Friendship Junior High School
Accelerated Math Program
Mr. Lavine (Room 102A)

A.T.I.M.

Advanced Topics In Mathematics

UNIT 20
Matrices

UNIT 21
Logarithms

UNIT 22
Conics

REVIEW I & II
Algebra Skills

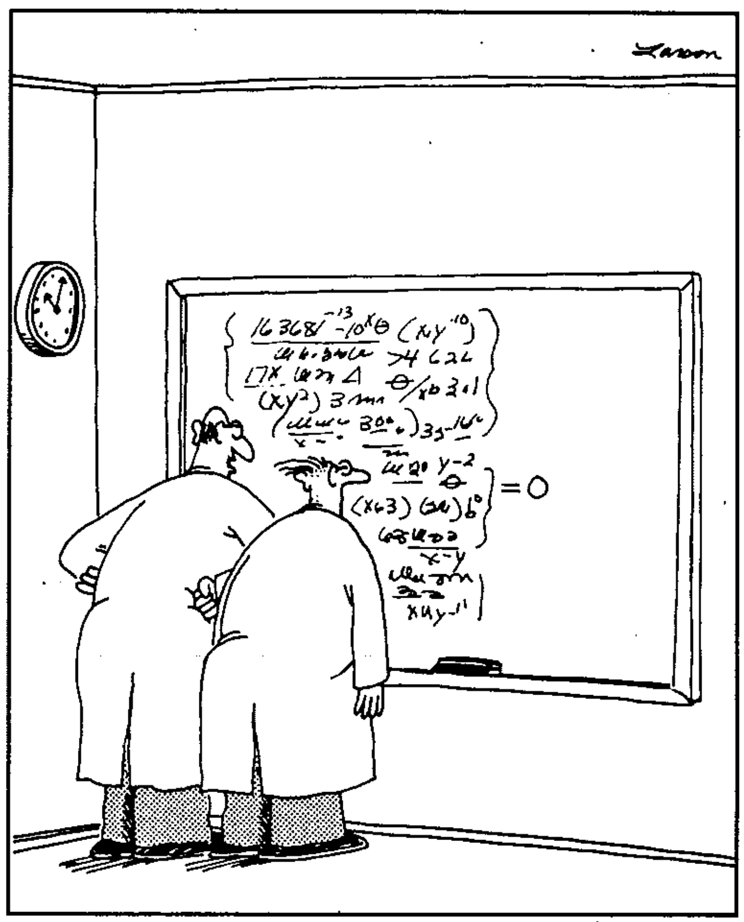
REVIEW III & IV
Algebra Skills

REVIEW V
Algebra Problem Solving

"Hey! What's this, Higgins? Logarithmic equations? Do you or don't you enjoy your job here as a cartoonist?"

UNIT 20

Matrices



"No doubt about it, Ellington—we've mathematically expressed the purpose of the universe. Gad, how I love the thrill of scientific discovery!"

Augmented Matrix Solutions

DEMONSTRATION 20.1

A matrix is a rectangular arrangement of terms in rows and columns enclosed in brackets. A system of equations can be represented in an augmented matrix.

Rules For Matrix Row Operations

A row can be replaced by any non-zero multiple of that row
A row can be replaced by the sum of that row and another

Solving 2nd Order Systems (Augmented Matrix Method)

1. $3x - 2y = 3$
   $5x - y = -2$

$$\begin{bmatrix} 3 & -2 & 3 \\ 5 & -1 & -2 \end{bmatrix} \begin{array}{l} \\ \times -2 \text{ (add to row 1)} \end{array}$$

$$\begin{bmatrix} -7 & 0 & 7 \\ 5 & -1 & -2 \end{bmatrix} \begin{array}{l} \times 5 \text{ (add to 2)} \\ \times 7 \end{array}$$

$$\begin{bmatrix} -7 & 0 & 7 \\ 0 & -7 & 21 \end{bmatrix} \begin{array}{l} \div -7 \\ \div -7 \end{array}$$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & -3 \end{bmatrix} \begin{array}{l} \rightarrow x = -1 \\ \rightarrow y = -3 \end{array} \quad \boxed{(-1, -3)}$$

$x = -1$, $y = -3$

Procedure:

1. Manipulate the rows to produce a "0" in row 1, column 2
2. Manipulate the rows to produce a "0" in row 2, column 1
3. Manipulate rows to produce "1" as follows:

$$\begin{bmatrix} 1 & 0 & x \\ 0 & 1 & y \end{bmatrix} \qquad (x, y)$$

Solving 3rd Order Systems (Augmented Matrix Method)

2. $3x + 4y - 2z = 5$
   $2x + y - z = 1$
   $-x - y - 2z = -9$

Use the matrix method and substitution
Produce "0's" $\rightarrow$

$$\begin{bmatrix} - & 0 & 0 & - \\ - & - & 0 & - \\ - & - & - & - \end{bmatrix}$$

Augmented Matrix Solutions

DEMONSTRATION 20.1

$$\begin{bmatrix} 3 & 4 & -2 & 5 \\ 2 & 1 & -1 & 1 \\ -1 & -1 & -2 & -9 \end{bmatrix} \begin{array}{l} \text{(add to row 2)} \\ \times\ -2 \\ \ \end{array}$$

$$\begin{bmatrix} 3 & 4 & -2 & 5 \\ -1 & 2 & 0 & 3 \\ -1 & -1 & -2 & -9 \end{bmatrix} \begin{array}{l} \ \\ \ \\ \times\ -1 \text{ (add to row 1)} \end{array}$$

$$\begin{bmatrix} 4 & 5 & 0 & 14 \\ -1 & 2 & 0 & 3 \\ -1 & -1 & -2 & -9 \end{bmatrix} \begin{array}{l} \times\ 2 \\ \times\ -5 \text{ (add to row 1)} \\ \ \end{array}$$

$$\begin{bmatrix} 13 & 0 & 0 & 13 \\ -1 & 2 & 0 & 3 \\ -1 & -1 & -2 & -9 \end{bmatrix}$$

$13x = 13$ $\quad x=1$
$-1(1)+2y=3$ $\quad y=2$
$-1(1)-1(2)-2z=-9$ $\quad z=3$

When carried to completion, substitution is not necessary. The goal is to produce "0's" and "1's" in the matrix shown below:

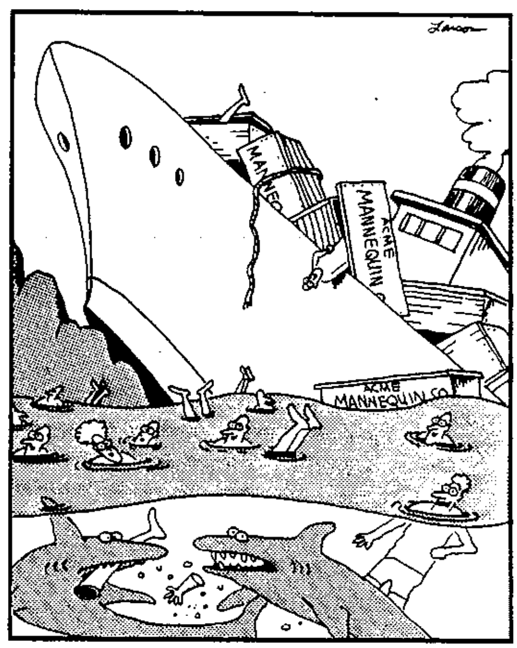
$$\begin{bmatrix} 1 & 0 & 0 & x \\ 0 & 1 & 0 & y \\ 0 & 0 & 1 & z \end{bmatrix} \qquad (x, y, z)$$

$$\begin{aligned} 1x + 0y + 0z &= x \\ 0x + 1y + 0z &= y \\ 0x + 0y + 1z &= z \end{aligned}$$

Procedure

1. Produce "0" in row 2, col 3
2. Use row 3 to help produce a "0" in row 1, col 3
3. Use row 2 to help produce a "0" in row 1, col 2
4. Use row 1 to solve for x
5. Use row 2 and substitution to solve for y
6. Use row 3 and substitution to solve for z

$\boxed{(1, 2, 3)}$



"What is this? ... Some kind of cruel hoax?"

Augmented Matrix Solutions

DEMONSTRATION 20.1

③ Use the matrix method with no substitution

$$\begin{aligned} x+2y+z&=0 \\ 2x+5y+4z&=-1 \\ x-y-9z&=-5 \end{aligned}$$

$$\left[\begin{array}{cccc} 1 & 2 & 1 & 0 \\ 2 & 5 & 4 & -1 \\ 1 & -1 & -9 & -5 \end{array}\right] \begin{array}{l} \times -4 \text{ (add to row 2)} \\ \\ \\ \end{array}$$

$$\left[\begin{array}{cccc} 1 & 2 & 1 & 0 \\ -2 & -3 & 0 & -1 \\ 1 & -1 & -9 & -5 \end{array}\right] \begin{array}{l} \times 9 \\ \\ \text{(add to row 1)} \end{array}$$

$$\left[\begin{array}{cccc} 10 & 17 & 0 & -5 \\ -2 & -3 & 0 & -1 \\ 1 & -1 & -9 & -5 \end{array}\right] \begin{array}{l} \times 3 \\ \times 17 \text{ (add to row1)} \\ \\ \end{array}$$

$$\left[\begin{array}{cccc} -4 & 0 & 0 & -32 \\ -2 & -3 & 0 & -1 \\ 1 & -1 & -9 & -5 \end{array}\right] \begin{array}{l} \\ \text{(add to row 3)} \\ \times -3 \end{array}$$

$$\left[\begin{array}{cccc} -4 & 0 & 0 & -32 \\ -2 & -3 & 0 & -1 \\ -5 & 0 & 27 & 14 \end{array}\right] \begin{array}{l} \times -5 \text{ (add to row 3)} \\ \\ \times 4 \end{array}$$

$$\left[\begin{array}{cccc} -4 & 0 & 0 & -32 \\ -2 & -3 & 0 & -1 \\ 0 & 0 & 108 & 216 \end{array}\right] \begin{array}{l} \text{(add to row 2)} \\ \times -2 \\ \\ \end{array}$$

$$\left[\begin{array}{cccc} -4 & 0 & 0 & -32 \\ 0 & 6 & 0 & -30 \\ 0 & 0 & 108 & 216 \end{array}\right] \begin{array}{l} \div -4 \\ \div 6 \\ \div 108 \end{array}$$

$$\left[\begin{array}{cccc} 1 & 0 & 0 & 8 \\ 0 & 1 & 0 & -5 \\ 0 & 0 & 1 & 2 \end{array}\right] \begin{array}{l} x=8 \\ y=-5 \\ z=2 \end{array}$$

<u>Procedure</u>

1. Produce "0" in row 2, col 3
2. Use row 3 to produce "0" in row 1, col 3
3. Use row 2 to produce "0" in row 1, col 2

Now produce "0's" at the bottom left of the matrix

4. Use row 2 to produce "0" in row 3, col 2
5. Use row 1 to produce "0" in row 3, col 1
6. Use row 1 to produce "0" in row 2, col 1

Complete the procedure

7. Divide each row to produce "1's" in the final matrix

$\boxed{(8, -5, 2)}$

Augmented Matrix Solutions

PROBLEM SET 20.1

Solve each system using the augmented matrix method:

① $3x + 2y = 0$
$6x + y = 9$

② $2a - 3b = 19$
$3a + 2b = 9$

③ $3x + 2y = 5$
$4x - 3y = 1$

④ $2x - y = 11$
$x + 3y = -12$

Use an augmented matrix and substitution to solve:

⑤
$$\begin{aligned} x + y + z &= 6 \\ 2x - 3y + 4z &= 3 \\ 4x - 8y + 4z &= 12 \end{aligned}$$

⑥
$$\begin{aligned} a + b + c &= 0 \\ 3a - 2b + 5c &= 1 \\ 2a + b + 2c &= -1 \end{aligned}$$

⑦
$$\begin{aligned} 2x + y + z &= 0 \\ 3x - 2y - 3z &= -21 \\ 4x + 5y + 3z &= -2 \end{aligned}$$

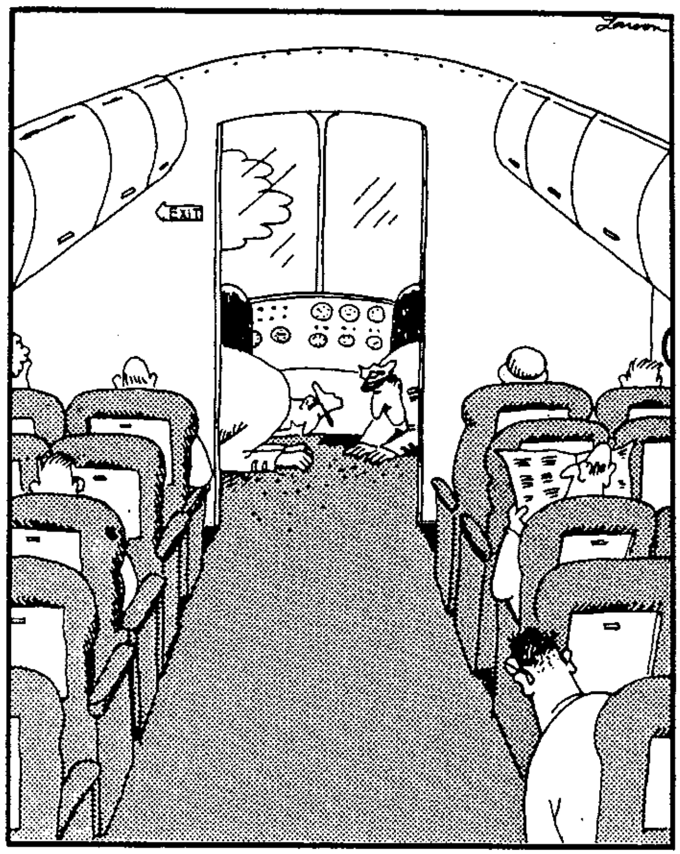
To solve the system, use an augmented matrix (only):

⑧
$$\begin{aligned} x + y + z &= -2 \\ 2x - 3y + z &= -11 \\ -x + 2y - z &= 8 \end{aligned}$$

<u>Review</u>

Solve for "y" using Cramer's Rule. Use diagonals for the numerator and expansion of minors for the denominator:

⑨
$$\begin{aligned} x + 2y + z &= 24 \\ 2x - 3y + z &= -1 \\ x - 2y + 2z &= 7 \end{aligned}$$



"Well there is some irony in all this, you know ... I mean we BOTH lose a lens at the same time?!!"

Multiplying Matrices

DEMONSTRATION 20.2

The product of a matrix and a constant is a new matrix of the same dimension with each element multiplied by the constant.

① $5\begin{bmatrix} -1 & 3 & 5 \\ 2 & 8 & -4 \end{bmatrix} = \begin{bmatrix} -5 & 15 & 25 \\ 10 & 40 & -20 \end{bmatrix}$

② $-\frac{2}{3}\begin{bmatrix} 3 & -12 \\ -6 & 15 \\ 8 & 9 \end{bmatrix} = \begin{bmatrix} -2 & 8 \\ 4 & -10 \\ -16/3 & -6 \end{bmatrix}$

Two matrices can be multiplied only if the number of columns in the first matrix is the same as the number of rows in the second matrix.

③ $\begin{bmatrix} 3 & -2 \\ -4 & 1 \end{bmatrix}\begin{bmatrix} 5 \\ -6 \end{bmatrix} \quad \begin{matrix} 3(5)+(-2)(-6) \\ -4(5)+(1)(-6) \end{matrix} = \begin{bmatrix} 27 \\ -26 \end{bmatrix}$

④ $\begin{bmatrix} 3 & 2 \\ -6 & 0 \end{bmatrix}\begin{bmatrix} 4 & -5 \\ 1 & 7 \end{bmatrix} \quad \begin{matrix} 3(4)+2(1) & 3(-5)+2(7) \\ -6(4)+0(1) & -6(-5)+0(7) \end{matrix} = \begin{bmatrix} 14 & -1 \\ -24 & 30 \end{bmatrix}$

⑤ $\begin{bmatrix} 3 & -4 & 2 \\ -2 & -5 & 6 \\ 0 & -1 & -3 \end{bmatrix}\begin{bmatrix} 4 & 0 \\ 1 & -8 \\ 7 & -6 \end{bmatrix} \quad \begin{matrix} 3(4)+(-4)(1)+2(7) & 3(0)+(-4)(-8)+2(-6) \\ -2(4)+(-5)(1)+6(7) & -2(0)+(-5)(-8)+6(-6) \\ 0(4)+(-1)(1)+(-3)(7) & 0(0)+(-1)(-8)+(-3)(-6) \end{matrix} = \begin{bmatrix} 22 & 20 \\ 29 & 4 \\ -22 & 26 \end{bmatrix}$

⑥ $\begin{bmatrix} 6 & 2 & 0 \\ -3 & -4 & 1 \end{bmatrix}\begin{bmatrix} -3 \\ -2 \\ 5 \end{bmatrix} \quad \begin{matrix} 6(-3)+2(-2)+0(5) \\ -3(-3)+(-4)(-2)+1(5) \end{matrix} = \begin{bmatrix} -22 \\ 22 \end{bmatrix}$

⑦ $-\frac{1}{2}\begin{bmatrix} -2 & 0 \\ 3 & -4 \\ 5 & 1 \end{bmatrix}\begin{bmatrix} -4 & -6 \\ 2 & 0 \end{bmatrix} \quad \begin{matrix} -2(-4)+0(2) & -2(-6)+0(0) \\ 3(-4)+(-4)(2) & 3(-6)+(-4)(0) \\ 5(-4)+1(2) & 5(-6)+1(0) \end{matrix} = -\frac{1}{2}\begin{bmatrix} 8 & 12 \\ -20 & -18 \\ -18 & -30 \end{bmatrix} = \begin{bmatrix} -4 & -6 \\ 10 & 9 \\ 9 & 15 \end{bmatrix}$

⑧ $-\frac{2}{3}\begin{bmatrix} 3 & -1 & 6 \\ 4 & 2 & -3 \\ 5 & 0 & 7 \end{bmatrix}\begin{bmatrix} 6 \\ -4 \\ 3 \end{bmatrix} \quad \begin{matrix} 3(6)+(-1)(-4)+6(3) \\ 4(6)+(2)(-4)+(-3)(3) \\ 5(6)+0(-4)+7(3) \end{matrix} = -\frac{2}{3}\begin{bmatrix} 40 \\ 7 \\ 51 \end{bmatrix} = \begin{bmatrix} -80/3 \\ -14/3 \\ -34 \end{bmatrix}$

Multiplication of matrices will be used in Lesson 20.4 when solving systems with inverse matrices.

Multiplying Matrices

PROBLEM SET 20.2

Multiply each matrix by a constant value:

① $3\begin{bmatrix}4 & 1\\ -2 & 3\end{bmatrix}$ ② $5\begin{bmatrix}-3 & 4 & 1\\ 2 & 7 & 0\end{bmatrix}$

③ $-4\begin{bmatrix}2 & 5\\ -3 & 2\\ 6 & -4\end{bmatrix}$ ④ $\frac{2}{3}\begin{bmatrix}9 & 18 & -15\\ -12 & 0 & 6\\ 3 & -9 & -3\end{bmatrix}$

Find the product of two matrices:

⑤ $\begin{bmatrix}3 & -1\\ 2 & 4\end{bmatrix}\begin{bmatrix}-1 & 0\\ 3 & 7\end{bmatrix}$

⑥ $\begin{bmatrix}-2 & -3\\ 4 & 0\end{bmatrix}\begin{bmatrix}-6 & -3\\ 8 & -1\end{bmatrix}$

⑦ $\frac{1}{2}\begin{bmatrix}2 & 3\\ 4 & -4\end{bmatrix}\begin{bmatrix}-2\\ 6\end{bmatrix}$

⑧ $\frac{2}{3}\begin{bmatrix}3 & 0\\ -5 & -1\end{bmatrix}\begin{bmatrix}-3\\ 6\end{bmatrix}$

⑨ $\frac{1}{2}\begin{bmatrix}3 & -1\\ 2 & 4\end{bmatrix}\begin{bmatrix}4 & 0 & -3\\ 7 & -5 & 9\end{bmatrix}$

⑩ $\frac{1}{4}\begin{bmatrix}-1 & 2\\ 0 & -3\end{bmatrix}\begin{bmatrix}4 & -2 & -12\\ -8 & 10 & -4\end{bmatrix}$

⑪ $\begin{bmatrix}2 & 0 & 2\\ -3 & -1 & 5\\ 1 & 3 & -4\end{bmatrix}\begin{bmatrix}-2\\ 1\\ -2\end{bmatrix}$

⑫ $-\frac{1}{2}\begin{bmatrix}2 & -5 & -2\\ -3 & 0 & 0\\ -1 & 3 & -4\end{bmatrix}\begin{bmatrix}-4\\ 2\\ -2\end{bmatrix}$

Review

Find the value of the determinant using expansion of minors:

⑬ $\begin{vmatrix}-4 & 3 & 2\\ 3 & 0 & 4\\ -2 & -3 & -1\end{vmatrix}$

Solve the system using the augmented matrix method (no substitution):

⑭ $2x - 3y = 15$
$x - 4y = 25$

Solve the system using an augmented matrix and substitution:

⑮ $x + 2y - z = 0$
$3x + y - 4z = 13$
$2x - 3y + 5z = -7$

As a challenge, you may wish to solve #15 without substitution

Inverse Values

DEMONSTRATION 20.3

The inverse of a matrix is the matrix that produces a product of "1" when multiplied by the original matrix. To find the inverse of a second order matrix, use the following pattern (if the determinant is non-zero):

If matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, then $A^{-1} = \left(\begin{matrix}\text{reciprocal of} \\ \text{determinant}\end{matrix}\right) \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$

Determine the inverse of each matrix:

① $\begin{bmatrix} -3 & 5 \\ 1 & -4 \end{bmatrix}$

determinant

$\begin{vmatrix} -3 & 5 \\ 1 & -4 \end{vmatrix} = 7$

(12)−(5)

inverse

$\frac{1}{7}\begin{bmatrix} -4 & -5 \\ -1 & -3 \end{bmatrix}$

② $\begin{bmatrix} 43 & 12 \\ -7 & -2 \end{bmatrix}$

determinant

$\begin{vmatrix} 43 & 12 \\ -7 & -2 \end{vmatrix} = -2$

(−86)−(−84)

inverse

$\frac{-1}{2}\begin{bmatrix} -2 & -12 \\ 7 & 43 \end{bmatrix}$

③ $\begin{bmatrix} 1 & 2 \\ 7 & 14 \end{bmatrix}$

(continued)

determinant

$\begin{vmatrix} 1 & 2 \\ 7 & 14 \end{vmatrix} = 0$

(14)−(14)

inverse is undefined

④ $\begin{bmatrix} -6 & 8 \\ 4 & 3 \end{bmatrix}$

determinant

$\begin{vmatrix} -6 & 8 \\ 4 & 3 \end{vmatrix} = -50$

(−18)−(32)

inverse

$\frac{-1}{50}\begin{bmatrix} 3 & -8 \\ -4 & -6 \end{bmatrix}$

To find the inverse of a 3rd order matrix with a non-zero determinant:

1. Replace each element with its signed minor
2. Find the transpose
3. The inverse is the product of the determinant reciprocal and the transpose

Inverse Values

DEMONSTRATION 20.3

Signed minor elements:

$$\begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

The transpose of a matrix is formed by interchanging the rows and columns:

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix} \xrightarrow{\text{Transpose}} \begin{bmatrix} 1 & 4 & 7 \\ 2 & 5 & 8 \\ 3 & 6 & 9 \end{bmatrix}$$

Determine the inverse for each matrix:

⑤ $$\begin{bmatrix} 2 & 3 & -1 \\ 2 & 4 & 5 \\ 0 & -3 & 7 \end{bmatrix}$$

$+\begin{vmatrix} 4 & 5 \\ -3 & 7 \end{vmatrix} = 43$ $-\begin{vmatrix} 2 & 5 \\ 0 & 7 \end{vmatrix} = -14$ $+\begin{vmatrix} 2 & 4 \\ 0 & -3 \end{vmatrix} = -6$

$(28)-(-15)$ $(14)-(0)$ $(-6)-(0)$

$-\begin{vmatrix} 3 & -1 \\ -3 & 7 \end{vmatrix} = -18$ $+\begin{vmatrix} 2 & -1 \\ 0 & 7 \end{vmatrix} = 14$ $-\begin{vmatrix} 2 & 3 \\ 0 & -3 \end{vmatrix} = 6$

$(21)-(3)$ $(14)-(0)$ $(-6)-(0)$

$+\begin{vmatrix} 3 & -1 \\ 4 & 5 \end{vmatrix} = 19$ $-\begin{vmatrix} 2 & -1 \\ 2 & 5 \end{vmatrix} = -12$ $+\begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} = 2$

$(15)-(-4)$ $(10)-(-2)$ $(8)-(6)$

$$\begin{bmatrix} 43 & -14 & -6 \\ -18 & 14 & 6 \\ 19 & -12 & 2 \end{bmatrix} \xrightarrow{\text{Transpose}} \begin{bmatrix} 43 & -18 & 19 \\ -14 & 14 & -12 \\ -6 & 6 & 2 \end{bmatrix}$$

Determinant Value

$$\left|\begin{array}{ccc|cc} 2 & 3 & -1 & 2 & 3 \\ 2 & 4 & 5 & 2 & 4 \\ 0 & -3 & 7 & 0 & -3 \end{array}\right| \quad \begin{array}{l} (56)+(0)+(6) \\ -(0)-(-30)-(42) \\ = 50 \end{array}$$

Inverse

$$\frac{1}{50}\begin{bmatrix} 43 & -18 & 19 \\ -14 & 14 & -12 \\ -6 & 6 & 2 \end{bmatrix}$$

⑥ $$\begin{bmatrix} 7 & 4 & 2 \\ 0 & 2 & 3 \\ 1 & 5 & -2 \end{bmatrix}$$

$+\begin{vmatrix} 2 & 3 \\ 5 & -2 \end{vmatrix} = -19$ $-\begin{vmatrix} 0 & 3 \\ 1 & -2 \end{vmatrix} = 3$ $+\begin{vmatrix} 0 & 2 \\ 1 & 5 \end{vmatrix} = -2$

$(-4)-(15)$ $(0)-(3)$ $(0)-(2)$

$-\begin{vmatrix} 4 & 2 \\ 5 & -2 \end{vmatrix} = 18$ $+\begin{vmatrix} 7 & 2 \\ 1 & -2 \end{vmatrix} = -16$ $-\begin{vmatrix} 7 & 4 \\ 1 & 5 \end{vmatrix} = -31$

$(-8)-(10)$ $(-14)-(2)$ $(35)-(4)$

$+\begin{vmatrix} 4 & 2 \\ 2 & 3 \end{vmatrix} = 8$ $-\begin{vmatrix} 7 & 2 \\ 0 & 3 \end{vmatrix} = -21$ $+\begin{vmatrix} 7 & 4 \\ 0 & 2 \end{vmatrix} = 14$

$(12)-(4)$ $(21)-(0)$ $(14)-(0)$

$$\begin{bmatrix} -19 & 3 & -2 \\ 18 & -16 & -31 \\ 8 & -21 & 14 \end{bmatrix} \xrightarrow{\text{Transpose}} \begin{bmatrix} -19 & 18 & 8 \\ 3 & -16 & -21 \\ -2 & -31 & 14 \end{bmatrix}$$

Determinant value

$$\left|\begin{array}{ccc|cc} 7 & 4 & 2 & 7 & 4 \\ 0 & 2 & 3 & 0 & 2 \\ 1 & 5 & -2 & 1 & 5 \end{array}\right| \quad \begin{array}{l} (-28)+(12)+(0)-(4) \\ -(105)-(0) \\ = -125 \end{array}$$

Inverse

$$-\frac{1}{125}\begin{bmatrix} -19 & 18 & 8 \\ 3 & -16 & -21 \\ -2 & -31 & 14 \end{bmatrix}$$

Inverse Values

PROBLEM SET 20.3

Find the inverse of each matrix:

① $\begin{bmatrix} -3 & 2 \\ 4 & -1 \end{bmatrix}$ ② $\begin{bmatrix} -6 & -2 \\ -3 & -1 \end{bmatrix}$

③ $\begin{bmatrix} 5 & 3 \\ -1 & 6 \end{bmatrix}$ ④ $\begin{bmatrix} 10 & -4 \\ -6 & 2 \end{bmatrix}$

⑤ $\begin{bmatrix} 0 & 4 \\ -3 & 6 \end{bmatrix}$ ⑥ $\begin{bmatrix} 8 & -3 \\ 4 & 2 \end{bmatrix}$

Find the transpose for each matrix;

⑦ $\begin{bmatrix} -2 & 6 & 8 \\ 3 & -1 & 0 \\ 4 & 2 & -1 \end{bmatrix}$ ⑧ $\begin{bmatrix} 4 & -2 & 5 \\ -1 & 3 & 7 \\ 0 & 1 & 3 \end{bmatrix}$

Find the signed minor for each element in the <u>2nd</u> row:

⑨ $\begin{bmatrix} 2 & -1 & -6 \\ 3 & 4 & 8 \\ 5 & 0 & -2 \end{bmatrix}$

Find the inverse:

⑩ $\begin{bmatrix} 1 & 1 & 3 \\ 0 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ ⑪ $\begin{bmatrix} 3 & 1 & 2 \\ -2 & 0 & 4 \\ 3 & 5 & 2 \end{bmatrix}$

⑫ $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 4 & 2 \\ 3 & 5 & 0 \end{bmatrix}$ ⑬ $\begin{bmatrix} 4 & -2 & 5 \\ 0 & 6 & 1 \\ 5 & -2 & 3 \end{bmatrix}$

Review

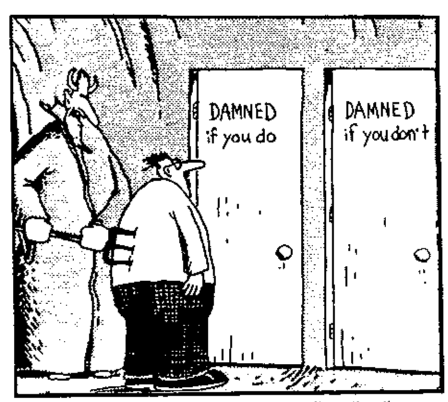
Determine the product:

⑭ $-\frac{1}{2}\begin{bmatrix} 3 & 1 \\ 5 & -3 \end{bmatrix}\begin{bmatrix} -4 \\ 8 \end{bmatrix}$

⑮ $-3\begin{bmatrix} 2 & 4 & -6 \\ -1 & -3 & -2 \\ 0 & 5 & 1 \end{bmatrix}\begin{bmatrix} -3 \\ 2 \\ 4 \end{bmatrix}$

Solve the system using an augmented matrix and substitution:

⑯ $$\begin{aligned} 2x + 3y - z &= -2 \\ 3x + y + z &= 9 \\ 2x - 4y - 3z &= 8 \end{aligned}$$



"C'mon, c'mon—it's either one or the other."

Inverse Matrix Solutions

DEMONSTRATION 20.4

A second order system can be represented by a matrix equation that can be solved by isolating the variable matrix.

(1) $5x + 3y = -5$
$7x + 5y = -11$

Coefficient Matrix	Variable Matrix	Constant Matrix

$$\begin{bmatrix} 5 & 3 \\ 7 & 5 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -5 \\ -11 \end{bmatrix}$$

Multiply both sides by the inverse of the coefficient matrix: (see right side)

$$\frac{1}{4}\begin{bmatrix} 5 & -3 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} 5 & 3 \\ 7 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 5 & -3 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} -5 \\ -11 \end{bmatrix}$$

When multiplied by its inverse, the coefficient matrix is cancelled out (product of "1"):

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 5 & -3 \\ -7 & 5 \end{bmatrix}\begin{bmatrix} -5 \\ -11 \end{bmatrix}$$

When the right side of the equation is mutiplied, the resulting matrix gives the solution (see above right):

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \end{bmatrix} \qquad \boxed{(2, -5)}$$

Reciprocal of Determinant:

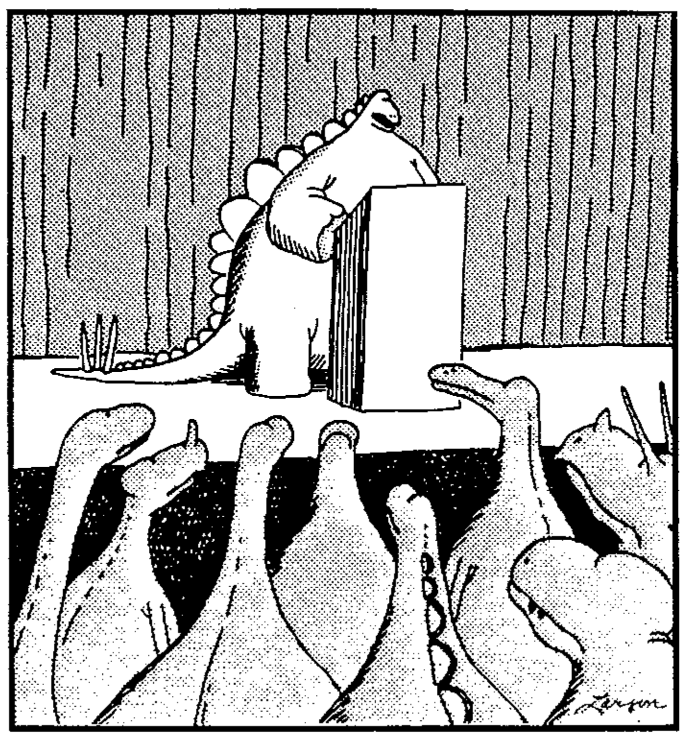
$$\begin{vmatrix} 5 & 3 \\ 7 & 5 \end{vmatrix} = (25) - (21) = 4 \qquad \text{Reciprocal } \tfrac{1}{4}$$

Inverse of Coefficient Matrix:

$$\frac{1}{4}\begin{bmatrix} 5 & -3 \\ -7 & 5 \end{bmatrix}$$

Multiplying the Right Side:

$$\begin{matrix} 5(-5) + (-3)(-11) \\ -7(-5) + (5)(-11) \end{matrix} \rightarrow \frac{1}{4}\begin{bmatrix} 8 \\ -20 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$$



"The picture's pretty bleak, gentlemen. ... The world's climates are changing, the mammals are taking over, and we all have a brain about the size of a walnut."

Inverse Matrix Solutions

DEMONSTRATION 20.4

A third order system can also be represented by a matrix equation that can be solved in the same fashion.

② $3x - 2y + z = 0$
$2x + 3y = 12$
$y + 4z = -18$

Coefficient Matrix, Variable Matrix, Constant Matrix

$$\begin{bmatrix} 3 & -2 & 1 \\ 2 & 3 & 0 \\ 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 12 \\ -18 \end{bmatrix}$$

Multiply by inverse:

$$\frac{1}{54}\begin{bmatrix} 12 & 9 & -3 \\ -8 & 12 & 2 \\ 2 & -3 & 13 \end{bmatrix} \begin{bmatrix} 3 & -2 & 1 \\ 2 & 3 & 0 \\ 0 & 1 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{54}\begin{bmatrix} 12 & 9 & -3 \\ -8 & 12 & 2 \\ 2 & -3 & 13 \end{bmatrix} \begin{bmatrix} 0 \\ 12 \\ -18 \end{bmatrix}$$

Inverse cancels coefficient matrix (product of "I"):

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{54}\begin{bmatrix} 12 & 9 & -3 \\ -8 & 12 & 2 \\ 2 & -3 & 13 \end{bmatrix} \begin{bmatrix} 0 \\ 12 \\ -18 \end{bmatrix}$$

Multiply the right side to produce a solution matrix:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$$

$$\boxed{(3, 2, -5)}$$

Reciprocal of Determinant:

$$\begin{vmatrix} 3 & -2 & 1 \\ 2 & 3 & 0 \\ 0 & 1 & 4 \end{vmatrix} \begin{matrix} 3 & -2 \\ 2 & 3 \\ 0 & 1 \end{matrix}$$

$(36) + (0) + (2) - (0) - (0) - (-16) = 54$

Reciprocal $= 1/54$

Inverse of Coefficient Matrix

$+\begin{vmatrix} 3 & 0 \\ 1 & 4 \end{vmatrix} = 12$ $\quad (12) - (0)$

$-\begin{vmatrix} 2 & 0 \\ 0 & 4 \end{vmatrix} = -8$ $\quad (8) - (0)$

$+\begin{vmatrix} 2 & 3 \\ 0 & 1 \end{vmatrix} = 2$ $\quad (2) - (0)$

$-\begin{vmatrix} -2 & 1 \\ 1 & 4 \end{vmatrix} = 9$ $\quad (-8) - (1)$

$+\begin{vmatrix} 3 & 1 \\ 0 & 4 \end{vmatrix} = 12$ $\quad (12) - 0$

$-\begin{vmatrix} 3 & -2 \\ 0 & 1 \end{vmatrix} = -3$ $\quad (3) - (0)$

$+\begin{vmatrix} -2 & 1 \\ 3 & 0 \end{vmatrix} = -3$ $\quad (0) - (3)$

$-\begin{vmatrix} 3 & 1 \\ 2 & 0 \end{vmatrix} = 2$ $\quad (0) - 2$

$+\begin{vmatrix} 3 & -2 \\ 2 & 3 \end{vmatrix} = 13$ $\quad (9) - (-4)$

$$\begin{bmatrix} 12 & -8 & 2 \\ 9 & 12 & -3 \\ -3 & 2 & 13 \end{bmatrix} \xrightarrow{\text{Transpose}} \begin{bmatrix} 12 & 9 & -3 \\ -8 & 12 & 2 \\ 2 & -3 & 13 \end{bmatrix}$$

$$\text{Inverse} \longrightarrow \frac{1}{54}\begin{bmatrix} 12 & 9 & -3 \\ -8 & 12 & 2 \\ 2 & -3 & 13 \end{bmatrix}$$

Multiply Right Side of Equation:

$12(0) + 9(12) + (-3)(-18)$
$-8(0) + 12(12) + 2(-18)$
$2(0) + (-3)(12) + 13(-18)$

$$\rightarrow \frac{1}{54}\begin{bmatrix} 162 \\ 108 \\ -270 \end{bmatrix} = \begin{bmatrix} 3 \\ 2 \\ -5 \end{bmatrix}$$

Inverse Matrix Solutions

PROBLEM SET 20.4

Solve each system using an inverse matrix solution:

① $3x - y = 13$
$3x + 4y = -22$

② $4x + 5y = -1$
$x + y = -1$

③ $3x + 2y = -8$
$x + 5y = 6$

④ $2x + 3y = -9$
$x - 2y = -8$

⑤ $x - 2y + 3z = 13$
$3x + y - 2z = 0$
$x - y + 3z = 9$

⑥ $2x - 2y + z = 0$
$x - 3y + 2z = 8$
$4x - y - z = -15$

⑦ $3x + y + 4z = -10$
$x - y + z = 2$
$2x - 3y - 2z = 8$

Review

Solve using the augmented matrix method:

⑧ $2x - y = 3$
$x - 3y = -11$

Use an augmented matrix and substitution to solve:

⑨ $x - 2y + 3z = 3$
$3x - y + z = 7$
$2x + y + 5z = -3$

The third most common cause of forest fires

Matrices

UNIT 20 REVIEW & PRACTICE

Solve using an augmented matrix:

① $3x + 4y = 12$
$5x + 2y = -8$

Solve using an augmented matrix and substitution:

② $2x + 3y - z = -7$
$4x - y - 6z = 10$
$x + 2y + 3z = -2$

Find each product:

③ $\begin{bmatrix} 2 & 3 \\ -4 & 0 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ -5 & -1 \end{bmatrix}$

④ $-\frac{1}{2} \begin{bmatrix} -2 & 3 & 4 \\ 0 & 2 & 5 \end{bmatrix} \begin{bmatrix} 6 \\ -4 \\ 10 \end{bmatrix}$

⑤ $\begin{bmatrix} -2 & -1 & 0 \\ 3 & 0 & 4 \\ 4 & 2 & -3 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 5 & -1 \\ 3 & 0 \end{bmatrix}$

Find the signed minor for each element:

⑥ In row 1

⑦ In row 2

$\begin{bmatrix} 6 & -1 & 0 \\ 3 & 4 & -5 \\ 2 & -3 & 7 \end{bmatrix}$

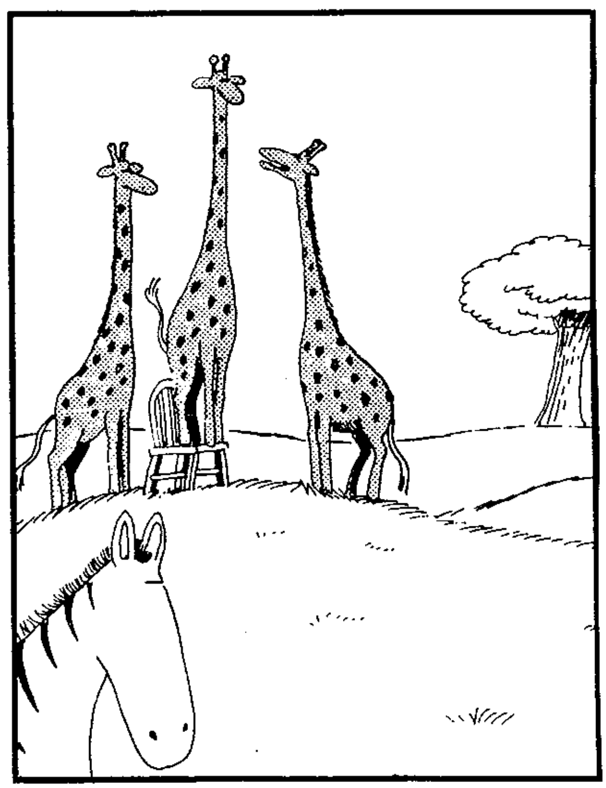
Find the inverse:

⑧ $\begin{bmatrix} 4 & 3 \\ -6 & 8 \end{bmatrix}$

⑨ $\begin{bmatrix} 2 & 0 & 6 \\ -4 & 3 & 8 \\ 5 & -1 & 7 \end{bmatrix}$

Solve using an inverse matrix:

⑩ $3x + 5y = 3$
$x - 3y = 15$



"No lions anywhere? ... Let me have the chair."

UNIT 21

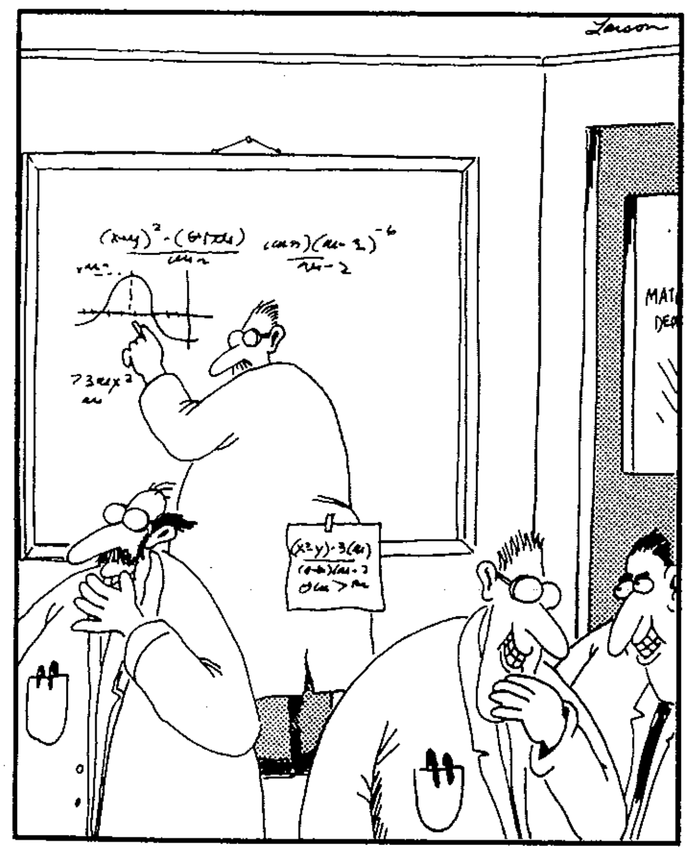
Logarithms

21.1
Exponents & Inverse Relations

21.2
Properties of Logarithms

21.3
Common Logarithms

21.4
Exponential Equations



Exponents & Inverse Relations

DEMONSTRATION 21.1

The following rules for exponents serve as a review from an earlier unit:

<u>Multiplication (same base)</u>

$a^n \cdot a^m = a^{n+m}$

<u>Division (same base)</u>

$a^n \div a^m = a^{n-m}$

<u>Raising A Power To A Power</u>

$(a^n)^m = a^{nm}$

Simplify each expression:

① $n^{\sqrt{2}} \cdot n^{\sqrt{8}}$

$n^{\sqrt{2}+\sqrt{8}} = n^{\sqrt{2}+2\sqrt{2}} = n^{3\sqrt{2}}$

② $8^n \div 4^{n-1}$

$2^{3n} \div 2^{2n-2} = 2^{3n-(2n-2)} = 2^{n+2}$

③ $(3^{\sqrt{2}})^{\sqrt{6}}$

$3^{\sqrt{2}\cdot\sqrt{6}} = 3^{\sqrt{12}} = 3^{2\sqrt{3}}$

④ $(8x^{-3})^{-2/3}$

$8^{-2/3}x^2 = \frac{1}{8^{2/3}} \cdot x^2 = \frac{x^2}{4}$

An equation with a base value greater than 0, not equal to 1, and a variable exponent is defined as an exponential function.

<u>Property of Equality: Exponential Functions</u> For $a > 0$ and $a \neq 1$, $a^x = a^y$ if and only if $x = y$

⑤ $3^5 = 3^{2n-1}$ solve for n

The equation is true only if:

$5 = 2n - 1$

$n = 3$

⑥ $9^{3r} = 27^{r-2}$ solve for r

$(3^2)^{3r} = (3^3)^{r-2}$

$3^{6r} = 3^{3r-6}$

$6r = 3r - 6 \longrightarrow r = -2$

Exponents & Inverse Relations

DEMONSTRATION 21.1

A logarithm is an inverse exponential value. In the equations below, 3 is the logarithm of 8 to base 2:

Exponential Equation: $2^3 = 8$ Logarithmic Equation: $\log_2 8 = 3$

Change to logarithmic form:

⑦ $3^4 = 81$
$\log_3 81 = 4$

⑧ $4^{-2} = 1/16$
$\log_4 (1/16) = -2$

Change to exponential form:

⑨ $\log_2 32 = 5$
$2^5 = 32$

⑩ $\log_{1/3} 9 = -2$
$(1/3)^{-2} = 9$

Evaluate each expression:

⑪ $\log_5 125$
$5^x = 125$
$x = 3$

⑫ $\log_{1/2} 32$
$(1/2)^x = 32$
$x = -5$

⑬ $\log_8 32$
$8^x = 32$
$2^{3x} = 2^5$
$3x = 5$
$x = 5/3$

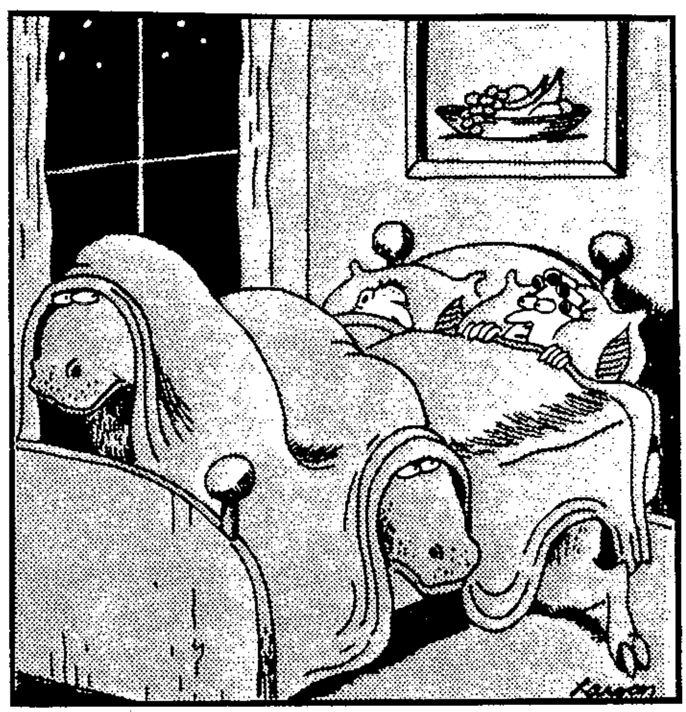
⑭ $\log_{25} 125$
$25^x = 125$
$5^{2x} = 5^3$
$2x = 3$
$x = 3/2$

Solve each equation:

⑮ $\log_2 64 = y$
$2^y = 64$ $y = 6$

⑯ $\log_9 x = 1/2$
$9^{1/2} = x$ $x = 3$

⑰ $\log_x \sqrt{8} = 3/4$
$x^{3/4} = \sqrt{8}$
$x^{3/4} = 2^{3/2}$
$(x^{3/4})^{4/3} = (2^{3/2})^{4/3}$ $x = 4$



"For heaven's sake! Harold! Wake up! We've got bed buffaloes!"

Exponents & Inverse Relations

PROBLEM SET 21.1

Use the rules for exponents to simplify:

1. $x^4 \cdot x^3$
2. $n^{3x} \cdot n^x$
3. $a^5 \div a^2$
4. $(2b^3)^2$
5. $(2^{\sqrt{3}})^{\sqrt{12}}$
6. $5^{\sqrt{3}} \cdot 5^{\sqrt{27}}$
7. $16^{\sqrt{7}} \div 16^{\sqrt{3}}$
8. $(m^{\sqrt{2}} \cdot p^{\sqrt{2}})^{\sqrt{2}}$

Use the rules for exponents to solve each equation:

9. $2^5 = 2^{2x-1}$
10. $3^y = 3^{3y+1}$
11. $9^{3y} = 27^{y+2}$
12. $8^{x-1} = 16^{3x}$
13. $\frac{1}{27} = 3^{x-5}$
14. $2^{n+3} = 1/16$
15. $(\frac{1}{3})^p = 3^{p-6}$
16. $4^{y-1} = 8^y$

Change to logarithmic form:

17. $3^3 = 27$
18. $4^2 = 16$
19. $2^{-3} = 1/8$
20. $5^{-2} = 1/25$

Change to exponential form:

21. $\log_4 64 = 3$
22. $\log_3 9 = 2$
23. $\log_9 27 = 3/2$
24. $\log_3 \frac{1}{81} = -4$
25. $\log_{\frac{1}{2}} 16 = -4$
26. $\log_{27} 3 = 1/3$
27. $\log_{10} \frac{1}{10} = -1$
28. $\log_{\frac{1}{3}} 81 = -4$

Evaluate each expression:

29. $\log_{10} 1000$
30. $\log_6 36$
31. $\log_3 81$
32. $\log_{12} 144$
33. $\log_7 1/343$
34. $\log_{\frac{1}{4}} 64$
35. $\log_4 2$
36. $\log_9 27$

Solve each equation:

37. $\log_6 x = 2$
38. $\log_9 x = -1$
39. $\log_{\frac{1}{2}} 16 = x$
40. $\log_3 27 = x$
41. $\log_n 81 = 4$
42. $\log_n 18 = 1$
43. $\log_5 x = -2$
44. $\log_3 x = -3$
45. $\log_{10} \sqrt{10} = x$
46. $\log_5 \sqrt{5} = x$
47. $\log_n \frac{1}{27} = -3$
48. $\log_n 36 = -2$
49. $\log_{\sqrt{3}} 27 = x$
50. $\log_x \sqrt{5} = 1/4$
51. $\log_x \sqrt[3]{7} = 1/3$
52. $\log_{10} \sqrt[3]{10} = n$

Properties of Logarithms

DEMONSTRATION 21.2

The domain of a logarithmic function cannot be negative. That would make the function undefined.

$\log_x y = z$	$y > 0$	Domain must be positive

For a positive base ($\neq 1$) and positive domains, the sum of logs of the same base is equal to the log of the product of the domains.

$\log_x y + \log_x z = \log_x yz$	Product Property

For a positive base ($\neq 1$) and positive domains, the difference of the logs of the same base is equal to the log of the quotient of the domains.

$\log_x y - \log_x z = \log_x \left(\frac{y}{z}\right)$	Quotient Property

For a positive base ($\neq 1$) and a positive domain, a real number coefficient for a log is equal to the log of the domain to that real number power.

$(n) \log_x y = \log_x y^n$	Power Property

For a positive base ($\neq 1$) and positive domains, the domains are equal if logs to the same base of those domains are equal.

If $\log_x y = \log_x z$, $y = z$	Property of Equality

Use the properties of logarithms to solve each equation:

① $\log_2 (x^2 - 1) = 3 \log_2 2$ $\quad x^2 - 1 = 2^3$ $\quad x^2 - 9 = 0$

$\log_2 (x^2 - 1) = \log_2 2^3$ $\quad x^2 - 1 = 8$ $\quad (x+3)(x-3) = 0$ $\quad x = \pm 3$

Properties of Logarithms

DEMONSTRATION 21.2

② $\log_{12} 72 - \frac{1}{2}\log_{12} 81 = \log_{12} 4m$

$\log_{12} 72 - \log_{12} 81^{1/2} = \log_{12} 4m$

$\log_{12} 72 - \log_{12} 9 = \log_{12} 4m$

$\log_{12}\left(\frac{72}{9}\right) = \log_{12} 4m$

$\frac{72}{9} = 4m$

$4m = 8 \longrightarrow m = 2$

③ $\log_3(y+4) + \log_3(y-4) = 2$

$\log_3 (y+4)(y-4) = 2$

$\log_3(y^2-16) = 2$

$3^2 = y^2 - 16$

$y^2 - 25 = 0$

$(y+5)(y-5) = 0$

$y = \pm 5 \longrightarrow y = 5$

Note: (−5) would make both log values undefined

⑤ Nested logarithms (Solve from the outside toward the inside):

$\log_9\left[\log_8(\log_2 x)\right] = -1/2$

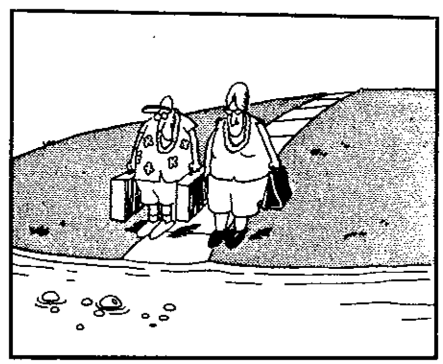
$9^{-1/2} = \log_8(\log_2 x)$

$\log_8(\log_2 x) = 1/3$

$8^{1/3} = \log_2 x$

$\log_2 x = 2$

$2^2 = x \longrightarrow x = 4$



Returning from vacation, Roy and Barbara find their house, their neighborhood, their friends—in fact, all of Atlantis—just plain gone.

④ $\frac{1}{2}\log_2 4x^2 + \log_2(2x+3) = \log_3 9 \longrightarrow$ Note: $\log_3 9 \rightarrow 3^n = 9$

$n = 2$

$\log_2(4x^2)^{1/2} + \log_2(2x+3) = 2$

$\log_2 2x + \log_2(2x+3) = 2$

$\log_2 (2x)(2x+3) = 2$

$2^2 = (2x)(2x+3)$

$4 = 4x^2 + 6x$

$4x^2 + 6x - 4 = 0 \longrightarrow 2(2x-1)(x+2) = 0 \longrightarrow x = \frac{1}{2}$ or $\cancel{-2} \rightarrow x = 1/2$

Properties of Logarithms

PROBLEM SET 21.2

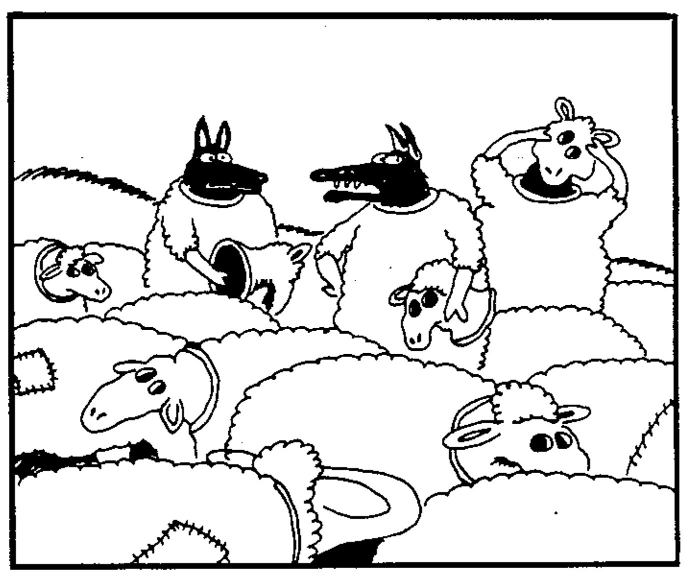
Solve each equation:

1. $\log_{10}(3n) = \log_{10}(n+2)$
2. $\log_4(2x-3) = \log_4(x+2)$
3. $\log_{10}(x^2+36) = \log_{10}100$
4. $\log_5(4x-4) = \log_5 100$
5. $\log_3 y - \log_3 2 = \log_3 12$
6. $\log_3 14 + \log_3 m = \log_3 42$
7. $\log_5 x = 3\log_5 7$
8. $\log_2 n = \frac{1}{2}\log_2 81$
9. $\log_9 x = \frac{1}{2}\log_9 144 - \frac{1}{3}\log_9 8$
10. $\log_7 m = \frac{1}{3}\log_7 64 + \frac{1}{2}\log_7 121$
11. $\log_{10}7 + \log_{10}(n-2) = \log_{10}6n$
12. $\log_{10}(m+3) - \log_{10}m = \log_{10}4$
13. $\log_{10}x + \log_{10}x + \log_{10}x = \log_{10}27$
14. $4\log_5 x - \log_5 4 = \log_5 4$
15. $\log_2 15 + \log_2 14 - \log_2 105 = \log_2 x$
16. $2\log_3 x + \log_3 \frac{1}{10} = \log_3 5 + \log_3 2$
17. $\log_{10}(y-1) + \log_{10}(y+2) = \log_7 7$
18. $\log_3(x+2) + \log_3(3x) = \log_6 36$
19. $\log_2(\log_4 x) = 2$
20. $\log_3(\log_{1/27} x) = -1$
21. $\log_2[\log_4(\log_3 x)] = -1$
22. $\log_{10}[\log_2(\log_7 x)] = 0$

Review

Change forms (exponential and logarithmic):

23. $\log_3 243 = 5$
24. $216^{2/3} = 36$



"Wait a minute! Isn't anyone here a real sheep?"

Common Logarithms

DEMONSTRATION 21.3

Logarithms to base 10 are called common logarithms. When a log is written without a base, the base is assumed to be 10.

Example: $\log 100 = 2$ (same as $\log_{10} 100 = 2$) $\rightarrow 10^2 = 100$

USING THE TABLE

The table includes common logarithms for numbers from 1 to 10. To find the value of log 1.23, follow these steps:

1. Under the "n" column, find 12 (which represents 1.2)
2. Follow values to the right under column 3 ($\log 1.23 = .0899$)

Every log has two parts: the "characteristic" and the "mantissa." The mantissa is the value in the table. The characteristic is a power of 10 used to move the decimal point.

To find the value of log 745,000, follow these steps:

1. Put the number in scientific notation: $745{,}000 = 7.45 \times 10^5$
2. Under the "n" column, find 74 (representing 7.4)
3. Follow values right to column 5 ($\log 7.45 = .8722$)
4. Scientific notation exponent (5) is the characteristic ($5 + .8722$)
   $\text{Log } 745{,}000 = 5.8722$

When using the table, it is customary to place a negative characteristic after the mantissa (instead of subtracting)

Example: $\log .000524$ $\quad .000524 = 5.24 \times 10^{-4}$ $\quad \boxed{.7193 - 4}$

Use the table in reverse to find the <u>antilog</u>:

Example: $\log 100 = 2$ $\quad$ antilog of $2 = 100$ $\quad$ ($\log x = 2$) antilog

Common Logarithms

DEMONSTRATION 21.3

To find an antilog, follow these steps: ($\log x = 3.5821$)

1. Use the mantissa to find antilog ($\log n = .5821$) $n = 3.82$
2. Use characteristic for scientific notation (3.82×10^3)
3. Antilog $3.5821 = \boxed{3820}$

USING A CALCULATOR

On a scientific calculator, the log key gives the value for common logarithms. Use INV (or 2nd function) first to find the antilog. When using a calculator, subtract the mantissa if you have a negative characteristic.

Use the table to find each value:

① $\log 4.61$ $\boxed{.6637}$

② $\log 6870$
6.87×10^3 $\boxed{3.8370}$

③ $\log .076$
7.6×10^{-2} $\boxed{.8808-2}$

④ $\log x = .8376 - 2$
mantissa $.8376 \rightarrow \log 6.88$
move decimal; antilog $= \boxed{.0688}$

⑤ $\log x = 3.8096$
mantissa $.8096 \rightarrow \log 6.45$
move decimal: antilog $= \boxed{6450}$

Use the calculator:

⑥ $\log 38.41$ (round to 4 places) $\boxed{1.5844}$

⑦ $\log .0345$ (round to 4 places) $\boxed{-1.4622}$

⑧ $\log x = .6263 - 3$ (round to 6 places) $.6263 - 3 = -2.3737$
antilog $-2.3737 = \boxed{.004230}$

⑨ $\log x = 2.1106$ (round to 4 places) antilog $2.1106 = \boxed{129.0031}$

Common Logarithms of Numbers

n	0	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396

Common Logarithms of Numbers

n	0	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996

Common Logarithms

PROBLEM SET 21.3

Use the table of mantissas to find the log of each:

1. 4.94
2. 7.63
3. 58.2
4. 715
5. 9.58
6. .000741
7. 7420
8. .3

Use the table of mantissas to find the antilog:

9. $\log x = .8698$
10. $\log x = .6580$
11. $\log x = 1.0899$
12. $\log x = .8727-2$
13. $\log x = 3.9581$
14. $\log x = .7846-1$
15. $\log x = .9542-2$
16. $\log x = 5.7451$
17. $\log x = .2014-1$
18. $\log x = 5.7168$

Use a calculator to find each log (round to 4 places):

19. 841,000
20. 62,700
21. .00211
22. .0385

Use a calculator to find each antilog (round as indicated)

23. $\log x = .1673-2$ (4 places)
24. $\log x = 1.3075$ (4 places)
25. $\log x = .6656-3$ (6 places)
26. $\log x = .6304$ (4 places)

Review

Evaluate each expression:

27. $\log_8 2$
28. $\log_{1/4} 64$

Solve each equation

29. $\log_4 (x+3) + \log_4 (x-3) = 2$
30. $\log_8 (m+1) - \log_8 m = \frac{1}{2} \log_8 16$
31. $\log_6 \left[\log_{1/2} (\log_4 x)\right] = 0$

Exponential Equations

DEMONSTRATION 21.4

Exponential equations feature variables as exponents. These equations can be solved using exponents.

Single Variable As Exponent

① $2^x = 27$

$\log 2^x = \log 27$

$(x)\log 2 = \log 27$

$x = \dfrac{\log 27}{\log 2}$

$x = 4.7549$

Using An Equation To Determine A Log

② $\log_3 35$

$\log_3 35 = x$

$3^x = 35$

$\log 3^x = \log 35$

$x \log 3 = \log 35$

$x = \dfrac{\log 35}{\log 3}$

$x = 3.2362$

Multiple Variable Exponents

③ $5^x = 2^{x-1}$

$\log 5^x = \log 2^{x-1}$

$(x)\log 5 = (x-1)\log 2$

$(x)\log 5 = (x)\log 2 - (1)\log 2$

$(x)\log 5 - (x)\log 2 = -\log 2$

$x(\log 5 - \log 2) = -\log 2$

$x = \dfrac{-\log 2}{(\log 5 - \log 2)} = -.7565$

Note: In problem ③, you must use parenthesis in your calculator.

④ $2^{3y} = 3^{5y+2}$

$\log 2^{3y} = \log 3^{5y+2}$

$(3y)\log 2 = (5y+2)\log 3$

$(3y)\log 2 = (5y)\log 3 + (2)\log 3$

$(3y)\log 2 - (5y)\log 3 = (2)\log 3$

$y(3\log 2 - 5\log 3) = 2\log 3$

$y = \dfrac{2\log 3}{3\log 2 - 5\log 3}$

$y = \dfrac{\log 3^2}{\log 2^3 - \log 3^5} = \dfrac{\log 9}{\log 8 - \log 243}$

$y = -.6437$

When completing the division, very careful use of ( ) and = in the calculator are required. It is also helpful to round the solution and check.

Exponential Equations

PROBLEM SET 21.4

Approximate each logarithm to three decimal places:

① $\log_3 7$ ② $\log_7 12$

③ $\log_4 22$ ④ $\log_6 11$

Solve each equation using logarithms (round to 4 places):

⑤ $9^{x-4} = 6.28$

⑥ $5^{y+2} = 15.3$

⑦ $x = \log_4 51.6$

⑧ $x = \log_3 19.8$

* ⑨ $32^{2y} = 5^{4y+1}$

* ⑩ $2^{5x-1} = 3^{2x+1}$

⑪ $5^{x-1} = 3^x$

⑫ $7^{x-2} = 5^x$

⑬ $5^{2x} = 9^{x-1}$

⑭ $12^{x-4} = 4^{2-x}$

⑮ $7^{x-2} = 5^{3-x}$

⑯ $3^{3x} = 2^{2x+3}$

Review

⑰ Evaluate : $\log_2 16$

⑱ Solve : $\log_4(y-1) + \log_4(y-1) = \log_3 27$

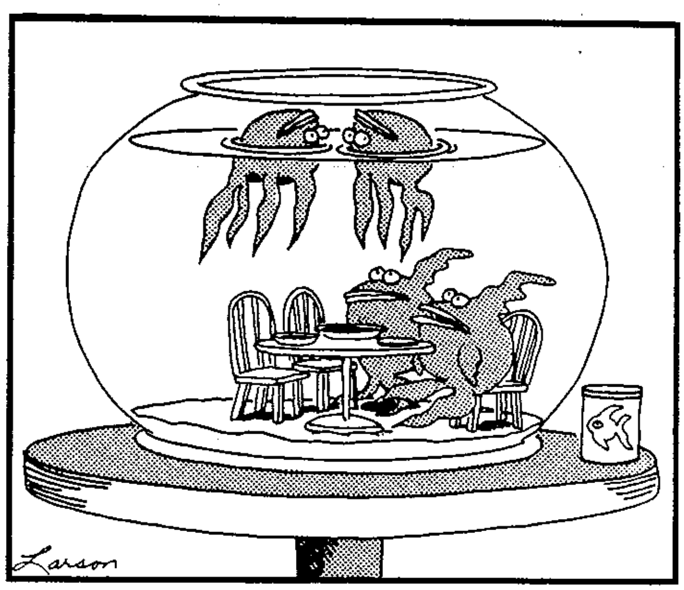
⑲ Solve: $3\log_2 2 - \log_2 32 = -\log_2 x$

⑳ Use the table to find: $\log 4630$

㉑ Use the table to find: $\log .0083$

㉒ Use a calculator to find: $\log 304.1$ (to 4 places)

㉓ Use a calculator to find: $\log x = .4541 - 2$ (to 4 places)



"Well, the Parkers are dead. ... You had to encourage them to take thirds, didn't you?"

Logarithms

UNIT 21 REVIEW & PRACTICE

Change to logarithmic form:

① $3^{-2} = 1/9$

Change to exponential form:

② $\log_4 2 = 1/2$

Solve a basic logarithmic equation:

③ $\log_4(1-2x) = \log_4(x+10)$

Use the properties of logarithms to solve:

④ $\log_6(n-3) + \log_6(n+2) = \log_3 3$

⑤ $2\log_2 x - \frac{1}{2}\log_2 16 = 4$

Use the table of mantissas to solve:

⑥ $\log 437$

⑦ $\log x = .5011 - 2$

Approximate the value as a common logarithm:

⑧ $\log_3 8$ (round to 4 places)

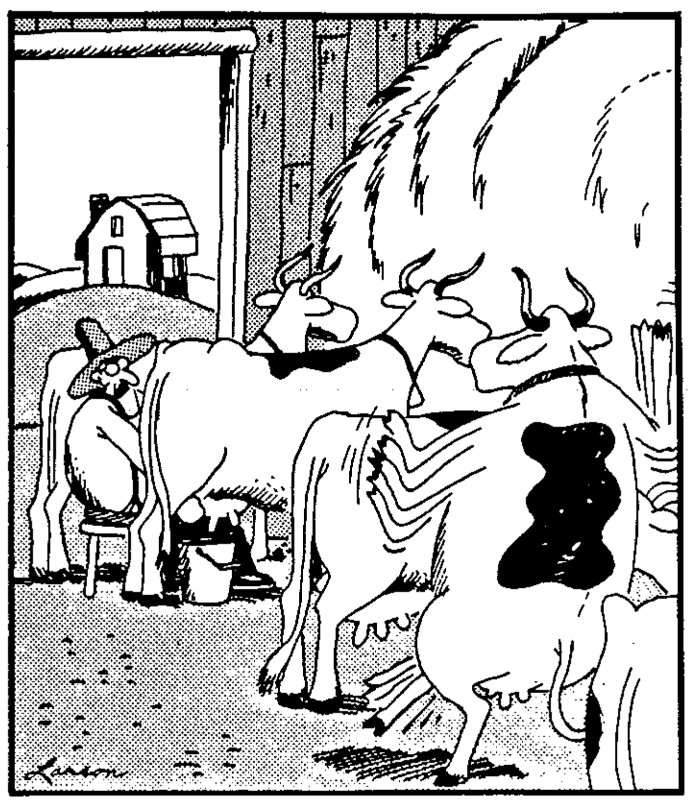
Solve an exponential equation:

⑨ $4^{3x-2} = 6^{2x+1}$

(round to 4 places)

Solve:

⑩ $\log_3[\log_8(\log_3 x)] = -1$



"Hey! I'm coming, I'm coming—just cross your legs and wait!"

UNIT 22

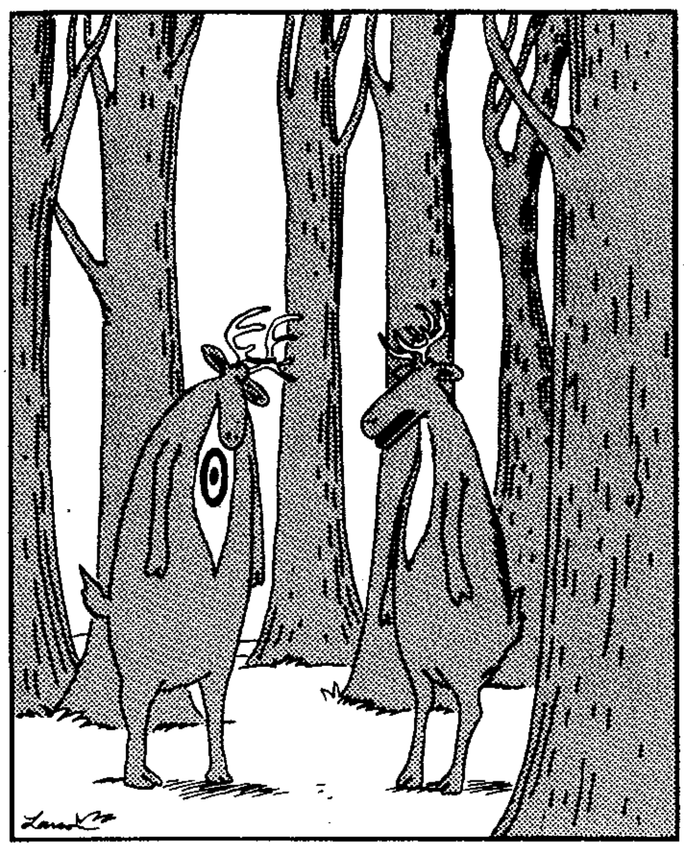
Conics

22.1
Parabolas

22.2
Circles

22.3
Ellipses

22.4
Hyperbolas



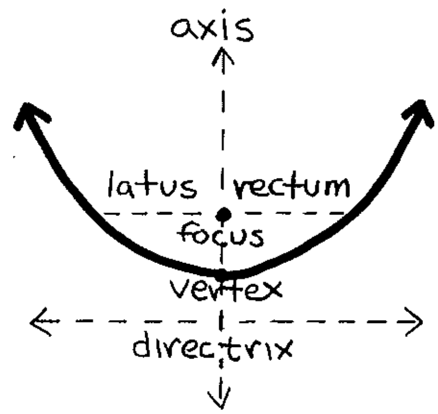
"Bummer of a birthmark, Hal."

Conics

REFERENCE PAGE

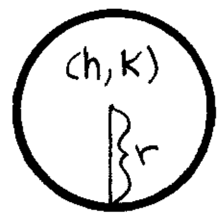
PARABOLA

	Horizontal	Vertical
EQUATION	$x = a(y-k)^2 + h$	$y = a(x-h)^2 + k$
AXIS	$y = k$	$x = h$
VERTEX	(h, k)	(h, k)
FOCUS	$(h + \frac{1}{4a}, k)$	$(h, k + \frac{1}{4a})$
DIRECTRIX	$x = h - \frac{1}{4a}$	$y = k - \frac{1}{4a}$
OPENING	right if $a > 0$	upward if $a > 0$
LATUS RECTUM	$\left\lvert \frac{1}{a} \right\rvert$	$\left\lvert \frac{1}{a} \right\rvert$



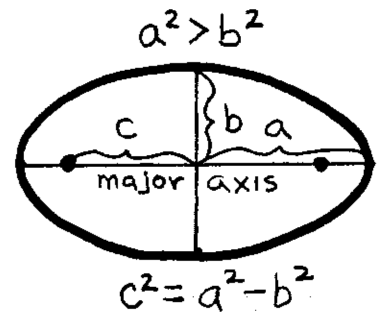
CIRCLE

EQUATION	$(x-h)^2 + (y-k)^2 = r^2$
DISTANCE FORMULA	$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$



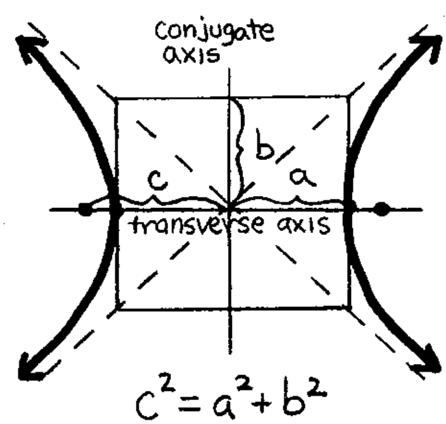
ELLIPSE

	Horizontal	Vertical
EQUATION	$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$	$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$
FOCI	$(h \pm c, k)$	$(h, k \pm c)$
CENTER	(h, k)	(h, k)



HYPERBOLA

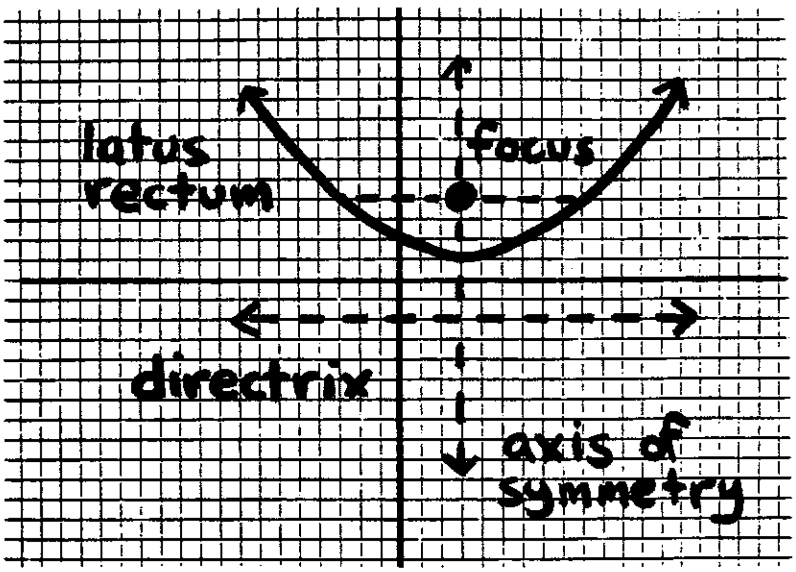
	Horizontal	Vertical
EQUATION	$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$	$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$
CENTER	(h, k)	(h, k)
FOCI	$(h \pm c, k)$	$(h, k \pm c)$
VERTICES	$(h \pm a, k)$	$(h, k \pm a)$
ASYMPTOTE SLOPE	$\pm \frac{b}{a}$	$\pm \frac{a}{b}$



Parabolas

DEMONSTRATION 22.1

A parabola is the set of all points in a plane that are the same distance from a given point and a given line in that plane. The point is called the focus. The line is called the directrix.



A line segment through the focus perpendicular to the axis of symmetry with endpoints on the parabola is called the latus rectum.

PARABOLA	Horizontal	Vertical
Equation	$x=a(y-k)^2+h$	$y=a(x-h)^2+k$
Axis	$y=k$	$x=h$
Vertex	(h,k)	(h,k)
Focus	$(h+1/4a, k)$	$(h, k+1/4a)$
Directrix	$x=h-1/4a$	$y=k-1/4a$
Opening	right if $a>0$	upward if $a>0$
Latus Rectum	$\lvert 1/a \rvert$	$\lvert 1/a \rvert$

Parabolas

DEMONSTRATION 22.1

Name the axis of symmetry, vertex, focus, directrix, direction of opening, and length of latus rectum. Draw the graph.

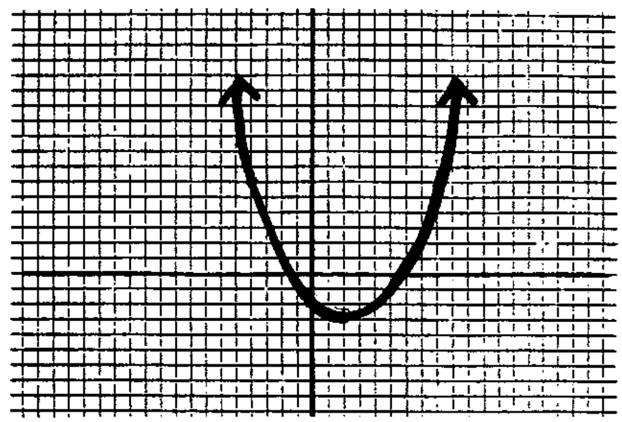
① $y = \frac{1}{4}(x-2)^2 - 3$

$a = 1/4$
$h = 2$
$k = -3$

Use relationships for a vertical parabola:

axis: $x = 2$
vertex: $(2, -3)$
focus: $(2, -2)$
directrix: $y = -4$
opening: upward
latus rectum: 4

x	y
2	-3
0	-2
-4	6



Choose values for "x" and substitute to determine "y"

② $8x = y^2 + 4y + 28$

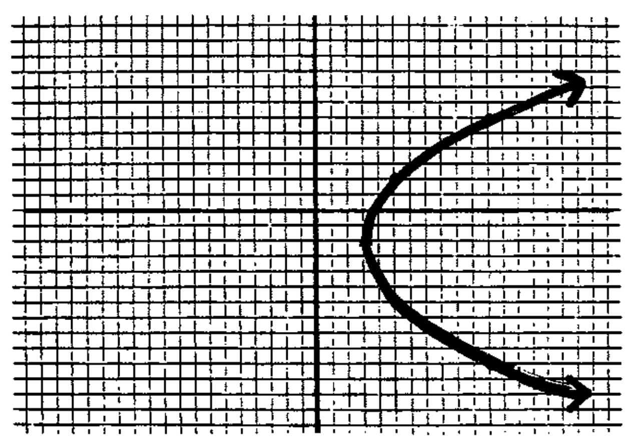
$8x = y^2 + 4y + \square + 28 - \square$
$8x = y^2 + 4y + 4 + 28 - 4$
$8x = (y+2)^2 + 24$
$x = 1/8\,(y+2)^2 + 3$

Use completing the square to put the equation in proper form

$a = 1/8$, $h = 3$, $k = -2$

axis: $y = -2$
vertex: $(3, -2)$
focus: $(5, -2)$
directrix: $x = 1$
opening: right
latus rectum: 8

x	y
3	-2
5	2
11	6



Choose values for "y" and substitute to determine "x"

Parabolas

DEMONSTRATION 22.1

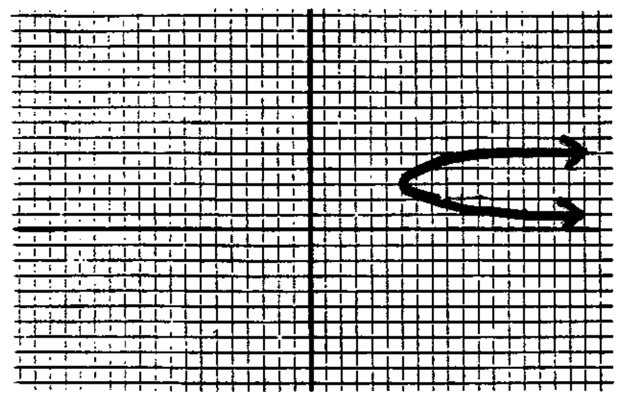
③ $x = 2y^2 - 12y + 24$

$\frac{1}{2}x = y^2 - 6y + \square + 12 - \square$

$\frac{1}{2}x = y^2 - 6y + 9 + 12 - 9$

$\frac{1}{2}x = (y-3)^2 + 3$

$x = 2(y-3)^2 + 6$



$a = 2$
$h = 6$
$k = 3$

use the horizontal relationships:

axis:	$y = 3$
vertex:	$(6, 3)$
focus:	$(6\frac{1}{8}, 3)$
directrix:	$x = 5\frac{7}{8}$
opening:	right
latus rectum:	$\frac{1}{2}$

x	y	
6	3	
8	2	$2((2)-3)^2+6 = 8$
14	1	$2((1)-3)^2+6 = 14$

Given the focus and directrix, indicate the direction of the opening for the parabola and write the equation.

④ focus $(1, 3)$
directrix $y = -1$

distance between focus and directrix is -4

add $\frac{1}{2}$ the distance to the y-coordinate of focus to determine the vertex: $(1, 1)$

$h = 1,\ K = 1$

directrix:

$y = K - \frac{1}{4a}$

$(-1) = (1) - \frac{1}{4a}$

$-2 = -\frac{1}{4a}$

$-8a = -1$

$a = \frac{1}{8}$

A quick sketch of focus and directrix shows $\boxed{\text{upward}}$ opening

$$\boxed{y = \frac{1}{8}(x-1)^2 + 1}$$

Parabolas

DEMONSTRATION 22.1

5 focus $(3,4)$
directrix $x = 11$

distance between focus and directrix is 8

add $\frac{1}{2}$ the distance to the x-coordinate of the focus to determine the vertex: $(7,4)$

$h = 7, \; k = 4$

directrix:

$x = h - 1/4a$

$(11) = (7) - 1/4a$

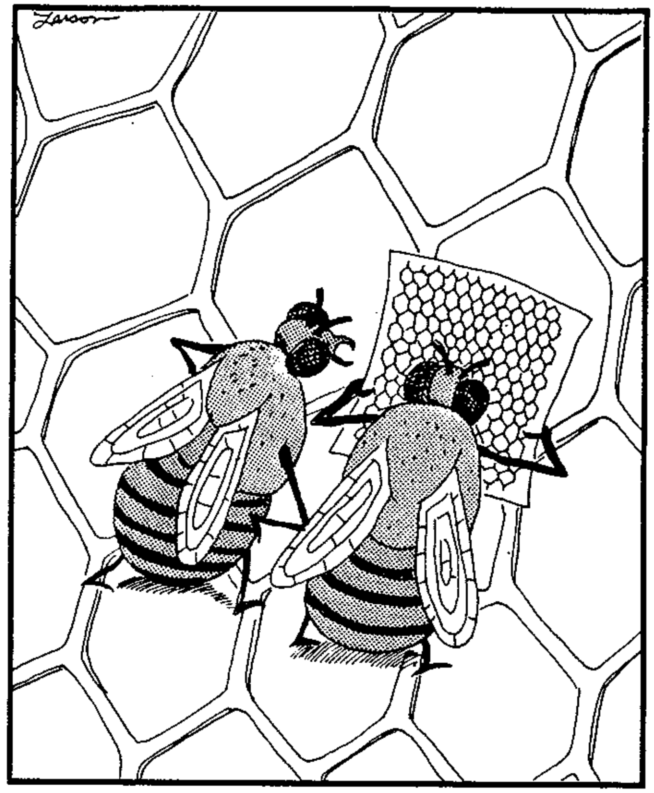
$4 = -1/4a$

$16a = -1$

$a = -1/16$

The parabola opens to the $\boxed{\text{left}}$

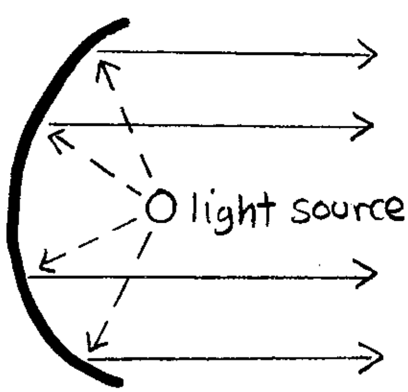
$$\boxed{x = \frac{-1}{16}(y-4)^2 + 7}$$



"Face it, Fred—you're lost!"

Applications

An automobile headlight has a parabolic reflector:



Radar antennas also use parabolic reflectors:

Sound waves collected by a parabolic reflector are concentrated by an amplifier at the focus to produce a stronger signal.

Parabolas

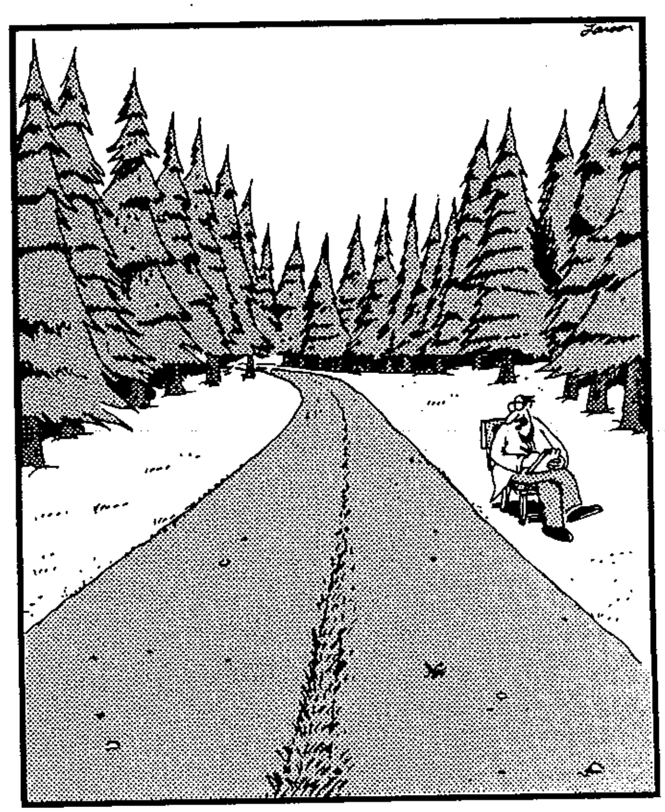
PROBLEM SET 22.1

Name the axis of symmetry, vertex, focus, directrix, direction of opening, and length of the latus rectum. Then graph the equation.

1. $y = x^2 + 4x + 1$
2. $y = 3x^2 - 24x + 50$
3. $x = y^2 - 14y + 25$
4. $x = \frac{1}{2}y^2 - 3y + \frac{19}{2}$
5. $(y-8)^2 = -4(x-4)$
6. $(x+3)^2 = -4(y-2)$
7. $y = -2x^2 + 4x + 2$
8. $y = 2x^2 + 8x + 2$

Given the focus and directrix, indicate the direction of opening for the parabola and write the equation.

9. $(3,5)$ $\quad y=1$
10. $(2,4)$ $\quad y=6$
11. $(2,2)$ $\quad x=4$
12. $(3,-1)$ $\quad x=-2$



"Now now now. ... You won't be a lonely road forever, you know."

Circles

DEMONSTRATION 22.2

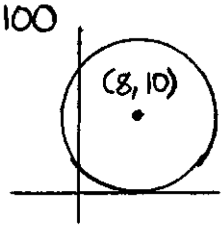
A circle is the set of all points in a plane equal distance from its center point. The equation of a circle is defined as:

Equation $(x-h)^2+(y-k)^2=r^2$ Center (h,k) Radius $=r$

Determine the center point, the length of the radius, and then graph the circle.

① $(x-8)^2+(y-10)^2=100$

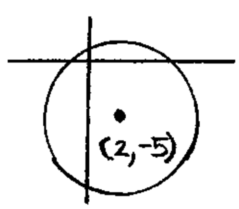
center $(8,10)$
radius $=\sqrt{100}=10$



② $x^2+y^2-4x+10y-9=0$

complete the square for each variable:

$x^2-4x+\square+y^2+10y+\triangle=9+\square+\triangle$
$x^2-4x+4+y^2+10y+25=9+4+25$
$(x-2)^2+(y+5)^2=38$



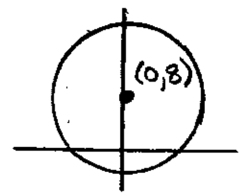
center $(2,-5)$
radius $=\sqrt{38}$

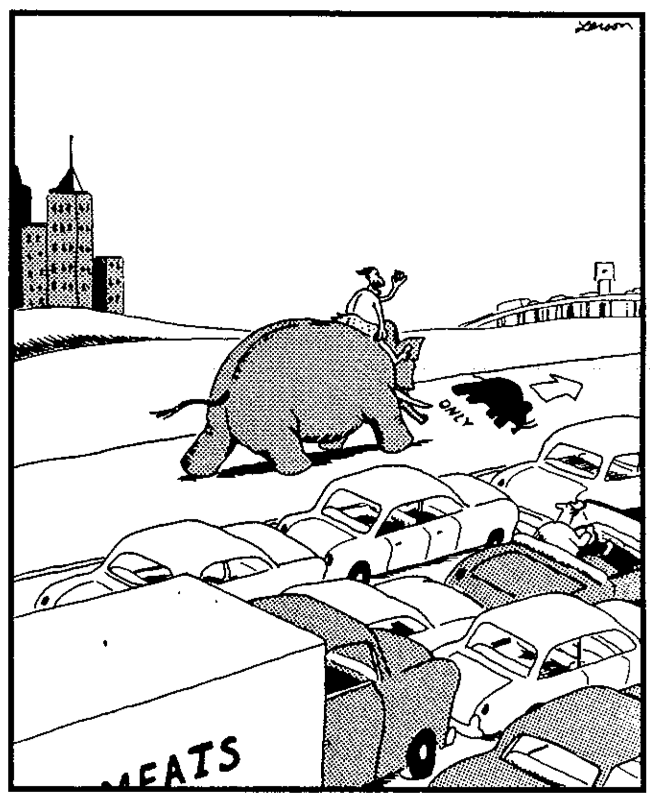
③ $x^2+y^2-16y+48=64$

Complete the square for each variable:

$x^2+y^2-16y+\square=16+\square$
$x^2+y^2-16y+64=16+64$
$(x-0)^2+(y-8)^2=80$

center $(0,8)$
radius $=\sqrt{80}$
radius $=4\sqrt{5}$





Special commuter lanes

Circles

DEMONSTRATION 22.2

Write an equation for each circle.

④ A circle with a center of (6,0) and a radius of 7

center (h, k) $h=6$, $k=0$

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-6)^2 + (y-0)^2 = 7^2$$

$$(x-6)^2 + y^2 = 49$$

⑤ A circle with a center of $(-3,-6)$ passing through $(-1,-2)$

Use the distance formula to determine the length of the radius:

$$d = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2}$$

$$r = \sqrt{((-3)-(-1))^2 + ((-6)-(-2))^2}$$

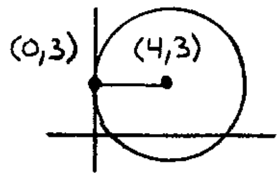
$$r = \sqrt{4+16} = \sqrt{20} = 2\sqrt{5}$$

$h=-3$ $k=-6$ $r=2\sqrt{5}$

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x+3)^2 + (y+6)^2 = 20$$

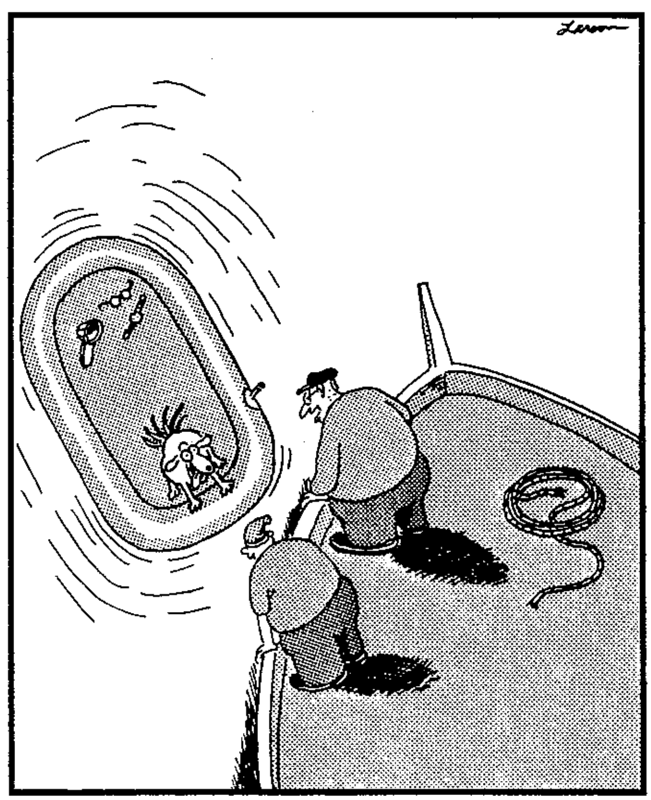
⑥ A circle with a center of (4,3) and tangent to the y-axis.

The radius of the circle is 4 because the point of tangency is (0,3):



$r=4$

$$(x-h)^2 + (y-k)^2 = r^2$$
$$(x-4)^2 + (y-3)^2 = 16$$



"Hey, boy! How ya doin'? ... Look at him, Dan. Poor guy's been floating out here for days but he's still just as fat and happy as ever."

Circles

PROBLEM SET 22.2

Identify the coordinate of the center point, the length of the radius, and graph each equation.

1. $(x-2)^2+(y-5)^2=16$
2. $(x+2)^2+(y-1)^2=81$
3. $(x+4)^2+y^2=49$
4. $x^2+(y+2)^2=4$
5. $x^2+y^2-12x-16y+84=0$
6. $x^2+y^2+14x+6y=23$
7. $x^2+y^2+4x=8$
8. $x^2+y^2+14x+6y=-50$

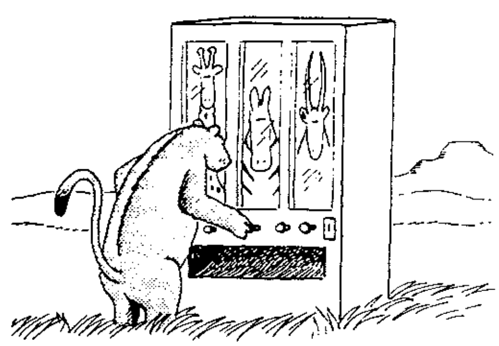
Write the equation for each of the following circles.

9. A circle with a center of $(-3,-5)$ and a radius of 5
10. A circle with a center of $(6,2)$ and a radius of 5
11. A circle with a center of $(4,-2)$ passing through $(9,-3)$
12. A circle with a center of $(1,5)$ passing through the origin
13. A circle with a center of $(-3,8)$ tangent to the x-axis
14. A circle with a center of $(4,-3)$ tangent to the y-axis

Review

Name the axis of symmetry, the vertex, the focus, the directrix, the direction of the opening, and the length of the latus rectum. Then graph each parabola.

15. $-6x=y^2+8y+40$
16. $y=-\frac{1}{12}(x-4)^2+2$



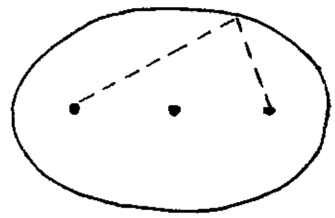
Vending machines of the Serengeti

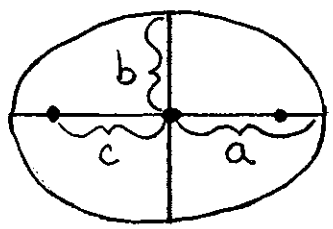
Ellipses

DEMONSTRATION 22.3

While a circle is defined by a center point and a given distance, an ellipse is defined by two points (foci) and the sum of two distances.

In an ellipse, the sum of the distances from the foci to all points on the ellipse remains constant.





center (h, k)

major axis $= 2a$
minor axis $= 2b$

	EQUATION	FOCI
Horizontal Ellipse	$\frac{(x-h)^2}{a^2}+\frac{(y-k)^2}{b^2}=1$	$(h \pm c, k)$
Vertical Ellipse	$\frac{(x-h)^2}{b^2}+\frac{(y-k)^2}{a^2}=1$	$(h, k \pm c)$

$a^2 > b^2$ and $c^2 = a^2 - b^2$

Find the center point, the foci, the lengths of the major and minor axis. Then graph the ellipse.

① $\frac{(x+2)^2}{36}+\frac{(y-3)^2}{9}=1$

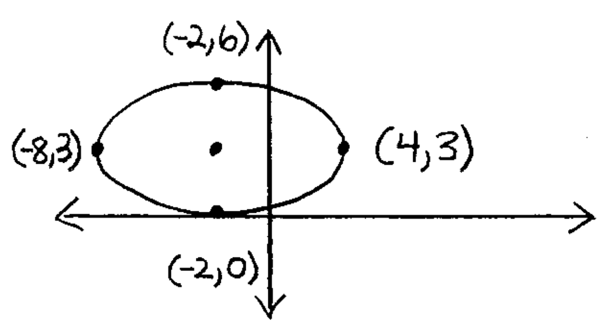
$a = \sqrt{36} = 6$ major axis $= 2a = 12$
$b = \sqrt{9} = 3$ minor axis $= 2b = 6$

Since $a^2 > b^2$, this is a horizontal ellipse.

center $(h, k) = (-2, 3)$

$c^2 = a^2 - b^2$
$c^2 = 36 - 9 = 27$
$c = \sqrt{27} = 3\sqrt{3}$

foci
$(-2 \pm 3\sqrt{3}, 3)$



Ellipses

DEMONSTRATION 22.3

② CENTER AT THE ORIGIN

$$\frac{x^2}{9} + \frac{y^2}{25} = 1$$ Vertical Ellipse

$(h, k) = (0,0)$ center

$a = 5$ $\quad c^2 = a^2 - b^2$

$b = 3$ $\quad c^2 = 25 - 9 = 16$

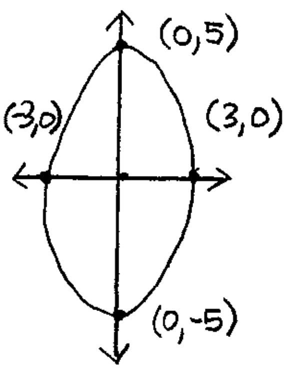
$c = \sqrt{16} = 4$

Foci $(0, 0 \pm 4)$

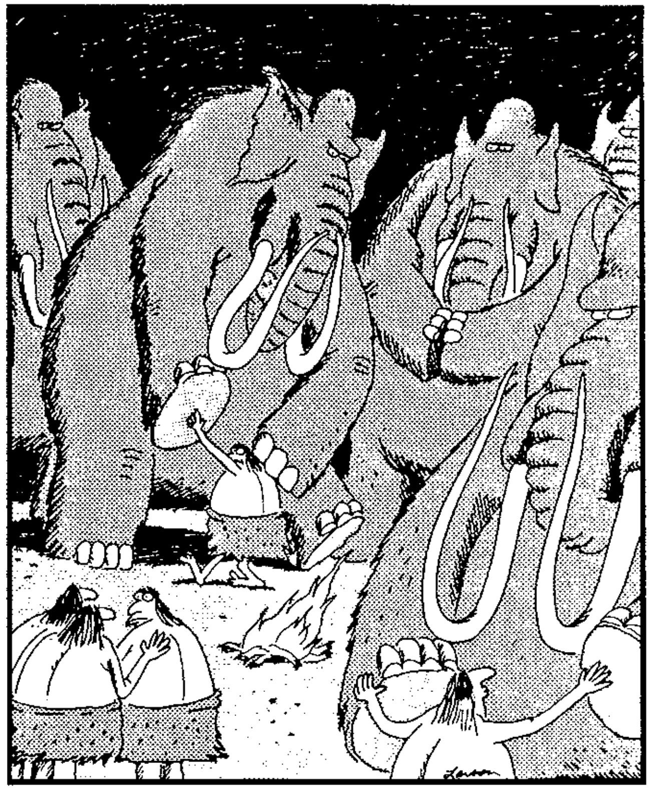
$(0, 4)$ and $(0, -4)$

major axis $= 2a = 10$

minor axis $= 2b = 6$



when h or k do not appear in the equation, their value is equal to zero



Tomorrow, they would be mortal enemies. But on the eve of the great hunt, feelings were put aside for the traditional Mammoth Dance.

③ COMPLETING THE SQUARE & APPROXIMATING AXIS

$2x^2 + 5y^2 + 8x - 30y = -43$

$2x^2 + 8x + 5y^2 - 30y = -43$

$2(x^2 + 4x + \square) + 5(y^2 - 6y + \Delta) = -43 + 2\square + 5\Delta$

$2(x^2 + 4x + 4) + 5(y^2 - 6y + 9) = -43 + 8 + 45$

$2(x+2)^2 + 5(y-3)^2 = 10$

$$\frac{(x+2)^2}{5} + \frac{(y-3)^2}{2} = 1$$

center $(h, k) = (-2, 3)$

$a = \sqrt{5}$ $\quad c^2 = 5 - 2 = 3$

$b = \sqrt{2}$ $\quad c = \sqrt{3}$

Foci $(-2 \pm \sqrt{3}, 3)$

major axis $= 2a = 2\sqrt{5}$

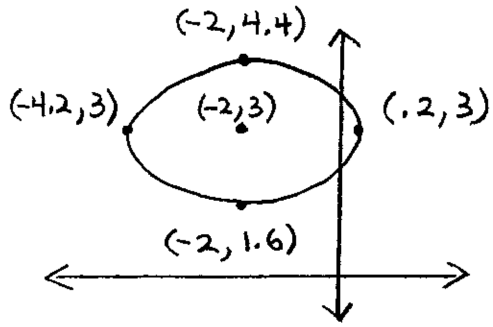
minor axis $= 2b = 2\sqrt{2}$

Ellipses

DEMONSTRATION 22.3

To graph the ellipse, it is necessary to approximate the values of a and b.

$a = \sqrt{5} \approx 2.2$
$b = \sqrt{2} \approx 1.4$



④ USE INFORMATION TO WRITE AN EQUATION

The foci is $(-2, 3 \pm 2\sqrt{5})$. The major axis is 12 units long.

Because of the direction of the major axis, the ellipse is vertical.

The center is $(-2,3)$.
$h = -2$, $k = 3$

Because the major axis is 12, $2a = 12$ and $a = 6$

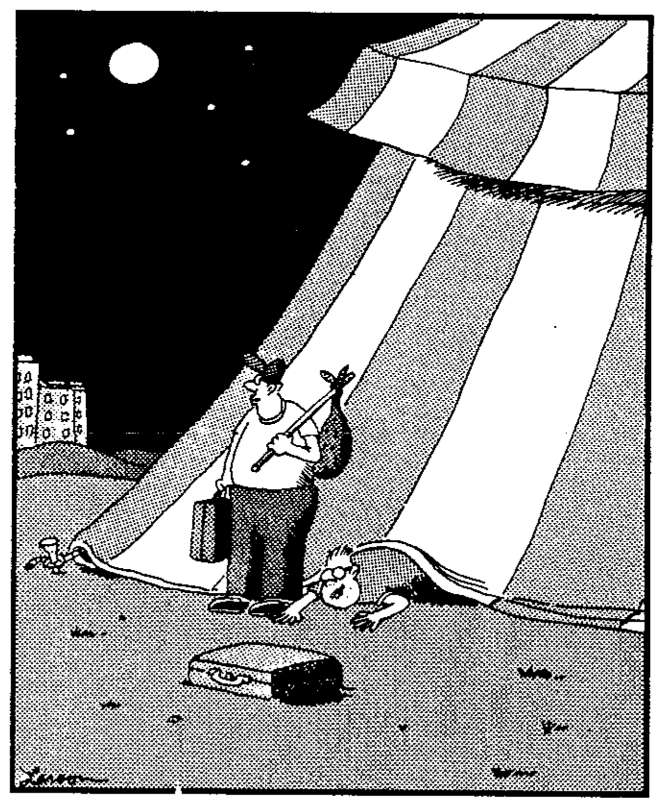
Because the center is $(h, k \pm c)$: $c = 2\sqrt{5}$.

To determine b:

$c^2 = a^2 - b^2$
$(2\sqrt{5})^2 = (6)^2 - b^2$
$20 = 36 - b^2$
$b = 4$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

$$\frac{(x+2)^2}{16} + \frac{(y-3)^2}{36} = 1$$



Ironically, Barnum's and Bailey's respective kids—Sid and Marty—both ran away one night to join corporate America.

Ellipses

PROBLEM SET 22.3

Find the center, foci, and the lengths of the major and minor axis. Then graph the ellipse.

① $\dfrac{(x+3)^2}{36} + \dfrac{(y-4)^2}{9} = 1$

② $\dfrac{(x+2)^2}{20} + \dfrac{(y+3)^2}{40} = 1$

③ $\dfrac{(x-4)^2}{121} + \dfrac{(y+5)^2}{64} = 1$

④ $\dfrac{(x-2)^2}{16} + \dfrac{(y-3)^2}{9} = 1$

⑤ $9x^2 + 4y^2 - 18x + 16y = 11$

⑥ $3x^2 + 7y^2 - 12x - 28y = -19$

⑦ $25x^2 + 4y^2 = 100$

⑧ $16x^2 + 25y^2 + 32x - 150y = 159$

Write the equation for each ellipse:

⑨ Foci $(0,8)$ and $(0,-8)$. The endpoints of the major axis $(0,10)$ and $(0,-10)$.

⑩ Center $(5,4)$. Major axis is 16 units long and parallel to x-axis. Minor axis is 8 units long.

Review

Name the axis, vertex, focus, directrix, direction of opening, and length of latus rectum. Then graph the parabola.

⑪ $-2x = y^2 + 12y + 32$

⑫ $8y = x^2 - 8x + 32$

Identify the center point and the length of the radius. Then graph the circle.

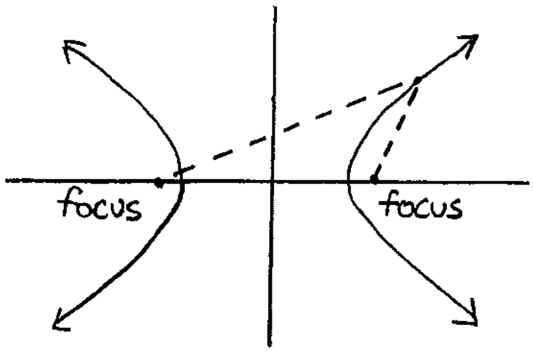
⑬ $x^2 + y^2 - 6y = 16$

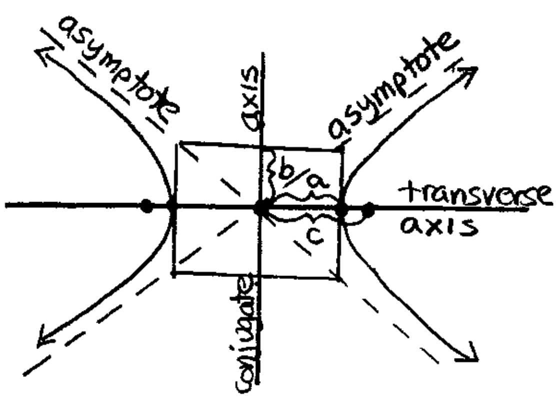
⑭ $x^2 + 2x + y^2 + 4y = 9$

Hyperbolas

DEMONSTRATION 22.4

A hyperbola is the set of all points in a plane such that the absolute value of the difference of the distances from the two foci remains constant.





	HORIZONTAL	VERTICAL
Equation	$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$	$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$
Center	(h,k)	(h,k)
Foci	$(h\pm c, k)$	$(h, k\pm c)$
Vertices	$(h\pm a, k)$	$(h, k\pm a)$
Asymptote slope	$\pm\frac{b}{a}$	$\pm\frac{a}{b}$

$\boxed{c^2=a^2+b^2}$ Dist. Betw. Vertices $=2a$

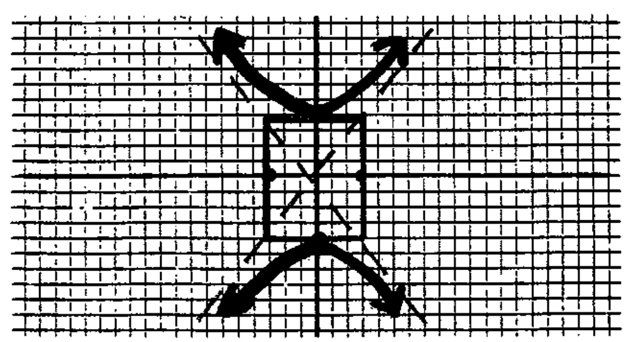
Find the center point, foci, vertices, slope of the asymptotes, and then graph the hyperbola.

① $\frac{y^2}{16}-\frac{x^2}{9}=1$ transverse axis is vertical   asymptotes $m=\pm 4/3$

center $(h,k)=(0,0)$
$a=4,\ b=3$

$c^2=a^2+b^2$
$c^2=16+9=25$ $c=\sqrt{25}=5$

foci $(0,\pm 5)$
vertices $(0,\pm 4)$



Hyperbolas

DEMONSTRATION 22.4

② $9x^2-4y^2-54x-40y-55=0$

$9x^2-54x-4y^2-40y=55$

$9(x^2-6x+\square)-4(y^2+10y+\Delta)=55+\square+\Delta$

$9(x^2-6x+9)-4(y^2+10y+25)=55+81-100$

$9(x-3)^2-4(y+5)^2=36$

$\frac{(x-3)^2}{4}-\frac{(y+5)^2}{9}=1$ transverse axis is horizontal

center $(h,k)=(3,-5)$

$a=2,\ b=3$

$c^2=a^2+b^2$

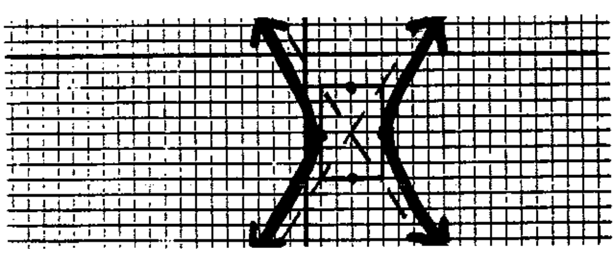
$c^2=4+9=13 \qquad c=\sqrt{13}$

foci $(3\pm\sqrt{13},-5)$

vertices $(3\pm2,-5)$:

$(5,-5)\ (1,-5)$

asymptotes $m=\pm 3/2$



③ $2y^2-4y-5x^2-20x=-22$

$2(y^2-2y+\square)-5(x^2+4x+\Delta)=-22+\square+\Delta$

$2(y-2y+1)-5(x^2+4x+4)=-22+2-20$

$2(y-1)^2-5(x+2)^2=-40$

$\frac{(y-1)^2}{-20}+\frac{(x+2)^2}{8}=1$

$\frac{(x+2)^2}{8}-\frac{(y-1)^2}{20}=1$ transverse axis is horizontal

center $(h,k)=(-2,1)$

$a=2\sqrt{2},\ b=2\sqrt{5}$

$c^2=a^2+b^2$

$c^2=8+20=28 \quad c=2\sqrt{7}$

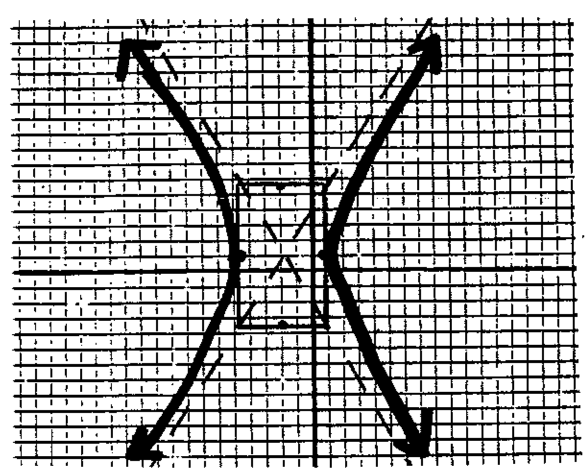
foci $(-2\pm2\sqrt{7},1)$

vertices $(-2\pm2\sqrt{2},1)$:

$(.8,1)\ (-4.8,1)$

asymptotes $m=\pm\frac{2\sqrt{5}}{2\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}$

$\pm\frac{2\sqrt{10}}{4}=\pm\frac{\sqrt{10}}{2}$



$a=2\sqrt{2}\approx 2.8$

$b=2\sqrt{5}\approx 4.5$

Hyperbolas

PROBLEM SET 22.4

Find the center point, foci, vertices, slope of the asymptotes, and then graph each hyperbola.

① $\frac{x^2}{16} - \frac{y^2}{4} = 1$

② $\frac{y^2}{25} - \frac{x^2}{9} = 1$

③ $\frac{(x+6)^2}{36} - \frac{(y+3)^2}{9} = 1$

④ $\frac{(y-3)^2}{25} - \frac{(x-2)^2}{16} = 1$

⑤ $x^2 - 4y^2 + 6x + 16y - 11 = 0$

⑥ $y^2 - 4x^2 - 2y - 16x = -1$

⑦ $y^2 - 3x^2 + 6y + 6x = 18$

⑧ $5x^2 - 4y^2 - 40x - 16y = 36$

Review

⑨ Name the axis, vertex, focus, directrix, opening, latus rectum, and then graph the parabola:

$4x = y^2 + 16y + 48$

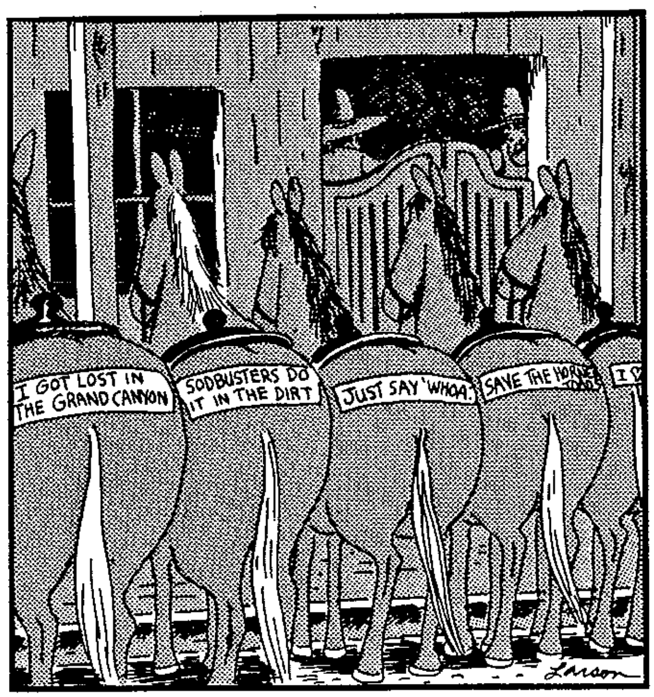
⑩ Determine the center point, length of the radius, and then graph the circle:

$x^2 - 4x + y^2 - 20y = -68$

⑪ Write the equation of a circle with center $(5,-1)$ passing through $(-2,-2)$

⑫ Determine the center, foci, length of major and minor axis, and then graph the ellipse:

$16x^2 - 64x + 9y^2 + 54y = -1$



Common butt stickers of the Old West

Conics

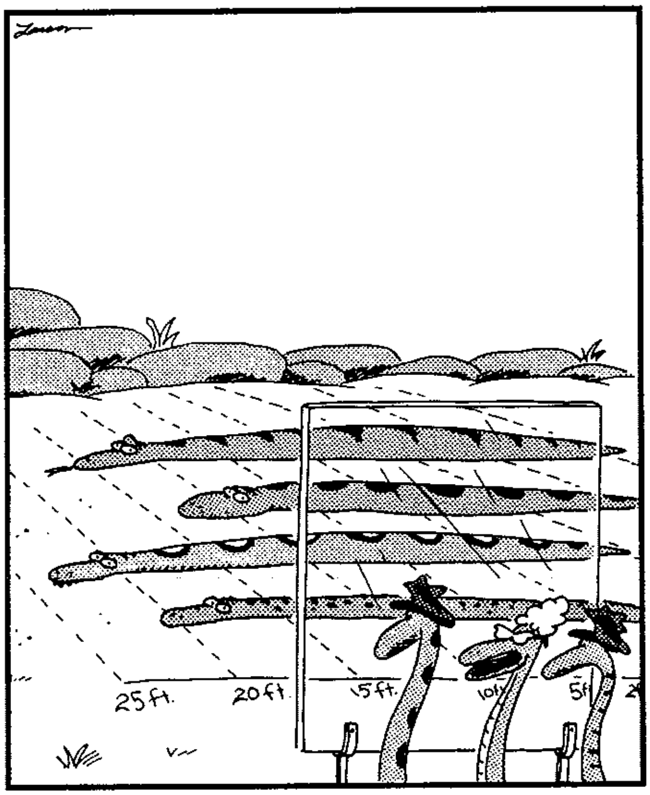
UNIT 22 REVIEW & PRACTICE

For each parabola, determine the axis, vertex, focus, directrix, opening, and length of latus rectum. Then draw the graph.

① $x = \frac{1}{2}y^2 - 5y + \frac{29}{2}$

② $y = -2x^2 - 12x - 24$

③ $x = -\frac{1}{4}y^2 + 2y + 4$



"That's him. Second from the end—the 12-footer!"

For each of the following circles, determine the center, the radius, and draw the graph.

④ $x^2 + y^2 - 6x + 12y + 20 = 0$

⑤ $x^2 + y^2 - 8y = 20$

Determine the length of the radius and write the equation for the following circles.

⑥ center $(-2, -4)$ passing through $(6, -2)$

⑦ center $(4, 10)$ passing through $(8, 16)$

Determine the center, foci, length of the major and minor axis, and draw the graph for each ellipse.

⑧ $\frac{(x-3)^2}{25} + \frac{(y-4)^2}{4} = 1$

⑨ $4x^2 + 16x + y^2 + 12y = -16$

⑩ $x^2 + 2x + 4y^2 - 8y = 59$

continued

Conics

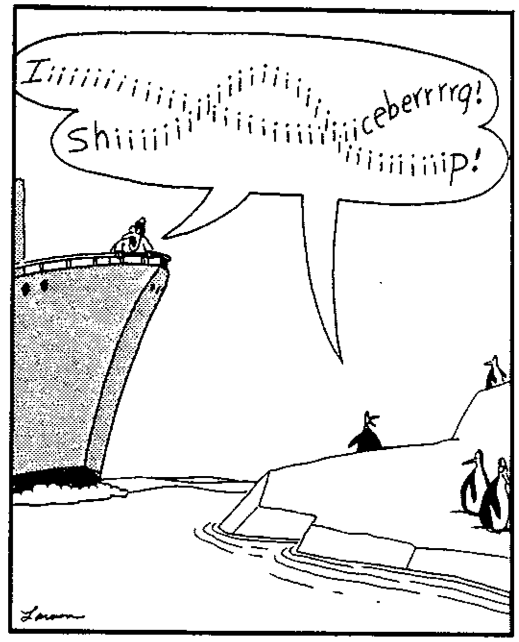
UNIT 22 REVIEW & PRACTICE

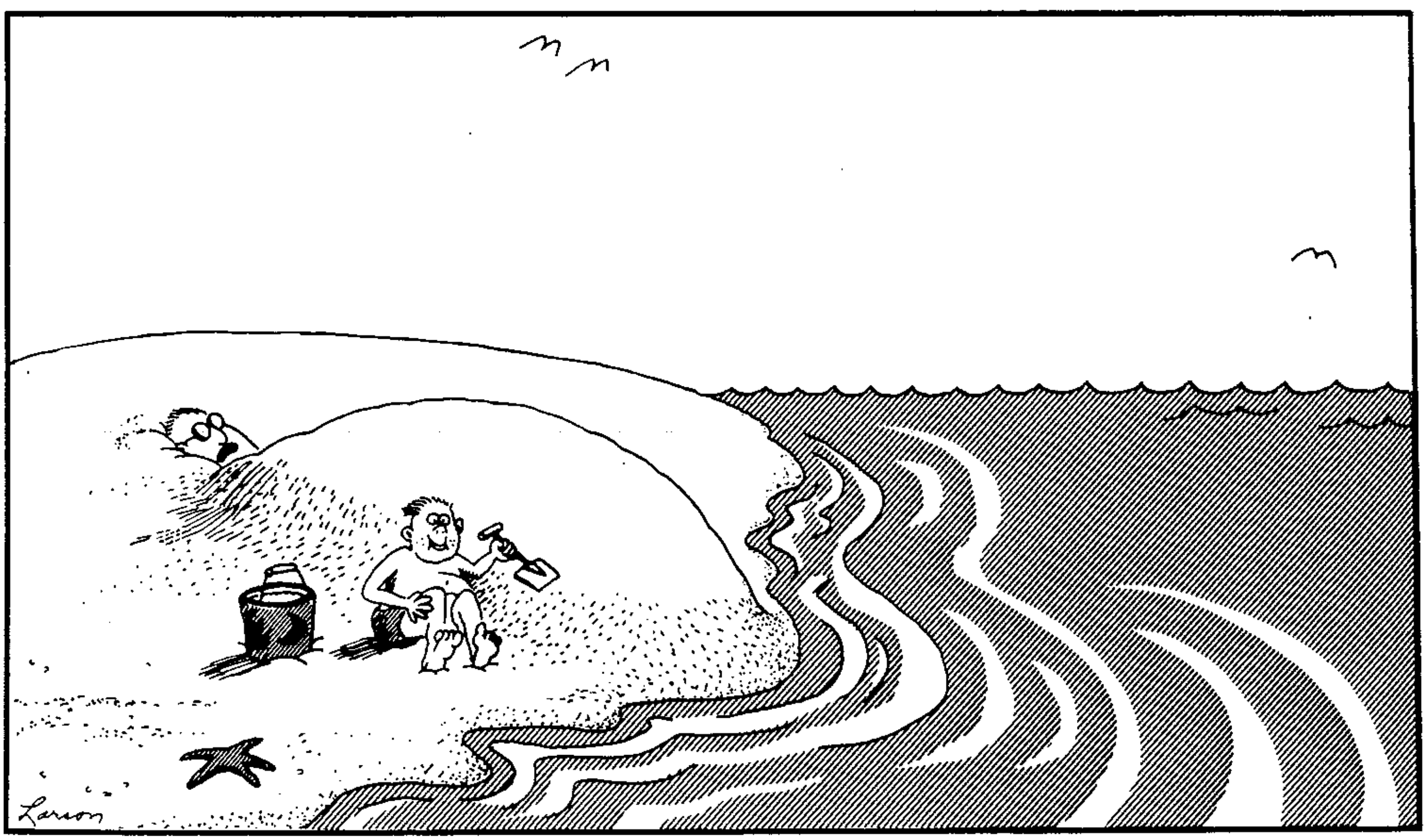
Find the center, foci, vertices, slope of the asymptotes, and then draw the graph of each hyperbola.

(11) $\frac{(x-6)^2}{16} - \frac{(y+4)^2}{49} = 1$

(12) $9x^2 - 36x - 4y^2 - 32y = 172$

(13) $y^2 - 6y - 4x^2 - 8x = 95$





"Okay, Billy . . . Tide's coming in now . . . Dig me out, Billy . . . Billy, I don't want to get angry."

ALGEBRA REVIEW

High School Placement Test Preparation

Part 1
Algebra Skills

Part 2
Algebra Skills

Part 3
Algebra Problem Solving

Hell's library

Algebra Skills Review

PROBLEM SET I

Identify the property:

① $\frac{-3a}{4} + \frac{3a}{4} = 0$

Evaluate: $a = -2$, $b = -3$

② $2a^2 - 3ab + a^3$

Simplify:

③ $3ab - 2a(a-3b) - 4a^2$

Solve the equation:

④ $\frac{2n}{3} - 3(n-1) = -4(n+4) - 1$

Solve the inequality:

⑤ $\frac{2x+9}{5} > \frac{x+4}{3}$

Solve for n:

⑥ $3nc - 2 = 5n$

Solve and graph on a number line:

⑦ $|a+2| - 4 \geq 2$

⑧ $|3a-9| < 6$ and $|a| \neq 2$

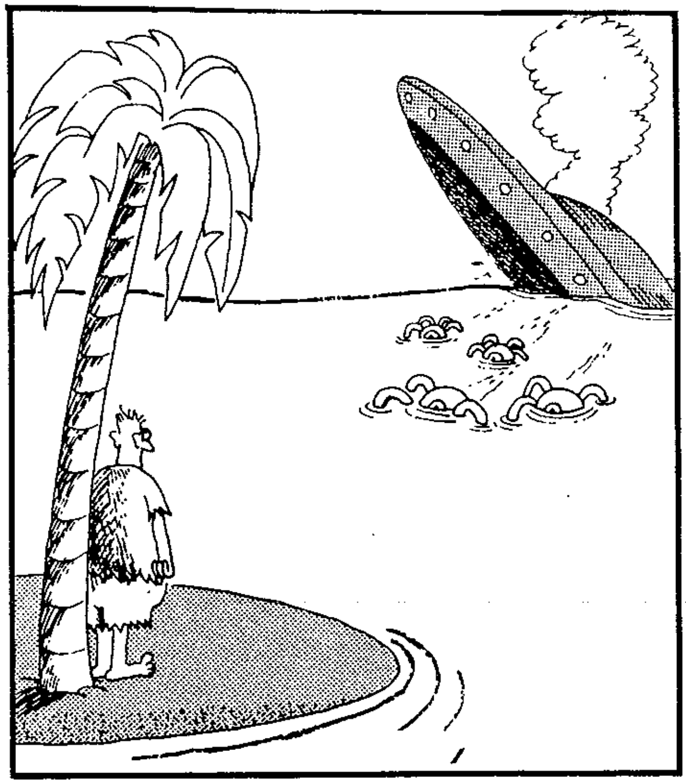
Simplify monomials:

⑨ $\left(\frac{-1}{2}x^2y^3z\right)^2 \left(-2xy^{-1}z^2\right)^3$

⑩ $\frac{-18a^2b^3c^{-3}}{8a^{-3}b^2c^{-1}}$

Evaluate in scientific notation:

⑪ $(.03 \times 10^{-3})(.14 \times 10^{-2})$



Multiply/Divide polynomials:

⑫ $(4x+7)(2x-5)$

⑬ $(2a^3 + a^2 - a - 5) \div (a-1)$

Algebra Skills Review

PROBLEM SET I

(14) $(2ab^{x+2} - 3c^{4x})^2$

(15) $(4x^n + y^{n-3})(4x^n - y^{n-3})$

Determine GCF and LCM:

(16) $12a^2b,\ 30ab^3c,\ 56abc^2$

Factor completely:

(17) $n^8 - 1$

(18) $6n^2 + 14n - 12$

(19) $(3n-2x)^2 - (n+5x)^2$

(20) $a^2b^2 - a^4 + a^2b^2 - b^4$

Solve the equation:

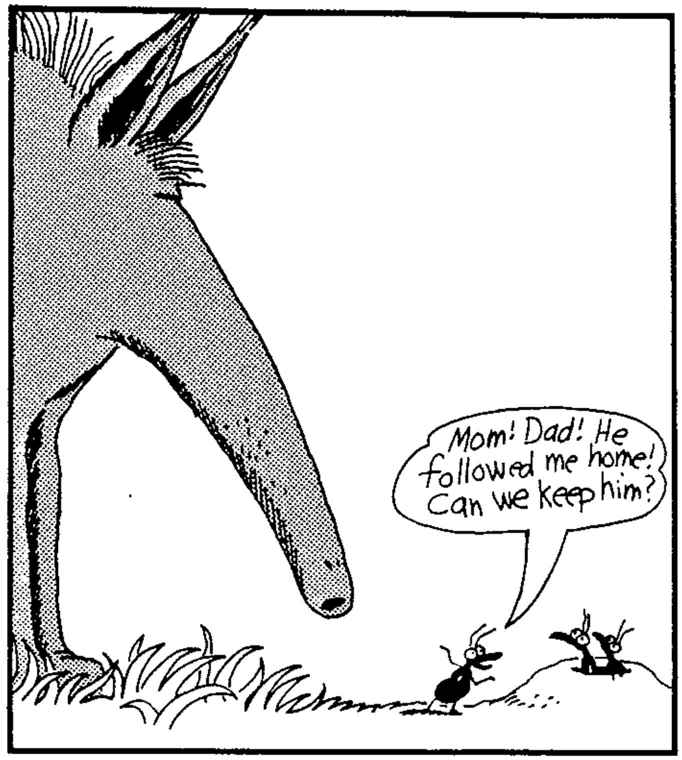
(21) $2n^3 + 6n = -8n^2$

Linear equations:

(22) Write an equation in standard form for a line that includes $(-2,-10)$ and $(4,-15)$.

(23) Write an equation in slope-intercept form for a line perpendicular to $2x - y = 12$ going through $(-3, 9)$.

(24) Write an equation in point-slope form parallel to $y = \frac{2}{5}x - 4$ passing through the point $(-2,-5)$.



Childhood innocence

(25) Change $4x + 4 = -2y$ to slope-intercept form. Determine the slope and both intercepts. Graph it.

Determine the midpoint:

(26) $(-9, 7)$ $(-3, 5)$

Determine the function value:

(27) $f(g(n+3))$ for $f(x) = x - x^2$ and $g(x) = x - 5$

Algebra Skills Review

PROBLEM SET II

Identify the property:

① $4y + ab = ab + 4y$

Evaluate: $a = -1$, $b = -4$

② $a^3 - 3b^2 + 3ab$

Simplify:

③ $4x(2x-y) - 3x(y+2x)$

Solve the equation:

④ $12 + \frac{3x}{5} = -3(x+8)$

Solve the inequality:

⑤ $\frac{3(2n+1)}{7} \geq \frac{12-n}{3}$

"Hey! Look at Red Bear! ... Waiiiiiiit ... THAT not real!"

Solve for x:

⑥ $2(3x+y) = 3y(x-1)$

Solve and graph on a number line:

⑦ $|n-1| + 3 < 9$

⑧ $|n+4| - 1 \geq 2$ and $|n| < 10$

Simplify monomials:

⑨ $\left(-\frac{1}{3}a^2b^3c\right)^2(-3ab^{-1}c)^3$

⑩ $\frac{-30a^{-3}bc^{-2}}{-6ab^4c^{-5}}$

Evaluate in scientific notation:

⑪ $\frac{.0063 \times 10^4}{9 \times 10^7}$

Multiply/Divide polynomials:

⑫ $(3a+5b)(a-6b)$

⑬ $(5a^3-2b^3) \div (a+2b)$

⑭ $(3c^{x+3} + d^{3x})^2$

⑮ $(2a^{n-1} + 3b^{n+4})(2a^{n-1} - 3b^{n+4})$

Determine GCF and LCM:

⑯ $75xyz$, $40x^2y$, $12y^2$

Algebra Skills Review

PROBLEM SET II

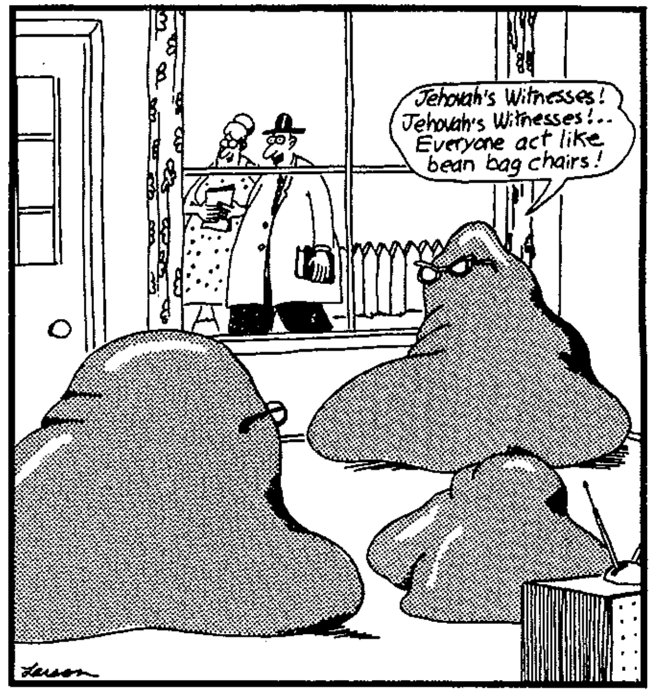
Factor completely (if possible):

(17) $3x^3 + 6x^2 + 15x$

(18) $4a^2 + 18ab + 8b^2$

(19) $(a+4)^2 + 3(a+4) - 10$

(20) $x^6 - x^4y^2 + 16y^2 - 16x^2$



The Blob family at home

Solve the equation:

(21) $4a^3 + 6a = 14a^2$

Linear equations:

(22) Write an equation in slope-intercept form for a line defined by:

x	y
-2	5
-4	9
-6	13

(23) Write an equation in standard form for a line parallel to $y = -\frac{1}{2}x + 3$ passing through the point $(8, -6)$.

(24) Write an equation in point-slope form for a line perpendicular to $2x - 3y = 8$ that passes through $(-2, -6)$.

(25) Change the equation $y = \frac{1}{4}x - 2$ to standard form. Determine the slope and both intercepts. Draw the graph.

Determine the endpoint:

(26) Find B if A $(-2, 9)$ is one endpoint and P$(5, -1)$ is the midpoint of $\overline{AB}$.

Determine the function value:

(27) $f[g(a+b)]$ for $f(x) = -2x$ and $g(x) = x^2$

Algebra Skills Review

ADDITIONAL PRACTICE PROBLEMS I & II

Identify the property:

① $1 = \frac{-2x}{5} \cdot \frac{-5}{2x}$

Evaluate: $x=-1, y=-5$

② $4x^2y - 2(x+2y)$

Simplify:

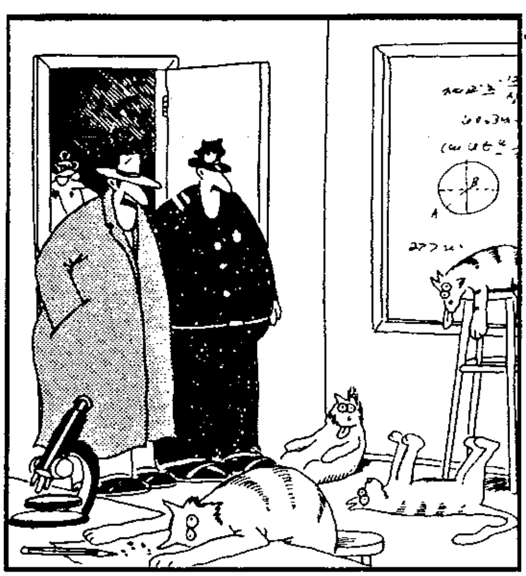
③ $3n^2 - 2n(n+m) + 2mn - n^2$

Solve the equation:

④ $5(n-2) = \frac{n}{2} + 3n - 1$

Solve the inequality:

⑤ $3(n+4) > \frac{4n+6}{3}$



"Notice all the computations, theoretical scribblings, and lab equipment, Norm. ... Yes, curiosity killed these cats."

Solve for x:

⑥ $xy - 3ab = 2a(x+b)$

Solve and graph on a number line:

⑦ $|2a-1| - 2 > 7$

⑧ $|a+4| \leq 7$ <u>and</u> $|a| \neq 2$

Simplify monomials:

⑨ $(2x^{-2}y^3)^5(-\frac{1}{2}x^4y^{-2})^3$

⑩ $\frac{-12x^3y^{-2}}{-20x^{-2}y^4z^{-2}}$

Evaluate in scientific notation:

⑪ $\frac{7000 \times 10^{-4}}{.5 \times 10^{-2}}$

Multiply/Divide polynomials:

⑫ $(2a-7b)(3a-5b)$

⑬ $(4x^4 - 5y^4) \div (x-3y)$

⑭ $(x^{3n-2} - 4y^{2n+1})^2$

⑮ $(3a^{3x} + b^{x-1})(3a^{3x} - b^{x-1})$

(continued)

Algebra Skills Review

ADDITIONAL PRACTICE PROBLEMS I & II

Determine GCF and LCM:

(16) $60a^3b^5c^2$, $100a^4b^2c^3$, $108a^2b^3c^4$

Factor completely (if possible):

(17) $3a^6b^2 - 3a^2b^6$

(18) $6a^2 - 3ab - 9b^2$

(19) $(2n+5)^2 - (n-4)^2$

(20) $a^6 - a^4b^2 - a^2b^4 + b^6$

Solve the equation:

(21) $9a^3 = 4a$

Linear equations:

(22) Write an equation in standard form for a line that includes the points $(-4,5)$ and $(-3,9)$.

(23) Write an equation in slope-intercept form for a line perpendicular to $3x+2y=4$ passing through $(-1,5)$.

(24) For the following equation: $y+6 = \frac{2}{3}(x-3)$ identify the slope, and then change the form of the equation to determine the intercepts.

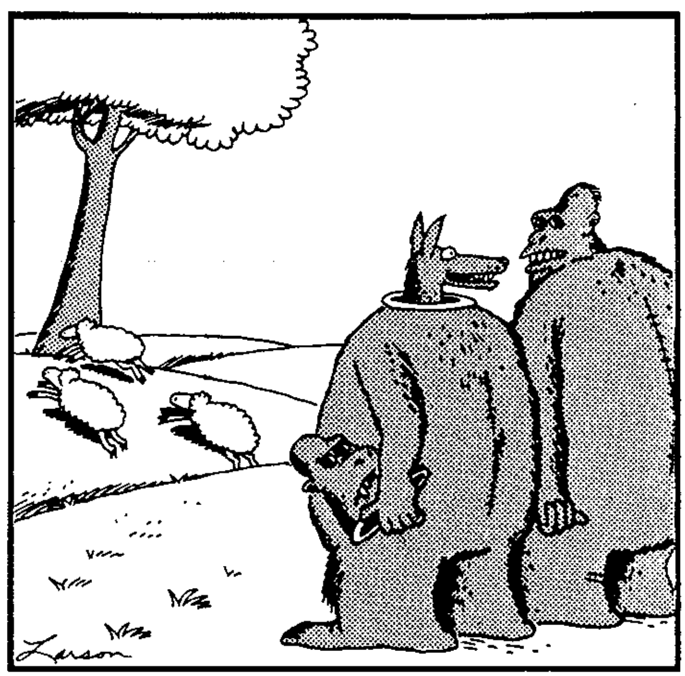
(25) Change $\frac{-4}{3}x + \frac{2}{3}y = 2$ to standard form. Determine the slope and both intercepts. Draw the graph.

Determine the midpoint:

(26) $\overline{AB}$ $A(5,-12)$ $B(-1,4)$

Determine the function value:

(27) $f[g(n-3)]$ for $f(x)=6-x^2$ and $g(x)=x-1$.

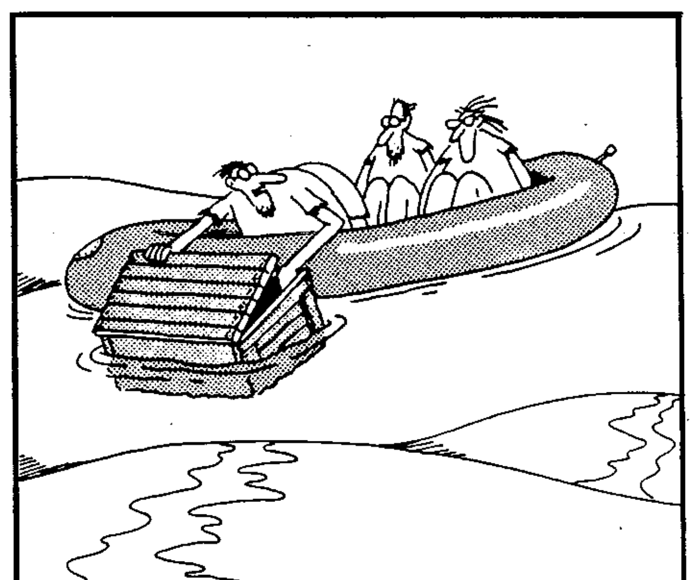


"Hey! I think you've hit on something there! Sheep's clothing! Sheep's clothing! ... Let's get out of these gorilla suits!"

SQUARES & SQUARE ROOTS

N	N^2	$\sqrt{N}$	N	N^2	$\sqrt{N}$
1	1	1.000	**51**	2601	7.141
2	4	1.414	**52**	2704	7.211
3	9	1.732	**53**	2809	7.280
4	16	2.000	**54**	2916	7.348
5	25	2.236	**55**	3025	7.416
6	36	2.449	**56**	3136	7.483
7	49	2.646	**57**	3249	7.550
8	64	2.828	**58**	3364	7.616
9	81	3.000	**59**	3481	7.681
10	100	3.162	**60**	3600	7.746
11	121	3.317	**61**	3721	7.810
12	144	3.464	**62**	3844	7.874
13	169	3.606	**63**	3969	7.937
14	196	3.742	**64**	4096	8.000
15	225	3.873	**65**	4225	8.062
16	256	4.000	**66**	4356	8.124
17	289	4.123	**67**	4489	8.185
18	324	4.243	**68**	4624	8.246
19	361	4.359	**69**	4761	8.307
20	400	4.472	**70**	4900	8.367
21	441	4.583	**71**	5041	8.426
22	484	4.690	**72**	5184	8.485
23	529	4.796	**73**	5329	8.544
24	576	4.899	**74**	5476	8.602
25	625	5.000	**75**	5625	8.660
26	676	5.099	**76**	5776	8.718
27	729	5.196	**77**	5929	8.775
28	784	5.292	**78**	6084	8.832
29	841	5.385	**79**	6241	8.888
30	900	5.477	**80**	6400	8.944
31	961	5.568	**81**	6561	9.000
32	1024	5.657	**82**	6724	9.055
33	1089	5.745	**83**	6889	9.110
34	1156	5.831	**84**	7056	9.165
35	1225	5.916	**85**	7225	9.220
36	1296	6.000	**86**	7396	9.274
37	1369	6.083	**87**	7569	9.327
38	1444	6.164	**88**	7744	9.381
39	1521	6.245	**89**	7921	9.434
40	1600	6.325	**90**	8100	9.487
41	1681	6.403	**91**	8281	9.539
42	1764	6.481	**92**	8464	9.592
43	1849	6.557	**93**	8649	9.644
44	1936	6.633	**94**	8836	9.695
45	2025	6.708	**95**	9025	9.747
46	2116	6.782	**96**	9216	9.798
47	2209	6.856	**97**	9409	9.849
48	2304	6.928	**98**	9604	9.899
49	2401	7.000	**99**	9801	9.950
50	2500	7.071	**100**	10000	10.000

"Say, there's something wrong here. ... We may have to move shortly."



"Well, we might as well put it on board—although I'm not sure what use we'll have for a box of rusty nails, broken glass, and throwing darts."

Algebra Skills Review

PROBLEM SET III

Classify the system and indicate number of solutions:

① $4x - y = 6$
$12x - 3y = 18$

Solve the system using any method:

② $5x - 2y = 17$
$x + 3y = 0$

Graph the inequality:

③ $2x + y > 6$

Graph the system:

④ $|y-1| + 3 \leq x$

⑤ $y < \frac{1}{2}x + 4$
$|x| > 3$

Simplify the radical:

⑥ $\sqrt{54x^2y^3z^8}$

Calculate to 1/10:

⑦ $\sqrt{204}$

Compute and simplify:

⑧ $\sqrt{2}\,(3\sqrt{6} - \sqrt{10}) - 2\sqrt{3}$

Although an unexplained phenomenon, there is a place on the outskirts of Mayfield, Nebraska, where the sun does not shine.

Simplify and rationalize:

⑨ $\dfrac{2\sqrt{3}}{3\sqrt{2}-\sqrt{3}}$

Interpolate:

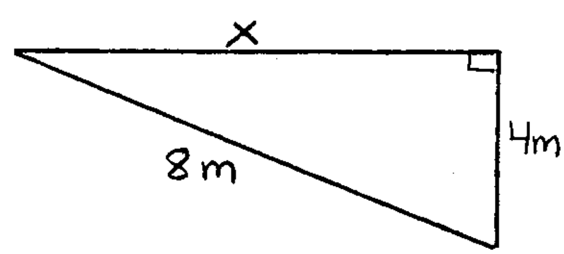
⑩ $\sqrt{1179}$

Radical equation:

⑪ $\sqrt{2x+5} - 1 = 2$

Pythagorean Theorem:

⑫



Algebra Skills Review

PROBLEM SET III

Find the distance:

(13) $(3,5)$ to $(5,9)$

Find the axis of symmetry and turning point. Graph the function:

(14) $y = x^2 - 10x + 21$

Factor this quadratic equation to find the roots:

(15) $2x^2 - 11x + 15 = 0$

Complete the square to determine the roots:

(16) $2n^2 - 12n + 14 = 0$

Use the formula to determine the roots:

(17) $-4n^2 + 8n = -3$

Use the discriminant to determine the nature of the roots:

(18) $x^2 + 5x + 3 = 0$

Multiply and simplify:

(19) $\dfrac{2x^2+x-1}{x^2+5x+6} \cdot \dfrac{x+3}{x+1}$

Divide and simplify:

(20) $\dfrac{n^2-16}{n^2-64} \div \dfrac{n+4}{n-8}$

Subtract and simplify:

(21) $\dfrac{4}{5-n} - \dfrac{3}{n^2-5n}$

"No, Zak ... it Wilga's turn lick bowl."

Simplify:

(22) $\dfrac{\frac{x+y}{a+b}}{\frac{x^2-y^2}{a^2-b^2}}$

Solve:

(23) $\dfrac{-2}{y+3} - \dfrac{2}{y} = -1$

Algebra Skills Review

PROBLEM SET IV

Classify the system and indicate the number of solutions:

① $y = 6x - 8$
$12x - 3y = 16$

Solve the system using any method:

② $2x + 3y = 6$
$3x + 4y = 7$

Graph the inequality:

③ $y < -4$

Graph the system:

④ $|y+2| + 5 \leq -2x$

⑤ $|y - 3x| > 6$
$|y| > 4$

Simplify the radical:

⑥ $\sqrt{72a^3b^4c^2}$

Calculate to 1/10:

⑦ $\sqrt{456}$

Compute and simplify:

⑧ $\sqrt{5}\left(3\sqrt{10} + 2\sqrt{15}\right) - \sqrt{27}$

At the Comedians' Cemetery

Simplify and rationalize:

⑨ $\dfrac{3\sqrt{5}}{\sqrt{5} + 2\sqrt{2}}$

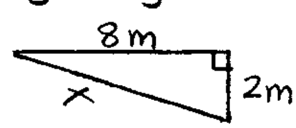
Interpolate:

⑩ $\sqrt{1488}$

Radical equation:

⑪ $\sqrt{4a+13} - 3 = 2$

Pythagorean Theorem:

⑫


Algebra Skills Review

PROBLEM SET IV

Find the distance:

(13) $(6,-2)$ to $(-2,-4)$

Find the axis of symmetry and turning point. Graph the function:

(14) $y = -x^2 + 8x - 12$

Factor this quadratic equation to find the roots:

(15) $3x^2 + 16x + 16 = 0$

Complete the square to determine the roots:

(16) $3n^2 - 24n = 36$

Use the formula to determine the roots:

(17) $4n^2 + 8n - 1 = 0$

Use the discriminant to determine the nature of the roots:

(18) $3x^2 + 6x + 3 = 0$

Multiply and simplify:

(19) $$\frac{n^2-16}{n^2-8n+16} \cdot \frac{n-4}{n^2+6n+8}$$

Divide and simplify:

(20) $$\frac{n^2+2n+1}{n-1} \div \frac{n+1}{2n^2-8n+6}$$

Add and simplify:

(21) $$\frac{3}{2n-4} + \frac{2}{2n-n^2}$$

Simplify:

(22) $$\frac{\dfrac{x+y}{x^2-y^2}}{\dfrac{x-y}{y^2-x^2}}$$

Solve:

(23) $$1 - \frac{1}{x-1} = \frac{2}{x+1}$$

"Well, Zoron. ... Is THIS a close enough look for you?"

Algebra Skills Review

ADDITIONAL PRACTICE PROBLEMS III & IV

Classify the system and indicate the number of solutions:

① $y = 2x + 6$
$8x - 4y = 10$

Solve the system using any method:

② $3x - 2y = 7$
$4x - 3y = 11$

Graph the inequality:

③ $x \geq 7$

Graph the system:

④ $|y-3| + 6 < 3x$

⑤ $|x| < y + 3$
$|x| > 4$

Simplify the radical:

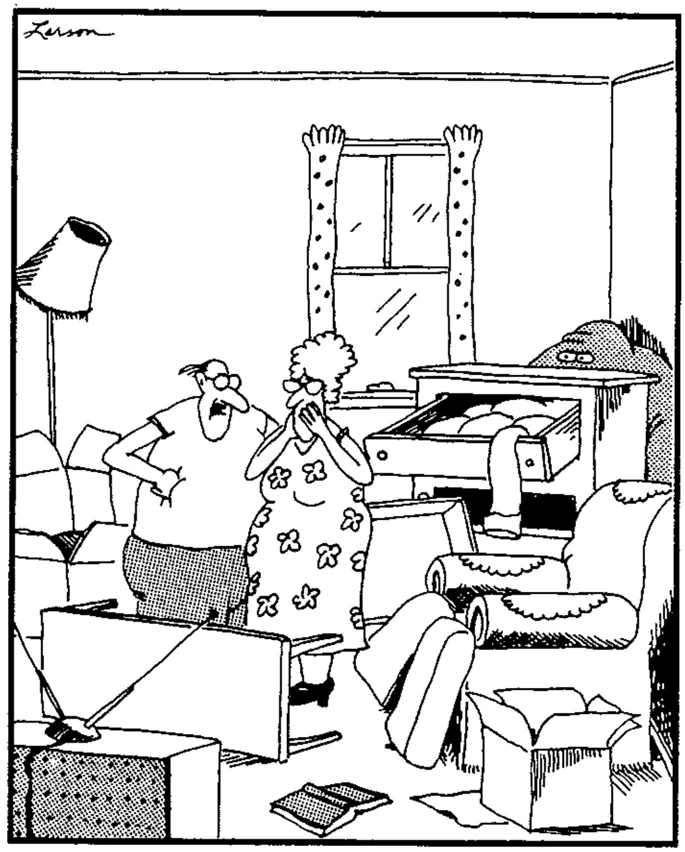
⑥ $\sqrt{125a^3b^3c^2}$

Calculate to $\frac{1}{10}$:

⑦ $\sqrt{642}$

Compute and simplify:

⑧ $4\sqrt{3}(\sqrt{6} - 2\sqrt{15}) - 2\sqrt{5}$



"Now calm down, Barbara. ... We haven't looked everywhere yet, and an elephant can't hide forever."

Simplify and rationalize:

⑨ $\dfrac{4\sqrt{3}}{2\sqrt{2} - \sqrt{3}}$

Interpolate:

⑩ $\sqrt{2264}$

Radical equation:

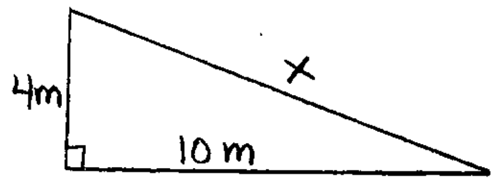
⑪ $\sqrt{4x+13} + 3 = 4$

Algebra Skills Review

ADDITIONAL PRACTICE PROBLEMS III & IV

Pythagorean Theorem:

(12)



Find the distance:

(13) $(4,5)$ to $(8,-1)$

Find the axis of symmetry and turning point. Graph the function:

(14) $y = x^2 - 6x - 16$

Factor this quadratic equation to determine the roots:

(15) $4x^2 + 15x - 4 = 0$

Complete the square to determine the roots:

(16) $3n^2 + 12 = 18n$

Use the formula to determine the roots:

(17) $n^2 - 6n + 1 = 0$

Use the discriminant to determine the nature of the roots:

(18) $2x^2 + 4x + 3 = 0$

Multiply and simplify:

(19) $\dfrac{2n^2 - 13n + 15}{2n - 3} \cdot \dfrac{n+1}{n^2 - 4n - 5}$

Divide and simplify:

(20) $\dfrac{n^2 - 81}{n^2 - 36} \div \dfrac{n-9}{n+6}$

Subtract and simplify:

(21) $\dfrac{-3}{5-a} - \dfrac{5}{a^2 - 25}$

Simplify:

(22) $\dfrac{n + 2 + \frac{2}{n+5}}{n + 6 + \frac{6}{n+1}}$

Solve:

(23) $\dfrac{3x}{x^2 - 5x + 4} = \dfrac{2}{x-4} + \dfrac{3}{x-1}$

Algebra Problem Solving Review

PROBLEM SET V

① Find the middle of three consecutive odd integers such that three times the largest decreased by five more than twice the smallest is six.

② Find a positive even integer such that four less than three times the integer is between −10 and 17.

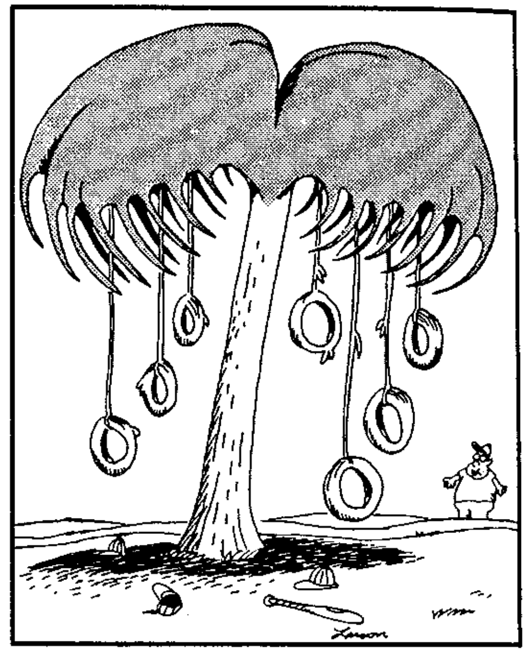
③ Ryan is 5/8 as old as Sam. In 5 years, Ryan will be 3/4 as old as Sam will be 4 years from now. How old was Ryan last year?

④ The sum of the squares of two consecutive integers is 61. Find the integers.

⑤ The sum of the digits of a two digit number is 7. If the digits are reversed, the new number is 25 less than twice the original. Find the original number.

⑥ Fran earns $210 per week plus 8½ % commission on all sales. If she takes home $1004 over a 3 week period, what are her sales?

⑦ Simon has 28 coins in nickels, dimes, and quarters. He has twice as many quarters as dimes and $3.65 in all. How many nickels?



Her tentacles swaying seductively in the breeze, the Venus Kidtrap was again poised and ready.

⑧ Willy paid the cashier $15.37 including 6% sales tax. What was the price before tax?

⑨ Merrill invested $18,000, part at 8% and part at 6% annual interest. After one year, he earned $240 less from the 8% investment. How much did he invest at 6%?

⑩ The scout troop spent $120 on 50 admission

Algebra Problem Solving Review

PROBLEM SET V

tickets to the carnival. If adult tickets sell for \$4.25 and child tickets sell for \$1.75, how many of each did they buy?

(11) How much water should be evaporated from 30ℓ of a 40% salt solution to raise the concentration to 60%?

(12) Amy leaves home at 1:30 PM riding her bicycle at a rate of 6 mph on her way to the beach. Her brother Rick leaves 30 minutes later and arrives 15 minutes earlier. If Rick rides 8 mph, how far is it to the beach and at what time will Amy arrive?

(13) A boat travels 42 miles downstream in 3 hours. The return trip takes 7 hours. What is the rate of the current?

(14) A rectangle has a length that is 1 cm more than 5 times its width. The perimeter is 38 cm. Determine the area.

(15) A photo that is 2 inches longer than it is wide fits exactly into a frame that has an area of 39 in^2. What are the dimensions of the photo if the frame is $1\frac{1}{2}$ inches all the way around?

(16) Gary has a 6 by 12 ft. garden. When he adds a uniform strip all the way around, he increases the area by 63 square feet. How wide is the strip?

(17) Five less than a number is multiplied by two more than twice the number. What is the number if the product is 54?

(18) Jim can clean the house in 3 hours. It takes Russ 6 hours. How long would it take if they worked together?

(19) Susan can wash the car in 12 minutes. Working together with Miwa, the two girls can do the job in 8 minutes. How long would it take Miwa to do the job herself?

(20) One pipe can load the tank in 2 hours. Another can do the job in 5 hours. The drain can empty the tank in 4 hours. If all three are open, how long (to the nearest minute) will it be until the tank is full?