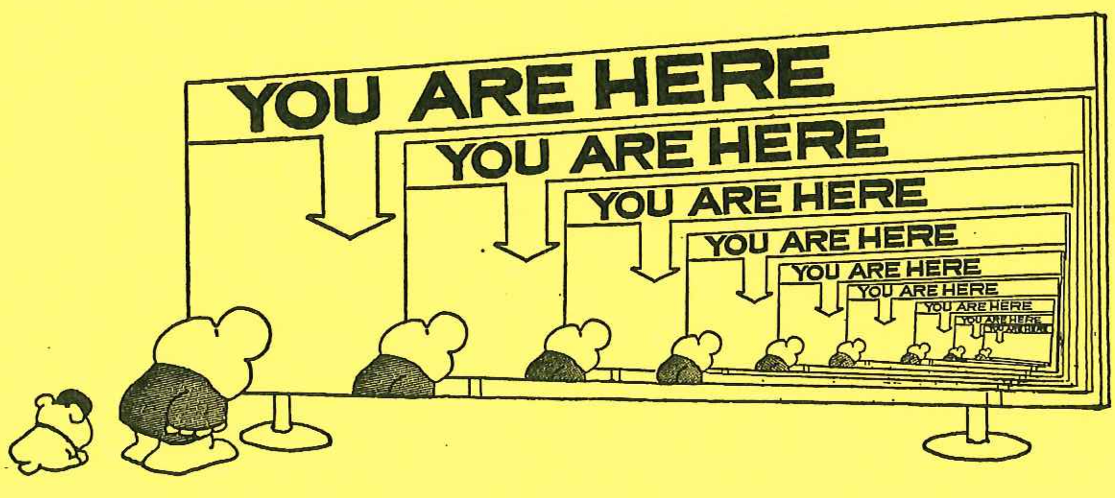
Friendship Junior High School
Sixth Grade Accelerated Math Program

Room 102A (Mr. Lavine)

Critical Thinking & Problem Solving



<u>Special 3rd Quarter Unit</u>

Problem Attack Skills
Illinois Math League
S.A.T.

Introduction to the S.A.T.

SUBSTITUTION PROBLEMS

① C

Substitute the answers for y. See which one gives you an integer for x:

(A) 2	$3x+2(2)=17$	$x = 13/3$
(B) 3	$3x+2(3)=17$	$x = 11/3$
(C) 4	$3x+2(4)=17$	$x = \boxed{3}$
(D) 5	$3x+2(5)=17$	$x = 7/3$
(E) 6	$3x+2(6)=17$	$x = 5/3$

② C

Quadrant I $(+,+)$ $+1/+1 = \boxed{1}$

Quadrant II $(-,+)$ $-1/+1 = -1$

Quadrant III $(-,-)$ $-1/-1 = \boxed{1}$

Quadrant IV $(+,-)$ $+1/-1 = -1$

ANGLE PROBLEMS

③ D

2y and y are supplementary angles.

$2y+y=180 \rightarrow y=60$

x and y are supplementary

$x+y=180 \rightarrow x=120$

④ B

If the area of ΔABC is 30, then $\frac{1}{2}(BC)(5)=30$. That makes $BC=12$. If $BC=12$, then $DC=8$.

Use the Pythagorean Theorem to solve for AC:

$(AC)^2 = 5^2+8^2$

$(AC)^2 = 89 \rightarrow (AC) = \sqrt{89}$

FUNCTION PROBLEMS

⑤ B

$(3x \odot 4) = 3(3x)+2(4) \rightarrow 9x+8$

$(9x+8) \odot 2x = 3(9x+8)+2(2x)$

$27x+24+4x$

$\boxed{31x+24}$

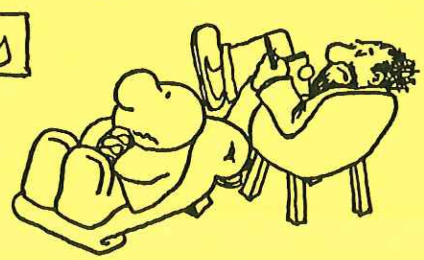
⑥ A

$$\underset{x}{\overset{3}{x \times 2}} - \underset{3}{\overset{y}{4 \times y}} + \underset{y}{\overset{3}{x \times 2}}$$

$(2x-3x)-(4y-3y)+(2x-3y)$

$2x-3x-4y+3y+2x-3y$

$\boxed{x-4y}$



CIRCLE PROBLEMS

⑦ B

The center of the circle is at $(0,0)$. By checking intercepts, you can determine that $r=5$.

$C = 2\pi r \rightarrow C = 2\pi(5) = 10\pi$

⑧ D

$C = 2\pi r \rightarrow$ if $C = 6\pi$ for one circle, therefore:

$6\pi = 2\pi r \rightarrow r = 3$

If the radius is 3, the <u>area</u> of one circle is:

$\pi r^2 \rightarrow \pi(3)^2 \rightarrow 9\pi$

If the radius is 3, a side of the square in the middle is 6. The <u>area</u> of the square is $6 \times 6 = 36$.

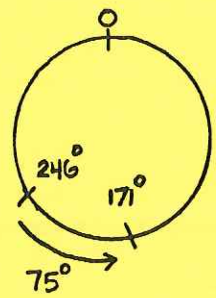
The shaded region is:

(sq. area) − (circle area)
36 − 9π

READING PROBLEMS

⑨ B

The turn (shown below) is $75°$



$246 - 171 = 75$

At 3 degrees per second:

$75 \div 3 = \boxed{25 \text{ sec}}$

⑩ E

Substitute for a and b. Pick random multiples of 3 from the set. (This problem combines careful reading and substitution).

Example: $a=3$, $b=6$

(A) $ab = (3)(6) = 18$

(B) $a+b = (3)+(6) = 9$

(C) $a-b = (3)-(6) = -3$

(D) $-a-b = -(3)-(6) = -9$

(E) $a/b = 3/6 = \boxed{1/2}$ (not in the set)

Be careful: If you had chosen to substitute $a=9$, $b=3 \rightarrow$ all 5 answers would be in the set.

Should this happen, choose a different pair of integer multiples of 3.

QUANTITATIVE PROBLEMS

(11) B

Since $\frac{1}{N} > 1$ N cannot be (-) or ≥ 1 or 0 (undefined)

Substitute a few values:

$0 < N < 1$

$N = 1/2 \quad N = 1/3 \quad N = 1/4$

$(\frac{1}{2})^2 < 1 \quad (\frac{1}{3})^2 < 1 \quad (\frac{1}{4})^2 < 1$

(12) D

Try values for x and y. Remember to include 0 and negative values. $x > y$

$\left.\begin{matrix} x=2 \\ y=1 \end{matrix}\right\} \quad \begin{matrix} x+y \\ 3 \end{matrix} \quad > \quad \begin{matrix} x-y \\ 1 \end{matrix}$

$\left.\begin{matrix} x=3 \\ y=-3 \end{matrix}\right\} \quad \begin{matrix} x+y \\ 0 \end{matrix} \quad < \quad \begin{matrix} x-y \\ 6 \end{matrix}$

$\left.\begin{matrix} x=4 \\ y=0 \end{matrix}\right\} \quad \begin{matrix} x+y \\ 4 \end{matrix} \quad = \quad \begin{matrix} x-y \\ 4 \end{matrix}$

It cannot be determined

AVERAGING

(1) Sum: $10x - 6$

Divide: $\frac{10x-6}{2} = \boxed{5x-3}$

(2) Sum: $12a + 3b + 9$

Divide: $\frac{12a+13b+9}{3} = \boxed{4a+b+3}$

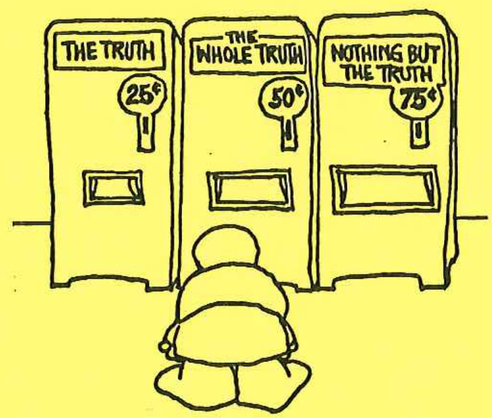
(3) Sum: $-4x + 12xy + 4y$

Divide: $\frac{-4x+12xy+4y}{4}$

$\boxed{-x+3xy+y}$

(4) Sum: $4a + 4b + 8$

Divide: $\frac{4a+4b+8}{4} = \boxed{a+b+2}$



(5) org. sum: $12 \times 11 = 132$

new sum: $10 \times 12 = 120$

missing numbers: $132 - 120 = 12$

avg: $12 \div 2 = \boxed{6}$

(6) sum: $1 \times 24 = 24$

$2 \times 30 = 60$

$4 \times 32 = 128$

$2 \times 38 = 76$

9 teams 288 books

$\frac{288}{9} = \boxed{\text{32 books per team}}$

(7) 5 tests 92 % average

Sum needed : $5 \times 92 = 460$

after 4 tests : 366

last test : $460 - 366 = \boxed{94\%}$

(8) 1st trip : $\dfrac{90 \text{ miles}}{45 \text{ mph}} = 2$ hrs.

2nd trip : $\dfrac{90 \text{ miles}}{30 \text{ mph}} = 3$ hrs.

Round trip average $= \dfrac{\text{Total Dist.}}{\text{Total Time}}$

$\dfrac{180 \text{ miles}}{5 \text{ hours}} = \boxed{36 \text{ mph}}$

(9) 1st trip : $\dfrac{120 \text{ miles}}{30 \text{ mph}} = 4$ hrs.

Round trip average : $40 = \dfrac{240 \text{ miles}}{x \text{ hrs}}$

$40x = 240$

$x = 6$ hrs. total time

6 hrs.(total) − 4 hrs. (1st trip) = return trip (2 hrs.)

Return trip : $\dfrac{120 \text{ miles}}{2 \text{ hrs.}} = \boxed{60 \text{ mph}}$

(10) 1st trip : $2 \times 35 = 70$

$2 \times 40 = 80$

$3 \times 50 = 150$

7 hrs. 300 miles

Round trip average $40 = \dfrac{600 \text{ miles}}{x \text{ hrs}}$

$40x = 600$

$x = 15$ hrs. total

15 hrs (total) − 7 hrs (1st trip)

return trip (8 hrs.)

Return trip : $\dfrac{300 \text{ miles}}{8 \text{ hrs}}$

$\boxed{37.5 \text{ mph}}$

S.A.T. Multiple Choice

Demonstration Problems

① E

Use cross multiplication to solve the equation:

$$\frac{1}{x-1} = \frac{1}{5}$$

$(5)(1) = (x-1)(1)$

$5 = x-1$

$6 = x$

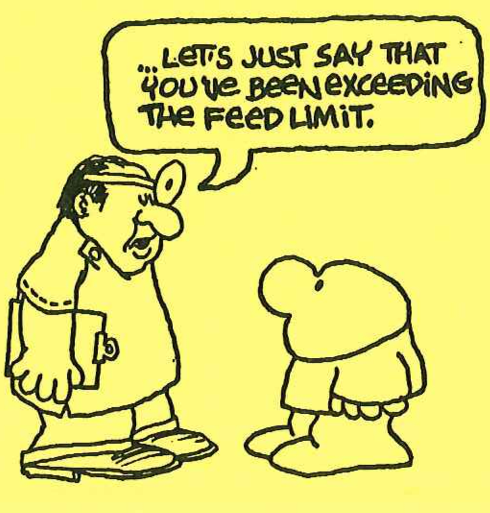
② E

If John was 25 minutes late, the boys were supposed to meet at:

3:40 −:25 = 3:15 pm

Bill was 50 minutes late:

3:15 + :50 = 4:05 pm



③ C

First solve for x:

$x = 2^2 \rightarrow x = 4$

Then substitute: $x^2 = 4^2 = 16$

④ D

$60 \times (.10) = 6 \rightarrow 60 - 6 = 54$

$54 \times (.10) = 5.4 \rightarrow 54 + 5.4 = 59.4$

⑤ D

Substitute and evaluate:

$$\frac{2ab}{a^2-b} = \frac{2(2)(2)}{(2)^2-(2)} = \frac{8}{2} = 4$$

⑥ E

If the marks are an equal distance apart, each mark is "3." x is 6 marks: 6x3 = 18

⑦ E

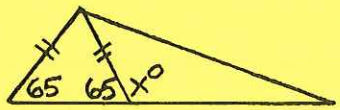
Be patient. Add. It's a complex fraction.

$\frac{\text{numerator}}{\text{denominator}}$ $\frac{12/3}{12/4}$ $\rightarrow \frac{4}{3}$

(8) D

$\triangle ABC$ is isosceles ($AB = BD$).
That makes $\angle ADB = 65°$
x is supplementary to 65

$x = 115$



(9) E

Pick a simple example to try: $m = 3$, $n = 4$

Given this example, none of the first four choices are divisible by 7.

(10) E

Sum needed: $6(x+1) = 6x+6$

Sum of first 5 games: $5x-3$

Score needed:

$(6x+6)-(5x-3) = x+9$

(11) C

Use your imagination and your ability to "picture" this problem in your mind. 8 blocks stacked in 2 rows of 4 will have a common vertex.

(12) B

Draw and label . All sides "1" (smallest positive integer).

Draw in height and label.

This is a 30-60-90 $\triangle$
The height is the long leg. $b = a\sqrt{3}$
$\frac{1}{2}\sqrt{3}$

Be sure to recognize special triangles.

(13) A

Plug in some values that fit easily: $K=5, T=30$

Using these values, it will take 2 hours to walk 20 Km.

Check to see which ans. equals 2 hours:

(A) $\frac{T}{3K} = \frac{30}{3(5)} = \frac{30}{15} = 2$

(14) D

$x+y$ must $=14$. Since both are single digit integers: $7+7$, $6+8$, $5+9$ are possibilities. 5 is the smallest possible integer.

(15) B

Solve the equation for "y" as if k was a coefficient:

$$ky = x$$

$$\left(\frac{1}{k}\right)(ky) = \left(\frac{1}{k}\right)(x)$$

$$y = \frac{x}{k}$$

(16) E

x and y are alternate interior angles. That means $x=y$. If $x+y=110$, both x and y must be equal to 55°

z is supplementary to x:

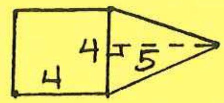
$z+55=180 \rightarrow z=125$

(17) C

To have an area of $16cm^2$, the square must be 4cm by 4cm. That means the height of the triangle must be 5cm

area of triangle $= \frac{1}{2}(\text{base})(\text{ht})$

$\frac{1}{2}(4)(5) = 10$



(18) E

To get the shaded area: Take the area of the square and subtract the 4 sectors. The 4 sectors make one complete circle with radius "r."

area of circle $= \pi r^2$

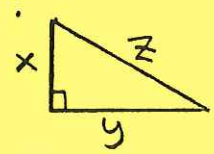
(square) $-$ (circle)

$(2r)^2 - (\pi r^2)$

$4r^2 - \pi r^2$

(19) A

Since the longest side of a rt. triangle is the hypotenuse, the triangle looks like this:



area of Δ

$\frac{1}{2}(\text{base})(\text{ht})$

$\frac{1}{2}xy$

20 C

Pick simple values to substitute:

$1 < a < b$ (A) $a^2 = 2^2 = 4$

$1 < 2 < 3$ (B) $1/ab = 1/6$

$a = 2, b = 3$ (C) $b^2 = \boxed{9}$

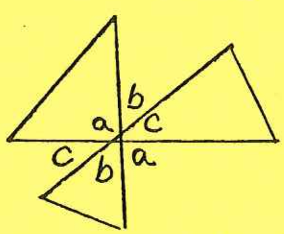
(D) $1/a^2 = 1/4$

(E) $ab = 6$

21 B

Add all 9 angles together and you get: $3(180) = 540°$

The three unmarked angles in the center $= 180°$ because they represent ½ of 360° when you consider vertical angles.



$540° - 180° = 360°$

marked ∠'s − unmarked ∠'s

$a+b+c = a+b+c$

22 C

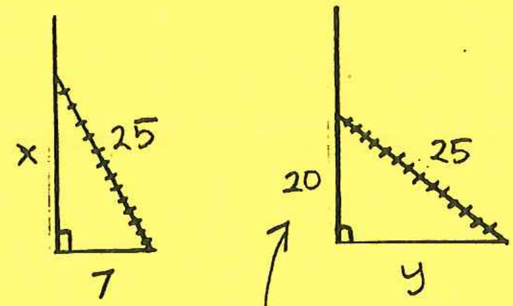
According to definition, choose the integer equal to or just greater than the boxed values:

$\boxed{-2.5} + \boxed{12}$

$(-2) + (12) = 10$

23 E

At first, you might think the answer is (A) 4 ft. But, be careful. If you draw a diagram, the Pythagorean theorem will help you:



$7^2 + x^2 = 25^2$

$x^2 = 576$

$x = 24$

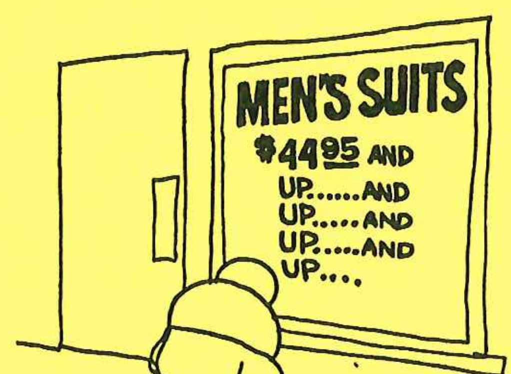
→ slide down 4ft to 20

$20^2 + y^2 = 25^2$

$y^2 = 225$

$y = 15$

Since the ladder is now 15 ft. from the building, it slides: $15 - 7 = 8$ ft.



PART II

① D

Plug 12 into the equation for x and solve for y:

$\frac{x}{y}=3 \quad 3y=x \quad 3y=12 \quad y=4$

Then plug in x=12 and y=4:

$x-y = 12-4 = 8$

② A

If you multiply a whole number by an even number, the result is even. Since "a" is multiplied by the large quantity, if "a" is even, the product is even.

③ A

The square's area is half the area of the triangle. The square is 4 by 4.

④ E

Since z is the largest, x and y must be less than 500g. For 1500 to be the answer, all three would have to equal 500g

⑤ C

Continued in next column

$\frac{\text{Turn toward B}}{\text{Arrive at A}} \quad \frac{7}{12}=\frac{n}{48} \quad n=28$

⑥ A

$\frac{\text{Turn toward B}}{\text{Arrive at A}} \quad \frac{7}{12}=\frac{n}{72} \quad n=42$

$\frac{\text{Turn toward D}}{\text{Arrive at B}} \quad \frac{3}{7}=\frac{n}{42} \quad n=18$

⑦ B

Experiment with the answers: Only (B) and (C) sum to 12. Only (B) meets the other two conditions.

⑧ D

Since $\overline{AC}$ is a side of both triangles, it can be ignored. Look at the other two sides. The large △ has sides of 22, the small △ has sides of 15.

$22-15=7$

(9) B

Sum of 4 : $4 \times 50 = 200$ kg

Sum of 3 : $3 \times 45 = 135$ kg

Missing object : $200 - 135 = 65$ kg

(10) B

I. is false $AB + CD \neq AD$

II. is true

III. is false $AC - AB = BC$

$AD - CD = AC$

(11) D

start by substituting $Q = 20$, $h = 5$:

$$Q = \frac{(x+y)h}{2} \qquad 20 = \frac{(x+y)5}{2}$$

$$(x+y) = 8$$

If $(x+y) = 8$, to get their average, add them together and divide by 2 :

$$\frac{(x+y)}{2} = \frac{8}{2} = 4$$

(12) E

The four marked angles add to 180°, but no other information can be determined to solve for y.

(13) C

Work backwards : After 20 min (1 left in room), after 15 min (2 left), after 10 min (4 left), after 5 min (8 left), at 0 min (16 people)

(14) A

Choose simple numbers to substitute (avoid "1") :

Example : $f = 3$, $l = 4$, $w = 5$

Using these numbers, each square meter has 3 clovers. A 4 by 5 rectangle will have 60 clovers (3×20).

The only solution = 60 is (A)

$flw = 3 \cdot 4 \cdot 5 = 60$

(15) D

The radius of the circle is 4. The circumference of the circle : $C = 2\pi r = 8\pi$

(16) C

According to definition :

(18) = 18 because 18 is (+)

(-18) = -17 because -18 is (-) add one

$18 + (-17) = 1$

S.A.T. Quantitative

Demonstration Problems

① A

4 tens in 48, 3 hundreds in 348

② A

x^2 will be positive even if x is negative. $1/x^2$ will be greater than 0.

③ C

$4(2x-2)$ and $2(4x-4)$ both equal $8x-8$. No matter what value x has, both will be the same.

④ B

Using the Pythagorean Theorem:

$6^2+8^2=100$ $\quad 12^2+5^2=169$

hyp = 10 $\quad$ hyp = 13

⑤ B

The numbers in question are 17 and 7. Even if you cannot determine those numbers quickly, 18 is too large:

$18+6=24 \rightarrow 18-6=12$

⑥ A

$8-(7-6-5)=12 \quad 8-7-(6-5)=0$

⑦ C

$V=1\cdot2\cdot3=6m^3 \quad V=1\cdot12\cdot(.5)=6m^3$

⑧ D

Unless all angles are 90°, the diagonals will not be equal.

⑨ D

Substitute a couple of times:

Example:

$x=1, y=2 \qquad x=2, y=2$

$(3^1)^2 \quad 3^1\cdot3^2 \qquad (3^2)^2 \quad 3^2\cdot3^2$

$9 < 27 \qquad 81 = 81$

Unless all values create the same result, the relationship cannot be determined.

⑩ C

Both circles have the same radius.

(11) D

x must be 0, but y could be positive <u>or</u> negative.

(12) B

Solve for x: $2x+2<1$

$2x<-1$

$x<-\frac{1}{2}$

(13) C

Using relationships learned when studying parallels and transversals, x and y can be shown to be supplementary. Thus: $x=180-y$.

(14) C

$(1\cdot2)+(2\cdot3)+(4\cdot3)+(1\cdot4)$ $\quad$ $(4\cdot1)+(1\cdot2)+(3\cdot2)+(4\cdot3)$

$24 = 24$

(15) B

$(0\cdot x)+(x\cdot0)+(y\cdot0)+(0\cdot y)$ $\quad$ $(x\cdot0)+(0\cdot0)+(y\cdot0)+(x\cdot y)$

$0 < xy$

xy is greater because x and y are both positive

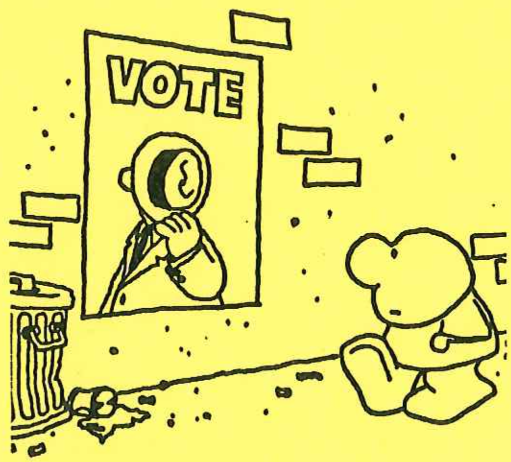
(16) D

For the equation to be true, $x+y$ must $=0$. But x could be $+6$ and y could be -6 <u>or</u> it could be the reverse. Therefore: $x-y$ could be positive or negative.

(17) C

Substitute any value for x and both columns will be equal. Example: $x=2$.

$\frac{3x^2+x}{x}$ $\qquad$ $3x+1=7$



(18) D

Substituting a positive value gives:

$4-\frac{1}{(2)}=3\frac{1}{2}$ $\qquad$ $3+\frac{3}{4}=3\frac{3}{4}$ *

Substituting a negative value gives:

$4-\frac{1}{(-2)}=4\frac{1}{2}$ * $\qquad$ $3+\frac{3}{4}=3\frac{3}{4}$

(19) D

x could be 42 or 84. The first is less, but the second is equal.

(20) A

It is not necessary to determine the exact value. Both columns start with the same value (10^6) and (A) subtracts a smaller amount (2^6) than B(3^6).

(21) C

Column A: $60 \cdot 30 = 1800$
Column B: 1800

(22) B

Column A: Since $y = 2x$,
$2x - y = 0$
Column B: Must be positive even if x and y are both (-).

(23) C

Vertical angles being equal, both chords will be "cut" by central angles of the same measure. That makes the chords equal.

(24) D

If a or b is negative, Column A will be negative and Column B will be positive. If a and b are both positive, $ab = a^2$.

(25) A

Column A: $\frac{1}{\frac{3}{2}-1} = \frac{1}{\frac{1}{2}} = 2$

Column B: $\frac{3}{2} - 1 = \frac{1}{2}$

(26) A

Even though x is negative, x^2 is positive. Since x^2+1 is greater than x^2, column A > column B.

(27) D

Substitute.

Examples:

$2(100) > 3(4)$

$2(6) < 3(10)$

(28) B

If x and y are positive and less than 10, the maximum value of $x-y$ is:

$9-1=8$

(29) C

Angle relationships using parallels and a transversal show a and b to be Supplementary. Therefore:

$a=180-b$

(30) A

Solve the equation:

$2x-7=13 \rightarrow x=10$

Substitute:

$2x+7=27$*　　21

(31) C

Column A: $6 \cdot 60 \cdot 24 = 8640$

Column B: 8640

(32) A

The greatest distance from one side of a rectangle to the other is at the diagonal.

PART II

(1) B

The triangle at the right shows $y=70$. On the left, if $y=70$ and $z>45$, x must be less than 65.

(2) A

Column A: $3 \bullet 2 = 3/2$

Column B: $3 \Delta 2 = 2/3$

(3) D

Example: $z=5$　$1 \bullet 5$　$1 \Delta 5$

$\frac{1}{5}$　$\boxed{5}$

Example: $z=-2$　$1 \bullet -2$　$1 \Delta -2$

$\boxed{-1/2}$　-2

(4) B

Since $x>30$, more than 60% of the circle is shaded.

Therefore:

$4y\% < 40\%$ and $y<10$

(5) B

The sum must be 48. Since $x > 0$, $y < 0$ to keep the sum at 48.

(6) D

h and k are completely independent of each other. Either could be larger than the other.

(7) C

90% of 10 = 9   9% of 100 = 9

(8) A

AC is 200 because C marks the middle of AE which is 400. BD is 198.

(9) B

The three angles add up to 90° because the two lines are perpendicular ($\perp$). x and 70 − x add up to 70°. That leaves 20° for y.

(10) B

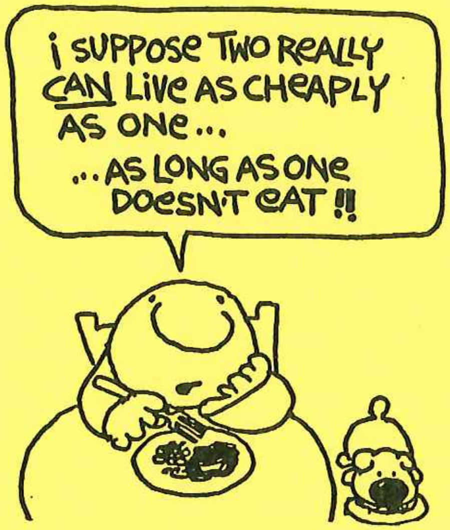
The first half takes 3 hours and the second half 1½.

$180 \div 4\frac{1}{2} = 40$

40 kph is the average

(11) D

Both coordinates will be negative, but either could be larger than the other.



(12) C

When a number ends in "1" (like 731), it will have a "1" in the units digit when raised to every positive power. When a number ends in "1," it will have a remainder of "1" when divided by 10.

S.A.T. (A)

SECTION 2

① B

Solve for x: $6x=12 \rightarrow x=2$.
Therefore: $2x-1 = 2(2)-1=3$.

② B

If $\frac{1}{8}$ of a number is 3, the number is 24. $\frac{1}{3}$ of 24 is 8.

③ E

Subtract \$.50 for the first $\frac{1}{5}$ mile. Divide the remaining \$2.00 by \$.10 to get 20.

$20 \times \frac{1}{5} = 4$ miles. Then add the first $\frac{1}{5} = 4\frac{1}{5}$.

④ D

The top angle of the triangle is 20° (vertical angles). That leaves
$2x = 160 \rightarrow x = 80$.

⑤ C

Calculate: $\left(\frac{2}{3} \div \frac{3}{4}\right) - \left(\frac{1}{9} \div \frac{1}{7}\right)$

$\frac{2}{3} \times \frac{4}{3} = \frac{8}{9}$ $\quad \frac{1}{9} \times \frac{7}{1} = \frac{7}{9}$

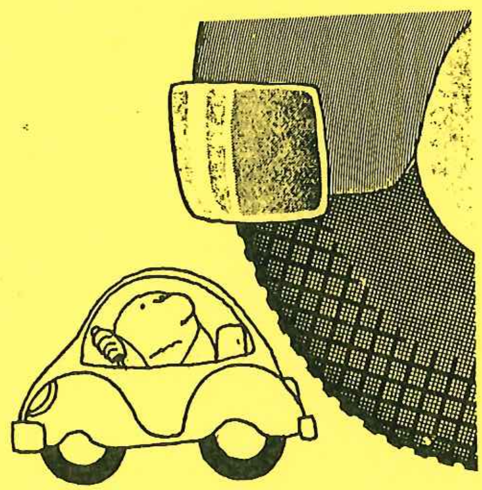
$\frac{8}{9} - \frac{7}{9} = \boxed{\frac{1}{9}}$

⑥ E

write down the relationships in equation form. Often, you will see something:

15 dops = 1 tif
10 dops = 1 decadop → mult. by 6

60 dops = 6 decadops
60 dops = 4 tifs



⑦ E

This is a reading problem. Be patient and go slowly. Check each possible value of k to see if it is in the set:

$\frac{27}{1} = 27$ $\quad \frac{27}{3} = 9$ $\quad \frac{27}{9} = 3$

$\frac{27}{27} = 1$ $\quad \frac{27}{81} = \frac{1}{3} \rightarrow$ not in the set

⑧ A

Because vertical angles are equal, the central angle that cuts arc STQ is 30°.

$\frac{30}{360}$ is equal to $\frac{1}{12}$

⑨ C

Substitute the possible answers. Notice:

$(C)\ x^3 = (2x)^2 \rightarrow 4^3 = (2 \cdot 4)^2$

$64 = 64$

⑩ B

Pick the smallest possible values for x−y (6−1) and check: 6+1=7

⑪ C

This figure is drawn to scale. You can quickly count 14 squares. Be careful: Since a square is defined as x (not x^2), do not choose $14x^2$.

⑫ E

8 ounces of soda. Subtract 6 ounces. 2 ounces are left in the bottle. 8 ounces are empty out of 10 capacity. 8/10 = 80%

⑬ C

In the center of the figure, there is a quadrilateral (where the two rectangles overlap). All quadrilaterals have 360°. Notice: The 4 marked angles are supplementary to the 4 angles of the quadrilateral. Therefore: the marked angles must also total 360°

⑭ E

With each condition (possible solution), try to find a reason it might not work.

Example: (A) $x<0$ will not work if y is less than x

$$\frac{(-2)-(-3)}{(-2)} = \frac{1}{-2} \quad \text{(negative)}$$

Only solution (E) produces a positive under all circumstances.

⑮ C

Since ℓ is straight, $\angle x$ and $\angle y$ form a linear pair. Since $x+y=180$, if $x=y$ they both must $= 90°$

⑯ D

Use any number to represent the number of marbles both have <u>now</u>. Example: John 10, Bill 10. Before the "gift," John had 16, Bill had 4. $16-4=12$.

(17) A

For a fraction to be >1, the numerator must be larger than the denominator. Therefore: $r>q>p$. Plug in values and try them: $r=4, q=2, p=1$.

(A) $\frac{p}{r}=\frac{1}{4}$ (not an integer)

(18) D

Take each condition one at a time and plug in numbers trying to show it to be false:

II $T=4$, $H=2$, $K=1$ $\quad \frac{p}{m}=\frac{2+1}{4+1}=\frac{3}{5}$

$m \neq 2p$

(19) A

Plug in a value for "a."
Example: $a=2$

$$\frac{a}{1-\frac{1}{a}}=\frac{2}{1-\frac{1}{2}}=\frac{2}{\frac{1}{2}}=4$$

The reciprocal of 4 is $\frac{1}{4}$. Plug $a=2$ into each solution until you get a value equal to $1/4$:

(A) $\frac{a-1}{a^2}=\frac{2-1}{2^2}=\frac{1}{4}$

(20) A

Don't be afraid of complex fractions:

$$\frac{7}{3}x=\frac{3}{7}y$$

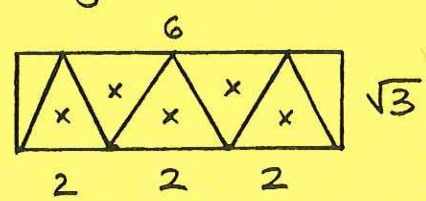
Divide both sides by y and by $7/3$:

$$\frac{x}{y}=\frac{3/7}{7/3} \rightarrow \frac{3}{7}\div\frac{7}{3}$$

$$\frac{x}{y}=\frac{9}{49}$$

(21) B

Make a diagram of the rectangle (6 by $\sqrt{3}$). 5 triangles can be made.



(22) D

If the average of <u>v</u> and <u>w</u> is p, you can replace v and w with p.

If the average of <u>x</u>, <u>y</u>, and <u>z</u> is q, you can replace x, y, and z with q.

The average of v, w, x, y, z:

$$\frac{v+w+x+y+z}{5} = \frac{p+p+q+q+q}{5}$$

$$\frac{2p+3q}{5}$$

(23) C

This is a difficult problem. The perpendicular height is the line segment that looks like a pole in the center.

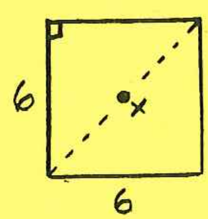
First, look at the square base and determine the length of a diagonal using the Pythagorean Theorem:

$6^2+6^2=x^2$

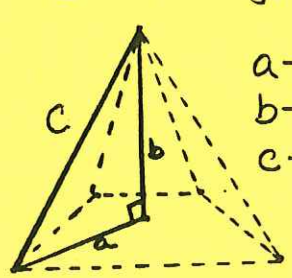
$x^2 = 72$

$x = \sqrt{72}$

$x = 6\sqrt{2}$



Now, picture a triangle with legs and hypotenuse below:



a → ½ diagonal (above)
b → ht. we solve for
c → edge = 6

"b" is the perpendicular height

$a^2+b^2=c^2$

$(3\sqrt{2})^2+b^2=6^2$

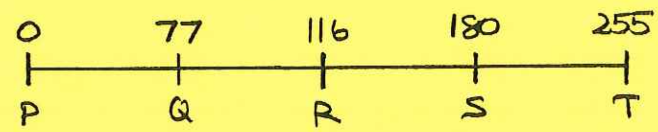
$18+b^2=36$

$b^2=\sqrt{18}$

$b = 3\sqrt{2}$

(24) B

Using the distances indicated below the line, you can label the points:



Using these points, calculate the midpoints:

midpoint QS

$180-77=103$

$\frac{103}{2} = 51\frac{1}{2}$

$77+51\frac{1}{2} = \boxed{128\frac{1}{2}}$

midpoint PT

$\frac{0+255}{2}$

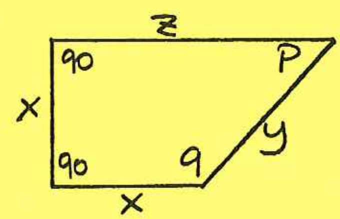
$\boxed{127\frac{1}{2}}$

$128\frac{1}{2}-127\frac{1}{2}=1$

(25) A

Try to disprove the conditions.

Condition I:



this example shows ABCD does not have to be a square or rectangle

Condition III:

In the diagram above $y \neq z$.

Condition II:

$p+q$ must $=180$ because a quadrilateral has $360°$

SECTION 5

① E

Substitute: $x^2+3 = (3)^2+3 = 12$

② C

Experiment a little. Pick numbers that 3 divides into evenly. Check any remainders when dividing by 6.

③ E

Plug in an even integer for x and check all of the answers.

④ D

Notice how cross reducing makes the problem easy to solve:

$\frac{2}{\cancel{3}}\times\frac{\cancel{3}}{\cancel{4}}\times\frac{\cancel{4}}{\cancel{5}}\times\frac{\cancel{5}}{\cancel{6}}\times\frac{\cancel{6}}{7}\times y=1$

$\frac{2}{7}y=1$

$y=\frac{7}{2}$

⑤ B

Start with $y^2=64$ and keep taking the square root of both sides:

$y^2=64$

$y=8$

$\sqrt{y}=2\sqrt{2}=x$

⑥ B

Using numbers or letters to represent colors, fill in different colors starting with 3 in the middle. You will find it possible to keep filling in the 3 colors without overlapping regions.

⑦ B

8 squares on each side + 4 more for the corners.

⑧ A

Do not determine the exact value. Column A adds a middle term that Column B subtracts.

⑨ C

Vertical angles are equal.

⑩ B

If $3x=6 \rightarrow x=2$. If $y+y=6 \rightarrow y=3$.

(11) A

The largest angle of any right triangle is the 90° ∠.

(12) D

The sets of integers are independent. Either could be very large or very small.

(13) A

Since x is larger, (A) will be positive and (B) negative.

(14) B

The series in Column (A) will total $31/32$. Even if that series continued $(+ 1/64 + 1/128 \ldots)$ it would never total as high as 1.

(15) B

Column A: 2 Column B: 3

(16) A

yz will be a positive fraction less than 1.

(17) D

If $w = -1$, Column A will be less. If $w = 0$, Column A will be greater.

(18) B

Both x and $1/x$ will be negative. $1/x$ will be closer to 0 (greater). $1/x$ will be a negative fraction.

(19) C

Solve for x: $\frac{5}{x} = \frac{1}{3} \rightarrow x = 15$.

$\frac{13}{15} = \frac{1}{5}$

(20) D

Since n could be positive or negative, there is no way to tell which column will be greater.

(21) B

Column A: $60 \cdot 24 \cdot 7$
Column B: $60 \cdot 60 \cdot 7$

There is no need to multiply it out. Column B is greater.

(22) C

Column A: $V = 6 \cdot 6 \cdot 6 = 216$
Column B: $V = 3 \cdot 3 \cdot 3 \times 8 = 216$

(23) B

Column A: 1–40 40% chance
Column B: 41–100 60% chance

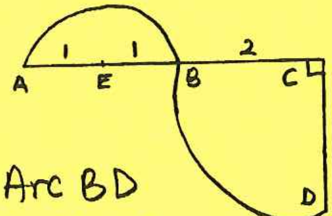
(24) A

Column A: Use the Pythagorean theorem
$10^2 + 15^2 = c^2 \quad c = \sqrt{325}$

Column B: If the area of a square is 324, a side would be $\sqrt{324}$

(25) C

Assign numbers to the original diagram:



Arc BD is 1/4 of the circumference of a circle with $r=2$

$\frac{1}{4}(2\pi)(2) = \boxed{\pi}$

Arc AB is 1/2 the circumference of a circle with $r=1$

$\frac{1}{2}(2\pi)(1)$

$\boxed{\pi}$

The arcs are equal.

(26) D

If $a=101$ and $b=999$

$\frac{101}{102} < \frac{999}{1001}$

If $a=199$ and $b=201$

$\frac{199}{200} > \frac{201}{203}$

(27) B

All coordinates on the line will have $x=y$. Within quadrant I, all points above the line will have $x<y$ and all points below the line will have $x>y$.

Therefore: $a<b$ and $d<c$

Since Column (A) has the two smaller values, Column (B) is greater.

(28) D

Check each answer. (D) moves 6 beads.

Left	Right
Black 2	Black 1
Total 8	Total 4

(29) B

There were 3 races. 9 points were awarded in each race (27 points in all). The other 2 teams totalled 16. $27-16=11$

(30) C

Since Lincoln totalled 9 points, they could have had $5+3+1=9$.

(31) B

Set up the following proportions and look at them closely:

$\frac{p}{q}=\frac{3}{5}$ $\frac{q}{r}=\frac{10}{13}$ In the 2nd proportion, q is twice as large as it is in the 1st proportion.

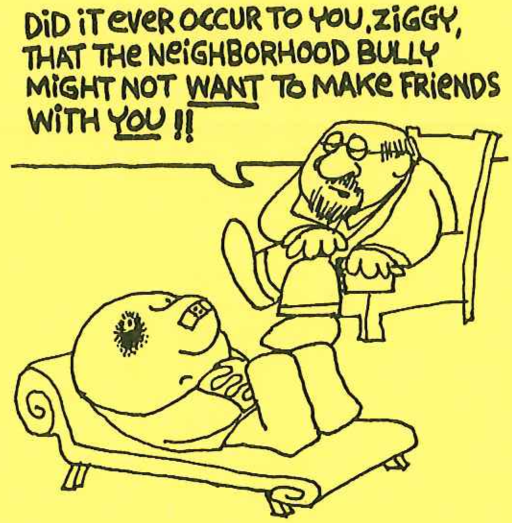
$\frac{p}{q}=\frac{6}{10}$ Double the 1st one to make $q=10$

$\frac{p}{r}=\frac{6}{13}$ Examine the values of p and r in these proportions when q is equal to 10.

(32) E

Since A and B are even and $B=2A$, here are the possibilities for AB: 24, 48.

If $AB=24$: $24+24=48$ and that makes $C=4$ and $B=4$. Since C and B must be different digits, try $AB=48$. $48+48=96$



(33) A

If you substitute different values for r, k, and m:

$r=2$, $k=3$, $m=1$

(Be careful to make J come out as an integer to keep things simple)

$J=2$

$$J=\frac{(2)(3)}{(1)+(2)}$$

Check all answers:

(A) $\frac{JM}{K-J} \rightarrow \frac{(2)(1)}{(3)-(2)}=2$

A quicker method involves factoring. This is an algebra skill we will cover during the second quarter of 7th grade.

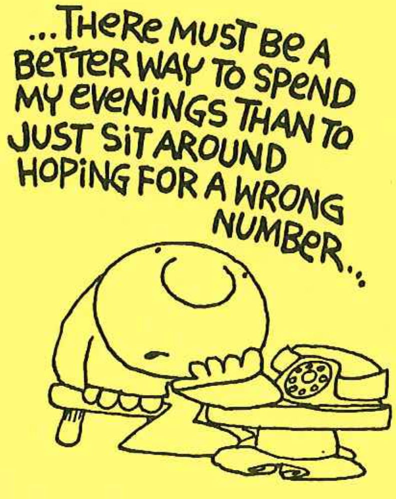
$J = \frac{rk}{m+r}$ mult. by $(m+r)$

$Jm + Jr = rk$ subtract Jr

$Jm = rk - Jr$ factor out the r

$Jm = r(k-J)$ divide by $(k-J)$

$\frac{Jm}{k-J} = r$



(34) D

To have a surface area of 54, each of the 6 faces must have an area of 9. That makes the first cube $3 \times 3 \times 3$.

To have a surface area of 216, each of the 6 faces must have an area of 36. That makes the second cube $6 \times 6 \times 6$.

Cube 1: $V = 3 \times 3 \times 3 = 27\ cm^3$

Cube 2: $V = 6 \times 6 \times 6 = 216\ cm^3$

Double Cube 2: $432\ cm^3$

It takes 16 of the first cube (16×27) to make $432\ cm^3$

(35) E

Pick any number.
Example: 3

$3+3+3+3+3+3+3+3+3+3 = 30$

$\frac{30}{10} = 3$ (average)

30 is 10 times 3.

$1 \times$ any number is 100% of the number. $10 \times$ any number is $100\% \times 10 = 1000\%$

S.A.T. (B)

SECTION 2

① D

If $x^3+y = x^3+5$, y must $=5$

② B

y and $2y$ form a linear pair.
$y+2y=180 \rightarrow 3y=180 \rightarrow$
$y=60$. Since x and y are vertical angles, $x=y$

③ D

Substitute values:

$x^2+y+\frac{y}{x} \rightarrow (-3)^2(0)+\frac{0}{-3}=0$

④ D

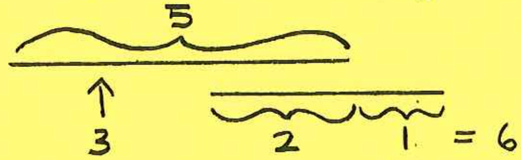
There is an odd number of 1's in 111. 3 1's cannot divide into 8 9's.

⑤ A

Count backwards from 15 to 1 on the calendar.

⑥ C

If the overlap is 2m, the section to the left is 3 and the section to the right is 1.



⑦ C

Sum of items: \$11.45
Avg: $11.45 \div 5 =$ \$2.29

⑧ A

Reduced by 10%, 1 saucer is reduced from \$1.90 to \$1.71.
Save 19¢ × 8 = \$1.52

⑨ E

In order: .1201, .11, .102, .101, .1001

(10) B

The circle can fit into the square - but has an area greater than half.

(11) E

Possibilities: (5,8) (7,6) (7,8)

(12) D

Try each condition with a variety of even integers trying to disprove them:

Condition I: Disproven

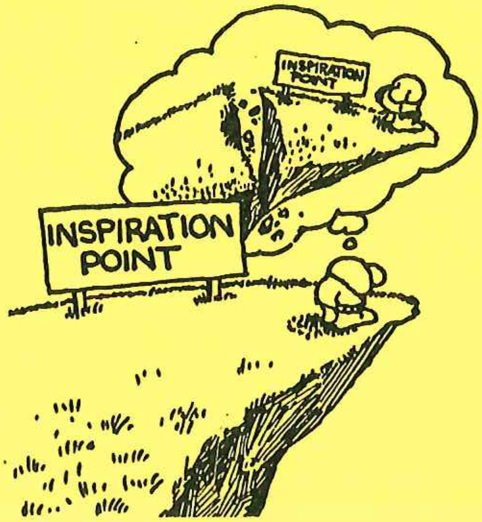
$\frac{(2)+(4)}{2} = 3$ (odd) $\frac{(2)+(6)}{2} = 4$ (even)

Condition II: Always works

$(6)-(2)=4$ (even) $(10)-(4)=6$ (even)

Condition III: Always works

$(6)+(2)=8$ (yes) $(12)+(10)=22$ (yes)



(13) E

This problem can be done quickly once you study factoring in 7th grade:

$x^2-y^2=(x+y)(x-y)$

$3(x+y)(x-y)=3(27)=81$

If do not understand how to factor, you can experiment to find values for x and y:

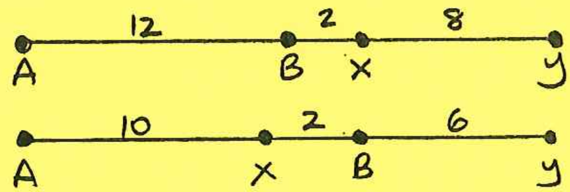
$x^2-y^2=27$ $x=6$ $y=3$

Therefore $3(x+y)(x-y)$:

$3(6+3)(6-3)=81$

(14) E

Look for possibilities that fit the conditions. Since more than one possibility exists, you must choose (E).



(15) A

Plug in a value that fits the condition.

Example: $x=11$

$11 \div 7 = 1$ r4, $2x=22$,

$22 \div 7 = 3$ r1

(16) C

The triangle at the left is a 45-45-90. Therefore: $x = 4$, $2x = 8$, $AC = 12$

(17) A

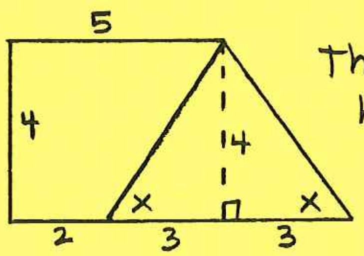
Pick a value for d. Example: $d = \$100$. Based on this example, each ticket costs \$1, 5 tickets cost \$5.

Plug in $d = 100$ to each solution to determine which is equal to \$5.

(A) $d/20 = 100/20 = 5$

(18) A

Drop an altitude in ΔABC:



The altitude helps you see that the base of the Δ is equal to 6. The area: $A = \frac{1}{2}(6)(4) = 12$

(19) D

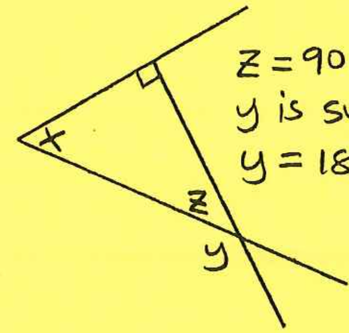
It is always important to label proportions:

$\frac{\text{read scripts}}{\text{total scripts}} \quad \frac{4}{20} \rightarrow \frac{\text{unread}}{\text{read}} \quad \frac{16}{4}$

(unread: $20 - 4 = 16$) 4:1

(20) A

In the diagram, the angle marked z is $90 - x$ (large Δ: $(90) + (x) + (90 - x) = 180$)



$z = 90 - x$

y is supplentary to z

$y = 180 - (90 - x)$

$y = 90 + x$

(21) C

The first person shakes hands with 9 people. The second shakes hands with 8 other people (one has already been counted)...

$9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 45$

(22) E

Write out the proportions and examine them.

$\frac{x}{y} = \frac{2}{3}$

$\frac{y}{z} = \frac{-3}{2} \rightarrow \frac{y}{z} = \frac{3}{-2}$

When $y = 3$, x and z are opposites. Use this when examining each condition.

Condition I: works

$\frac{x}{z} \rightarrow \frac{\text{number}}{\text{opposite}} = -1$

Condition II: Does not work

$xy \rightarrow$ values can change

Condition III: Works

$(x+z)^2 \rightarrow (\text{numb} + \text{opposite})^2 = 0$

(23) B

If the arcs are equal, Arc CDE is 2/5 of the ○.

If the area is 25π, the radius is 5 ($\pi r^2 \rightarrow \pi 5^2$)

The circumference of the whole circle is:

$2\pi r = 2\pi(5) = \boxed{10\pi}$

Arc CDE is 2/5 of 10π:

$(2/5)(10\pi) = \boxed{4\pi}$

(24) B

Plug in numbers that fit the condition. x is odd.
Example: $x=5$, $3x+1=16$

Next 2 odds: $17+19=36$

Examine the solutions looking for 36:

(A) $6x+8=38$ (D) $6x+4=34$
(B) $6x+6=\boxed{36}$ (E) $6x+3=33$
(C) $6x+5=35$

(25) D

Choose numbers that fit the conditions:

Alice 20 (common denom.)
Bob 16 (4/5 of Alice)
Chris 12 (3/4 of Bob)

Avg. of Bob and Chris
$(16+12) \div 2 = 14$

Alice ÷ (avg)
$20 \div 14 = 20/14 = 10/7$

SECTION 5

(1) B

Solve for x. Multiply by 5 to start: $9+x=10$, $x=1$

(2) D

$4+8+9=21$

$21 \div 3 = 7$

③ C

Take the second equation and solve for x.
$10x = 14$. $x = 14/10 = 7/5$.
Then substitute into the first equation:

$$x = \frac{y}{5} \rightarrow \frac{7}{5} = \frac{y}{5} \rightarrow y = 7$$

④ E

Set up equations to solve for x and y.

$$\left.\begin{array}{ll} 14 = x+5 & 12 = y-3 \\ x = 9 & y = 15 \end{array}\right\} \begin{array}{l} x-y \\ 9-15=-6 \end{array}$$

⑤ C

32K is 8 times as much as 4K.

⑥ D

Check each possible solution set. It is correct if each number in the set is 1/2 of one of the elements of p.

⑦ B

Sum for first 3 days: 186
Sum for 4 days: $4 \times 63 = 252$
Temp on last day: $252 - 186 = 66$

⑧ C

Anything times 0 is still 0

⑨ A

No matter what value is assigned to "a": 25 more than "a" is greater than 5 less than "a."

⑩ D

AB+CD must equal 60, but either one could be longer than the other. That makes it impossible to compare.

⑪ C

If 3 cups is half, 6 cups is the capacity.

⑫ D

Because you do not know which costs more before the discount, it is impossible to compare.

⑬ A

Evaluate the expression:
$3y^2 - 2x$
$3(1)^2 - 2(-2) = (3) - (-4) = 7$

⑭ A

$x + z = 80°$ to make up $180°$ in the triangle. It is not possible for x to be equal to or greater than $90°$.

⑮ D

Both columns include (b+c),

but "a" might be positive, negative, or a fraction. When mutiplied, "a" could make the product larger or smaller.

(16) B

Since x is positive, $2x+3$ will be greater than x. That makes y larger.

(17) C

x is supplementary to $110°$.
$x + 110 = 180$, $x = 70°$

y is equal to $110°$ (alternate interior angles)
$x + 40 \rightarrow (70) + 40 = 110°$, $y = 110°$

(18) B

Following this pattern, the 34th number will be -1. $34 \div 3 = 11$ r 1. This means that the number will be the first in the sequence.

(19) A

One meter per second = 60m/min. This much faster than 60m/hr.

(20) B

Column: $(x-2)+5+(x+9)=2x+12$
Row: $5+(w+2)+(w+4)=2w+11$
If $2x+12 = 2w+11$, w must be larger.

(21) B

Assign values in proportion:
Tina 30, Rita 20
Since Rita : Maria is 1:2, Maria is 40

(22) C

All sides of ABCD are radius measures. Since the diameter is x, the radius is equal to $\frac{1}{2}x$. Per ABCD $= 4(\frac{1}{2}x) = 2x$

(23) B

Plug in values for $x > 1$, $x = 1$, $x < 1$ and check the results. the system of inequalities will not work unless $x < 1$.

(24) B

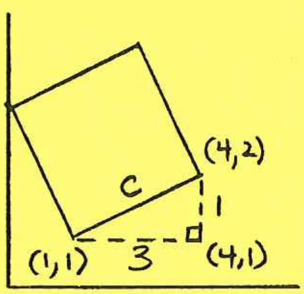
Line segment AE is common to both Δ's so we will check the remaining sides. The second Δ has a greater perimeter because AD > BE and ED > AB.

(25) D

Plugging in values for n shows n = 4 and n = 7 will produce values for 2n+1 that are multiples of 3.

(26) C

To find the area of ABCD, it is necessary to determine the measure of the sides. Use the Pythagorean Theorem.



The point (4,1) is the vertex of a right Δ. The legs can be determined using the x and y values.

The hypotenuse can then be determined: $1^2+3^2=c^2$

$c^2=10 \quad c=\sqrt{10}$

If a side of the square is $\sqrt{10}$, the area is $(\sqrt{10})^2 = 10$.

(27) C

Plug in values. You will find that 1, 2, 3, 4, 5 are perfect hypercubes. 6 is not.

(28) D

Choose numbers that can serve as proportional co-efficients. The largest number needed is the common denominator of the two fractions.

Soph	$12x$	$12x+9x+6x=360$
Jun	$9x$	$27x=360$
Sen	$6x$	$x=13.\overline{3}$

Seniors: $6x=6(13.\overline{3})=80$

(29) C

Substitute 3 for a, 4 for b.
Evaluate ab+a:
$(3)(4)+(3)=15$

(30) B

Solve the left side of the equation: $(4)(6)+(4)=28$

Substitute x for a, 5 for b.

$28=(x)(5)+x$

$28=6x \qquad x=14/3$

(31) E

All points above the angled line have coordinates with $x > y$. Points below the line have coordinates with $y > x$.

Since $r > x$, it must be E or D.

For E: $y > s$ For D: $y < s$

(32) A

Substitute a value for b and for x. Ex: $b=3$, $x=1$

$3 = (3)^{(1)} \rightarrow 3b \rightarrow 3(3) = 9$

Plug in the same values into the solutions. Two of them work:

(A) $b^{x+1} \rightarrow 3^{1+1} = 3^2 = 9$

(D) $b^{2x} \rightarrow 3^{2(1)} = 3^2 = 9$

Try another set of values:

Ex: $b = \sqrt{3}$, $x = 2$

$3 = (\sqrt{3})^{2+1} = (\sqrt{3})^3 = 3\sqrt{3}$

Plug in:

(A) $b^{x+1} \rightarrow (\sqrt{3})^{2+1} = (\sqrt{3})^3 = \boxed{3\sqrt{3}}$

(D) $b^{2x} \rightarrow (\sqrt{3})^{2(1)} = (\sqrt{3})^2 = 3$

(33) D

Surface area of rt. Δ prism:

$A = 2(\text{base area}) + (\text{per})(\text{ht})$

$2\left[\frac{1}{2}(2\cdot 8)\right] + (10 + \sqrt{68})(3)$

$16 + (30 + 3\sqrt{68}) = 46 + 3\sqrt{68}$

(34) B

Set up a chart with a value for n: $n=20$

	now	In 2 yrs
Jim	18	← 20
Polly	9	→ 11

(A) $\frac{20}{2} = 10$ (B) $\frac{20}{2} + 1 = \boxed{11}$

(C) $\frac{20}{2} + 2 = 12$ (D) $20 + 2 = 22$

(E) $2(20) = 40$

(35) B

Pick an original price: \$10

Reduced by 20% = \$8

Substitute $p = 8$ into the solutions looking for 10:

(A) $1.8(P) = 1.8(8)$

(B) $1.25(P) = 1.25(8) = \boxed{10}$

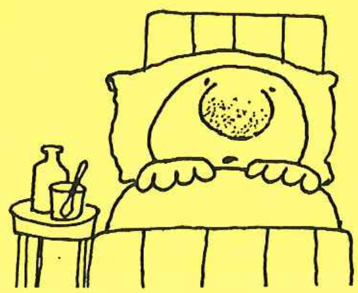
(C) $1.2(P) = 1.2(8)$

(D) $.8(P) = .8(8)$

(E) $.75(P) = .75(8)$

I'VE GOT ONE OF THOSE DISAGREEABLE COLDS

...SOMETIMES THE EYES HAVE IT, SOMETIMES THE NOSE!



S.A.T. (C)

SECTION 2

① A

$22 \cdot 3 \cdot Q = 6$, $66Q = 6$, $Q = 1/11$

② B

(B) is just right of 2nd line
(P) is just left of 2nd to last line.

③ A

$100 \div 1/10 = 1000$

④ E

x is supplementary to $40°$
$x = 140°$
e is vertical to $140°$, $e = 140°$

⑤ C

Since he uses oranges twice as fast, the oranges will run out first after 50 bags: $50 \times 2 = 100$

⑥ E

Since the figure is not drawn to scale and there is no additional information, no assumptions can be made about the sides that are not labelled.

⑦ D

Estimate to determine (C) is the answer:

$50^2 + 2(50)(50) + 50^2$

$2500 + 5000 + 2500 = 10{,}000$

This problem can be solved quickly once you have learned about factoring in 7th grade:

$x^2 + 2xy + y^2 = (x+y)^2$

$(45+55)^2 = 100^2 = 10{,}000$

⑧ B

$8x + 2x = 10x \rightarrow 10x + 5$

Substitute a few positive integers:

$$\left.\begin{array}{l} 10x+5 = 10(1)+5 = 15 \\ 10x+5 = 10(2)+5 = 25 \\ 10x+5 = 10(3)+5 = 35 \end{array}\right\} \text{multiples of 5}$$

⑨ C

A quick sketch will help.
Another method:
(12, 8) is up 12, over 8 from the origin. Slope $12/8 = 3/2$.
(3, 2) is up 3, over 2

⑩ D

Substitute: Example $x = -10$
All answers are true except (D)

$$\left.\begin{array}{l} 2 - x < 2 - 2 \\ 2 - (-10) < 2 - 2 \\ 12 < 0 \end{array}\right\} \text{false inequality}$$

⑪ D

If $2\,3/8 = 1 + \text{fraction}$, the fraction must equal $1\,3/8$

$1\,3/8 = 11/8 \qquad 11/8 = \boxed{33}/24$

⑫ C

If you determine Δ, you can find $\square$ through multiplication.

Notice:

$235 \cdot \Delta = 1 \square 10$

Δ must be 0, 4, 6, or 8 to produce a zero in the units digit. Δ must be 6 to produce 1 in the tens digit.

That makes $\square = 4$.

⑬ D

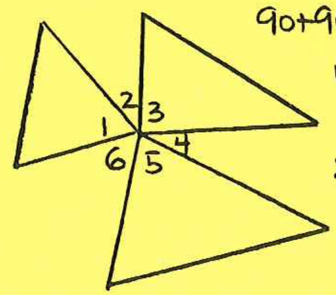
Since the new ratio has z with the least number of marbles, experiment by taking marbles out of z.

Transfer 3 from z to x:

x y z → 9 6 3

⑭ A

The six middle angles add up to 360°



$90 + 90 + 90 + x + y + z$ must $= 360°$

$x + y + z = 90°$

⑮ E

If $px = p^3x$ and $x \neq 0$, that means $p = p^3$
you can cancel x on each side

For p to equal p^3, p can be:
0, 1, −1

(16) C

Be careful, it's <u>not</u> $\frac{1}{3} \times \frac{1}{2}$.

After $\frac{1}{3}$ has been eaten, that leaves $\frac{2}{3}$.

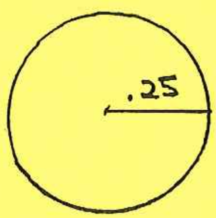
Now $\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$

↳ remainder of sandwich

(17) A

The center of the wheel will travel forward the length of the circumference each revolution

$d = 2r$, $r = .25$



$C = 2\pi r$

$C = 2\pi(.25)$

$C = \pi/2$

3 revolutions: $3(\pi/2) = 3\pi/2$

(18) C

Substitute each answer to see which works:

(A) $7+5+2+4+(4) = 22$

(B) $7+5+2+4+(4.5) = 22.5$

(C) $7+5+2+4+(6) = \boxed{24}$ $\quad \frac{1}{4}(24) = 6$

(D) $7+5+2+4+(18) = 36$

(E) $7+5+2+4+(24) = 42$

(19) D

Since the largest number in the chart is 10 (dist. Polo to Greco), those cities are farthest apart. Only (D) shows them first and last.

(20) A

Pick the highest and lowest possible values for x and y. Try those combinations:

$\underline{x} - \underline{y}$

$3 - 4 = -1$

$3 - 7 = \boxed{-4}$

$7 - 4 = \boxed{3}$

$7 - 7 = 0$

$-4 < x - y < 3$

(21) B

The figure is an octagon.

$(8-2) \times 180 = 1080°$

$1080 \div 8 = 135°$

(22) E

Try each combination:

	square	reciprocal
I)	$.25^2 = 1/16$	$2 \rightarrow 1/2$
	$2^2 = 4$	$4 \rightarrow 1/4$ *
II)	$1^2 = 1$	$1 \rightarrow 1/1$ *
III)	$.5^2 = 1/4$	$4 \rightarrow 1/4$ *
	$4^2 = 16$	$.5 \rightarrow 2$

(23) E

25% of 300 = $(.25)(300) = 75$

$\frac{\text{part}}{\text{whole}}$ $\quad \frac{75}{n} = \frac{7.5}{100}$ $\quad n = 1000$

(24) B

If the sum of the 6 faces is $54x^2$: $54x^2 \div 6 = 9x^2$

That makes each face $3x$ by $3x$

$V = (3x)(3x)(3x) = 27x^3$

(25) C

Set up a chart based on the relationship $r \times t = d$ (rate × time = distance)

	rate	×	time	=	distance
Trip 1	40	·	n	=	$40n$
Trip 2	30	·	$1-n$	=	$30-30n$

Total time is 1 hr.

Distances are equal

$40n = 30-30n$

$n = 30/7$

One way distance:

$40n = 40(30/7) = 120/7$

Round trip: $2(120/7) = 34\frac{2}{7}$ miles

SECTION 5

(1) C

$22{,}222 + (5 \cdot 10^3)$

$22{,}222 + (5000) = 27{,}222$

(2) A

Set up a proportion:

$\frac{\text{acres plowed}}{\text{gallons}} \quad \frac{3}{7} = \frac{n}{16} \quad 7n = 48$

$n = 6\,6/7$

(3) E

The bottom two angles are Supplements:

$\left.\begin{array}{l}130^\circ \rightarrow 50^\circ \\ 150^\circ \rightarrow 30^\circ\end{array}\right\}$ $80^\circ + x = 180$, $x = 100^\circ$

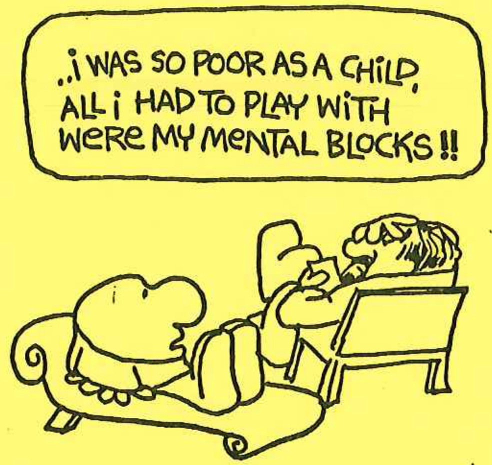
(4) B

The largest product in question is 99 x 99. Rather than multiply it out, try this:

$100 \cdot 100 = 10{,}000$

smallest possible 5 digit numb.

$99 \cdot 99$ must be only 4

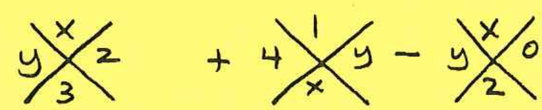


(5) D

$(5)(x) - (3)(x) = 10$

$5x - 3x = 10$, $2x = 10$, $x = 5$

(6) B

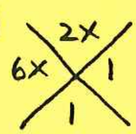


$(3x-2y) + (1x-4y) - (2x-0)$

$3x - 2y + x - 4y - 2x + 0$

$2x - 6y$

(B)



$= 2x - 6y$

7 D

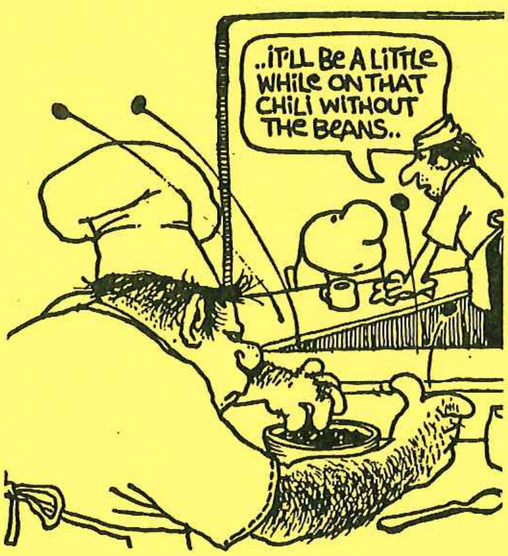
The bottom two angles sum to 120°. They are equal (opposite sides are equal). All angles are 60°. It is an equilateral triangle.
$5+5+5=15$

8 C

$\frac{2}{3} \cdot N = \frac{2N}{3} \rightarrow \frac{1}{3} \cdot 2N = \frac{2N}{3}$

9 B

without taking time to do the entire long division, the first digit of the quotient will be "2," and it will be a 3-digit quotient.
$x > 200$



10 B

Column A: $100 \div 2$
50 km/hr

Column B: $100 \div 1/2$
200 km/hr

11 D

If $n<12$, Column B is larger
If $n=12$, both columns are the same

12 B

A quadrilateral has 360°
$90+50+70+x=360$
$x=150$

13 A

Substitute a fraction, 1, and a number greater than 1:

$$\frac{(\frac{1}{2})^2+2(\frac{1}{2})}{\frac{1}{2}} = \frac{\frac{1}{4}+1}{\frac{1}{2}} = 2\frac{1}{2}$$

$$\frac{(1)^2+2(1)}{1} = 3$$

$$\frac{(2)^2+2(2)}{2} = 4$$

As x gets larger, Column A gets larger

14 C

x and y are equal (alt. interior angles)
$x+y = x+x = 2x$

15 B

Since $k<4$, $10-k$ will be greater than 6

16 B

If $w=120$, z is 60° because it is supplementary.
Since $x>y$, the largest possible value for y is still $<60°$.

(17) A

x can only equal 5

(18) D

Look "past" the drawings because they are not drawn to scale. Either triangle could have a very large or very small base.

(19) C

Solve the proportion

$\frac{.8}{.04} = \frac{.04}{x}$ $\quad .8x = .0016$

$x = .002$

(20) A

Since p is positive and n is negative: Adding n will make Column B smaller.

(21) A

Divisors are factors:

Column A: $12 \rightarrow 1, 2, 3, 4, 6, 12$

Column B: $16 \rightarrow 1, 2, 4, 8, 16$

(22) D

Column A: $(p+q)^2$

Column B: $(p-q)^2$

Both will be positive when squared, but p or q could be negative making it impossible to determine which is larger.

(23) C

Look at the 2nd equation

$(x+y)-z=x$

$x+y-z-x=0$

$y-z=0 \rightarrow y=z$

(24) B

Column A is a complex fraction:

$$\frac{\frac{2}{3}}{\frac{3}{2}} = \frac{2}{3} \div \frac{3}{2} = \frac{2}{3} \cdot \frac{2}{3} = 4/9$$

Column A: 4/9 $\quad$ Column B: 1

(25) C

Column A: $(.15)(2000) = 300$

Column B: $(20)(15) = 300$

(26) D

Substitute some possible values: $x > y > 1$

Ex: $x=3, y=2$

Column A: $x^y = 3^2 = \boxed{9}$

Column B: $y^x = 2^3 = 8$

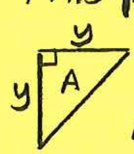
Ex: $x=5, y=2$

Column A: $x^y = 5^2 = 25$

Column B: $y^x = 2^5 = \boxed{32}$

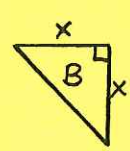
(27) C

This problem is tricky:



$A = \frac{1}{2}y^2$

$A = \boxed{y^2/2}$

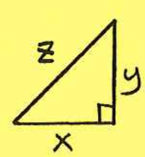


$A = \frac{1}{2}x^2$

$A = \boxed{x^2/2}$

Area of both triangles $= \frac{x^2+y^2}{2}$

Middle triange: Use Pyth. Theorem



$x^2+y^2=z^2$

Therefore: $\frac{x^2+y^2}{2} = \frac{z^2}{2}$

(28) B

Average is sum divided by 2:

$\frac{x+3x}{2}=8 \quad \frac{4x}{2}=8 \quad x=4$

(29) E

Substitute for n: $n=3$, $n=4$

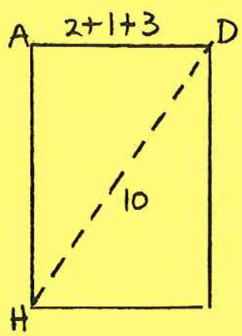
(A) n^2	$3^2=9$	$4^2=16$
(B) $n(n-1)$	$3(3-1)=6$	$4(4-1)=12$
(C) $n-1$	$3-1=2$	$4-1=3$
(D) $3n+1$	$3(3)+1=10$	$3(4)+1=13$
(E) $4n+3$	$4(3)+3=\boxed{15}$	$4(4)+3=\boxed{19}$

It is important to try one even and one odd value.

Notice: (E) must be odd because 4n has to be even. Then add 3 makes it odd

(30) A

If you know the Pythagorean triples, this can be solved quickly:



legs hye

$AD^2+AH^2 = DH^2$

$6^2+AH^2 = 10^2$

$AH=8$

(triple 3-4-5)·2

If $AH=8$, the rectangle BCFG has dimensions 1 by 8

$A=1\cdot 8$

$A=8$

(31) B

One tree on each point of intersection would make: 4 on each line segment Total of 10

(32) C

Substitute a value for x to determine a value for k:

Ex: $x=1$

$\frac{1}{1+\frac{1}{1}}=\frac{1}{2}$ then $k=1/2$

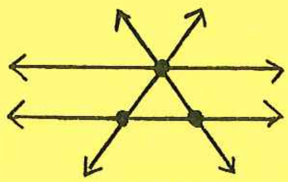
Now, since $2k=2(1/2)=1$: Check the answers to see which one equals 1 when $x=1$

(C) $\frac{1}{\frac{1}{2}+\frac{1}{2x}}=\frac{1}{\frac{1}{2}+\frac{1}{2(1)}}=\frac{1}{\frac{1}{2}+\frac{1}{2}}$

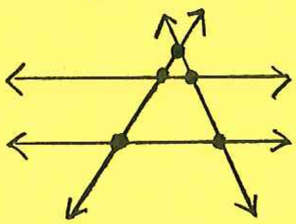
$\frac{1}{1}=\boxed{1}$

(33) D

Make some diagrams - all of which have exactly 2 parallel lines:



3 points



5 points

4 points of intersection is not possible

(34) D

Pick values for B and C that fit the condition (B is 125% of C):

EX: B=10, C=8

What % of B is C?

$\frac{\text{part}}{\text{whole}} \quad \frac{C}{B} \rightarrow \frac{8}{10} = 80\%$

(35) B

1/8 inch marks: 9 including end points

1/10 inch marks: 11 including end points

$9+11=20$

Common points: 2 end points
(3)
$\frac{4}{8}$ and $\frac{5}{10}$ } same point

$20-3=17$ points