Friendship Junior High School
Accelerated Math Program
Mr. Lavine (Room 102A)

A.T.I.M.

Advanced Topics In Mathematics

UNIT 20
Matrices

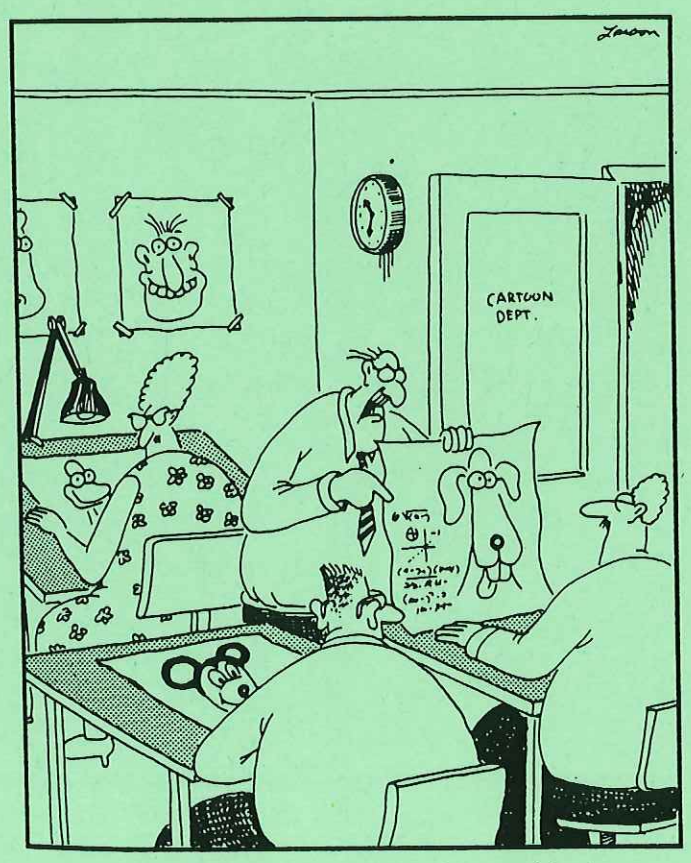
UNIT 21
Logarithms

UNIT 22
Conics

REVIEW I & II
Algebra Skills

REVIEW III & IV
Algebra Skills

REVIEW V
Algebra Problem Solving



"Hey! What's this, Higgins? Logarithmic equations? Do you or don't you enjoy your job here as a cartoonist?"

Augmented Matrix Solutions

ANSWER KEY 20.1

① $\begin{bmatrix} 3 & 2 & 0 \\ 6 & 1 & 9 \end{bmatrix}$ × -2 (add to row 2)
× -2 (add to row 1)

$\begin{bmatrix} -9 & 0 & -18 \\ 0 & -3 & 9 \end{bmatrix}$ ÷ -9
÷ -3

$\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & -3 \end{bmatrix}$ **(2, -3)**

② $\begin{bmatrix} 2 & -3 & 19 \\ 3 & 2 & 9 \end{bmatrix}$ × 2
× 3 (add to row 1)

$\begin{bmatrix} 13 & 0 & 65 \\ 3 & 2 & 9 \end{bmatrix}$ org row 1 × -3 (add to 2)
× 2

$\begin{bmatrix} 13 & 0 & 65 \\ 0 & 13 & -39 \end{bmatrix}$ ÷ 13
÷ 13

$\begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & -3 \end{bmatrix}$ **(5, -3)**

③ $\begin{bmatrix} 3 & 2 & 5 \\ 4 & -3 & 1 \end{bmatrix}$ × 3
× 2 (add to row 1)

$\begin{bmatrix} 17 & 0 & 17 \\ 4 & -3 & 1 \end{bmatrix}$ ÷ 17, × -4 (add to 2)

$\begin{bmatrix} 1 & 0 & 1 \\ 0 & -3 & -3 \end{bmatrix}$ ÷ -3

$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix}$ **(1, 1)**

④ $\begin{bmatrix} 2 & -1 & 11 \\ 1 & 3 & -12 \end{bmatrix}$ × 3
(add to row 1)

$\begin{bmatrix} 7 & 0 & 21 \\ 1 & 3 & -12 \end{bmatrix}$ ÷ -7 (add to row 2)

$\begin{bmatrix} -1 & 0 & -3 \\ 0 & 3 & -15 \end{bmatrix}$ × -1
÷ 3

$\begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & -5 \end{bmatrix}$ **(3, -5)**

⑤ $\begin{bmatrix} 1 & 1 & 1 & 6 \\ 2 & -3 & 4 & 3 \\ 4 & -8 & 4 & 12 \end{bmatrix}$ × -1 (add to row 2)

$\begin{bmatrix} 1 & 1 & 1 & 6 \\ -2 & 5 & 0 & -9 \\ 4 & -8 & 4 & 12 \end{bmatrix}$ × 4
× -1 (add to row 1)

$\begin{bmatrix} 0 & 12 & 0 & 12 \\ -2 & 5 & 0 & -9 \\ 4 & -8 & 4 & 12 \end{bmatrix}$ ÷ -12, × 5
(add to row 1)

$\begin{bmatrix} -2 & 0 & 0 & -14 \\ -2 & 5 & 0 & -9 \\ 4 & -8 & 4 & 12 \end{bmatrix}$

$-2x = -14$ $x = 7$

$-2(7) + 5y = -9$ $y = 1$

$4(7) - 8(1) + 4z = 12$ $z = -2$

(7, 1, -2)

Augmented Matrix Solutions

ANSWER KEY 20.1

⑥
$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ 3 & -2 & 5 & 1 \\ 2 & 1 & 2 & -1 \end{bmatrix} \quad x-5 \text{ (add to row 2)}$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ -2 & -7 & 0 & 1 \\ 2 & 1 & 2 & -1 \end{bmatrix} \quad \times -2$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 \\ -2 & -7 & 0 & 1 \\ 2 & 1 & 2 & -1 \end{bmatrix} \quad \text{(add to row 1)}$$

$$\begin{bmatrix} 0 & -1 & 0 & -1 \\ -2 & -7 & 0 & 1 \\ 2 & 1 & 2 & -1 \end{bmatrix} \quad \times -7$$

$$\begin{bmatrix} 0 & -1 & 0 & -1 \\ -2 & -7 & 0 & 1 \\ 2 & 1 & 2 & -1 \end{bmatrix} \quad \text{(add to row 1)}$$

$$\begin{bmatrix} -2 & 0 & 0 & 8 \\ -2 & -7 & 0 & 1 \\ 2 & 1 & 2 & -1 \end{bmatrix}$$

$\boxed{(-4, 1, 3)}$

$-2a = 8 \qquad a = -4$
$-2(-4) - 7b = 1 \qquad b = 1$
$2(-4) + (1) + 2c = -1 \qquad c = 3$

⑦
$$\begin{bmatrix} 2 & 1 & 1 & 0 \\ 3 & -2 & -3 & -21 \\ 4 & 5 & 3 & -2 \end{bmatrix} \quad \times 3 \text{ (add to row 2)}$$

$$\begin{bmatrix} 2 & 1 & 1 & 0 \\ 9 & 1 & 0 & -21 \\ 4 & 5 & 3 & -2 \end{bmatrix} \quad \times -3$$

$$\begin{bmatrix} 2 & 1 & 1 & 0 \\ 9 & 1 & 0 & -21 \\ 4 & 5 & 3 & -2 \end{bmatrix} \quad \text{(add to row 1)}$$

$$\begin{bmatrix} -2 & 2 & 0 & -2 \\ 9 & 1 & 0 & -21 \\ 4 & 5 & 3 & -2 \end{bmatrix} \quad \times -2 \text{ (add to row 1)}$$

$$\begin{bmatrix} -20 & 0 & 0 & 40 \\ 9 & 1 & 0 & -21 \\ 4 & 5 & 3 & -2 \end{bmatrix}$$

$-20x = 40 \qquad x = -2$
$9(-2) + y = -21 \qquad y = -3$
$4(-2) + 5(-3) + 3z = -2 \qquad z = 7$

$\boxed{(-2, -3, 7)}$

⑧
$$\begin{bmatrix} 1 & 1 & 1 & -2 \\ 2 & -3 & 1 & -11 \\ -1 & 2 & -1 & 8 \end{bmatrix} \quad \times -1 \text{ (add to row 2)}$$

$$\begin{bmatrix} 1 & 1 & 1 & -2 \\ 1 & -4 & 0 & -9 \\ -1 & 2 & -1 & 8 \end{bmatrix} \quad \text{(add to row 1)}$$

$$\begin{bmatrix} 0 & 3 & 0 & 6 \\ 1 & -4 & 0 & -9 \\ -1 & 2 & -1 & 8 \end{bmatrix} \quad \times 4$$
$\times 3$ (add to row 1)

$$\begin{bmatrix} 3 & 0 & 0 & -3 \\ 1 & -4 & 0 & -9 \\ -1 & 2 & -1 & 8 \end{bmatrix} \quad \text{(add to row 3)}$$
$\times 2$

$$\begin{bmatrix} 3 & 0 & 0 & -3 \\ 1 & -4 & 0 & -9 \\ -1 & 0 & -2 & 7 \end{bmatrix} \quad \text{(add to row 3)}$$
$\times 3$

$$\begin{bmatrix} 3 & 0 & 0 & -3 \\ 1 & -4 & 0 & -9 \\ 0 & 0 & -6 & 18 \end{bmatrix} \quad \text{(add to row 2)}$$
$\times -3$

$$\begin{bmatrix} 3 & 0 & 0 & -3 \\ 0 & 12 & 0 & 24 \\ 0 & 0 & -6 & 18 \end{bmatrix} \quad \div 3$$
$\div 12$
$\div -6$

$$\begin{bmatrix} 1 & 0 & 0 & -1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & -3 \end{bmatrix}$$

$\boxed{(-1, 2, -3)}$

Augmented Matrix Solutions

ANSWER KEY 20.1

⑨

$$y = \frac{\begin{vmatrix} 1 & 24 & 1 \\ 2 & 1 & -1 \\ 1 & 7 & 7 \end{vmatrix}}{\begin{vmatrix} 1 & 2 & 1 \\ 2 & -3 & 1 \\ 1 & -2 & 2 \end{vmatrix}}$$

$$(-2) + (24) + (14) - (-1) - (7) - (96) = -66$$

$$1\begin{vmatrix} -3 & 1 \\ -2 & 2 \end{vmatrix} - 2\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} + 1\begin{vmatrix} 2 & -3 \\ 1 & -2 \end{vmatrix}$$

$(-6)-(-2)$ $\quad$ $(4)-(1)$ $\quad$ $(-4)-(-3)$

$$(1)(-4) - (2)(3) + (0)(-1) = -11$$

$$y = \frac{-66}{-11} = \boxed{6}$$

"It's this new boyfriend, dear.... I'm just afraid one day your father's going to up and blow him away."

Multiplying Matrices

ANSWER KEY 20.2

① $3\begin{bmatrix}4 & 1\\ -2 & 3\end{bmatrix} = \begin{bmatrix}12 & 3\\ -6 & 9\end{bmatrix}$

② $5\begin{bmatrix}-3 & 4 & 1\\ 2 & 7 & 0\end{bmatrix} = \begin{bmatrix}-15 & 20 & 5\\ 10 & 35 & 0\end{bmatrix}$

③ $-4\begin{bmatrix}2 & 5\\ -3 & 2\\ 6 & -4\end{bmatrix} = \begin{bmatrix}-8 & -20\\ 12 & -8\\ -24 & 16\end{bmatrix}$

④ $\frac{2}{3}\begin{bmatrix}9 & 18 & -15\\ -12 & 0 & 6\\ 3 & -9 & -3\end{bmatrix} = \begin{bmatrix}6 & 12 & -10\\ -8 & 0 & 4\\ 2 & -6 & -2\end{bmatrix}$

⑤ $\begin{matrix}3(-1)+(-1)(3) & 3(0)+(-1)(7)\\ 2(-1)+4(3) & 2(0)+4(7)\end{matrix} = \begin{bmatrix}-6 & -7\\ 10 & 28\end{bmatrix}$

⑥ $\begin{matrix}-2(-6)+(-3)(8) & -2(-3)+(-3)(-1)\\ 4(-6)+0(8) & 4(-3)+0(-1)\end{matrix} = \begin{bmatrix}-12 & 9\\ -24 & -12\end{bmatrix}$

⑦ $\begin{matrix}2(-2)+3(6)\\ 4(-2)+(-4)(6)\end{matrix} = \begin{matrix}14\\ -32\end{matrix}$ $\quad \frac{1}{2}\begin{bmatrix}14\\ -32\end{bmatrix} = \begin{bmatrix}7\\ -16\end{bmatrix}$

⑧ $\begin{matrix}3(-3)+0(6)\\ -5(-3)+(-1)(6)\end{matrix} = \begin{matrix}-9\\ 9\end{matrix}$ $\quad \frac{2}{3}\begin{bmatrix}-9\\ 9\end{bmatrix} = \begin{bmatrix}-6\\ 6\end{bmatrix}$

⑨ $\begin{matrix}3(4)+(-1)(7) & 3(0)+(-1)(-5) & 3(-3)+(-1)(9)\\ 2(4)+4(7) & 2(0)+4(-5) & 2(-3)+4(9)\end{matrix}$

$\frac{1}{2}\begin{bmatrix}5 & 5 & -18\\ 36 & -20 & 30\end{bmatrix} = \begin{bmatrix}5/2 & 5/2 & -9\\ 18 & -10 & 15\end{bmatrix}$

⑩ $\begin{matrix}-1(4)+2(-8) & -1(-2)+2(10) & -1(-12)+2(-4)\\ 0(4)+(-3)(-8) & 0(-2)+(-3)(10) & 0(-12)+(-3)(-4)\end{matrix}$

$\frac{1}{4}\begin{bmatrix}-20 & 22 & 4\\ 24 & -30 & 12\end{bmatrix} = \begin{bmatrix}-5 & 11/2 & 1\\ 6 & -15/2 & 3\end{bmatrix}$

⑪ $\begin{matrix}2(-2)+0(1)+2(-2)\\ -3(-2)+(-1)(1)+5(-2)\\ 1(-2)+(3)(1)+(-4)(-2)\end{matrix}$ $\quad \begin{bmatrix}-8\\ -5\\ 9\end{bmatrix}$

⑫ $\begin{matrix}2(-4)+(-5)(2)+(-2)(-2)\\ -3(-4)+0(2)+0(-2)\\ -1(-4)+3(2)+(-4)(-2)\end{matrix}$ $\quad -\frac{1}{2}\begin{bmatrix}-14\\ 12\\ 18\end{bmatrix} = \begin{bmatrix}7\\ -6\\ -9\end{bmatrix}$

⑬ $(-4)\begin{vmatrix}0 & 4\\ -3 & -1\end{vmatrix} - (3)\begin{vmatrix}3 & 4\\ -2 & -1\end{vmatrix} + (2)\begin{vmatrix}3 & 0\\ -2 & -3\end{vmatrix}$

$(0)-(-12)$ $\quad (-3)-(-8)$ $\quad (-9)-(0)$

$(-4)(12) - (3)(5) + (2)(-9) = \boxed{-81}$

⑭ $\begin{bmatrix}2 & -3 & 15\\ 1 & -4 & 25\end{bmatrix}$ $\times 4$

$\times -3$ (add to row 1)

$\begin{bmatrix}5 & 0 & -15\\ 1 & -4 & 25\end{bmatrix}$ $\div 5$ (add to row 2)

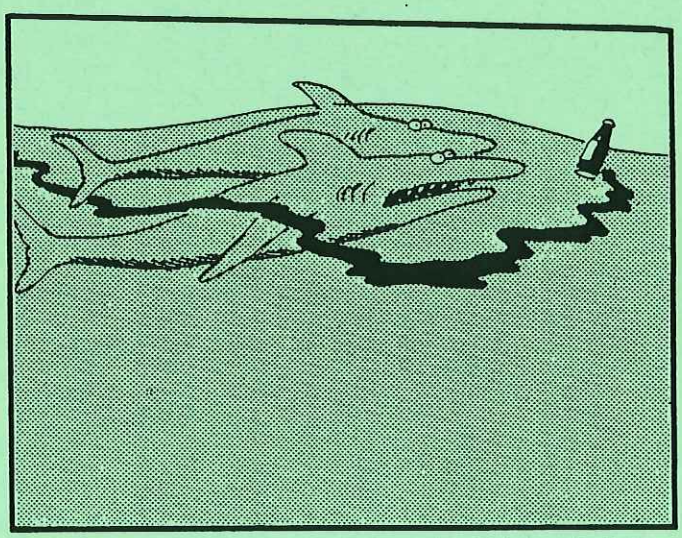
$\times -1$

$\begin{bmatrix}5 & 0 & -15\\ 0 & 4 & -28\end{bmatrix}$ $\div 5$

$\div 4$

$\begin{bmatrix}1 & 0 & -3\\ 0 & 1 & -7\end{bmatrix}$ $\rightarrow x=-3$

$\rightarrow y=-7$ $\quad \boxed{(-3,-7)}$



"What the—? Ketchup? We followed a *ketchup* trail for three miles?"

Multiplying Matrices

ANSWER KEY 20.2

(15) $\begin{bmatrix} 1 & 2 & -1 & 0 \\ 3 & 1 & -4 & 13 \\ 2 & -3 & 5 & -7 \end{bmatrix}$ $\times -4$ (add to row 2)

$\begin{bmatrix} 1 & 2 & -1 & 0 \\ -1 & -7 & 0 & 13 \\ 2 & -3 & 5 & -7 \end{bmatrix}$ $\times 5$ (add to row 1)

$\begin{bmatrix} 7 & 7 & 0 & -7 \\ -1 & -7 & 0 & 13 \\ 2 & -3 & 5 & -7 \end{bmatrix}$ (add to row 1)

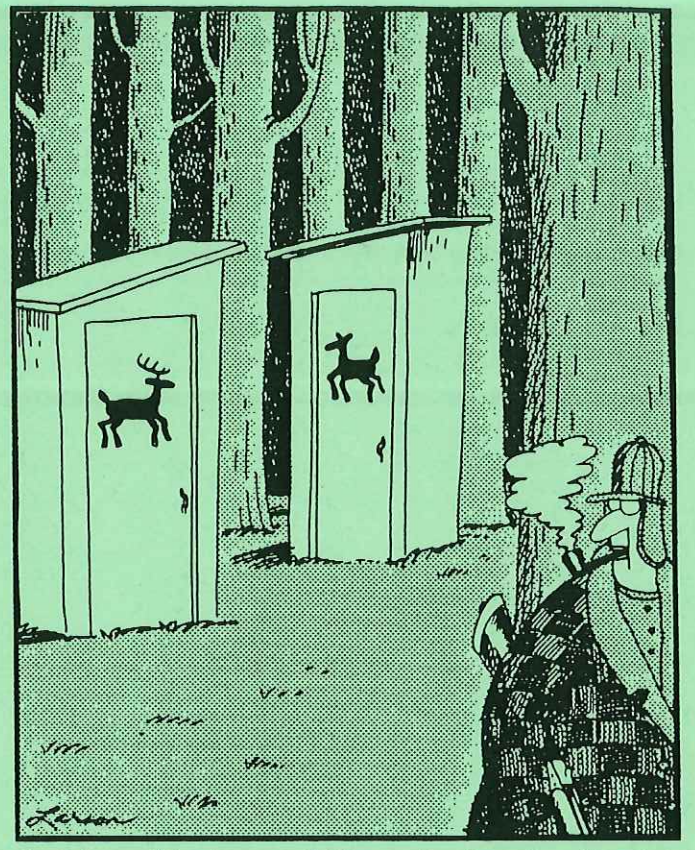
$\begin{bmatrix} 6 & 0 & 0 & 6 \\ -1 & -7 & 0 & 13 \\ 2 & -3 & 5 & -7 \end{bmatrix}$

$(+1, -2, -3)$

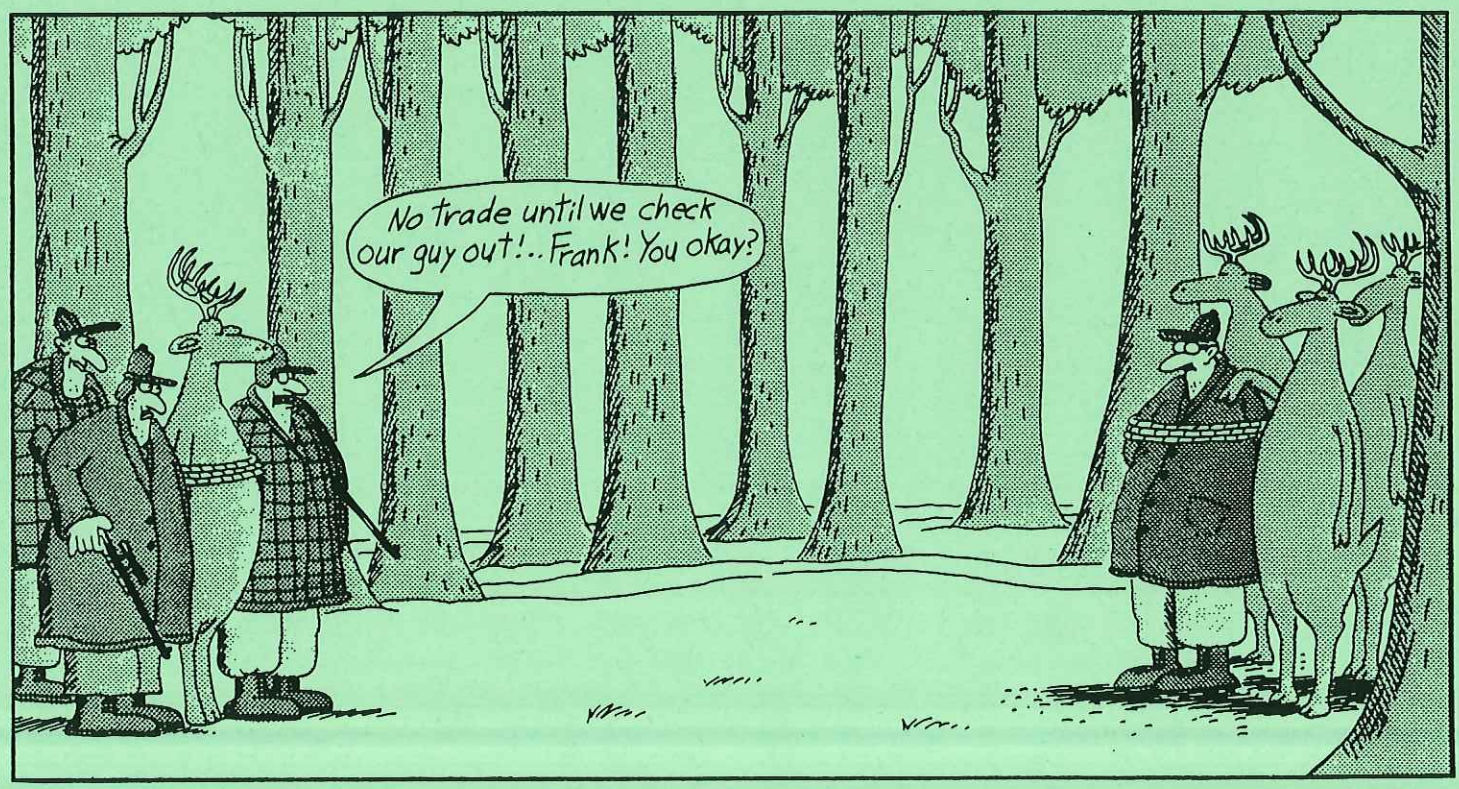
$6x = 6 \qquad x = 1$

$-1(1) - 7y = 13 \qquad y = -2$

$2(1) - 3(-2) + 5z = -7 \qquad z = -3$



Hank knew this place well. He need only wait. ... The deer would come, the deer would come.



Inverse Values

ANSWER KEY 20.3

① $\begin{bmatrix} -3 & 2 \\ 4 & -1 \end{bmatrix}^{-1} = \frac{-1}{5}\begin{bmatrix} -1 & -2 \\ -4 & -3 \end{bmatrix}$

(3) − (8) = −5

② $\begin{bmatrix} -6 & -2 \\ -3 & -1 \end{bmatrix}^{-1}$ = undefined

(6) − (6) = 0

③ $\begin{bmatrix} 5 & 3 \\ -1 & 6 \end{bmatrix}^{-1} = \frac{1}{33}\begin{bmatrix} 6 & -3 \\ 1 & 5 \end{bmatrix}$

(30) − (−3) = 33

④ $\begin{bmatrix} 10 & -4 \\ -6 & 2 \end{bmatrix}^{-1} = \frac{-1}{4}\begin{bmatrix} 2 & 4 \\ 6 & 10 \end{bmatrix}$

(20) − (24) = −4

⑤ $\begin{bmatrix} 0 & 4 \\ -3 & 6 \end{bmatrix}^{-1} = \frac{1}{12}\begin{bmatrix} 6 & -4 \\ 3 & 0 \end{bmatrix}$

(0) − (−12) = 12

⑥ $\begin{bmatrix} 8 & -3 \\ 4 & 2 \end{bmatrix}^{-1} = \frac{1}{28}\begin{bmatrix} 2 & 3 \\ -4 & 8 \end{bmatrix}$

(16) − (−12) = 28

⑦ $\begin{bmatrix} -2 & 3 & 4 \\ 6 & -1 & 2 \\ 8 & 0 & -1 \end{bmatrix}$

⑧ $\begin{bmatrix} 4 & -1 & 0 \\ -2 & 3 & 1 \\ 5 & 7 & 3 \end{bmatrix}$

⑨ $3 \rightarrow (-)\begin{vmatrix} -1 & -6 \\ 0 & -2 \end{vmatrix} = \boxed{-2}$

(2) − (0) = 2

$4 \rightarrow (+)\begin{vmatrix} 2 & -6 \\ 5 & -2 \end{vmatrix} = \boxed{26}$

(−4) − (−30) = 26

$8 \rightarrow (-)\begin{vmatrix} 2 & -1 \\ 5 & 0 \end{vmatrix} = \boxed{-5}$

(0) − (−5) = 5

⑩ $+\begin{vmatrix} 2 & -1 \\ -1 & 2 \end{vmatrix} = 3$ $-\begin{vmatrix} 0 & -1 \\ 1 & 2 \end{vmatrix} = -1$ $+\begin{vmatrix} 0 & 2 \\ 1 & -1 \end{vmatrix} = -2$

(4) − (1) (0) − (−1) (0) − (2)

$-\begin{vmatrix} 1 & 3 \\ -1 & 2 \end{vmatrix} = -5$ $+\begin{vmatrix} 1 & 3 \\ 1 & 2 \end{vmatrix} = -1$ $-\begin{vmatrix} 1 & 1 \\ 1 & -1 \end{vmatrix} = 2$

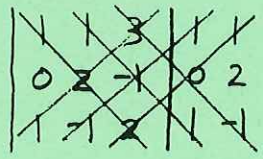
(2) − (−3) (2) − (3) (−1) − (1)

$+\begin{vmatrix} 1 & 3 \\ 2 & -1 \end{vmatrix} = -7$ $-\begin{vmatrix} 1 & 3 \\ 0 & -1 \end{vmatrix} = 1$ $+\begin{vmatrix} 1 & 1 \\ 0 & 2 \end{vmatrix} = 2$

(−1) − (6) (−1) − (0) (2) − (0)

$\begin{bmatrix} 3 & -1 & -2 \\ -5 & -1 & 2 \\ -7 & 1 & 2 \end{bmatrix}$ Transpose → $\begin{bmatrix} 3 & -5 & -7 \\ -1 & -1 & 1 \\ -2 & 2 & 2 \end{bmatrix}$

Determinant Value;



(4) + (−1) + (0) − (6) − (1) − (0)

= −4 reciprocal = −1/4

Inverse of original matrix:

$\frac{-1}{4}\begin{bmatrix} 3 & -5 & -7 \\ -1 & -1 & 1 \\ -2 & 2 & 2 \end{bmatrix}$

Inverse Values

ANSWER KEY 20.3

(11) $+\begin{vmatrix} 0 & 4 \\ 5 & 2 \end{vmatrix} = -20$ $-\begin{vmatrix} -2 & 4 \\ 3 & 2 \end{vmatrix} = 16$ $+\begin{vmatrix} -2 & 0 \\ 3 & 5 \end{vmatrix} = -10$

$(0)-(20)$ $(-4)-(12)$ $(-10)-(0)$

$-\begin{vmatrix} 1 & 2 \\ 5 & 2 \end{vmatrix} = 8$ $+\begin{vmatrix} 3 & 2 \\ 3 & 2 \end{vmatrix} = 0$ $-\begin{vmatrix} 3 & 1 \\ 3 & 5 \end{vmatrix} = -12$

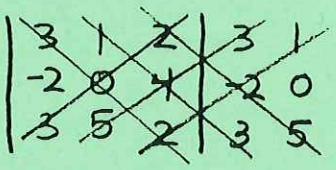
$(2)-(10)$ $(6)-(6)$ $(15)-(3)$

$+\begin{vmatrix} 1 & 2 \\ 0 & 4 \end{vmatrix} = 4$ $-\begin{vmatrix} 3 & 2 \\ -2 & 4 \end{vmatrix} = -16$ $+\begin{vmatrix} 3 & 1 \\ -2 & 0 \end{vmatrix} = 2$

$(4)-(0)$ $(12)-(-4)$ $(0)-(-2)$

$$\begin{bmatrix} -20 & 16 & -10 \\ 8 & 0 & -12 \\ 4 & -16 & 2 \end{bmatrix} \xrightarrow{\text{Transpose}} \begin{bmatrix} -20 & 8 & 4 \\ 16 & 0 & -16 \\ -10 & -12 & 2 \end{bmatrix}$$

Determinant value:



$(0)+(12)+(-20)$
$-(0)-(60)-(-4)$
$= -64$

Inverse of original matrix:

$$\frac{-1}{64}\begin{bmatrix} -20 & 8 & 4 \\ 16 & 0 & -16 \\ -10 & -12 & 2 \end{bmatrix}$$

(12) $+\begin{vmatrix} 4 & 2 \\ 5 & 0 \end{vmatrix} = -10$ $-\begin{vmatrix} 0 & 2 \\ 3 & 0 \end{vmatrix} = 6$ $+\begin{vmatrix} 0 & 4 \\ 3 & 5 \end{vmatrix} = -12$

$(0)-(10)$ $(0)-(6)$ $(0)-(12)$

$-\begin{vmatrix} 0 & 2 \\ 5 & 0 \end{vmatrix} = 10$ $+\begin{vmatrix} 1 & 2 \\ 3 & 0 \end{vmatrix} = -6$ $-\begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = -5$

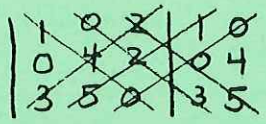
$(0)-(10)$ $(0)-(6)$ $(5)-(0)$

$+\begin{vmatrix} 0 & 2 \\ 4 & 2 \end{vmatrix} = -8$ $-\begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = -2$ $+\begin{vmatrix} 1 & 0 \\ 0 & 4 \end{vmatrix} = 4$

$(0)-(8)$ $(2)-(0)$ $(4)-(0)$

$$\begin{bmatrix} -10 & 6 & -12 \\ 10 & -6 & -5 \\ -8 & -2 & 4 \end{bmatrix} \xrightarrow{\text{Transpose}} \begin{bmatrix} -10 & 10 & -8 \\ 6 & -6 & -2 \\ -12 & -5 & 4 \end{bmatrix}$$

Determinant value:



$(0)+(0)+(0)$
$-(24)-(10)-(0)$
$= -34$

Inverse of original matrix:

$$\frac{-1}{34}\begin{bmatrix} -10 & 10 & -8 \\ 6 & -6 & -2 \\ -12 & -5 & 4 \end{bmatrix}$$

(13) $+\begin{vmatrix} 6 & 1 \\ -2 & 3 \end{vmatrix} = 20$ $-\begin{vmatrix} 0 & 1 \\ 5 & 3 \end{vmatrix} = 5$ $+\begin{vmatrix} 0 & 6 \\ 5 & -2 \end{vmatrix} = -30$

$(18)-(-2)$ $(0)-(5)$ $(0)-(30)$

$-\begin{vmatrix} -2 & 5 \\ -2 & 3 \end{vmatrix} = -4$ $+\begin{vmatrix} 4 & 5 \\ 5 & 3 \end{vmatrix} = -13$ $-\begin{vmatrix} 4 & -2 \\ 5 & -2 \end{vmatrix} = -2$

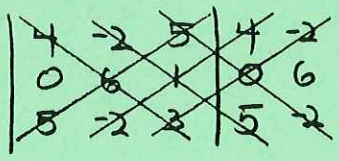
$(-6)-(-10)$ $(12)-(25)$ $(-8)-(-10)$

$+\begin{vmatrix} -2 & 5 \\ 6 & 1 \end{vmatrix} = -32$ $-\begin{vmatrix} 4 & 5 \\ 0 & 1 \end{vmatrix} = -4$ $+\begin{vmatrix} 4 & -2 \\ 0 & 6 \end{vmatrix} = 24$

$(-2)-(30)$ $(4)-(0)$ $(24)-(0)$

$$\begin{bmatrix} 20 & 5 & -30 \\ -4 & -13 & -2 \\ -32 & -4 & 24 \end{bmatrix} \xrightarrow{\text{Transpose}} \begin{bmatrix} 20 & -4 & -32 \\ 5 & -13 & -4 \\ -30 & -2 & 24 \end{bmatrix}$$

Determinant value:



$(72)+(-10)+(0)$
$-(150)-(-8)-(0)$
$= -80$

Inverse of original matrix:

$$\frac{-1}{80}\begin{bmatrix} 20 & -4 & -32 \\ 5 & -13 & -4 \\ -30 & -2 & 24 \end{bmatrix}$$

Inverse Values

ANSWER KEY 20.3

(14) $\frac{-1}{2}\begin{bmatrix}3 & 1\\ 5 & -3\end{bmatrix}\begin{bmatrix}-4\\ 8\end{bmatrix} = \begin{matrix}3(-4)+(1)(8)\\ 5(-4)+(-3)(8)\end{matrix} = \frac{-1}{2}\begin{bmatrix}-4\\ -44\end{bmatrix} = \begin{bmatrix}2\\ 22\end{bmatrix}$

(15) $-3\begin{bmatrix}2 & 4 & -6\\ -1 & -3 & -2\\ 0 & 5 & 1\end{bmatrix}\begin{bmatrix}-3\\ 2\\ 4\end{bmatrix} = \begin{matrix}2(-3)+4(2)+(-6)(4)\\ -1(-3)+(-3)(2)+(-2)(4)\\ 0(-3)+(5)(2)+(1)(4)\end{matrix} = -3\begin{bmatrix}-22\\ -11\\ 14\end{bmatrix} = \begin{bmatrix}66\\ 33\\ -42\end{bmatrix}$

(16) $2x+3y-z=-2$
$3x+y+z=9$
$2x-4y-3z=8$

$\begin{bmatrix}2 & 3 & -1 & -2\\ 3 & 1 & 1 & 9\\ 2 & -4 & -3 & 8\end{bmatrix}$ (add to row 2)

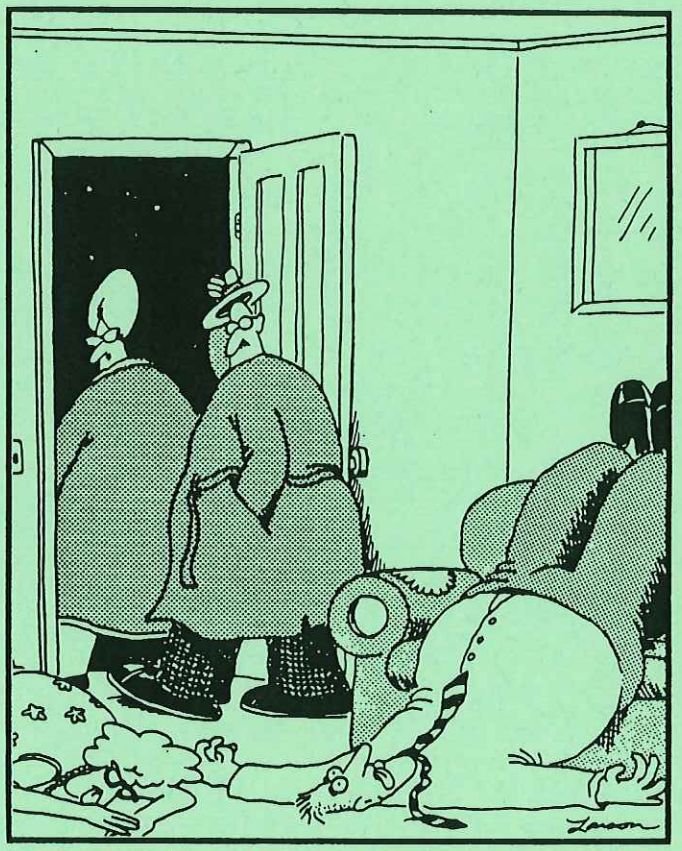
$\begin{bmatrix}2 & 3 & -1 & -2\\ 5 & 4 & 0 & 7\\ 2 & -4 & -3 & 8\end{bmatrix}$ $\times(-3)$ (add to row 1)

$\begin{bmatrix}-4 & -13 & 0 & 14\\ 5 & 4 & 0 & 7\\ 2 & -4 & -3 & 8\end{bmatrix}$ $\times 4$ $\times 13$ (add to row 1)

$\begin{bmatrix}49 & 0 & 0 & 147\\ 5 & 4 & 0 & 7\\ 2 & -4 & -3 & 8\end{bmatrix}$

$49x=147$ $\quad x=3$
$5(3)+4y=7$ $\quad y=-2$
$2(3)-4(-2)-3z=8$ $\quad z=2$

$\boxed{(3,-2,2)}$



The Arnolds feign death until the Wagners, sensing the sudden awkwardness, are compelled to leave.

Inverse Matrix Solutions

ANSWER KEY 20.4

① Matrix Equation:

$$\begin{bmatrix} 3 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 13 \\ -22 \end{bmatrix}$$

$$\frac{1}{15}\begin{bmatrix} 4 & 1 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} 3 & -1 \\ 3 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{15}\begin{bmatrix} 4 & 1 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} 13 \\ -22 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{15}\begin{bmatrix} 4 & 1 \\ -3 & 3 \end{bmatrix}\begin{bmatrix} 13 \\ -22 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \end{bmatrix} \qquad \boxed{(2, -7)}$$

Reciprocal of Determinant:

$$\begin{vmatrix} 3 & -1 \\ 3 & 4 \end{vmatrix} \quad (12)-(-3) = 15 \qquad \text{Reciprocal } \frac{1}{15}$$

Inverse of Coefficient Matrix:

$$\frac{1}{15}\begin{bmatrix} 4 & 1 \\ -3 & 3 \end{bmatrix}$$

Multiply Right Side of Equation:

$$\begin{matrix} 4(13)+(1)(-22) \\ -3(13)+(3)(-22) \end{matrix} \rightarrow \frac{1}{15}\begin{bmatrix} 30 \\ -105 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \end{bmatrix}$$

② Matrix Equation:

$$\begin{bmatrix} 4 & 5 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$-1\begin{bmatrix} 1 & -5 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} 4 & 5 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = -1\begin{bmatrix} 1 & -5 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = -1\begin{bmatrix} 1 & -5 \\ -1 & 4 \end{bmatrix}\begin{bmatrix} -1 \\ -1 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \end{bmatrix} \qquad \boxed{(-4, 3)}$$

Reciprocal of Determinant:

$$\begin{vmatrix} 4 & 5 \\ 1 & 1 \end{vmatrix} = (4)-(5) = -1 \qquad \text{Reciprocal } -1$$

Inverse of Coefficient Matrix:

$$-1\begin{bmatrix} 1 & -5 \\ -1 & 4 \end{bmatrix}$$

Multiply Right Side of the Equation:

$$\begin{matrix} (1)(-1)+(-5)(-1) \\ (-1)(-1)+(4)(-1) \end{matrix} \rightarrow -1\begin{bmatrix} 4 \\ -3 \end{bmatrix} = \begin{bmatrix} -4 \\ 3 \end{bmatrix}$$

③ Matrix Equation:

$$\begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -8 \\ 6 \end{bmatrix}$$

$$\frac{1}{13}\begin{bmatrix} 5 & -2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 5 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{13}\begin{bmatrix} 5 & -2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} -8 \\ 6 \end{bmatrix}$$

Reciprocal of Determinant:

$$\begin{vmatrix} 3 & 2 \\ 1 & 5 \end{vmatrix} = (15)-(2) = 13 \qquad \text{Reciprocal } \frac{1}{13}$$

Inverse Coefficient Matrix:

$$1/13\begin{bmatrix} 5 & -2 \\ -1 & 3 \end{bmatrix}$$

Inverse Matrix Solutions

ANSWER KEY 20.4

$$\begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{13}\begin{bmatrix} 5 & -2 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} -8 \\ 6 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \end{bmatrix} \qquad \boxed{(-4, 2)}$$

Multiply Right Side of the Equation:

$$\begin{matrix}(5)(-8)+(-2)(6) \\ (-1)(-8)+(3)(6)\end{matrix} \rightarrow \frac{1}{13}\begin{bmatrix} -52 \\ 26 \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \end{bmatrix}$$

④ Matrix Equation:

$$\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -9 \\ -8 \end{bmatrix}$$

$$-\frac{1}{7}\begin{bmatrix} -2 & -3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 2 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{7}\begin{bmatrix} -2 & -3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -9 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{7}\begin{bmatrix} -2 & -3 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} -9 \\ -8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6 \\ 1 \end{bmatrix} \qquad \boxed{(-6, 1)}$$

Reciprocal of Determinant:

$$\begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = (-4)-(3) = -7$$

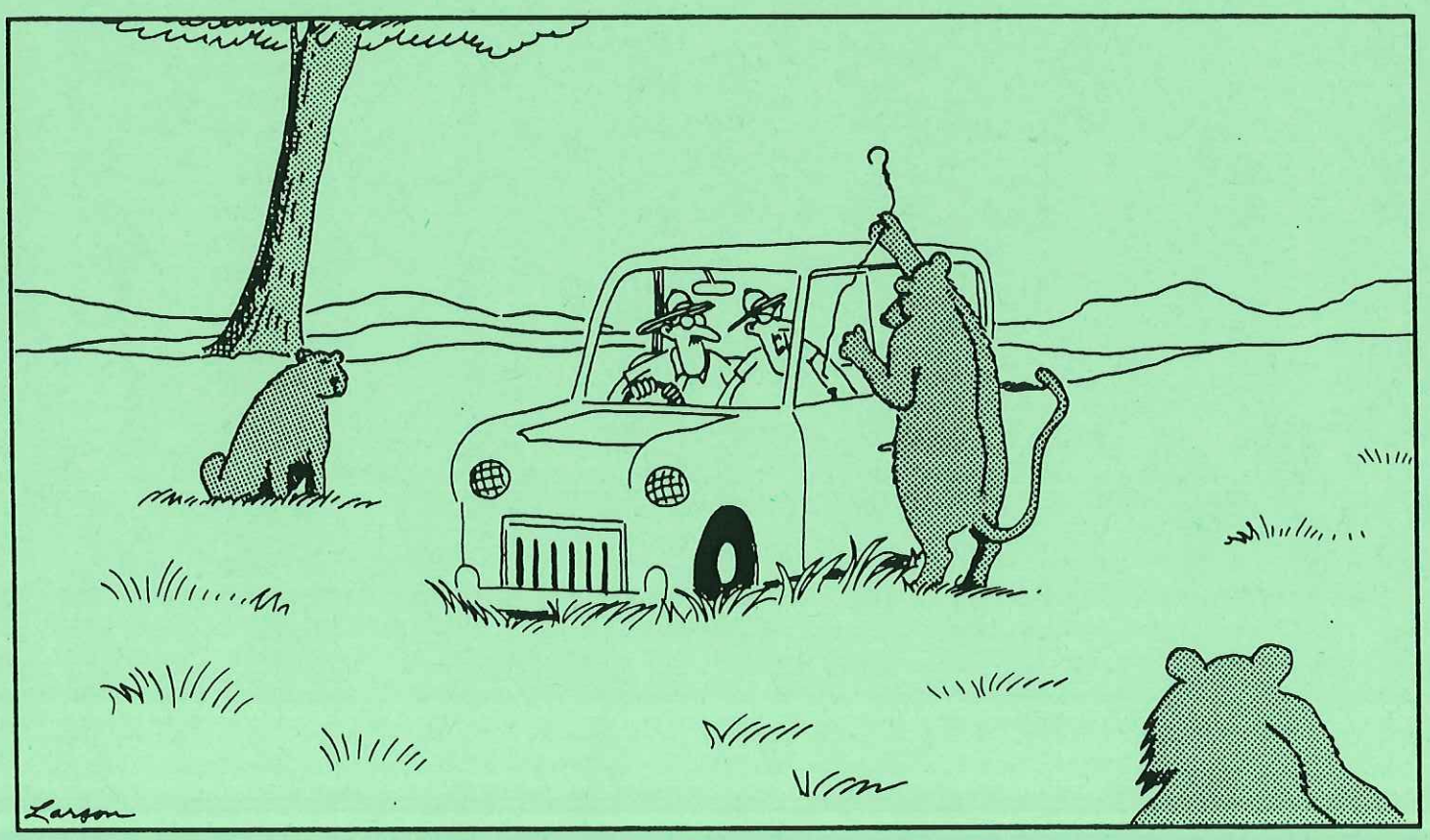
Reciprocal $-1/7$

Inverse of Coefficient Matrix:

$$-\frac{1}{7}\begin{bmatrix} -2 & -3 \\ -1 & 2 \end{bmatrix}$$

Multiply Right Side of the Equation:

$$\begin{matrix}(-2)(-9)+(-3)(-8) \\ (-1)(-9)+(2)(-8)\end{matrix} \rightarrow -\frac{1}{7}\begin{bmatrix} 42 \\ -7 \end{bmatrix} = \begin{bmatrix} -6 \\ +1 \end{bmatrix}$$



"Drive, George, drive! This one's got a coat hanger!"

Inverse Matrix Solutions

ANSWER KEY 20.4

⑤ Matrix Equation:

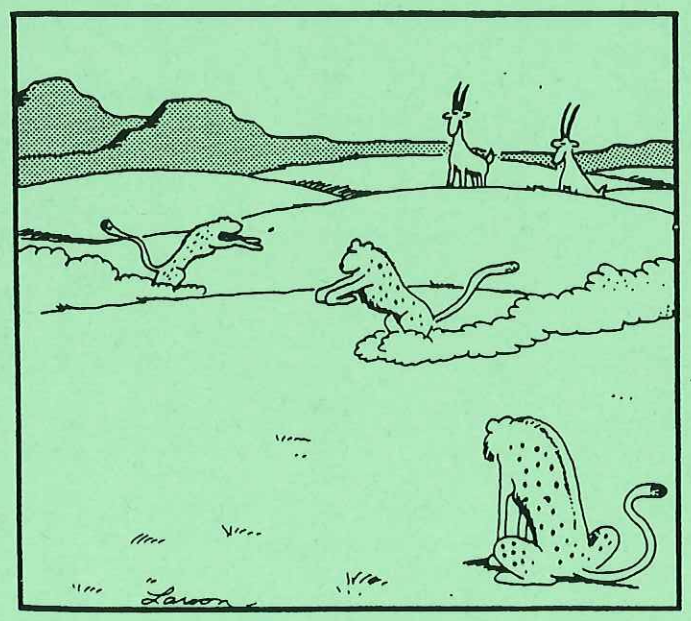
$$\begin{bmatrix} 1 & -2 & 3 \\ 3 & 1 & -2 \\ 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 13 \\ 0 \\ 9 \end{bmatrix}$$

$$\frac{1}{11}\begin{bmatrix} 1 & 3 & 1 \\ -11 & 0 & 11 \\ -4 & -1 & 7 \end{bmatrix} \begin{bmatrix} 1 & -2 & 3 \\ 3 & 1 & -2 \\ 1 & -1 & 3 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 1 & 3 & 1 \\ -11 & 0 & 11 \\ -4 & -1 & 7 \end{bmatrix} \begin{bmatrix} 13 \\ 0 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 1 & 3 & 1 \\ -11 & 0 & 11 \\ -4 & -1 & 7 \end{bmatrix} \begin{bmatrix} 13 \\ 0 \\ 9 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}$$

$\boxed{(2, -4, 1)}$



Cheetah wheelies

Reciprocal of Determinant:

$(3) + (4) + (-9) - (3) - (2)$
$-(-18) = 11$
Reciprocal $= 1/11$

Inverse of Coefficient Matrix:

$+\begin{vmatrix} 1 & -2 \\ -1 & 3 \end{vmatrix} = 1$ $(3)-(2)$ $\quad -\begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} = -11$ $(9)-(-2)$ $\quad +\begin{vmatrix} 3 & 1 \\ 1 & -1 \end{vmatrix} = -4$ $(-3)-(1)$

$-\begin{vmatrix} -2 & 3 \\ -1 & 3 \end{vmatrix} = 3$ $(-6)-(-3)$ $\quad +\begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix} = 0$ $(3)-(3)$ $\quad -\begin{vmatrix} 1 & -2 \\ 1 & -1 \end{vmatrix} = -1$ $(-1)-(-2)$

$+\begin{vmatrix} -2 & 3 \\ 1 & -2 \end{vmatrix} = 1$ $(4)-(3)$ $\quad -\begin{vmatrix} 1 & 3 \\ 3 & -2 \end{vmatrix} = 11$ $(-2)-(9)$ $\quad +\begin{vmatrix} 1 & -2 \\ 3 & 1 \end{vmatrix} = 7$ $(1)-(-6)$

$$\begin{bmatrix} 1 & -11 & -4 \\ 3 & 0 & -1 \\ 1 & 11 & 7 \end{bmatrix} \xrightarrow{\text{Transpose}} \begin{bmatrix} 1 & 3 & 1 \\ -11 & 0 & 11 \\ -4 & -1 & 7 \end{bmatrix}$$

$$\text{Inverse} \rightarrow \frac{1}{11}\begin{bmatrix} 1 & 3 & 1 \\ -11 & 0 & 11 \\ -4 & -1 & 7 \end{bmatrix}$$

Mutiply Right Side of Equation:

$$\begin{matrix} 1(13) + 3(0) + 1(9) \\ -11(13) + 0(0) + 11(9) \\ -4(13) + (-1)(0) + 7(9) \end{matrix} \rightarrow \frac{1}{11}\begin{bmatrix} 22 \\ -44 \\ 11 \end{bmatrix}$$

$$\frac{1}{11}\begin{bmatrix} 22 \\ -44 \\ 11 \end{bmatrix} = \begin{bmatrix} 2 \\ -4 \\ 1 \end{bmatrix}$$

Inverse Matrix Solutions

ANSWER KEY 20.4

⑥ Matrix Equation:

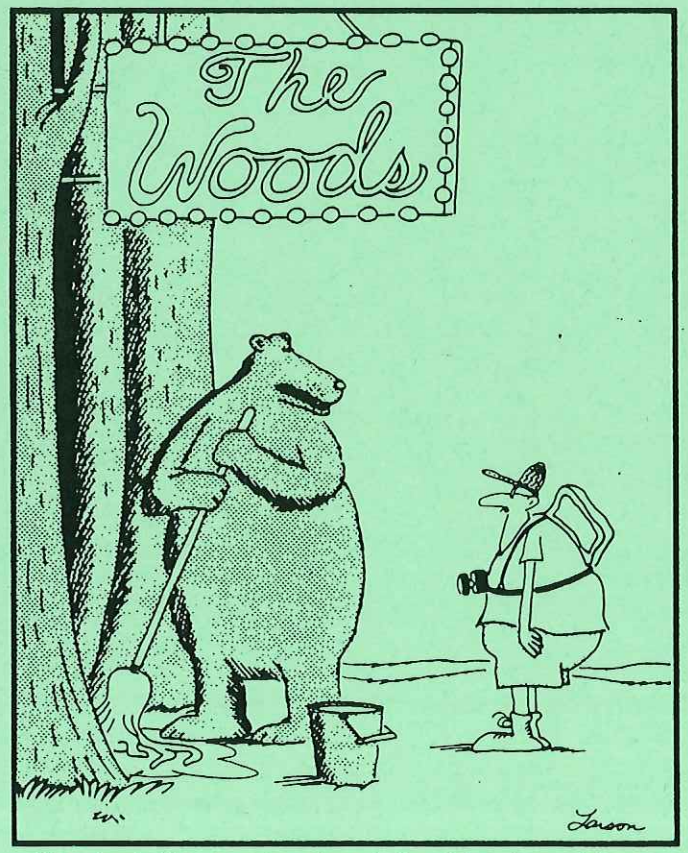
$$\begin{bmatrix} 2 & -2 & 1 \\ 1 & -3 & 2 \\ 4 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 8 \\ -15 \end{bmatrix}$$

$$\frac{1}{3}\begin{bmatrix} 5 & -3 & -1 \\ 9 & -6 & -3 \\ 11 & -6 & -4 \end{bmatrix} \begin{bmatrix} 2 & -2 & 1 \\ 1 & -3 & 2 \\ 4 & -1 & -1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 5 & -3 & -1 \\ 9 & -6 & -3 \\ 11 & -6 & -4 \end{bmatrix} \begin{bmatrix} 0 \\ 8 \\ -15 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 5 & -3 & -1 \\ 9 & -6 & -3 \\ 11 & -6 & -4 \end{bmatrix} \begin{bmatrix} 0 \\ 8 \\ -15 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ 4 \end{bmatrix}$$

$\boxed{(-3, -1, 4)}$



"Hey, look. No. 1, we're closed, No. 2, I only work here, and No. 3, we don't like your kind in here anyway."

Reciprocal of the Determinant:

$(+6) + (-16) + (-1) - (-12) - (-4)$
$-(+2) = 3$
Reciprocal $= 1/3$

Inverse of Coefficient Matrix:

$+\begin{vmatrix} -3 & 2 \\ -1 & -1 \end{vmatrix} = 5$ $\quad (3) - (-2)$

$-\begin{vmatrix} 1 & 2 \\ 4 & -1 \end{vmatrix} = 9$ $\quad (-1) - (8)$

$+\begin{vmatrix} 1 & -3 \\ 4 & -1 \end{vmatrix} = 11$ $\quad (-1) - (-12)$

$-\begin{vmatrix} -2 & 1 \\ -1 & -1 \end{vmatrix} = -3$ $\quad (2) - (-1)$

$+\begin{vmatrix} 2 & 1 \\ 4 & -1 \end{vmatrix} = -6$ $\quad (-2) - (4)$

$-\begin{vmatrix} 2 & -2 \\ 4 & -1 \end{vmatrix} = -6$ $\quad (-2) - (-8)$

$+\begin{vmatrix} -2 & 1 \\ -3 & 2 \end{vmatrix} = -1$ $\quad (-4) - (-3)$

$-\begin{vmatrix} 2 & 1 \\ 1 & 2 \end{vmatrix} = -3$ $\quad (4) - (1)$

$+\begin{vmatrix} 2 & -2 \\ 1 & -3 \end{vmatrix} = -4$ $\quad (-6) - (-2)$

$$\begin{bmatrix} 5 & 9 & 11 \\ -3 & -6 & -6 \\ -1 & -3 & -4 \end{bmatrix} \xrightarrow{\text{Transpose}} \begin{bmatrix} 5 & -3 & -1 \\ 9 & -6 & -3 \\ 11 & -6 & -4 \end{bmatrix}$$

$$\xrightarrow{\text{Inverse}} \frac{1}{3}\begin{bmatrix} 5 & -3 & -1 \\ 9 & -6 & -3 \\ 11 & -6 & -4 \end{bmatrix}$$

Multiply Right Side of Equation:

$$\begin{matrix} 5(0) + (-3)(8) + (-1)(-15) \\ 9(0) + (-6)(8) + (-3)(-15) \\ 11(0) + (-6)(8) + (-4)(-15) \end{matrix} \rightarrow \frac{1}{3}\begin{bmatrix} -9 \\ -3 \\ 12 \end{bmatrix}$$

$$\frac{1}{3}\begin{bmatrix} -9 \\ -3 \\ 12 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \\ 4 \end{bmatrix}$$

Inverse Matrix Solutions

ANSWER KEY 20.4

⑦ Matrix Equation:

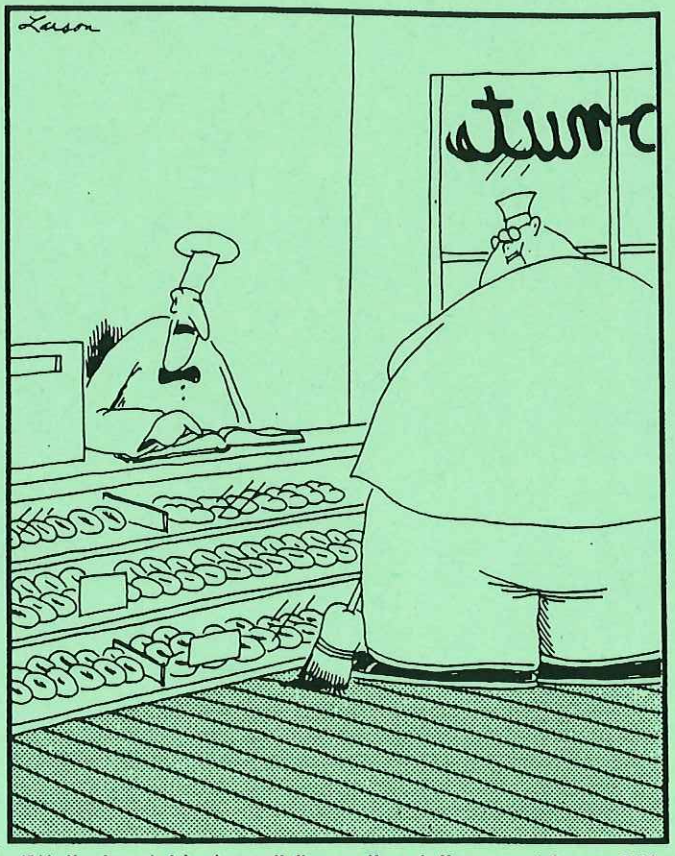
$$\begin{bmatrix} 3 & 1 & 4 \\ 1 & -1 & 1 \\ 2 & -3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -10 \\ 2 \\ 8 \end{bmatrix}$$

$$\frac{1}{15} \begin{bmatrix} 5 & -10 & 5 \\ 4 & -14 & 1 \\ -1 & 11 & -4 \end{bmatrix} \begin{bmatrix} 3 & 1 & 4 \\ 1 & -1 & 1 \\ 2 & -3 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{15} \begin{bmatrix} 5 & -10 & 5 \\ 4 & -14 & 1 \\ -1 & 11 & -4 \end{bmatrix} \begin{bmatrix} -10 \\ 2 \\ 8 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{15} \begin{bmatrix} 5 & -10 & 5 \\ 4 & -14 & 1 \\ -1 & 11 & -4 \end{bmatrix} \begin{bmatrix} -10 \\ 2 \\ 8 \end{bmatrix}$$

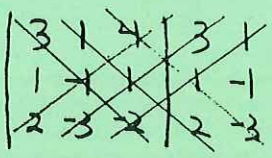
$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \\ 0 \end{bmatrix}$$

$\boxed{(-2, -4, 0)}$



"Well, shoot. I just can't figure it out. I'm movin' over 500 doughnuts a day, but I'm still just barely squeakin' by."

Reciprocal of the Determinant:



$(6) + (2) + (-12) - (-8) - (-9) - (-2) = 15$

Reciprocal $= 1/15$

Inverse of Coefficient Matrix:

$+\begin{vmatrix} -1 & 1 \\ -3 & -2 \end{vmatrix} = 5$ $\quad (2) - (3)$ $\qquad -\begin{vmatrix} 1 & 1 \\ 2 & -2 \end{vmatrix} = 4$ $\quad (-2) - (2)$ $\qquad +\begin{vmatrix} 1 & -1 \\ 2 & -3 \end{vmatrix} = -1$ $\quad (-3) - (-2)$

$-\begin{vmatrix} 1 & 4 \\ -3 & -2 \end{vmatrix} = -10$ $\quad (-2) - (-12)$ $\qquad +\begin{vmatrix} 3 & 4 \\ 2 & -2 \end{vmatrix} = -14$ $\quad (-6) - (8)$ $\qquad -\begin{vmatrix} 3 & 1 \\ 2 & -3 \end{vmatrix} = 11$ $\quad (-9) - (2)$

$+\begin{vmatrix} 1 & 4 \\ -1 & 1 \end{vmatrix} = 5$ $\quad (1) - (-4)$ $\qquad -\begin{vmatrix} 3 & 4 \\ 1 & 1 \end{vmatrix} = 1$ $\quad (3) - (4)$ $\qquad +\begin{vmatrix} 3 & 1 \\ 1 & -1 \end{vmatrix} = -4$ $\quad (-3) - (1)$

$$\begin{bmatrix} 5 & 4 & -1 \\ -10 & -14 & 11 \\ 5 & 1 & -4 \end{bmatrix} \xrightarrow{\text{Transpose}} \begin{bmatrix} 5 & -10 & 5 \\ 4 & -14 & 1 \\ -1 & 11 & -4 \end{bmatrix}$$

$$\xrightarrow{\text{Inverse}} \frac{1}{15} \begin{bmatrix} 5 & -10 & 5 \\ 4 & -14 & 1 \\ -1 & 11 & -4 \end{bmatrix}$$

Multiply Right Side of Equation:

$$\begin{matrix} 5(-10) + (-10)(2) + 5(8) \\ 4(-10) + (-14)(2) + 1(8) \\ -1(-10) + 11(2) + (-4)(8) \end{matrix} \rightarrow \frac{1}{15} \begin{bmatrix} -30 \\ -60 \\ 0 \end{bmatrix}$$

$$\frac{1}{15} \begin{bmatrix} -30 \\ -60 \\ 0 \end{bmatrix} = \begin{bmatrix} -2 \\ -4 \\ 0 \end{bmatrix}$$

Inverse Matrix Solutions

ANSWER KEY 20.4

⑧ $\begin{bmatrix} 2 & -1 & 3 \\ 1 & -3 & -11 \end{bmatrix}$ $\times -3$ (add to row 1)

$\begin{bmatrix} -5 & 0 & -20 \\ 1 & -3 & -11 \end{bmatrix}$ (add to row 2) $\times 5$

$\begin{bmatrix} -5 & 0 & -20 \\ 0 & -15 & -75 \end{bmatrix}$ $\div -5$ $\div -15$

$\begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 5 \end{bmatrix}$ $x = 4$ $y = 5$ **(4, 5)**

⑨ $\begin{bmatrix} 1 & -2 & 3 & 3 \\ 3 & -1 & 1 & 7 \\ 2 & 1 & 5 & -3 \end{bmatrix}$ (add to row 2) $\times(-3)$

$\begin{bmatrix} 1 & -2 & 3 & 3 \\ -8 & 1 & 0 & -18 \\ 2 & 1 & 5 & -3 \end{bmatrix}$ $\times -5$ $\times 3$ (add to row 1)

$\begin{bmatrix} 1 & 13 & 0 & -24 \\ -8 & 1 & 0 & -18 \\ 2 & 1 & 5 & -3 \end{bmatrix}$ $\times(-13)$ (add to row 1)

$\begin{bmatrix} 105 & 0 & 0 & 210 \\ -8 & 1 & 0 & -18 \\ 2 & 1 & 5 & -3 \end{bmatrix}$

$105x = 210$ $\quad x = 2$

$-8(2) + y = -18$ $\quad y = -2$

$2(2) + (-2) + 5z = -3$ $\quad z = -1$

(2, -2, -1)

Dog endorsements

Matrices

ANSWER KEY: UNIT 20 REVIEW & PRACTICE

① $\begin{bmatrix} 3 & 4 & 12 \\ 5 & 2 & -8 \end{bmatrix}$ $\times -2$ (add to row 1)

$\begin{bmatrix} -7 & 0 & 28 \\ 5 & 2 & -8 \end{bmatrix}$ $\times 5$ (add to row 2)
$\times 7$

$\begin{bmatrix} -7 & 0 & 28 \\ 0 & 14 & 84 \end{bmatrix}$ $\div -7$
$\div 14$

$\begin{bmatrix} 1 & 0 & -4 \\ 0 & 1 & 6 \end{bmatrix}$ $x = -4$
$y = 6$ $\boxed{(-4, 6)}$

② $\begin{bmatrix} 2 & 3 & -1 & -7 \\ 4 & -1 & -6 & 10 \\ 1 & 2 & 3 & -2 \end{bmatrix}$ $\times -6$ (add to row 2)

$\begin{bmatrix} 2 & 3 & -1 & -7 \\ -8 & -19 & 0 & 52 \\ 1 & 2 & 3 & -2 \end{bmatrix}$ $\times 3$
(add to row 1)

$\begin{bmatrix} 7 & 11 & 0 & -23 \\ -8 & -19 & 0 & 52 \\ 1 & 2 & 3 & -2 \end{bmatrix}$ $\times 19$
$\times 11$ (add to row 1)

$\begin{bmatrix} 45 & 0 & 0 & 135 \\ -8 & -19 & 0 & 52 \\ 1 & 2 & 3 & -2 \end{bmatrix}$ $\boxed{(3, -4, 1)}$

$45x = 135$ $\quad x = 3$
$-8(3) - 19y = 52$ $\quad y = -4$
$(3) + 2(-4) + 3z = -2$ $\quad z = 1$

③ $\begin{matrix} 2(-3) + 3(-5) & 2(1) + 3(-1) \\ -4(-3) + 0(-5) & -4(1) + 0(-1) \end{matrix}$ $\begin{bmatrix} -21 & -1 \\ 12 & -4 \end{bmatrix}$

④ $\begin{matrix} -2(6) + 3(-4) + 4(10) \\ 0(6) + 2(-4) + 5(10) \end{matrix} \rightarrow \frac{-1}{2}\begin{bmatrix} 16 \\ 42 \end{bmatrix} = \begin{bmatrix} -8 \\ -21 \end{bmatrix}$

⑤ $\begin{matrix} -2(-2) + (-1)(5) + 0(3) & -2(3) + (-1)(-1) + 0(0) \\ 3(-2) + (0)(5) + 4(3) & 3(3) + 0(-1) + 4(0) \\ 4(-2) + (2)(5) + (-3)(3) & 4(3) + 2(-1) + (-3)(0) \end{matrix}$

$\begin{bmatrix} -1 & -5 \\ 6 & 9 \\ -7 & 10 \end{bmatrix}$

⑥ $+\begin{vmatrix} 4 & -5 \\ -3 & 7 \end{vmatrix} = 13$ $\quad -\begin{vmatrix} 3 & -5 \\ 2 & 7 \end{vmatrix} = -31$ $\quad +\begin{vmatrix} 3 & 4 \\ 2 & -3 \end{vmatrix} = -17$

$(28) - (15)$ $\quad (21) - (-10)$ $\quad (-9) - (8)$

$\boxed{13 \quad -31 \quad -17}$

⑦ $-\begin{vmatrix} -1 & 0 \\ -3 & 7 \end{vmatrix} = 7$ $\quad +\begin{vmatrix} 6 & 0 \\ 2 & 7 \end{vmatrix} = 42$ $\quad -\begin{vmatrix} 6 & -1 \\ 2 & -3 \end{vmatrix} = 16$

$(-7) - (0)$ $\quad (42) - (0)$ $\quad (-18) - (-2)$

$\boxed{7 \quad 42 \quad 16}$

⑧ $\begin{vmatrix} 4 & 3 \\ -6 & 8 \end{vmatrix} = 50$ Reciprocal $1/50$

$(32) - (-18)$

$\frac{1}{50}\begin{bmatrix} 8 & -3 \\ 6 & 4 \end{bmatrix}$

Continued

Matrices

ANSWER KEY: UNIT 20 REVIEW & PRACTICE

⑨

$\begin{vmatrix} 2 & 0 & 6 & 2 & 0 \\ -4 & 3 & 8 & 4 & 3 \\ 5 & -1 & 7 & 5 & -1 \end{vmatrix}$

(42) + (0) + (24) − (90) − (−16) − (0) = −8

Reciprocal = $-\frac{1}{8}$

$+\begin{vmatrix} 3 & 8 \\ -1 & 7 \end{vmatrix} = 29$ (21) − (−8)

$-\begin{vmatrix} -4 & 8 \\ 5 & 7 \end{vmatrix} = 68$ (−28) − (40)

$+\begin{vmatrix} -4 & 3 \\ 5 & -1 \end{vmatrix} = -11$ (4) − (15)

$-\begin{vmatrix} 0 & 6 \\ -1 & 7 \end{vmatrix} = -6$ 0 − (−6)

$+\begin{vmatrix} 2 & 6 \\ 5 & 7 \end{vmatrix} = -16$ (14) − (30)

$-\begin{vmatrix} 2 & 0 \\ 5 & -1 \end{vmatrix} = 2$ (−2) − 0

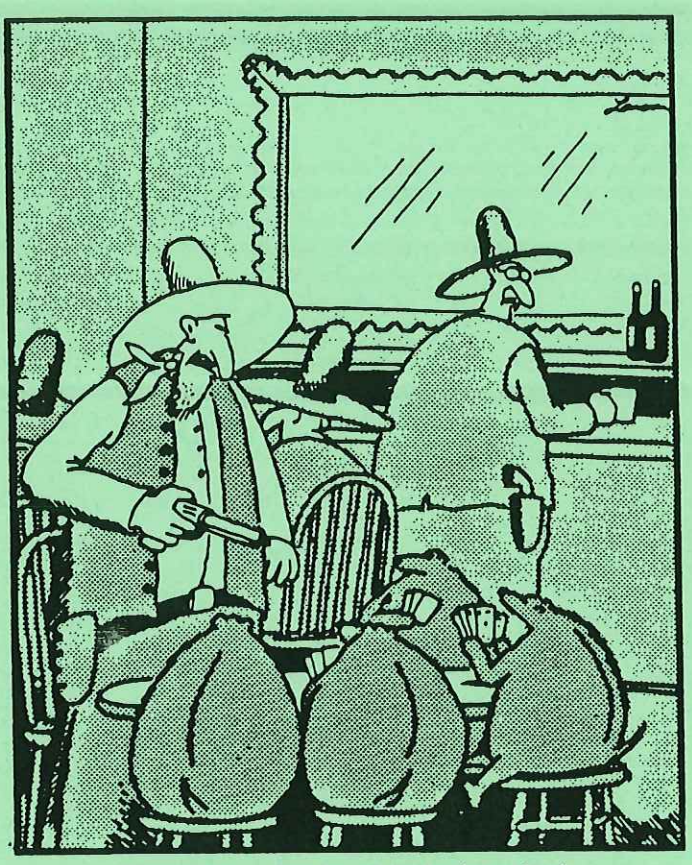
$+\begin{vmatrix} 0 & 6 \\ 3 & 8 \end{vmatrix} = -18$ 0 − (18)

$-\begin{vmatrix} 2 & 6 \\ -4 & 8 \end{vmatrix} = -40$ (16) − (−24)

$+\begin{vmatrix} 2 & 0 \\ -4 & 3 \end{vmatrix} = 6$ (6) − 0

$\begin{bmatrix} 29 & 68 & -11 \\ -6 & -16 & 2 \\ -18 & -40 & 6 \end{bmatrix}$ Transpose → $\begin{bmatrix} 29 & -6 & -18 \\ 68 & -16 & -40 \\ -11 & 2 & 6 \end{bmatrix}$

Inverse → $\frac{1}{8}\begin{bmatrix} 29 & -6 & -18 \\ 68 & -16 & -40 \\ -11 & 2 & 6 \end{bmatrix}$



"Varmints! ... You're all just a bunch of cheatin' varmints!"

⑩

$\begin{bmatrix} 3 & 5 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 \\ 15 \end{bmatrix}$

$-\frac{1}{14}\begin{bmatrix} -3 & -5 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 3 & 5 \\ 1 & -3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{14}\begin{bmatrix} -3 & -5 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 15 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{14}\begin{bmatrix} -3 & -5 \\ -1 & 3 \end{bmatrix}\begin{bmatrix} 3 \\ 15 \end{bmatrix}$

$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \end{bmatrix}$ **(6, −3)**

Reciprocal of Determinant:

$\begin{vmatrix} 3 & 5 \\ 1 & -3 \end{vmatrix} = (-9) - (5) = -14$ Reciprocal $-\frac{1}{14}$

Inverse of Coefficient Matrix:

$-\frac{1}{14}\begin{bmatrix} -3 & -5 \\ -1 & 3 \end{bmatrix}$

Multiplying Right side of Equation:

$-3(3) + (-5)(15) = -84$

$-1(3) + (3)(15) = 42$

$\rightarrow \frac{-1}{14}\begin{bmatrix} -84 \\ 42 \end{bmatrix} = \begin{bmatrix} 6 \\ -3 \end{bmatrix}$

Exponents & Inverse Relations

ANSWER KEY 21.1

(1) $x^4 \cdot x^3 = x^{4+3} = \boxed{x^7}$

(2) $n^{3x} \cdot n^x = n^{3x+x} = \boxed{n^{4x}}$

(3) $a^5 \div a^2 = a^{5-2} = \boxed{a^3}$

(4) $(2b^3)^2 = 2^2 b^{3\cdot 2} = \boxed{4b^6}$

(5) $(2^{\sqrt{3}})^{\sqrt{12}} = 2^{\sqrt{36}} = 2^6 = \boxed{64}$

(6) $5^{\sqrt{3}} \cdot 5^{\sqrt{27}} = 5^{\sqrt{3}+3\sqrt{3}} = \boxed{5^{4\sqrt{3}}}$

(7) $16^{\sqrt{7}} \div 16^{\sqrt{3}} = 2^{4\sqrt{7}} \div 2^{4\sqrt{3}} = \boxed{2^{4\sqrt{7}-4\sqrt{3}}}$

(8) $(m^{\sqrt{2}} \cdot p^{\sqrt{2}})^{\sqrt{2}} = \boxed{m^2 p^2}$

(9) $2^5 = 2^{2x-1}$
$5 = 2x-1 \rightarrow \boxed{x=3}$

(10) $3^y = 3^{3y+1}$
$y = 3y+1 \rightarrow \boxed{y=-1/2}$

(11) $9^{3y} = 27^{y+2}$
$3^{6y} = 3^{3y+6}$
$6y = 3y+6 \rightarrow \boxed{y=2}$

(12) $8^{x-1} = 16^{3x}$
$2^{3x-3} = 2^{12x}$
$3x-3 = 12x \rightarrow \boxed{x=-1/3}$

(13) $\frac{1}{27} = 3^{x-5}$
$3^{-3} = 3^{x-5}$
$-3 = x-5 \rightarrow \boxed{x=2}$

(14) $2^{n+3} = 1/16$
$2^{n+3} = 2^{-4}$
$n+3 = -4$
$\boxed{n=-7}$

(15) $(\frac{1}{3})^p = 3^{p-6}$
$3^{-p} = 3^{p-6}$
$-p = p-6$
$\boxed{p=3}$

(16) $4^{y-1} = 8^y$
$2^{2y-2} = 2^{3y}$
$2y-2 = 3y$
$\boxed{y=-2}$

(17) $3^3 = 27$
$\boxed{\log_3 27 = 3}$

(18) $4^2 = 16$
$\boxed{\log_4 16 = 2}$

(19) $2^{-3} = 1/8$
$\boxed{\log_2 1/8 = -3}$

(20) $5^{-2} = 1/25$
$\boxed{\log_5 1/25 = -2}$

(21) $\log_4 64 = 3$
$\boxed{4^3 = 64}$

(22) $\log_3 9 = 2$
$\boxed{3^2 = 9}$

(23) $\log_9 27 = 3/2$
$\boxed{9^{3/2} = 27}$

(24) $\log_3 1/81 = -4$
$\boxed{3^{-4} = 1/81}$

(25) $\log_{1/2} 16 = -4$
$\boxed{(1/2)^{-4} = 16}$

(26) $\log_{27} 3 = 1/3$
$\boxed{27^{1/3} = 3}$

(27) $\log_{10} 1/10 = -1$
$\boxed{10^{-1} = 1/10}$

(28) $\log_{1/3} 81 = -4$
$\boxed{(1/3)^{-4} = 81}$

(29) $\log_{10} 1000$
$10^x = 1000$
$x = \boxed{3}$

(30) $\log_6 36$
$6^x = 36$
$x = \boxed{2}$

(31) $\log_3 81$
$3^x = 81$
$x = \boxed{4}$

(32) $\log_{12} 144$
$12^x = 144$
$x = \boxed{2}$

(33) $\log_7 1/343$
$7^x = 1/343$
$x = \boxed{-3}$

Exponents & Inverse Relations

ANSWER KEY 21.1

(34) $\log_{1/4} 64$
$(1/4)^x = 64$
$x = \boxed{-3}$

(35) $\log_4 2$
$4^x = 2$
$2^{2x} = 2$
$x = \boxed{1/2}$

(36) $\log_9 27$
$9^x = 27$
$3^{2x} = 3^3$
$x = \boxed{3/2}$

(37) $\log_6 x = 2$
$6^2 = x$
$\boxed{x = 36}$

(38) $\log_9 x = -1$
$9^{-1} = x$
$\boxed{x = 1/9}$

(39) $\log_{1/2} 16 = x$
$(1/2)^x = 16$
$2^{-x} = 2^4$
$\boxed{x = -4}$

(40) $\log_3 27 = x$
$3^x = 27$
$3^x = 3^3$
$\boxed{x = 3}$

(41) $\log_n 81 = 4$
$n^4 = 81$
$n^4 = 3^4$
$\boxed{n = 3}$

(42) $\log_n 18 = 1$
$n^1 = 18$
$\boxed{n = 18}$

(43) $\log_5 x = -2$
$5^{-2} = x$
$\boxed{x = 1/25}$

(44) $\log_3 x = -3$
$3^{-3} = x$
$\boxed{x = 1/27}$

(45) $\log_{10} \sqrt{10} = x$
$10^x = \sqrt{10}$
$10^x = 10^{1/2}$
$\boxed{x = 1/2}$

(46) $\log_5 \sqrt{5} = x$
$5^x = \sqrt{5}$
$5^x = 5^{1/2}$
$\boxed{x = 1/2}$

(47) $\log_n 1/27 = -3$
$n^{-3} = 1/27$
$n^{-3} = 3^{-3}$
$\boxed{n = 3}$

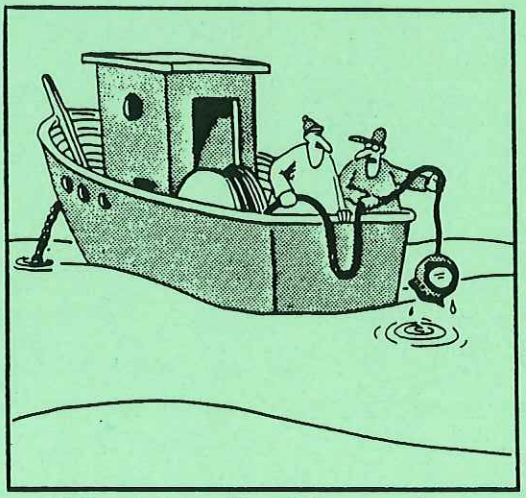
(48) $\log_n 36 = -2$
$n^{-2} = 36$
$(n^{-2})^{-1/2} = 36^{-1/2} \rightarrow$ $\boxed{n = 1/6}$

(49) $\log_{\sqrt{3}} 27 = x$
$\sqrt{3}^x = 27$
$3^{x/2} = 3^3 \rightarrow$ $\boxed{x = 6}$

(50) $\log_x \sqrt{5} = 1/4$
$x^{1/4} = \sqrt{5}$
$x^{1/4} = 5^{1/2}$
$(x^{1/4})^4 = (5^{1/2})^4 \rightarrow$ $\boxed{x = 25}$

(51) $\log_x \sqrt[3]{7} = 1/3$
$x^{1/3} = \sqrt[3]{7}$
$x^{1/3} = 7^{1/3} \rightarrow$ $\boxed{x = 7}$

(52) $\log_{10} \sqrt[3]{10} = n$
$10^n = \sqrt[3]{10}$
$10^n = 10^{1/3} \rightarrow$ $\boxed{n = 1/3}$



"Well, so that's it. ... I thought he was coming up awfully easy."

Properties of Logarithms

ANSWER KEY 21.2

① $\log_{10}(3n) = \log_{10}(n+2)$

$3n = n+2 \rightarrow \boxed{n=1}$

② $\log_4(2x-3) = \log_4(x+2)$

$2x-3 = x+2 \rightarrow \boxed{x=5}$

③ $\log_{10}(x^2+36) = \log_{10}100$

$x^2+36 = 100$

$x^2 = 64 \rightarrow \boxed{x = \pm 8}$

④ $\log_5(4x-4) = \log_5 100$

$4x-4 = 100 \rightarrow \boxed{x=26}$

⑤ $\log_3 y - \log_3 2 = \log_3 12$

$\log_3\left(\frac{y}{2}\right) = \log_3 12$

$\frac{y}{2} = 12 \rightarrow \boxed{y=24}$

⑥ $\log_3 14 + \log_3 m = \log_3 42$

$\log_3(14m) = \log_3 42$

$14m = 42 \rightarrow \boxed{m=3}$

⑦ $\log_5 x = 3\log_5 7$

$\log_5 x = \log_5 7^3$

$\log_5 x = \log_5 343$

$\boxed{x=343}$

⑧ $\log_2 n = \frac{1}{2}\log_2 81$

$\log_2 n = \log_2 81^{1/2}$

$\log_2 n = \log_2 9$

$\boxed{n=9}$

⑨ $\log_9 x = \frac{1}{2}\log_9 144 - \frac{1}{3}\log_9 8$

$\log_9 x = \log_9 144^{1/2} - \log_9 8^{1/3}$

$\log_9 x = \log_9 12 - \log_9 2$

$\log_9 x = \log_9\left(\frac{12}{2}\right)$

$x = 12/2 \rightarrow \boxed{x=6}$

⑩ $\log_7 m = \frac{1}{3}\log_7 64 + \frac{1}{2}\log_7 121$

$\log_7 m = \log_7 64^{1/3} + \log_7 121^{1/2}$

$\log_7 m = \log_7 4 + \log_7 11$

$\log_7 m = \log_7(4\cdot 11) \rightarrow \boxed{m=44}$

⑪ $\log_{10} 7 + \log_{10}(n-2) = \log_{10} 6n$

$\log_{10}(7)(n-2) = \log_{10} 6n$

$7n-14 = 6n \rightarrow \boxed{n=14}$

⑫ $\log_{10}(m+3) - \log_{10} m = \log_{10} 4$

$\log_{10}\left(\frac{m+3}{m}\right) = \log_{10} 4$

$\frac{m+3}{m} = 4$

$m+3 = 4m \rightarrow \boxed{m=1}$

⑬ $\log_{10} x + \log_{10} x + \log_{10} x = \log_{10} 27$

$\log_{10}(x)(x)(x) = \log_{10} 27$

$x^3 = 27 \rightarrow \boxed{x=3}$

⑭ $4\log_5 x - \log_5 4 = \log_5 4$

$4\log_5 x = \log_5 4 + \log_5 4$

$\log_5 x^4 = \log_5(4)(4)$

$x^4 = 16$

$\boxed{x=2}$

Properties of Logarithms

ANSWER KEY 21.2

(15) $\log_2 15 + \log_2 14 - \log_2 105 = \log_2 x$

$\log_2 \frac{(15)(14)}{105} = \log_2 x$

$\log_2 2 = \log_2 x \rightarrow \boxed{x=2}$

(16) $2\log_3 x + \log_3 \frac{1}{10} = \log_3 5 + \log_3 2$

$\log_3 x^2 + \log_3 \frac{1}{10} = \log_3 5 + \log_3 2$

$\log_3 (x^2)(\frac{1}{10}) = \log_3 (5)(2)$

$\frac{x^2}{10} = 10 \rightarrow x^2 = 100 \rightarrow \boxed{x=10}$

Remember: x cannot be negative (log would be undefined)

(17) $\log_{10}(y-1) + \log_{10}(y+2) = \log_7 7$

$\log_{10}(y-1)(y+2) = 1$ ← ↙ $7^x = 7$

$\log_{10}(y^2+y-2) = 1$

$10^1 = y^2+y-2$

$y^2+y-12 = 0$

$(y+4)(y-3) = 0 \rightarrow \boxed{y=3}$

Note: $y \neq -4$ (log would be undef.)

(18) $\log_3 (x+2) + \log_3 (3x) = \log_6 36$

$\log_3 (x+2)(3x) = 2$ ← ↙ $6^n = 36$

$\log_3 (3x^2+6x) = 2$

$3^2 = 3x^2+6x$ $\boxed{x=1}$

$3x^2+6x-9 = 0$

$x^2+2x-3 = 0 \rightarrow (x+3)(x-1) = 0$

(19) $\log_2 (\log_4 x) = 2$

$2^2 = \log_4 x$

$\log_4 x = 4$

$4^4 = x \rightarrow \boxed{x=256}$

(20) $\log_3 (\log_{1/27} x) = -1$

$3^{-1} = \log_{1/27} x$

$\log_{1/27} x = \frac{1}{3}$

$(\frac{1}{27})^{1/3} = x$

$\sqrt[3]{1/27} = x \rightarrow \boxed{x = 1/3}$

(21) $\log_2 [\log_4 (\log_3 x)] = -1$

$2^{-1} = \log_4 (\log_3 x)$

$\log_4 (\log_3 x) = 1/2$

$4^{1/2} = \log_3 x$

$\log_3 x = 2$

$3^2 = x \rightarrow \boxed{x=9}$

(22) $\log_{10} [\log_2 (\log_7 x)] = 0$

$10^0 = \log_2 (\log_7 x)$

$\log_2 (\log_7 x) = 1$

$2^1 = \log_7 x$

$\log_7 x = 2$

$7^2 = x \rightarrow \boxed{x=49}$

(23) $\log_3 243 = 5 \rightarrow \boxed{3^5 = 243}$

(24) $216^{2/3} = 36 \rightarrow \boxed{\log_{216} 36 = 2/3}$

Common Logarithms

ANSWER KEY 21.3

① .6937

② .8825

③ 1.7649

④ 2.8543

⑤ .9814

⑥ .8698 − 4

⑦ 3.8704

⑧ .4771 − 1

⑨ 7.41

⑩ 4.55

⑪ 12.3

⑫ .0746

⑬ 9080

⑭ .609

⑮ .09

⑯ 556,000

⑰ .159

⑱ 521,000

⑲ 5.9248

⑳ 4.7973

㉑ −2.6757

㉒ −1.4145

㉓ .0147

㉔ 20.3002

㉕ .004630

㉖ 4.2697

㉗ $8^x = 2 \rightarrow \boxed{x = 1/3}$

㉘ $(\frac{1}{4})^x = 64 \rightarrow \boxed{x = -3}$

㉙ $\log_4 (x+3) + \log_4 (x-3) = 2$

$\log_4 (x+3)(x-3) = 2$

$4^2 = x^2 - 9$

$x^2 - 25 = 0$

$(x+5)(x-5) = 0$ $\boxed{x = 5}$

㉚ $\log_8 (m+1) - \log_8 m = \frac{1}{2} \log_8 16$

$\log_8 (\frac{m+1}{m}) = \log_8 16^{1/2}$

$\log_8 (\frac{m+1}{m}) = \log_8 4$

$\frac{m+1}{m} = 4 \rightarrow m+1 = 4m \rightarrow \boxed{m = 1/3}$

㉛ $\log_6 [\log_{1/2} (\log_4 x)] = 0$

$6^0 = \log_{1/2} (\log_4 x)$

$\log_{1/2} (\log_4 x) = 1$

$(\frac{1}{2})^1 = \log_4 x$

$\log_4 x = \frac{1}{2}$

$4^{1/2} = x \rightarrow \boxed{x = 2}$

Exponential Equations

ANSWER KEY 21.4

① $\log_3 7$

$3^x = 7$

$\log 3^x = \log 7$

$x \log 3 = \log 7$

$x = \dfrac{\log 7}{\log 3}$

$x = \boxed{1.771}$

② $\log_7 12$

$7^x = 12$

$\log 7^x = \log 12$

$x \log 7 = \log 12$

$x = \dfrac{\log 12}{\log 7}$

$x = \boxed{1.277}$

③ $\log_4 22$

$4^x = 22$

$\log 4^x = \log 22$

$x \log 4 = \log 22$

$x = \dfrac{\log 22}{\log 4}$

$x = \boxed{2.230}$

④ $\log_6 11$

$6^x = 11$

$\log 6^x = 11$

$x \log 6 = \log 11$

$x = \dfrac{\log 11}{\log 6}$

$x = \boxed{1.338}$

⑤ $9^{x-4} = 6.28$

$\log 9^{x-4} = \log 6.28$

$(x-4)\log 9 = \log 6.28$

$x - 4 = \dfrac{\log 6.28}{\log 9}$

$x = \dfrac{\log 6.28}{\log 9} + 4$

$\boxed{x = 4.8362}$

⑥ $5^{y+2} = 15.3$

$\log 5^{y+2} = \log 15.3$

$(y+2)\log 5 = \log 15.3$

$y = \dfrac{\log 15.3}{\log 5} - 2$

$\boxed{y = -.3051}$

⑦ $x = \log_4 51.6$

$4^x = 51.6$

$\log 4^x = \log 51.6$

$x \log 4 = \log 51.6$

$x = \dfrac{\log 51.6}{\log 4}$

$\boxed{x = 2.8446}$

⑧ $x = \log_3 19.8$

$3^x = 19.8$

$x \log 3 = \log 19.8$

$x = \dfrac{\log 19.8}{\log 3}$

$\boxed{x = 2.7177}$

⑨ $32^{2y} = 5^{4y+1}$

$\log 32^{2y} = \log 5^{4y+1}$

$(2y)\log 32 = (4y+1)\log 5$

$(2y)\log 32 = (4y)\log 5 + (1)\log 5$

$(2y)\log 32 - (4y)\log 5 = \log 5$

$y[2\log 32 - 4\log 5] = \log 5$

$y = \dfrac{\log 5}{\log 32^2 - \log 5^4}$

$y = \dfrac{\log 5}{\log 1024 - \log 625}$

$\boxed{y = 3.2598}$

⑩ $2^{5x-1} = 3^{2x+1}$

$\log 2^{5x-1} = \log 3^{2x+1}$

$(5x-1)\log 2 = (2x+1)\log 3$

$(5x)\log 2 - \log 2 = (2x)\log 3 + \log 3$

$(5x)\log 2 - (2x)\log 3 = \log 3 + \log 2$

$x(5\log 2 - 2\log 3) = \log 3 + \log 2$

$x = \dfrac{\log 3 + \log 2}{\log 2^5 - \log 3^2}$

$x = \dfrac{\log 3 + \log 2}{\log 32 - \log 9}$

$\boxed{x = 1.4125}$

continued

Exponential Equations

ANSWER KEY 21.4

(11) $5^{x-1} = 3^x$

$\log 5^{x-1} = \log 3^x$

$(x-1)\log 5 = (x)\log 3$

$(x)\log 5 - (1)\log 5 = (x)\log 3$

$(x)\log 5 - (x)\log 3 = \log 5$

$x(\log 5 - \log 3) = \log 5$

$x = \dfrac{\log 5}{\log 5 - \log 3}$

$\boxed{x = 3.1507}$

(12) $7^{x-2} = 5^x$

$\log 7^{x-2} = \log 5^x$

$(x-2)\log 7 = (x)\log 5$

$(x)\log 7 - (2)\log 7 = (x)\log 5$

$(x)\log 7 - (x)\log 5 = (2)\log 7$

$x(\log 7 - \log 5) = \log 7^2$

$x = \dfrac{\log 49}{\log 7 - \log 5}$

$\boxed{x = 11.5665}$

(13) $5^{2x} = 9^{x-1}$

$\log 5^{2x} = \log 9^{x-1}$

$(2x)\log 5 = (x-1)\log 9$

$(2x)\log 5 = (x)\log 9 - \log 9$

$(2x)\log 5 - (x)\log 9 = -\log 9$

$x(2\log 5 - \log 9) = -\log 9$

$x = \dfrac{-\log 9}{\log 25 - \log 9}$

$\boxed{x = -2.1507}$

(14) $12^{x-4} = 4^{2-x}$

$\log 12^{x-4} = \log 4^{2-x}$

$(x-4)\log 12 = (2-x)\log 4$

$(x)\log 12 - (4)\log 12 = (2)\log 4 - (x)\log 4$

$(x)\log 12 + (x)\log 4 = (2)\log 4 + (4)\log 12$

$x(\log 12 + \log 4) = \log 4^2 + \log 12^4$

$x = \dfrac{\log 16 + \log 20736}{\log 12 + \log 4}$

$\boxed{x = 3.2838}$

(15) $7^{x-2} = 5^{3-x}$

$\log 7^{x-2} = \log 5^{3-x}$

$(x-2)\log 7 = (3-x)\log 5$

$(x)\log 7 - (2)\log 7 = (3)\log 5 - (x)\log 5$

$(x)\log 7 + (x)\log 5 = (3)\log 5 + (2)\log 7$

$x(\log 7 + \log 5) = \log 5^3 + \log 7^2$

$x = \dfrac{\log 125 + \log 49}{\log 7 + \log 5}$

$\boxed{x = 2.4527}$

(16) $3^{3x} = 2^{2x+3}$

$\log 3^{3x} = \log 2^{2x+3}$

$(3x)\log 3 = (2x+3)\log 2$

$(3x)\log 3 = (2x)\log 2 + (3)\log 2$

$(3x)\log 3 - (2x)\log 2 = (3)\log 2$

$x(3\log 3 - 2\log 2) = \log 2^3$

$x = \dfrac{\log 8}{\log 27 - \log 4}$

$\boxed{x = 1.0890}$

Exponential Equations

ANSWER KEY 21.4

(17) $\log_2 16$

$2^x = 16 \rightarrow \boxed{x = 4}$

(18) $\log_4 (y-1) + \log_4 (y-1) = \log_3 27$

$\log_4 (y-1)(y-1) = 3$

$4^3 = y^2 - 2y + 1$

$y^2 - 2y - 63 = 0$

$(y-9)(y+7) = 0$

$y = 9$ or ~~-7~~ $\boxed{y = 9}$

(19) $3\log_2 2 - \log_2 32 = -\log_2 x$

$\log_2 2^3 - \log_2 32 = \log_2 x^{-1}$

$\log_2 8 - \log_2 32 = \log_2 \frac{1}{x}$

$\log_2 \left(\frac{8}{32}\right) = \log_2 \frac{1}{x}$

$\frac{8}{32} = \frac{1}{x} \rightarrow \frac{1}{4} = \frac{1}{x} \rightarrow \boxed{x = 4}$

(20) $\log 4630$

$\log 4.63 \times 10^3$

mantissa $4.63 = .6656$

characteristic $= 3$

$\log 4630 = \boxed{3.6656}$

(21) $\log .0083$

$\log 8.3 \times 10^{-3}$

continued

mantissa $8.3 = .9191$

characteristic $= -3$

$\log .0083 = \boxed{.9191 - 3}$

(22) $\log 304.1 = \boxed{2.4830}$

(23) $\log x = .4541 - 2$

$\log x = -1.5459$

$x = \boxed{.0285}$

antilog (use inverse key)

When a tree falls in the forest and no one is around.

Logarithms

ANSWER KEY: UNIT 21 REVIEW & PRACTICE

① $3^{-2} = 1/9 \rightarrow \boxed{\log_3 1/9 = -2}$

② $\log_4 2 = 1/2 \rightarrow \boxed{4^{1/2} = 2}$

③ $\log_4(1-2x) = \log_4(x+10)$

$1-2x = x+10$

$-3x = 9 \rightarrow \boxed{x = -3}$

④ $\log_6(n-3) + \log_6(n+2) = \log_3 3$

$\log_6(n-3)(n+2) = 1$

$6^1 = n^2 - n - 6$

$n^2 - n - 12 = 0$

$(n-4)(n+3) = 0$

$n = 4$ or -3 $\boxed{n=4}$

⑤ $2\log_2 x - \frac{1}{2}\log_2 16 = 4$

$\log_2 x^2 - \log_2 16^{1/2} = 4$

$\log_2\left(\frac{x^2}{4}\right) = 4$

$2^4 = \frac{x^2}{4}$

$16 = \frac{x^2}{4}$ $\boxed{x = 8}$

⑥ $\log 437 \rightarrow 437 = 4.37 \times 10^2$

mantissa $4.37 = .6405$

characteristic $= 2$ $\boxed{2.6405}$

⑦ $\log x = .5011 - 2$

mantissa $.5011 = 3.17$

characteristic -2 $\boxed{.0317}$

⑧ $\log_3 8 = x$

$3^x = 8$

$\log 3^x = \log 8$

$x \log 3 = \log 8$

$x = \frac{\log 8}{\log 3} = \boxed{1.8928}$

⑨ $4^{3x-2} = 6^{2x+1}$

$\log 4^{3x-2} = \log 6^{2x+1}$

$(3x-2)\log 4 = (2x+1)\log 6$

$(3x)\log 4 - (2)\log 4 = (2x)\log 6 + \log 6$

$(3x)\log 4 - (2x)\log 6 = 2\log 4 + \log 6$

$x(3\log 4 - 2\log 6) = 2\log 4 + \log 6$

$x = \frac{\log 4^2 + \log 6}{\log 4^3 - \log 6^2} = \frac{\log 16 + \log 6}{\log 64 - \log 36}$

$\boxed{x = 7.9330}$

⑩ $\log_3[\log_8(\log_3 x)] = -1$

$3^{-1} = \log_8(\log_3 x)$

$\log_8(\log_3 x) = 1/3$

$8^{1/3} = \log_3 x$

$\log_3 x = 2$

$3^2 = x \rightarrow \boxed{x = 9}$

Parabolas

ANSWER KEY 22.1

① $y = x^2 + 4x + 1$

$y = x^2 + 4x + \square + 1 - \square$

$y = x^2 + 4x + 4 + 1 - 4$

$y = (x+2)^2 - 3 \quad h = -2, k = -3$

axis: $x = -2$

vertex: $(-2, -3)$

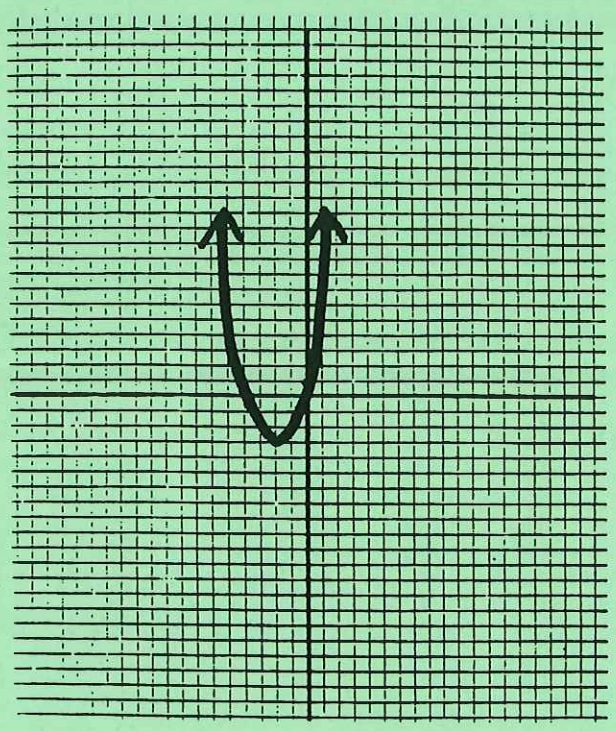
focus: $(-2, -2\frac{3}{4})$

directrix: $y = -3\frac{1}{4}$

opening: upward

latus rectum: 1

x	y
-2	-3
0	1
1	6



② $y = 3x^2 - 24x + 50$

$\frac{1}{3}y = x^2 - 8x + \square + 5\frac{2}{3} - \square$

$\frac{1}{3}y = x^2 - 8x + 16 + 5\frac{2}{3} - 16$

$\frac{1}{3}y = (x-4)^2 + \frac{2}{3}$

$y = 3(x-4)^2 + 2 \quad h = 4, k = 2$

axis: $x = 4$

vertex: $(4, 2)$

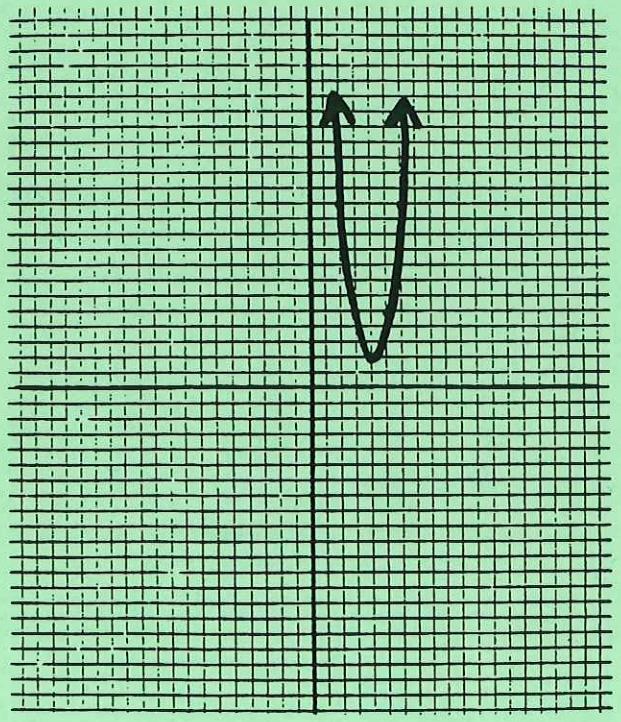
focus: $(4, 2\frac{1}{12})$

directrix: $y = 1\frac{11}{12}$

opening: upward

latus rectum: $\frac{1}{3}$

x	y
4	2
3	5
2	14



Parabolas

ANSWER KEY 22.1

③ $x = y^2 - 14y + 25$

$x = y^2 - 14y + \square + 25 - \square$

$x = y^2 - 14y + 49 + 25 - 49$

$x = (y-7)^2 - 24$ $\quad h = -24, K = 7$

axis: $y = 7$

vertex: $(-24, 7)$

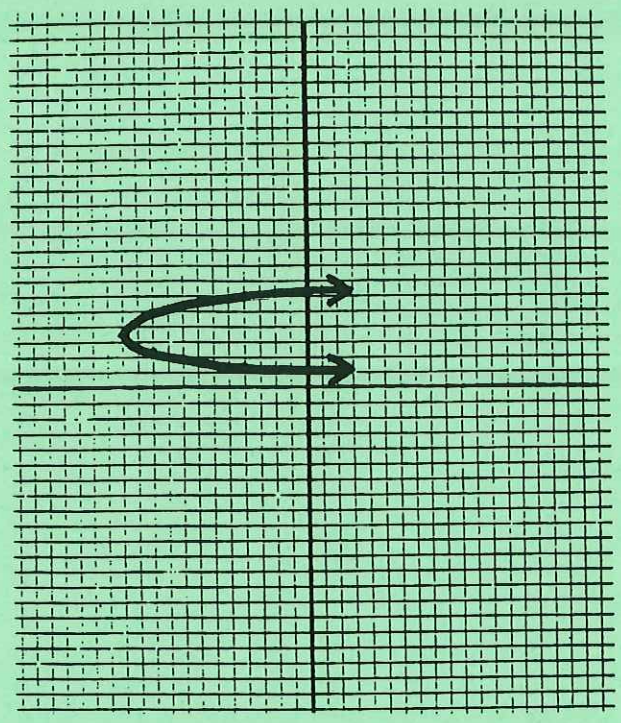
focus: $(-23\frac{3}{4}, 7)$

directrix: $x = -24\frac{1}{4}$

opening: right

latus rectum: 1

x	y
-24	7
-20	5
-8	3



Each square = 2 units above

④ $x = \frac{1}{2}y^2 - 3y + \frac{19}{2}$

$2x = y^2 - 6y + \square + 19 - \square$

$2x = y^2 - 6y + 9 + 19 - 9$

$2x = (y-3)^2 + 10$

$x = \frac{1}{2}(y-3)^2 + 5$ $\quad h = 5, K = 3$

axis: $y = 3$

vertex: $(5, 3)$

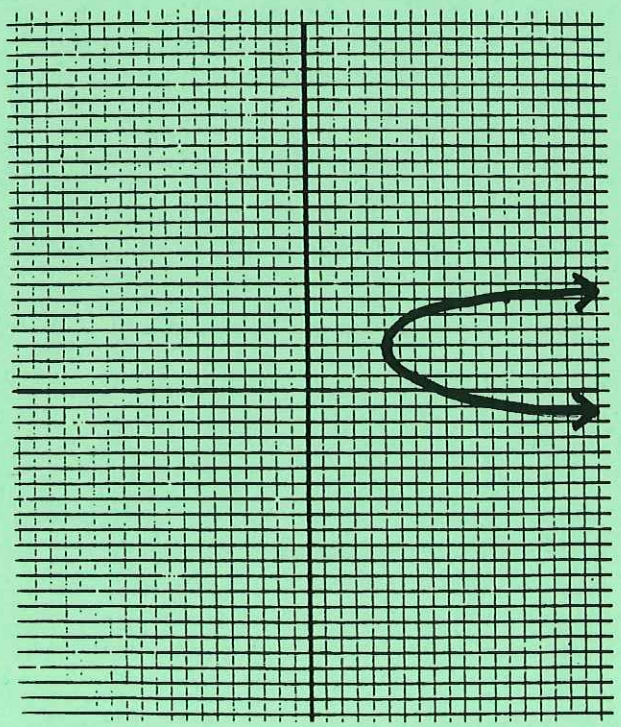
focus: $(5\frac{1}{2}, 3)$

directrix: $x = 4\frac{1}{2}$

opening: right

latus rectum: 2

x	y
5	3
7	1
13	-1



Parabolas

ANSWER KEY 22.1

⑤ $(y-8)^2 = -4(x-4)$

$(y-8)^2 = -4x+16$

$4x = -(y-8)^2+16$

$x = -\frac{1}{4}(y-8)^2+4$ $\quad h=4,\ k=8$

axis: $y=8$

vertex: $(4,8)$

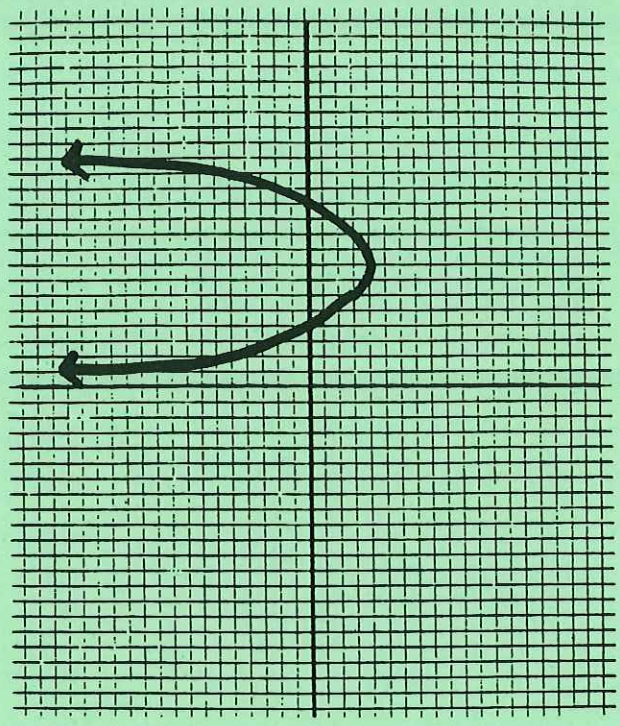
focus: $(3,8)$

directrix: $x=5$

opening: left

latus rectum: 4

x	y
4	8
0	4
-5	2



⑥ $(x+3)^2 = -4(y-2)$

$(x+3)^2 = -4y+8$

$4y = -(x+3)^2+8$

$y = -\frac{1}{4}(x+3)^2+2$ $\quad h=-3,\ k=2$

axis: $x=-3$

vertex: $(-3,2)$

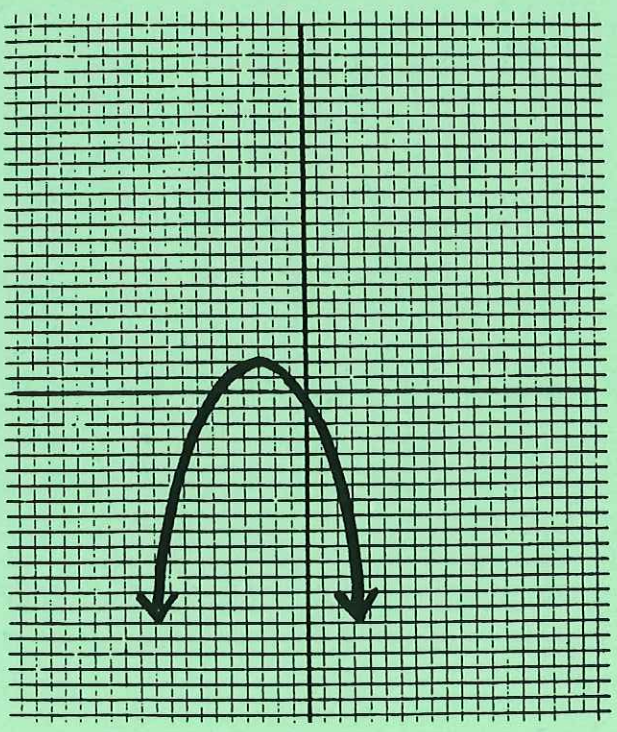
focus: $(-3,1)$

directrix: $y=3$

opening: downward

latus rectum: 4

x	y
-3	2
-1	1
1	-2



Parabolas

ANSWER KEY 22.1

⑦ $y = -2x^2 + 4x + 2$

$-\frac{1}{2}y = x^2 - 2x + \square - 1 - \square$

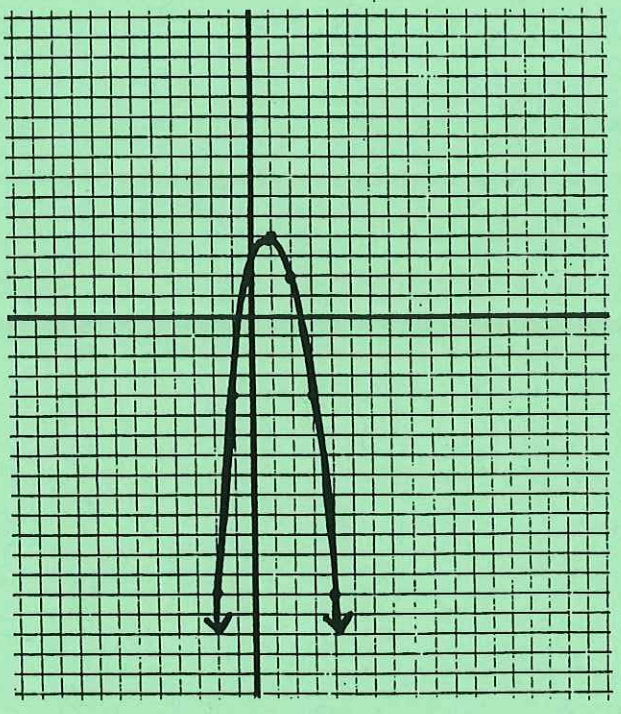
$-\frac{1}{2}y = x^2 - 2x + 1 - 1 - 1$

$-\frac{1}{2}y = (x-1)^2 - 2$

$y = -2(x-1)^2 + 4$

axis:	$x = 1$
vertex:	$(1, 4)$
focus:	$(1, 3\frac{7}{8})$
directrix:	$y = 4\frac{1}{8}$
opening:	downward
latus rectum:	$\frac{1}{2}$

x	y
0	2
-1	-4
-2	-14



⑧ $y = 2x^2 + 8x + 2$

$\frac{1}{2}y = x^2 + 4x + 1$

$\frac{1}{2}y = x^2 + 4x + \square + 1 - \square$

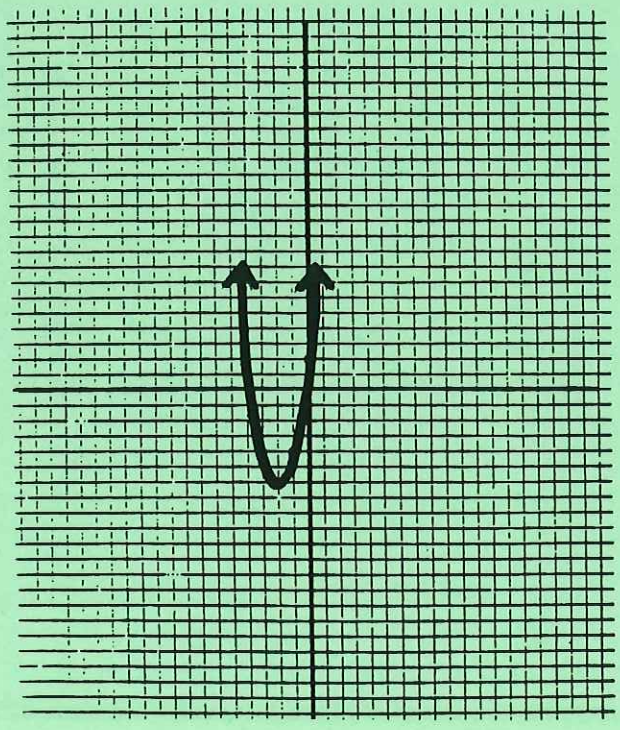
$\frac{1}{2}y = x^2 + 4x + 4 + 1 - 4$

$\frac{1}{2}y = (x+2)^2 - 3$

$y = 2(x+2)^2 - 6 \quad h = -2,\ k = -6$

axis:	$x = -2$
vertex:	$(-2, -6)$
focus:	$(-2, -5\frac{7}{8})$
directrix:	$y = -6\frac{1}{8}$
opening:	upward
latus rectum:	$\frac{1}{2}$

x	y
-2	-6
-1	-4
0	2



Parabolas

ANSWER KEY 22.1

⑨ (3, 5) $y = 1$

distance between focus and directrix is −4

parabola opens **upward**

add −2 to the y-coordinate in focus to obtain vertex (3, 3)

$h = 3$
$k = 3$

directrix:
$y = k - 1/4a$
$(1) = (3) - 1/4a$
$-2 = -1/4a$
$-8a = -1$
$a = 1/8$

$$y = 1/8\,(x-3)^2 + 3$$

⑩ (2, 4) $y = 6$

dist. focus to directrix is 2
parabola opens **downward**

vertex is (2, 5)
$h = 2$, $k = 5$

directrix:
$y = k - 1/4a$
$(6) = (5) - 1/4a$
$1 = -1/4a$
$4a = -1$
$a = -1/4$

$$y = -1/4\,(x-2)^2 + 5$$

⑪ (2, 2) $x = 4$

distance between focus and directrix is 2

parabola opens **left**

add 1 to x-coordinate in focus to obtain vertex (3, 2)

$h = 3$
$k = 2$

directrix:
$x = h - 1/4a$
$(4) = (3) - 1/4a$
$1 = -1/4a$
$4a = -1$
$a = -1/4$

$$x = -1/4\,(y-2)^2 + 3$$

⑫ (3, −1) $x = -2$

dist. focus to directrix is −5
parabola opens **right**

vertex is $(1/2, -1)$
$h = 1/2$, $k = -1$

directrix:
$x = h - 1/4a$
$(-2) = (1/2) - 1/4a$
$-5/2 = -1/4a$
$-10a = -1$
$a = 1/10$

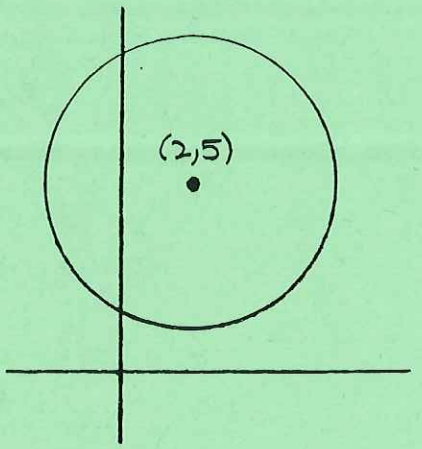
$$x = 1/10\,(y+1)^2 + 1/2$$

Circles

ANSWER KEY 22.2

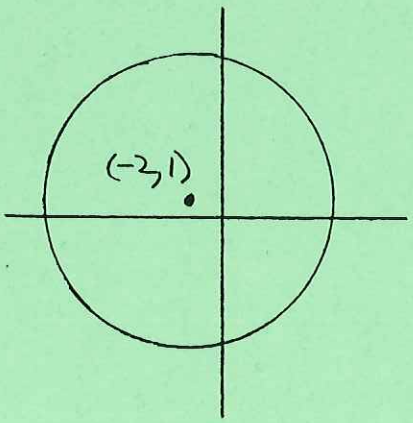
① $(x-2)^2+(y-5)^2=16$

center $(2,5)$
radius $=\sqrt{16}=4$



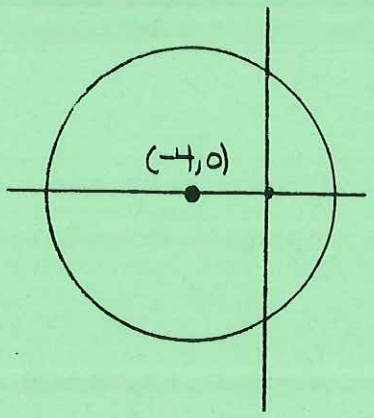
② $(x+2)^2+(y-1)^2=81$

center $(-2,1)$
radius $=\sqrt{81}=9$



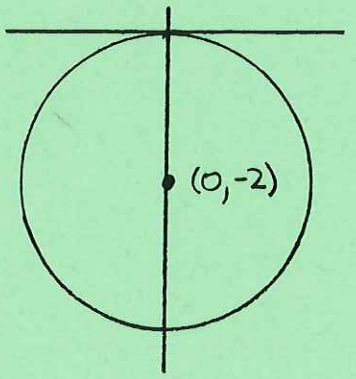
③ $(x+4)^2+y^2=49$

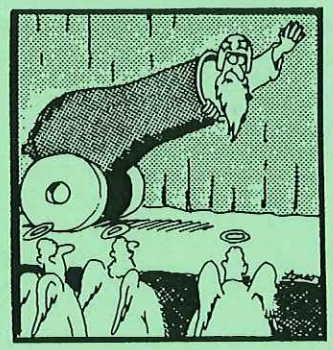
center $(-4,0)$
radius $=\sqrt{49}=7$

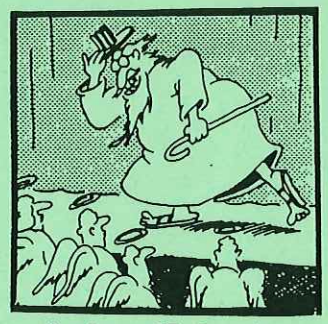


④ $x^2+(y+2)^2=4$

center $(0,-2)$
radius $=\sqrt{4}=2$







Acts of God

⑤ $x^2+y^2-12x-16y+84=0$

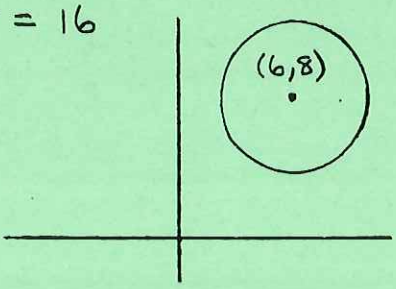
$x^2-12x+\square+y^2-16y+\triangle=-84+\square+\triangle$

$x^2-12x+36+y^2-16y+64=-84+36+64$

$(x-6)^2+(y-8)^2=16$

center
$(6,8)$

radius $=\sqrt{16}=4$



Circles

ANSWER KEY 22.2

⑥ $x^2+y^2+14x+6y=23$

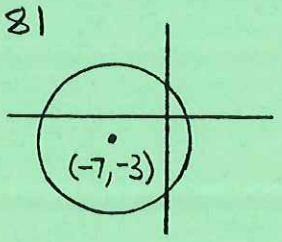
$x^2+14x+\square+y^2+6y+\triangle=23+\square+\triangle$

$x^2+14x+49+y^2+6y+9=23+49+9$

$(x+7)^2+(y+3)^2=81$

center $(-7,-3)$

radius $=\sqrt{81}=9$



⑦ $x^2+y^2+4x=8$

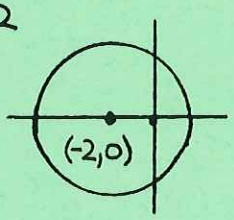
$x^2+4x+\square+y^2=8+\square$

$x^2+4x+4+y^2=8+4$

$(x+2)^2+(y+0)^2=12$

center $(-2,0)$

radius $=\sqrt{12}=2\sqrt{3}$



⑧ $x^2+y^2+14x+6y=-50$

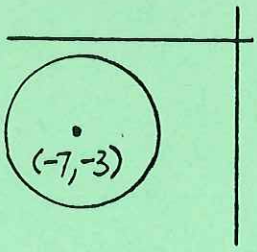
$x^2+14x+\square+y^2+6y+\triangle=-50+\square+\triangle$

$x^2+14x+49+y^2+6y+9=-50+49+9$

$(x+7)^2+(y+3)^2=8$

center $(-7,-3)$

radius $=\sqrt{8}=2\sqrt{2}$



⑨ $(-3,-5)$ $r=5$

$(x+3)^2+(y+5)^2=25$

⑩ $(6,2)$ $r=5$

$(x-6)^2+(y-2)^2=25$

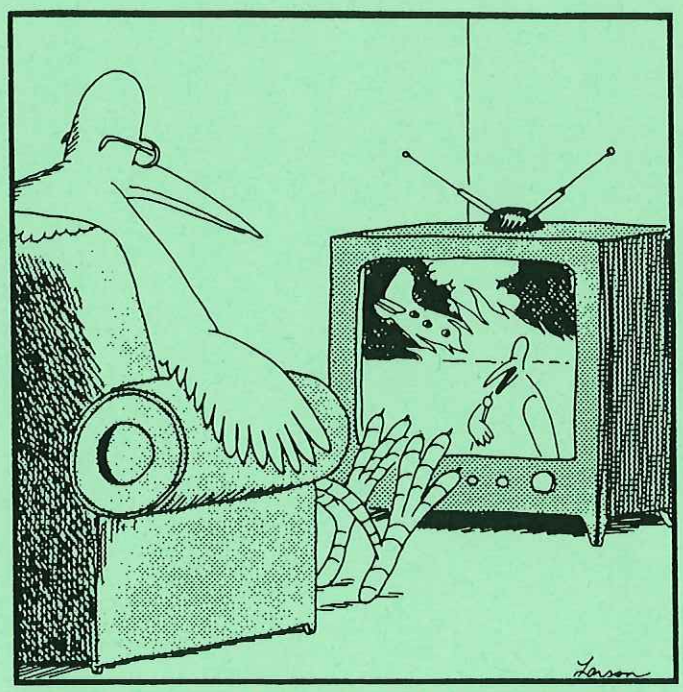
⑪ center $(4,-2)$

passing through $(9,-3)$

$\sqrt{(4-9)^2+(-2-(-3))^2}$

$\sqrt{25+1}=\sqrt{26}$ radius

$(x-4)^2+(y+2)^2=26$



"Details are still sketchy, but we think the name of the bird sucked into the jet's engines was Harold Meeker."

⑫ center $(1,5)$

passing through $(0,0)$

-continued-

Circles

ANSWER KEY 22.2

$\sqrt{(1-0)^2+(5-0)^2}$

$\sqrt{1+25} = \sqrt{26}$ radius

$(x-1)^2+(y-5)^2 = 26$

(13) center $(-3,8)$
passing through $(-3,0)$ since it is tangent to the x-axis

$\sqrt{(-3-(-3))^2+(8-0)^2}$

$\sqrt{0+64} = \sqrt{64} = 8$ radius

$(x+3)^2+(y-8)^2 = 64$

(14) center $(4,-3)$
passing through $(0,-3)$ since it is tangent to the y-axis

$\sqrt{(4-0)^2+(-3-(-3))^2}$

$\sqrt{16+0} = \sqrt{16} = 4$ radius

$(x-4)^2+(y+3)^2 = 16$

(15) $-6x = y^2+8y+40$

$-6x = y^2+8y+\square+40-\square$

$-6x = y^2+8y+16+40-16$

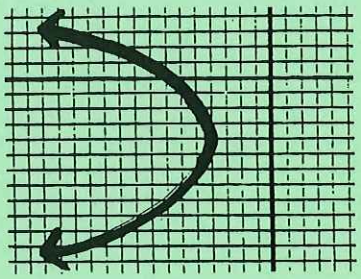
$-6x = (y+4)^2+24$

$x = -\frac{1}{6}(y+4)^2-4$

$a = -1/6 \quad h = -4 \quad K = -4$

axis:	$y = -4$
vertex:	$(-4,-4)$
focus:	$(-5\frac{1}{2},-4)$
directrix:	$x = -2\frac{1}{2}$
opening:	left
latus rectum:	6

x	y
-4	-4
-10	2
$-5\frac{1}{2}$	-1

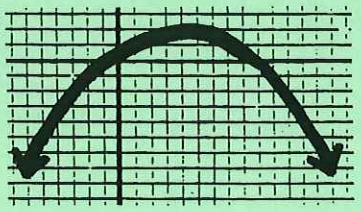


(16) $y = -\frac{1}{12}(x-4)^2+2$

$a = -1/12 \quad h = 4 \quad K = 2$

axis:	$x = 4$
vertex:	$(4,2)$
focus:	$(4,-1)$
directrix:	$y = 5$
opening:	downward
latus rectum:	12

x	y
4	2
-2	-1
-6	$-6\frac{1}{3}$



Ellipses

ANSWER KEY 22.3

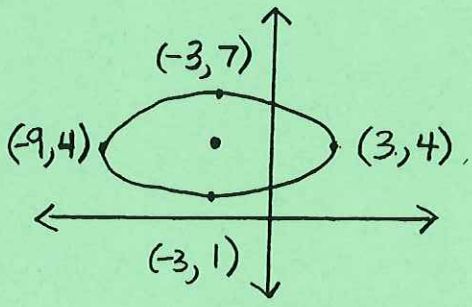
① $\frac{(x+3)^2}{36} + \frac{(y-4)^2}{9} = 1$

$a = 6$, $b = 3$

center $(-3, 4)$

$c^2 = 36 - 9 = 27$
$c = \sqrt{27} = 3\sqrt{3}$
foci $(-3 \pm 3\sqrt{3}, 4)$

major axis $= 2a = 12$
minor axis $= 2b = 6$



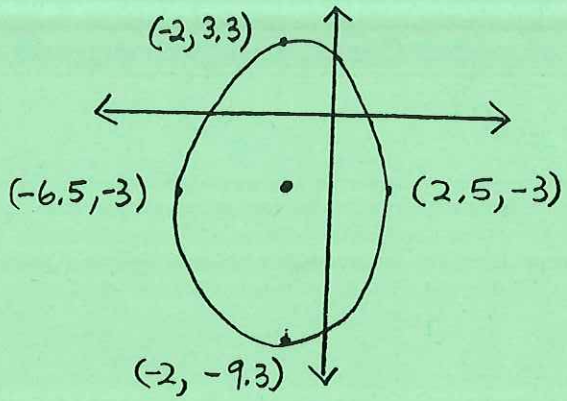
② $\frac{(x+2)^2}{20} + \frac{(y+3)^2}{40} = 1$

$a = \sqrt{40}$, $b = \sqrt{20}$

center $(-2, -3)$

$c^2 = 40 - 20 = 20$
$c = \sqrt{20} = 2\sqrt{5}$
foci $(-2, -3 \pm 2\sqrt{5})$

major axis $= 2a = 4\sqrt{10}$
minor axis $= 2b = 4\sqrt{5}$



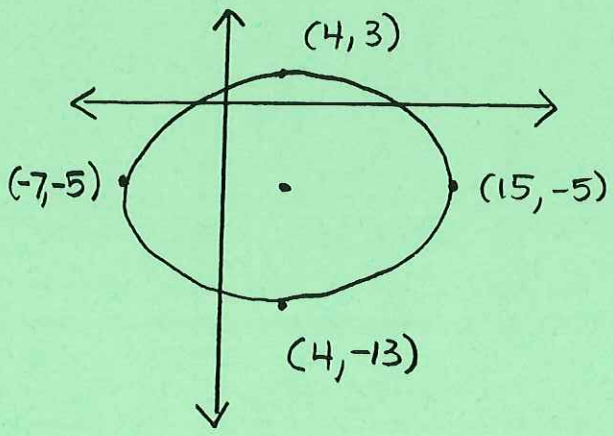
③ $\frac{(x-4)^2}{121} + \frac{(y+5)^2}{64} = 1$

$a = 11$, $b = 8$

center $(4, -5)$

$c^2 = 121 - 64 = 57$
$c = \sqrt{57}$
foci $(4 \pm \sqrt{57}, -5)$

major axis $= 2a = 22$
minor axis $= 2b = 16$



Ellipses

ANSWER KEY 22.3

④ $\frac{(x-2)^2}{16} + \frac{(y-3)^2}{9} = 1$

$a=4,\ b=3$

center $(2,3)$

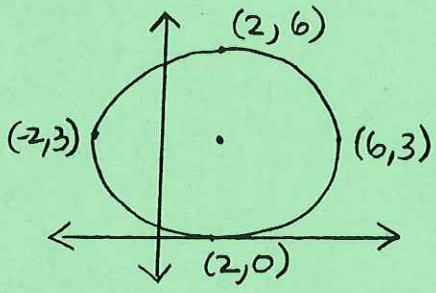
$c^2 = 16-9 = 7$

$c = \sqrt{7}$

foci $(2\pm\sqrt{7}, 3)$

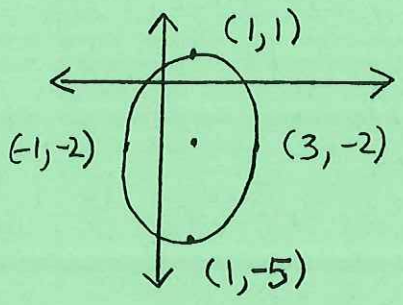
major axis $= 2a = 8$

minor axis $= 2b = 6$



major axis $= 2a = 6$

minor axis $= 2b = 4$



⑤ $9x^2+4y^2-18x+16y = 11$

$9x^2-18x+4y^2+16y = 11$

$9(x^2-2x+\square)+4(y^2+4y+\triangle) = 11+9\square+4\triangle$

$9(x^2-2x+1)+4(y^2+4y+4) = 11+9+16$

$9(x-1)^2+4(y+2)^2 = 36$

$\frac{(x-1)^2}{4} + \frac{(y+2)^2}{9} = 1$ $\quad a=3,\ b=2$

center $(1,-2)$

$c^2 = 9-4 = 5 \quad c = \sqrt{5}$

foci $(1, -2\pm\sqrt{5})$

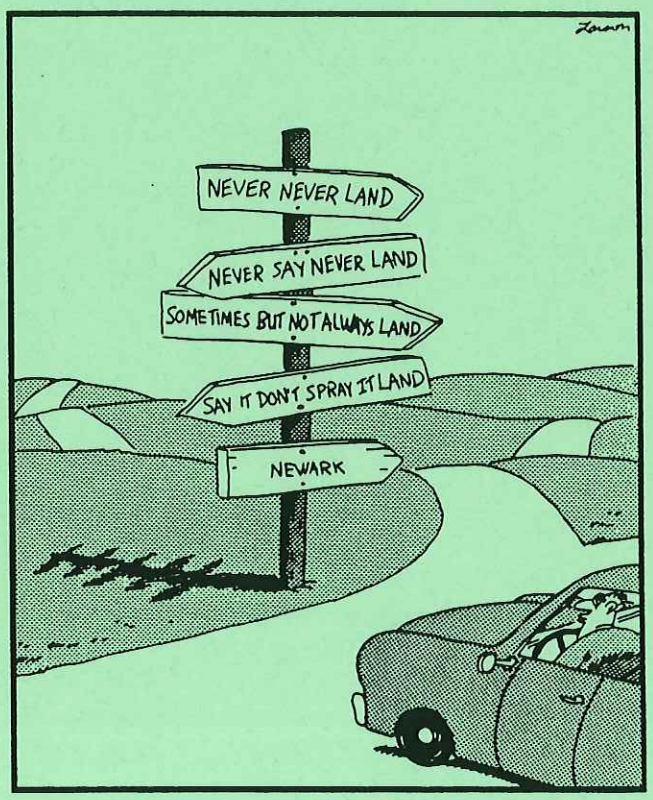
⑥ $3x^2+7y^2-12x-28y = -19$

$3(x^2-4x)+7(y^2-4y) = -19$

$3(x^2-4x+\square)+7(y^2-4y+\triangle) = -19+3\square+7\triangle$

$3(x^2-4x+4)+7(y^2-4y+4) = -19+12+28$

$3(x-2)^2+7(y-2)^2 = 21$



Crucial decisions along life's highway

Ellipses

ANSWER KEY 22.3

$\frac{(x-2)^2}{7} + \frac{(y-2)^2}{3} = 1$ $\quad a = \sqrt{7}$, $b = \sqrt{3}$

center $(2,2)$

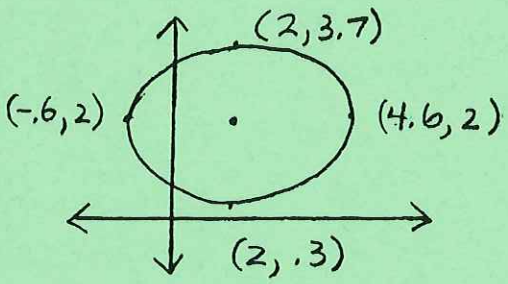
$c^2 = 7-3 = 4$

$c = \sqrt{4} = 2$

foci $(4,2)$ and $(0,2)$

major axis $= 2a = 2\sqrt{7}$

minor axis $= 2b = 2\sqrt{3}$



⑦ $25x^2 + 4y^2 = 100$

$\frac{x^2}{4} + \frac{y^2}{25} = 1$ $\quad a = 5$, $b = 2$

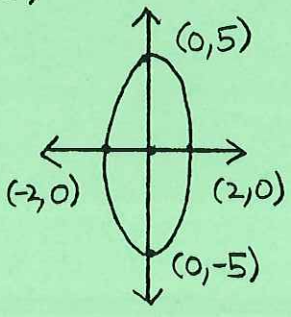
center $(0,0)$

$c^2 = 25-4 = 21$

$c = \sqrt{21}$

foci $(0, \pm\sqrt{21})$

major axis $2a = 10$ minor axis $2b = 4$



⑧ $16x^2 + 25y^2 + 32x - 150y = 159$

$16x^2 + 32x + 25y^2 - 150y = 159$

$16(x^2 + 2x + \square) + 25(y^2 - 6y + \Delta) = 159 + 16\square + 25\Delta$

$16(x^2 + 2x + 1) + 25(y^2 - 6y + 9) = 159 + 16 + 225$

$16(x+1)^2 + 25(y-3)^2 = 400$

$\frac{(x+1)^2}{25} + \frac{(y-3)^2}{16} = 1$ $\quad a = 5$, $b = 4$

center $(-1,3)$

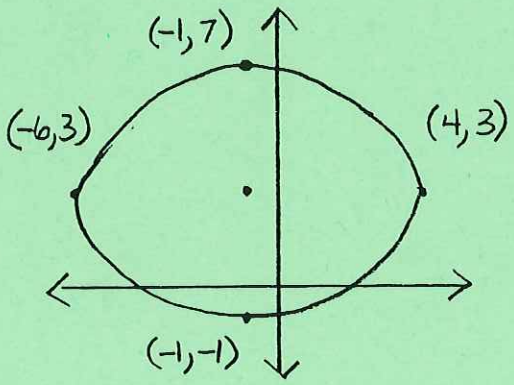
$c^2 = 25-16 = 9$

$c = \sqrt{9} = 3$

foci $(2,3)$ and $(-4,3)$

major axis $= 2a = 10$

minor axis $= 2b = 8$



⑨ Since the foci are lined up vertically, this is a vertical ellipse. $c = 8$

The major axis runs $(0,10)$ to

Ellipses

ANSWER KEY 22.3

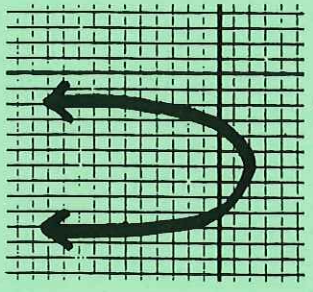
$(0,-10)$ and it is 20 units long.
$a = 10$

$c^2 = a^2 - b^2$ center point
$(8)^2 = (10)^2 - b^2$ is $(0,0)$
$64 = 100 - b^2$
$b = 6$

$\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad \frac{x^2}{36} + \frac{y^2}{100} = 1$

$a = -\frac{1}{2} \quad h = 2 \quad k = -6$

axis: $y = -6$
vertex: $(2, -6)$
focus: $(1\frac{1}{2}, -6)$
directrix: $x = 2\frac{1}{2}$
opening: left
latus rectum: 2



x	y
2	-6
0	-4
-6	-2

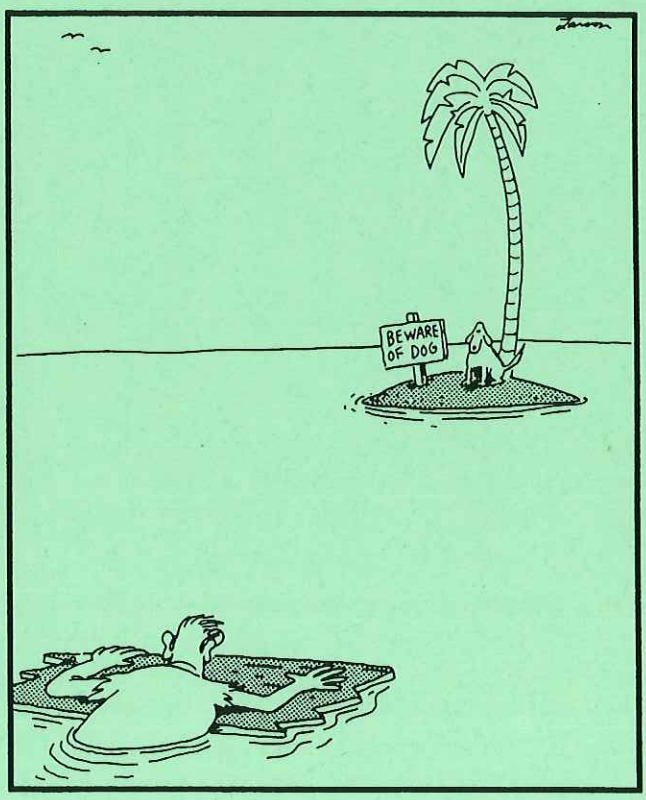
(10) Since the major axis is parallel to the x-axis, this is a horizontal ellipse.

Center $(5,4)$ $h = 5$, $k = 4$

major axis $= 2a = 16$ $a = 8$
minor axis $= 2b = 8$ $b = 4$

$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

$\frac{(x-5)^2}{64} + \frac{(y-4)^2}{16} = 1$



(11) $-2x = y^2 + 12y + 32$
$-2x = y^2 + 12y + \square + 32 - \square$
$-2x = y^2 + 12y + 36 + 32 - 36$
$-2x = (y+6)^2 - 4$
$x = -\frac{1}{2}(y+6)^2 + 2$

Ellipses

ANSWER KEY 22.3

(12) $8y = x^2 - 8x + 32$

$8y = x^2 - 8x + \square + 32 - \square$

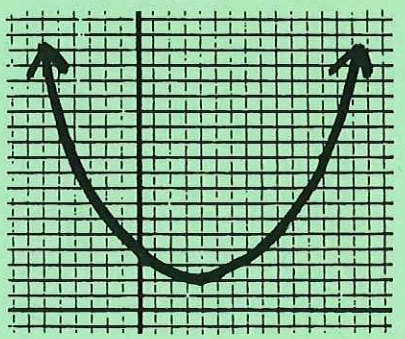
$8y = x^2 - 8x + 16 + 32 - 16$

$8y = (x-4)^2 + 16$

$y = \frac{1}{8}(x-4)^2 + 2$

$a = \frac{1}{8} \quad h = 4 \quad k = 2$

axis:	$x = 4$
vertex:	$(4, 2)$
focus:	$(4, 4)$
directrix:	$y = 0$
opening:	upward
latus rectum:	8



x	y
4	2
0	4
−4	10

(13) $x^2 + y^2 - 6y = 16$

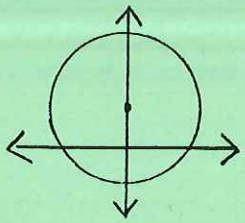
$x^2 + y^2 - 6y + \square = 16 + \square$

$x^2 + y^2 - 6y + 9 = 16 + 9$

$x^2 + (y-3)^2 = 25$

center $(0, 3)$

radius $= \sqrt{25} = 5$



(14) $x^2 + 2x + y^2 + 4y = 9$

$x^2 + 2x + \square + y^2 + 4y + \triangle = 9 + \square + \triangle$

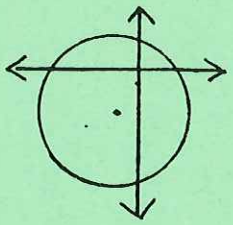
$x^2 + 2x + 1 + y^2 + 4y + 4 = 9 + 1 + 4$

$(x+1)^2 + (y+2)^2 = 14$

center $(-1, -2)$

radius $= \sqrt{14}$

≈ 3.74



"Oh, man! The coffee's *cold*!
They thought of *everything*!"

(continued)

Hyperbolas

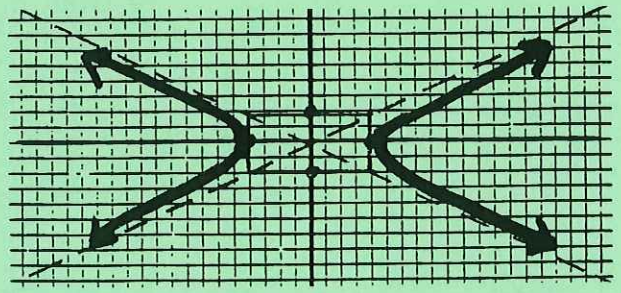
ANSWER KEY 22.4

① $\frac{x^2}{16} - \frac{y^2}{4} = 1$ transverse axis is horizontal

center $(h,k) = (0,0)$
$a = 4,\ b = 2$

$c^2 = a^2 + b^2$
$c^2 = 16 + 4 = 20 \quad c = \sqrt{20} = 2\sqrt{5}$

foci $(\pm 2\sqrt{5}, 0)$
vertices $(\pm 4, 0)$
asymptotes $m = \pm \frac{1}{2}$

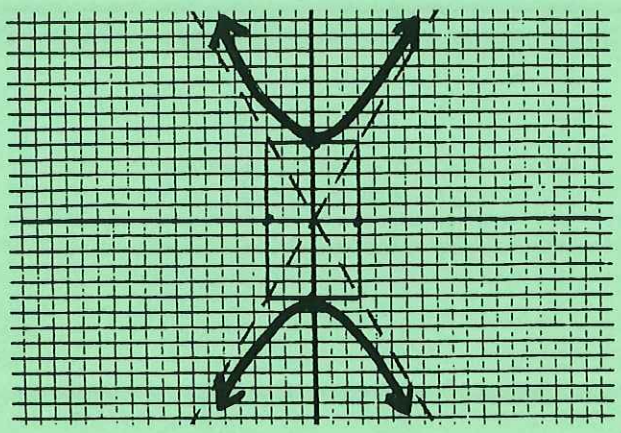


② $\frac{y^2}{25} - \frac{x^2}{9} = 1$ transverse axis is vertical

center $(h,k) = (0,0)$
$a = 5,\ b = 3$

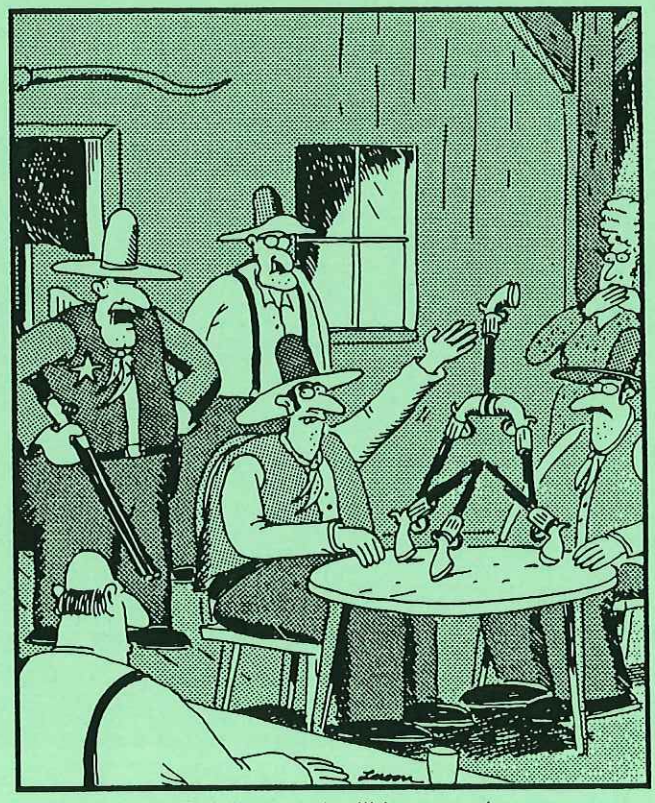
$c^2 = a^2 + b^2$
$c^2 = 25 + 9 = 34 \quad c = \sqrt{34}$

foci $(0, \pm\sqrt{34})$
vertices $(0, \pm 5)$
asymptotes $m = \pm \frac{5}{3}$



③ $\frac{(x+6)^2}{36} - \frac{(y+3)^2}{9} = 1$ transverse axis is horizontal

center $(h,k) = (-6,-3)$
$a = 6,\ b = 3$



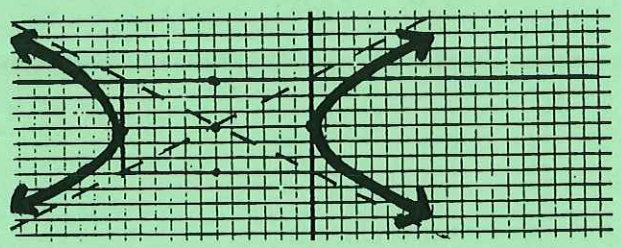
"OK, boys—that'll be enough.
We don't allow any gunplay in this town."

Hyperbolas

ANSWER KEY 22.4

$c^2 = a^2 + b^2$
$c^2 = 36 + 9 = 45 \quad c = \sqrt{45} = 3\sqrt{5}$

foci $(-6 \pm 3\sqrt{5}, -3)$
vertices $(-6 \pm 6, -3) \rightarrow (0, -3)\ (-12, -3)$
asymptotes $m = \pm \frac{1}{2}$

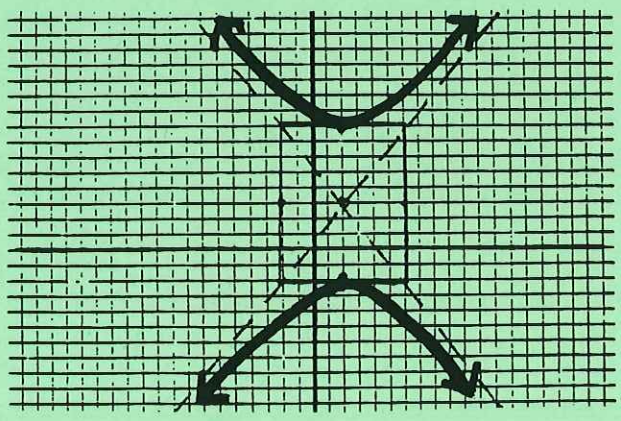


④ $\frac{(y-3)^2}{25} - \frac{(x-2)^2}{16} = 1$ transverse axis is vert.

center $(h, k) = (2, 3)$
$a = 5,\ b = 4$

$c^2 = a^2 + b^2$
$c^2 = 25 + 16 = 41 \quad c = \sqrt{41}$

foci $(2, 3 \pm \sqrt{41})$
vertices $(2, 3 \pm 5) \rightarrow (2, 8)\ (2, -2)$
asymptotes $m = \pm \frac{5}{4}$



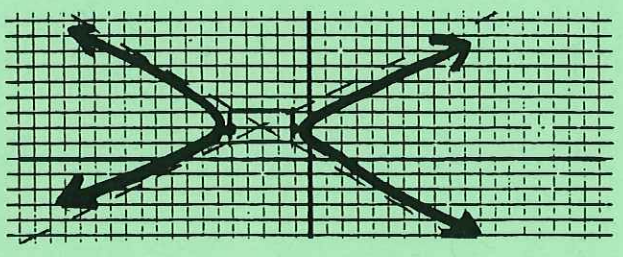
⑤ $x^2 - 4y^2 + 6x + 16y - 11 = 0$
$x^2 + 6x - 4y^2 + 16y = 11$
$(x^2 + 6x + \square) - 4(y^2 - 4y + \triangle) = 11 + \square - 4\triangle$
$(x^2 + 6x + 9) - 4(y^2 - 4y + 4) = 11 + 9 - 16$
$(x+3)^2 - 4(y-2)^2 = 4$

$\frac{(x+3)^2}{4} - \frac{(y-2)^2}{1} = 1$ transverse axis is horizontal

center $(h, k) = (-3, 2)$
$a = 2 \quad b = 1$

$c^2 = a^2 + b^2$
$c^2 = 4 + 1 = 5 \quad c = \sqrt{5}$

foci $(-3 \pm \sqrt{5}, 2)$
vertices $(-3 \pm 2, 2) \rightarrow (-1, 2)\ (-5, 2)$
asymptotes $m = \pm \frac{1}{2}$



⑥ $y^2 - 4x^2 - 2y - 16x = -1$
$(y^2 - 2y + \square) - 4(x^2 + 4x + \triangle) = -1 + \square - 4\triangle$
$(y^2 - 2y + 1) - 4(x^2 + 4x + 4) = -1 + 1 - 16$
$(y-1)^2 - 4(x+2)^2 = -16$

$\frac{(y-1)^2}{-16} + \frac{(x+2)^2}{4} = 1$ (continued)

Hyperbolas

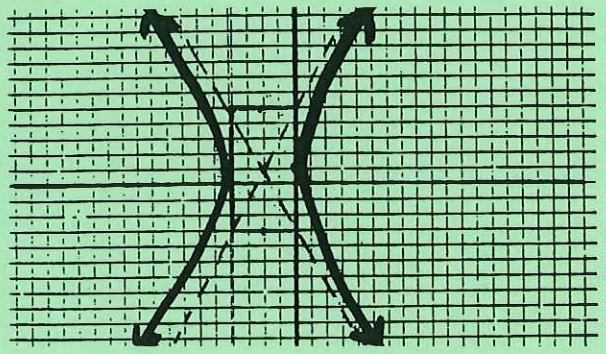
ANSWER KEY 22.4

$\frac{(x+2)^2}{4} - \frac{(y-1)^2}{16} = 1$ transverse axis is horizontal

center $(h,k) = (-2,1)$
$a = 2$, $b = 4$

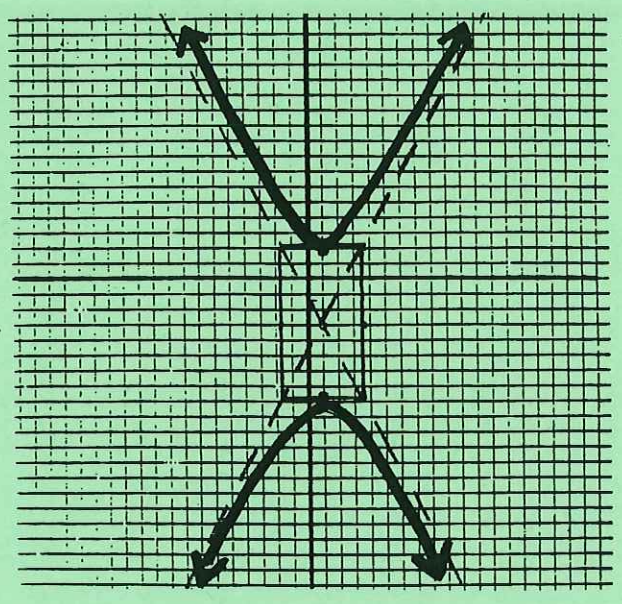
$c^2 = a^2 + b^2$
$c^2 = 4 + 16 = 20$ $c = 2\sqrt{5}$

foci $(-2 \pm 2\sqrt{5}, 1)$
vertices $(-2 \pm 2, 1) \rightarrow (0,1)\ (-4,1)$
asymptotes $m = \pm 2$



foci $(1, -3 \pm 4\sqrt{2})$
vertices $(1, -3 \pm 2\sqrt{6})$ $2\sqrt{6} \approx 4.9$
$(1, 1.9)\ (1, -7.9)$
asymptotes $m = \pm a/b \rightarrow \pm\sqrt{3}$

$\frac{a}{b} = \frac{2\sqrt{6}}{2\sqrt{2}} = \frac{\sqrt{6}}{\sqrt{2}} = \sqrt{3}$

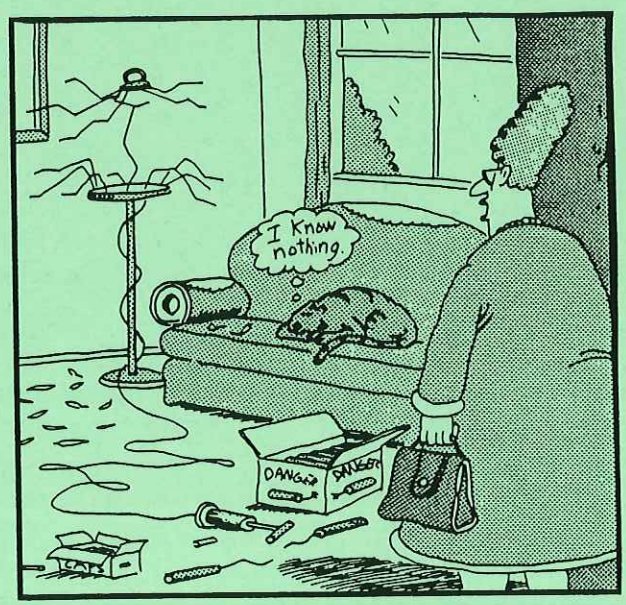


(7) $y^2 - 3x^2 + 6y + 6x = 18$
$(y^2 + 6y + \square) - 3(x^2 - 2x + \triangle) = 18 + \square - 3\triangle$
$(y^2 + 6y + 9) - 3(x^2 - 2x + 1) = 18 + 9 - 3$
$(y+3)^2 - 3(x-1)^2 = 24$

$\frac{(y+3)^2}{24} - \frac{(x-1)^2}{8} = 1$ transverse axis is vertical

center $(h,k) = (1,-3)$
$a = \sqrt{24} = 2\sqrt{6}$, $b = \sqrt{8} = 2\sqrt{2}$

$c^2 = a^2 + b^2$
$c^2 = 24 + 8 = 32$ $c = 4\sqrt{2}$



Hyperbolas

ANSWER KEY 22.4

⑧ $5x^2-4y^2-40x-16y=36$

$5(x^2-8x+\square)-4(y^2+4y+\triangle)=36+5\square-4\triangle$

$5(x^2-8x+16)-4(y^2+4y+4)=36+80-16$

$5(x-4)^2-4(y+2)^2=100$

$\frac{(x-4)^2}{20}-\frac{(y+2)^2}{25}=1$ transverse axis is horizontal

center $(h,k)=(4,-2)$

$a=\sqrt{20}=2\sqrt{5}$, $b=5$

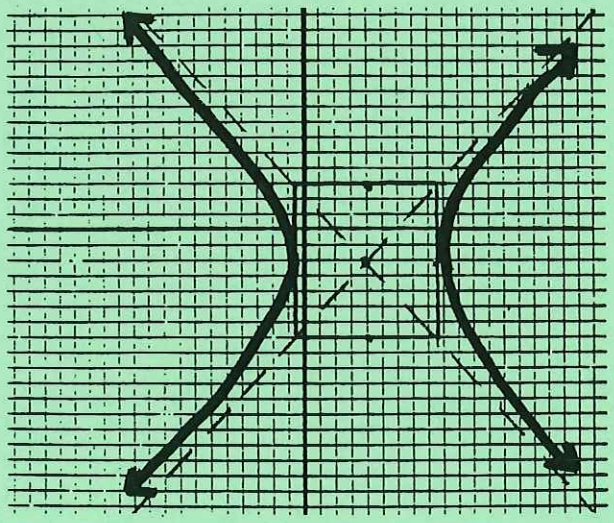
$c^2=a^2+b^2$

$c^2=20+25=45$ $c=\sqrt{45}=3\sqrt{5}$

foci $(4\pm3\sqrt{5},-2)$

vertices $(4\pm2\sqrt{5},-2)\rightarrow(8.5,-2)(-.5,-2)$

asymptotes $m=\pm\frac{5}{2\sqrt{5}}=\pm\frac{\sqrt{5}}{2}$



axis: $y=-8$

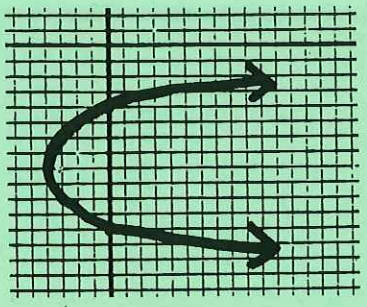
vertex: $(-4,-8)$

focus: $(-3,-8)$

directrix: $x=-5$

opening: right

latus rectum: 4



x	y
-4	-8
-3	-6
0	-4

⑨ $4x=y^2+16y+48$

$4x=y^2+16y+\square+48-\square$

$4x=y^2+16y+64+48-64$

$4x=(y+8)^2-16$

$x=\frac{1}{4}(y+8)^2-4$

⑩ $x^2-4x+y^2-20y=-68$

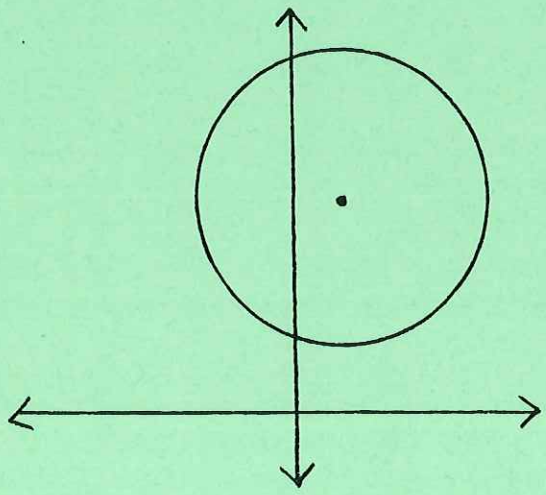
$(x^2-4x+\square)+(y^2-20y+\triangle)=-68+\square+\triangle$

$(x^2-4x+4)+(y^2-20y+100)=-68+4+100$

$(x-2)^2+(y-10)^2=36$

center $(h,k)=(2,10)$

radius $=6$



Hyperbolas

ANSWER KEY 22.4

(11) center $(5,-1)$
passing through $(-2,-2)$

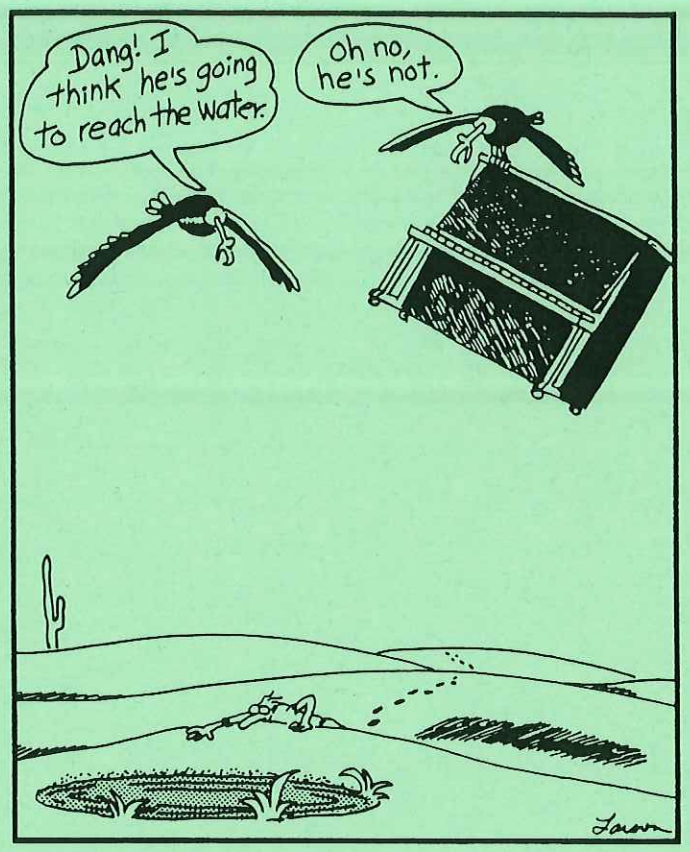
Use the distance formula to determine the radius:

$d = \sqrt{(x-x_1)^2+(y-y_1)^2}$

$r = \sqrt{(5-(-2))^2+(-1-(-2))^2}$

$r = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$

$(x-5)^2+(y+1)^2 = 50$



(12) $16x^2-64x+9y^2+54y = -1$

$16(x^2-4x+\square)+9(y^2+6y+\triangle) = -1+16\square+9\triangle$

$16(x^2-4x+4)+9(y^2+6y+9) = -1+64+81$

$16(x-2)^2+9(y+3)^2 = 144$

$\frac{(x-2)^2}{9} + \frac{(y+3)^2}{16} = 1$ $\quad a=4$ $\quad b=3$

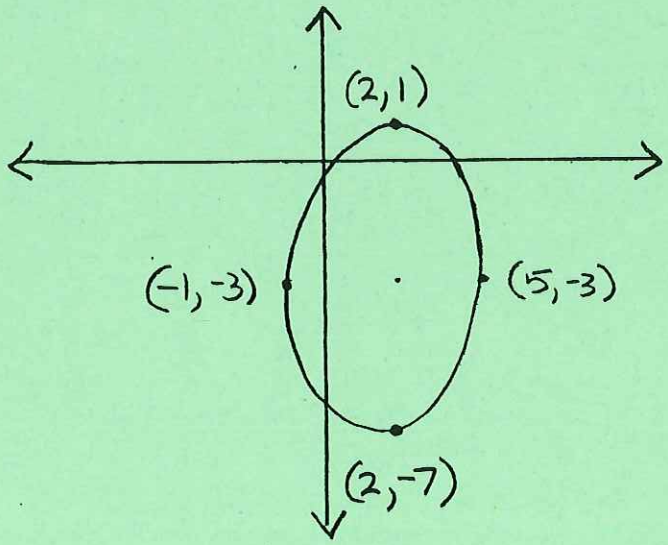
Center $(h,k) = (2,-3)$

$c^2 = a^2-b^2$

$c^2 = 16-9 = 7 \quad c=\sqrt{7}$

Foci $(2, -3\pm\sqrt{7})$

major axis $= 2a = 8$
minor axis $= 2b = 6$



ANSWER KEY: UNIT 22 REVIEW & PRACTICE

① $x = \frac{1}{2}y^2 - 5y + \frac{29}{2}$

$2x = y^2 - 10y + 29$

$2x = y^2 - 10y + \square + 29 - \square$

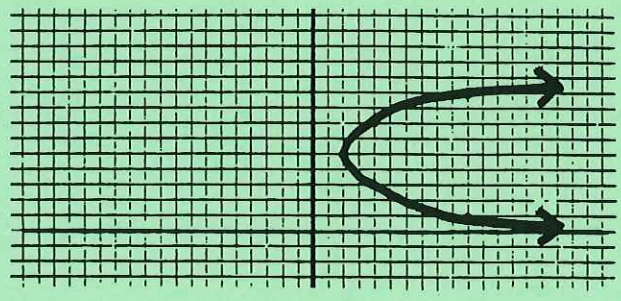
$2x = y^2 - 10y + 25 + 29 - 25$

$2x = (y-5)^2 + 4$

$x = \frac{1}{2}(y-5)^2 + 2$

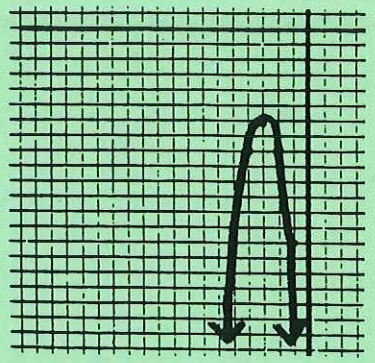
axis:	$y = 5$
vertex:	$(2, 5)$
focus:	$(2\frac{1}{2}, 5)$
directrix:	$x = 1\frac{1}{2}$
opening:	right
latus rectum:	2

x	y
2	5
4	3
10	1



axis:	$x = -3$
vertex:	$(-3, -6)$
focus:	$(-3, -6\frac{1}{8})$
directrix:	$y = -5\frac{7}{8}$
opening:	downward
latus rectum:	$\frac{1}{2}$

x	y
-3	-6
-2	-8
-1	-14



② $y = -2x^2 - 12x - 24$

$y = -2(x^2 + 6x + 12)$

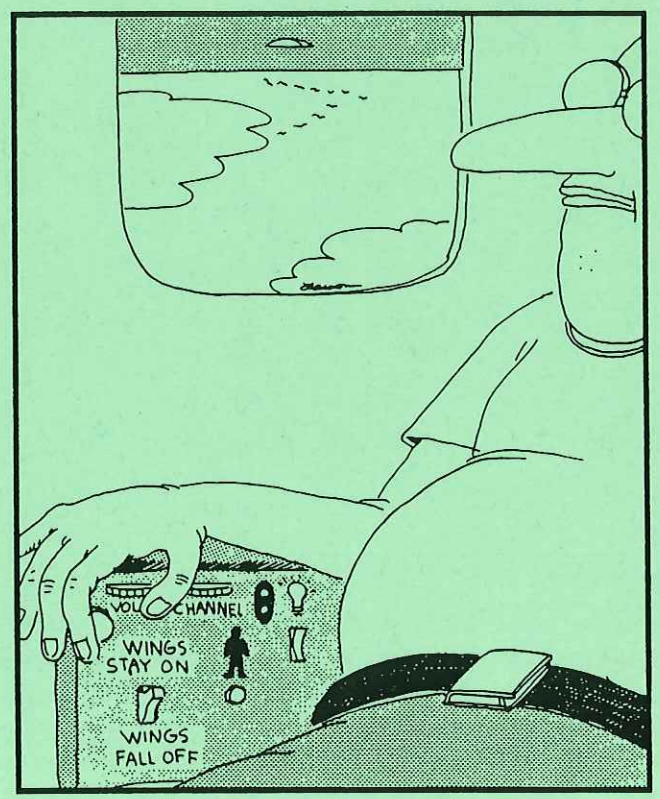
$-\frac{1}{2}y = x^2 + 6x + \square + 12 - \square$

$-\frac{1}{2}y = x^2 + 6x + 9 + 12 - 9$

$-\frac{1}{2}y = (x+3)^2 + 3$

$y = -2(x+3)^2 - 6$

$a = -2 \quad h = -3 \quad k = -6$



Fumbling for his recline button, Ted unwittingly instigates a disaster.

Conics

ANSWER KEY: UNIT 22 REVIEW & PRACTICE

③ $x = -\frac{1}{4}y^2 + 2y + 4$

$-4x = y^2 - 8y - 16$

$-4x = y^2 - 8y + \square - 16 - \square$

$-4x = (y^2 - 8y + 16) - 16 - 16$

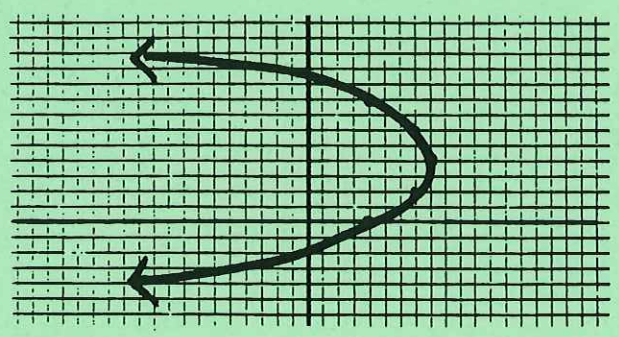
$-4x = (y-4)^2 - 32$

$x = -\frac{1}{4}(y-4)^2 + 8$

$a = -\frac{1}{4}$ $h = 8$ $k = 4$

axis:	$y = 4$
vertex:	$(8, 4)$
focus:	$(7, 4)$
directrix:	$x = 9$
opening:	left
latus rectum:	4

x	y
8	4
7	6
4	8



④ $x^2 + y^2 - 6x + 12y + 20 = 0$

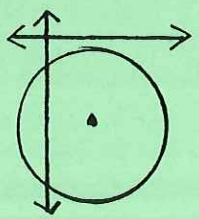
$x^2 - 6x + \square + y^2 + 12y + \triangle = -20 + \square + \triangle$

$x^2 - 6x + 9 + y^2 + 12y + 36 = -20 + 9 + 36$

$(x-3)^2 + (y+6)^2 = 25$

center $(3, -6)$

radius 5



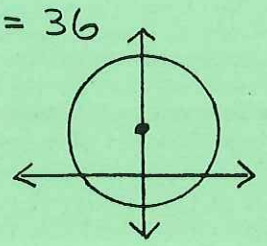
⑤ $x^2 + y^2 - 8y = 20$

$x^2 + y^2 - 8y + \triangle = 20 + \triangle$

$(x-0)^2 + (y-4)^2 = 36$

center $(0, 4)$

radius 6



⑥ center $(-2, -4)$ passing through $(6, -2)$

$r = \sqrt{(-2-6)^2 + (-4-(-2))^2}$

$r = \sqrt{64 + 4} = \sqrt{68} = 2\sqrt{17}$

$(x+2)^2 + (y+4)^2 = 68$

⑦ center $(4, 10)$ passing through $(8, 16)$

$r = \sqrt{(4-8)^2 + (10-16)^2}$

$r = \sqrt{16+36} = \sqrt{52} = 2\sqrt{13}$

$(x-4)^2 + (y-10)^2 = 52$

⑧ $\frac{(x-3)^2}{25} + \frac{(y-4)^2}{4} = 1$

center $(h, k) = (3, 4)$

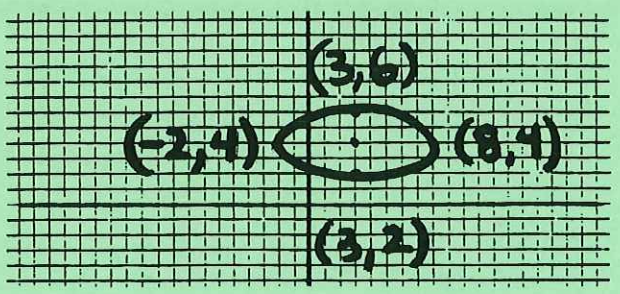
continued

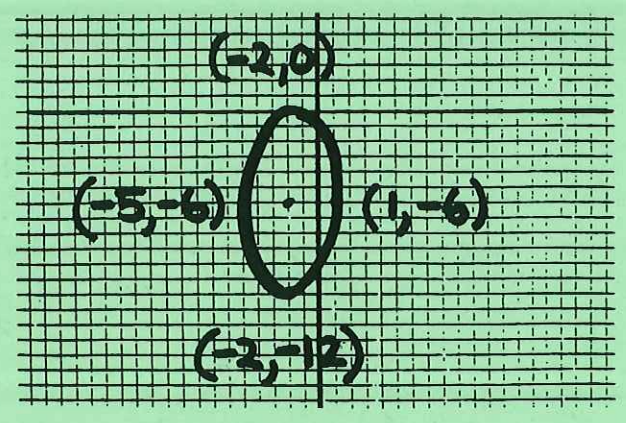
ANSWER KEY: UNIT 22 REVIEW & PRACTICE

$c^2 = a^2 - b^2$
$c^2 = 25 - 4 = 21 \quad c = \sqrt{21}$

foci $(3 \pm \sqrt{21}, 4)$

$a = \sqrt{25} = 5$ maj. axis $2a = 10$
$b = \sqrt{4} = 2$ min. axis $2b = 4$





⑨ $4x^2 + 16x + y^2 + 12y = -16$

$4(x^2 + 4x + \square) + (y^2 + 12y + \triangle) = -16 + 4\square + \triangle$
$4(x^2 + 4x + 4) + (y^2 + 12y + 36) = -16 + 16 + 36$
$4(x+2)^2 + (y+6)^2 = 36$

$\frac{(x+2)^2}{9} + \frac{(y+6)^2}{36} = 1$

center $(h, k) = (-2, -6)$

$c^2 = a^2 - b^2$
$c^2 = 36 - 9 = 27 \quad c = 3\sqrt{3}$

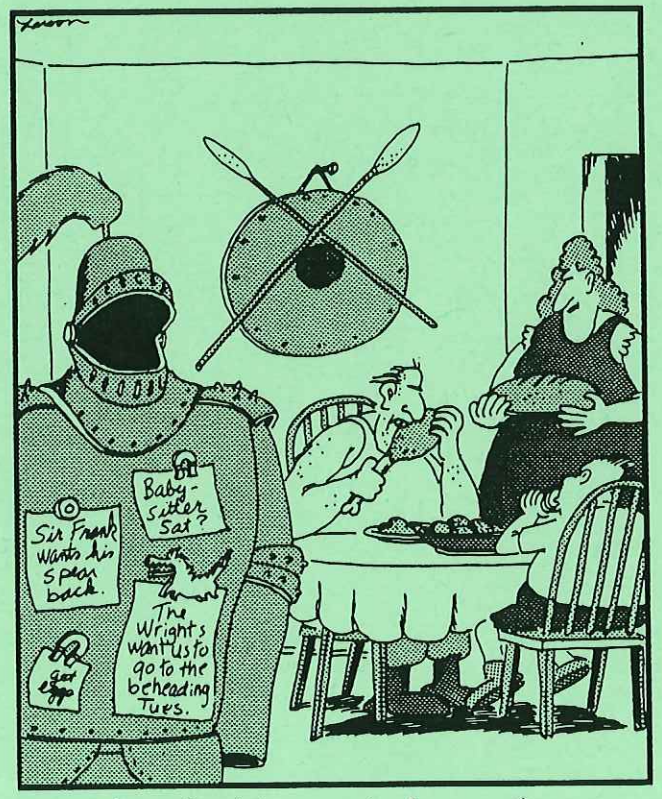
foci $(-2, -6 \pm 3\sqrt{3})$

$a = \sqrt{36} = 6$ maj. axis $2a = 12$
$b = \sqrt{9} = 3$ min. axis $2b = 6$

⑩ $x^2 + 2x + 4y^2 - 8y = 59$

$(x^2 + 2x + \square) + 4(y^2 - 2y + \triangle) = 59 + \square + 4\triangle$
$(x^2 + 2x + 1) + 4(y^2 - 2y + 1) = 59 + 1 + 4$

continued



In medieval times, a suit of armor often served as a family's message center.

Conics

ANSWER KEY: UNIT 22 REVIEW & PRACTICE

$(x+1)^2 + 4(y-1)^2 = 64$

$\frac{(x+1)^2}{64} + \frac{(y-1)^2}{16} = 1$

center $(h,k) = (-1,1)$

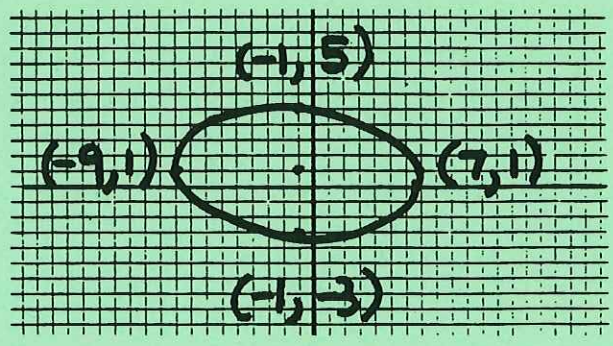
$c^2 = a^2 - b^2$

$c^2 = 64 - 16 = 48 \quad c = 4\sqrt{3}$

foci $(-1 \pm 4\sqrt{3}, 1)$

$a = \sqrt{64} = 8$ maj. axis $2a = 16$

$b = \sqrt{16} = 4$ min. axis $2b = 8$



(11) $\frac{(x-6)^2}{16} - \frac{(y+4)^2}{49} = 1$

transverse axis is horizontal

center $(h,k) = (6,-4)$

$a = 4 \quad b = 7$

$c^2 = a^2 + b^2$

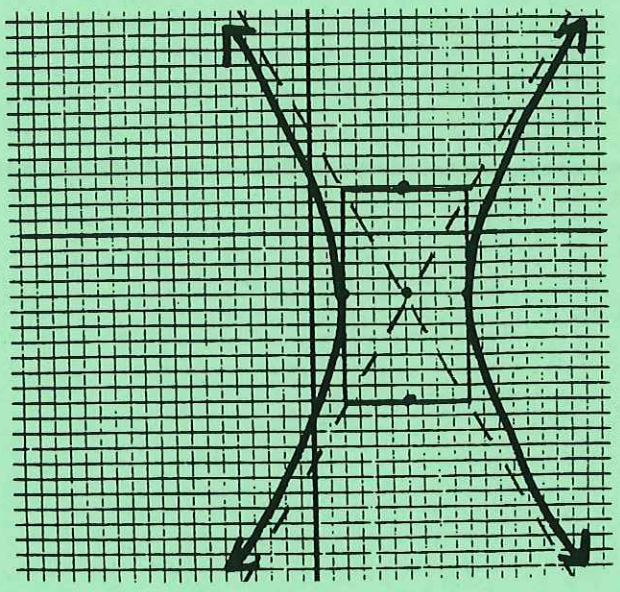
$c^2 = 16 + 49 = 65 \quad c = \sqrt{65}$

foci $(6 \pm \sqrt{65}, -4)$

vertices $(6 \pm 4, -4)$

$(10,-4) \; (2,-4)$

asymptote slope $\pm 7/4$



(12) $9x^2 - 36x - 4y^2 - 32y = 172$

$9(x^2 - 4x + \square) - 4(y^2 + 8y + \triangle) = 172 + 9\square - 4\triangle$

$9(x^2 - 4x + 4) - 4(y^2 + 8y + 16) = 172 + 36 - 64$

$9(x-2)^2 - 4(y+4)^2 = 144$

$\frac{(x-2)^2}{16} - \frac{(y+4)^2}{36} = 1$ transverse axis is horizontal

center $(h,k) = (2,-4)$

$a = 4 \quad b = 6$

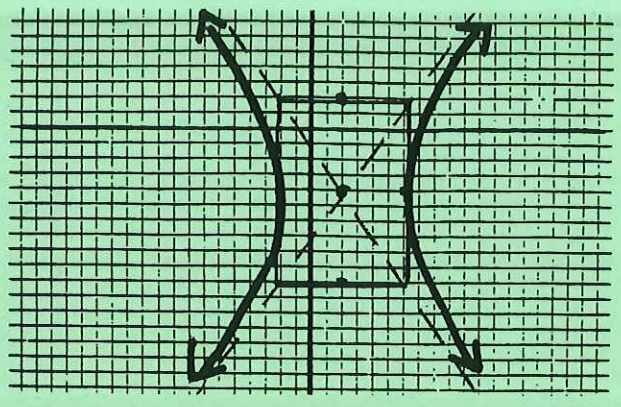
$c^2 = 16 + 36 = 52 \quad c = 2\sqrt{13}$

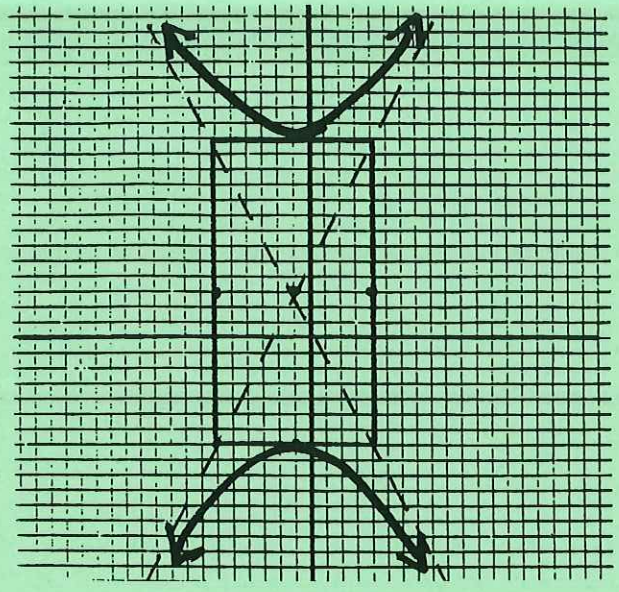
foci $(2 \pm 2\sqrt{13}, -4)$

vertices $(2 \pm 4, -4) \rightarrow (-2,-4) \; (6,-4)$

asymptote slope $\pm 6/4 = \pm 3/2$

ANSWER KEY: UNIT 22 REVIEW & PRACTICE





(13) $y^2-6y-4x^2-8x=95$

$(y^2-6y+\square)-4(x^2+2x+\triangle)=95+\square-4\triangle$

$(y^2-6y+9)-4(x^2+2x+1)=95+9-4$

$(y-3)^2-4(x+1)^2=100$

$\frac{(y-3)^2}{100}-\frac{(x+1)^2}{25}=1$

transverse axis is vertical

center $(h,k)=(-1,3)$

$a=10 \quad b=5$

$c^2=a^2+b^2$

$c^2=100+25=125 \quad c=5\sqrt{5}$

foci $(-1, 3\pm5\sqrt{5})$

vertices $(-1, 3\pm10)$

$(-1,-7)\ (-1,13)$

asymptote slope $\pm\frac{10}{5}=\pm2$