3})$ 35 35 21 7 1

$(2a)^7 - 7(2a)^6b + 21(2a)^5b^2 - 35(2a)^4b^3 + 35(2a)^3b^4 - 21(2a)^2b^5 + 7(2a)b^6 - b^7$

$\boxed{128a^7 - 448a^6b + 672a^5b^2 - 560a^4b^3 + 280a^3b^4 - 84a^2b^5 + 14ab^6 - b^7}$

(22) $\sum_{x=3}^{12} -4x+7$

$n=10$
$d=-4$
$a_1=-5$

-5
-9
-13

$\frac{10}{2}[2(-5)+9(-4)]$

$\boxed{\text{Sum} = -230}$

(23) geometric

$n=6$
$r=-3$
$a_1=1/3$

$\sum_{x=1}^{6} \frac{-1}{9}(-3)^x$

(24) $(x-2y)^{11}$ 9th term

$\frac{11!}{8!\,3!}(x)^3(-2y)^8 = 42{,}240x^3y^8$

(25) $-64, -58, -52$ 21st term
arithmetic sequence $d=6$

$(-64)+(20)(6) = 56$ -or-

$\sum_{x=1}^{21} 6x-70 \rightarrow 6(21)-70 = 56$

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REVIEW & PRACTICE - ANSWER KEY

(26) $\begin{bmatrix} 2 & -1 & 3 & 9 \\ 1 & 1 & 4 & 9 \\ 3 & -2 & -1 & -2 \end{bmatrix}$ $\times 4$ add to row 2

$\begin{bmatrix} 2 & -1 & 3 & 9 \\ 13 & -7 & 0 & 1 \\ 3 & -2 & -1 & -2 \end{bmatrix}$ $\times 3$ add to row 1

$\begin{bmatrix} 11 & -7 & 0 & 3 \\ 13 & -7 & 0 & 1 \\ 3 & -2 & -1 & -2 \end{bmatrix}$ $\times -1$ add to row 1

$\begin{bmatrix} -2 & 0 & 0 & 2 \\ 13 & -7 & 0 & 1 \\ 3 & -2 & -1 & -2 \end{bmatrix}$ $\div -2$

$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 13 & -7 & 0 & 1 \\ 3 & -2 & -1 & -2 \end{bmatrix}$

$a = -1$

$13(-1) - 7b = 1 \quad b = -2$

$3(-1) - 2(-2) - c = -2 \quad c = 3$

$\boxed{(-1, -2, 3)}$

(27) $\begin{bmatrix} 3 & 0 & -2 \\ 2 & 5 & 3 \\ -1 & 2 & 4 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 0 & 4 \\ -1 & -2 \end{bmatrix} = \begin{bmatrix} 11 & 10 \\ 3 & 18 \\ -7 & -2 \end{bmatrix}$

$3(3) + 0(0) - 2(-1) \quad 3(2) + 0(4) - 2(-2)$

$2(3) + 5(0) + 3(-1) \quad 2(2) + 5(4) + 3(-2)$

$-1(3) + 2(0) + 4(-1) \quad -1(2) + 2(4) + 4(-2)$

(28) $\begin{bmatrix} 3 & -2 \\ 1 & -3 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 13 \end{bmatrix}$ inverse $\frac{-1}{7}\begin{bmatrix} -3 & 2 \\ -1 & 3 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{-1}{7}\begin{bmatrix} -3 & 2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 13 \end{bmatrix}$

$-3(4) + 2(13) = 14$

$-1(4) + 3(13) = 35$

$\frac{-1}{7}\begin{bmatrix} 14 \\ 35 \end{bmatrix} = \begin{bmatrix} -2 \\ -5 \end{bmatrix}$

(29) $\log_4 18 = \frac{\log 18}{\log 4} \approx \boxed{2.0850}$

(30) $\log_2(x+3) - \log_2(5x+7) = -2$

$\log_2 \frac{x+3}{5x+7} = -2$

$2^{-2} = \frac{x+3}{5x+7} \qquad \frac{x+3}{5x+7} = \frac{1}{4}$

$4x + 12 = 5x + 7 \qquad \boxed{x = 5}$

(31) $4^{3x+1} = 3^{x+5}$

$\log 4^{3x+1} = \log 3^{x+5}$

$(3x+1)\log 4 = (x+5)\log 3$

$3x \log 4 + \log 4 = x \log 3 + 5 \log 3$

$3x \log 4 - x \log 3 = 5 \log 3 - \log 4$

continued

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REVIEW & PRACTICE - ANSWER KEY

$x(3\log 4 - \log 3) = 5\log 3 - \log 4$

$x = \dfrac{\log 3^5 - \log 4}{\log 4^3 - \log 3}$ $\boxed{x \approx 1.342}$

(32) horizontal parabola: center is equidistant between $x=1$ and $x=7$ $(h,k) = (4,5)$. It will open left. Distance from vertex to focus is 3 so the latus is 12.

$\boxed{x = \frac{-1}{12}(y-5)^2 + 4}$

(33) $9x^2 + 4y^2 + 54x - 48y + 189 = 0$

$9x^2 + 54x \quad + 4y^2 - 48y \quad = -189$

$9(x^2+6x+9) + 4(y^2-12y+36) = -189 + 81 + 144$

$9(x+3)^2 + 4(y-6)^2 = 36$

a) $\boxed{\dfrac{(x+3)^2}{4} + \dfrac{(y-6)^2}{9} = 1}$

b) $\boxed{(-3, 6)}$

c) $a = 3 \quad b = 2$

$\boxed{\begin{array}{l}\text{major axis} = 6 \\ \text{minor axis} = 4\end{array}}$

d) $c = \sqrt{5}$

ellipse is vertical

$\boxed{\text{foci } (-3, 6 \pm \sqrt{5})}$

(34) $3y^2 - 2x^2 + 6y + 4x - 17 = 0$

$3y^2 + 6y \quad -2x^2 + 4x \quad = 17$

$3(y^2+2y+1) - 2(x^2-2x+1) = 17 + 3 - 2$

$3(y+1)^2 - 2(x-1)^2 = 18$

a) $\boxed{\dfrac{(y+1)^2}{6} - \dfrac{(x-1)^2}{9} = 1}$

b) $a = \sqrt{6} \quad b = 3$

transverse axis is vert.

$\boxed{2a = 2\sqrt{6}}$

c) $\boxed{2b = 6}$

d) $\boxed{((1, -1 \pm \sqrt{6}))}$ $(h, k) = (1, -1)$

e) $c = \sqrt{15}$

$\boxed{\text{Foci} = (1, -1 \pm \sqrt{15})}$