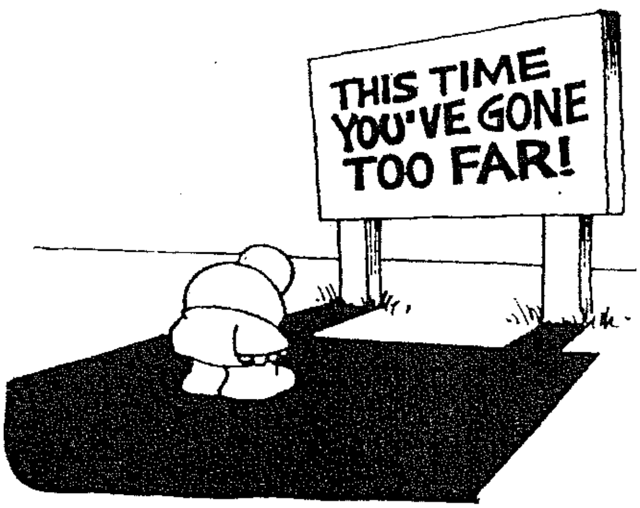
Friendship Junior High School
Seventh Grade Advanced Math Packet



Exploring Topics In Algebra

Algebra Units 14-18

Working With Polynomials
Linear Equations
Linear Systems
Factoring
Quadratic Equations

UNIT 14

Working With Polynomials

1. MULTIPLYING BINOMIALS

An expression that can be written as the sum of monomials is called a polynomial.

A polynomial with two terms is called a <u>binomial</u>.

To multiply two binomials together, you use a process called: FOIL.

F First
O Outer
I Inner
L Last

FOIL indicates an order for multiplying the terms of a binomial.

Demonstration

Use FOIL to multiply:

A) $(a+b)(a+b)$

First $a \cdot a = a^2$
Outer $a \cdot b = ab$
Inner $b \cdot a = ab$
Last $b \cdot b = b^2$

$a^2+2ab+b^2$

B) $(x+3)(x-5)$

$x^2-5x+3x-15 = x^2-2x-15$

C) $(2n-1)(3n+4)$

$6n^2+8n-3n-4 = 6n^2+5n-4$

D) $(3a+2b)(a-5b)$

$3a^2-15ab+2ab-10b^2$
$3a^2-13ab-10b^2$

E) $(4x-y)(x+2y)$

$4x^2+8xy-xy-2y^2$
$4x^2+7xy-2y^2$

UNIT 14

Working With Polynomials

Problem Set 14.1

Multiplying Binomials

Use FOIL to multiply:

1. $(x+1)(x+3)$
2. $(n-5)(n-2)$
3. $(3x+2)(x-3)$
4. $(a-4)(3a-1)$
5. $(a+b)(2a+b)$
6. $(x-y)(x-2y)$
7. $(3a+2b)(a+b)$
8. $(x+3y)(2x-y)$
9. $(2n-m)(n-2m)$
10. $(4x+y)(2x+y)$
11. $(2n+5)(3n-2)$
12. $(4x-3)(3x+5)$
13. $(x-y)(2x-3y)$
14. $(2a+b)(3a-b)$
15. $(2n-m)(3n-4m)$
16. $(2a+3b)(a-b)$
17. $(x+2y)(3x+4y)$
18. $(2n+7)(5n-3)$
19. $(4x-1)(3x-8)$
20. $(5a-3)(4a+5)$

2. SPECIAL PRODUCTS

The patterns used to form two special binomial products are worth remembering.

Binomial Squared

$$(a+b)^2 = a^2+2ab+b^2$$

The product is formed as follows:

$$a^2 \quad + \quad 2ab \quad + \quad b^2$$

first term squared	double the product of the two terms	second term squared

The result is called a perfect square trinomial.

Product of Sum & Differen

$$(a+b)(a-b) = a^2-b^2$$

When multiplying the sum and difference of two identical binomial terms, the product is formed as follows:

$$a^2 \quad - \quad b^2$$

First term squared	minus	second term squared

The two binomials $(a+b)$ and $(a-b)$ are called conjugates. Their product has no middle term.

UNIT 14

Working With Polynomials

Demonstration

Use the patterns for special products to multiply:

A) $(x-3)^2$

$(x-3)^2 = x^2-6x+9$

B) $(2a+b)^2$

$(2a+b)^2 = 4a^2+4ab+b^2$

C) $(3x-5y)^2$

$(3x-5y)^2 = 9x^2-30xy+25y^2$

D) $(x+5)(x-5)$

$(x+5)(x-5) = x^2-25$

E) $(a+2b)(a-2b)$

$(a+2b)(a-2b) = a^2-4b^2$

F) $(3n+4m)(3n-4m)$

$(3n+4m)(3n-4m) = 9n^2-16m^2$

Problem Set 14.2
Special Products

Use the patterns to determine the product:

1. $(n+2)^2$
2. $(x-5)^2$
3. $(3x+y)^2$
4. $(n-4m)^2$
5. $(2a-3b)^2$
6. $(4x+5y)^2$
7. $(a-7b)^2$
8. $(4x+3)^2$
9. $(n+1)(n-1)$
10. $(x+3)(x-3)$
11. $(x+y)(x-y)$
12. $(2a+b)(2a-b)$
13. $(3n+4)(3n-4)$
14. $(a+5b)(a-5b)$
15. $(2x+7y)(2x-7y)$
16. $(n+3m)(n-3m)$

UNIT 14

Working With Polynomials

Multiply using FOIL:

⑰ $(3x+2y)(x-6y)$

⑱ $(2n-m)(5n+4m)$

⑲ $(8a+2b)(3a+4b)$

⑳ $(x-5y)(3x-7y)$

3. DIVIDING POLYNOMIALS

Using polynomial division you can determine how many times a binomial divides into a polynomial.

The process is very similar to long division.

You can check your quotient using FOIL.

Demonstration

A) $(15x^2+26x+8) \div (5x+2)$

$$
\begin{array}{r|l}
 & \;\;3x\;+4 \\ \hline
5x+2 & 15x^2+26x+8 \\
 & \underline{15x^2+\;6x} \\
 & \qquad\;\; 20x+8 \\
 & \qquad\;\; \underline{20x+8}
\end{array}
$$

- Divide $15x^2$ by $5x$. It goes in $3x$ times.
- Multiply $(3x)\cdot(5x+2)$.
- Subtract (change signs and add). The difference is $20x$.
- Bring down the next term (8).
- Divide $20x$ by $5x$. It goes in 4 times.
- Multiply $(4)\cdot(5x+2)$.
- Subtract. There is no remainder.

B) $(6y^2+7y-20) \div (2y+5)$

$$
\begin{array}{r|l}
 & \;\;3y\;-4 \\ \hline
2y+5 & 6y^2+7y-20 \\
 & \underline{6y^2+15y} \\
 & \qquad -8y-20 \\
 & \qquad \underline{-8y-20}
\end{array}
$$

UNIT 14

Working With Polynomials

c) $(2x^2 - 11x - 20) \div (2x+3)$

$$\begin{array}{r|l} & x - 7 + \frac{1}{2x+3} \\ 2x+3 & 2x^2 - 11x - 20 \\ & \underline{2x^2 + 3x} \\ & \quad -14x - 20 \\ & \quad \underline{-14x - 21} \\ & \qquad\qquad 1 \end{array}$$

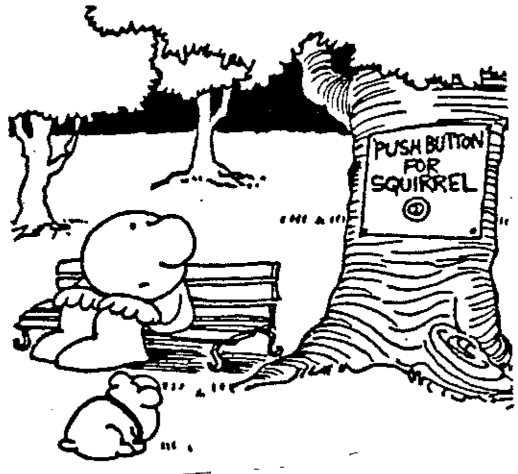
Note: Put the remainder (1) over the divisor (2x+3) and add it to the quotient.

d) $(4x^3 - 4x^2 + 7x - 4) \div (2x+1)$

$$\begin{array}{r|l} & 2x^2 - 3x + 5 - \frac{9}{2x+1} \\ 2x+1 & 4x^3 - 4x^2 + 7x - 4 \\ & \underline{4x^3 + 2x^2} \\ & \quad -6x^2 + 7x \\ & \quad \underline{-6x^2 - 3x} \\ & \qquad\quad 10x - 4 \\ & \qquad\quad \underline{10x + 5} \\ & \qquad\qquad\quad -9 \end{array}$$

When subtracting, change the signs of the bottom number and add:

$$\left.\begin{array}{r} -6x^2 + 7x \\ -6x^2 - 3x \end{array}\right\} \quad \begin{array}{r} -6x^2 + 7x \\ \underline{+6x^2 + 3x} \\ 10x \end{array}$$



Problem Set 14.3
Dividing Polynomials

1. $(x^2 + 12x + 36) \div (x+6)$
2. $(x^2 + 7x + 12) \div (x+3)$
3. $(a^2 - 2a - 35) \div (a-7)$
4. $(2x^2 - 3x - 35) \div (2x+7)$
5. $(3n^2 - 14n - 24) \div (3n+4)$
6. $(10x^2 + x - 21) \div (5x-7)$
7. $(30x^2 - 32x - 14) \div (6x+2)$
8. $(2a^2 + 5ab - 12b^2) \div (2a-3b)$
9. $(4n^3 - 11n^2 - 31n - 7) \div (4n+1)$
10. $(6x^3 - 11x^2 - x + 6) \div (2x-3)$

UNIT 14

Working With Polynomials

Challenge Problems

The following division problems involve a remainder:

11. $(10x^2-3x-15) \div (2x-3)$
12. $(14x^2+65x+8) \div (7x+1)$
13. $(3n^3+8n^2+n-7) \div (n+2)$
14. $(a^3+a^2+a+2) \div (a+1)$

REVIEW & PRACTICE

use FOIL to multiply:

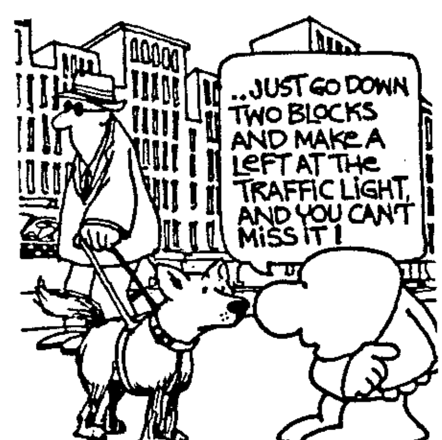
1. $(n+7)(n-5)$
2. $(x-y)(x-2y)$
3. $(3a+4)(2a+6)$
4. $(4a+3b)(3a-2b)$
5. $(8x+3)(3x-5)$
6. $(2a+b)(3a+b)$

Use special product patterns to multiply:

7. $(n-5)^2$
8. $(3x+1)^2$
9. $(2x-y)^2$
10. $(x-4y)^2$
11. $(3n+4)^2$
12. $(x+5)(x-5)$
13. $(2n+3)(2n-3)$
14. $(3a+b)(3a-b)$
15. $(4x+3y)(4x-3y)$

Use polynomial division:

16. $(4x^2-9x+5) \div (x-1)$
17. $(6n^2-n-12) \div (2n-3)$
18. $(15x^2+8x-16) \div (3x+4)$
19. $(2x^3+9x^2+11x+3) \div (2x+3)$
20. $(2x^3-15x^2+28x-15) \div (x-5)$
21. $(10x^2+19x-12) \div (2x+5)$
22. $(3n^3+11n^2+4n-4) \div (n+3)$



UNIT 14

Working With Polynomials

PRACTICE TEST #1

Use FOIL to multiply:

① $(n-4)(n-5)$

② $(3x-4)(2x+7)$

Use special product patterns to multiply:

③ $(n-3)^2$

④ $(2x+5)(2x-5)$

Divide:

⑤ $(3n^2+7n-20) \div (n+4)$

⑥ $(6x^3+17x^2+18x+11) \div (2x+3)$

PRACTICE TEST #2

Use FOIL to multiply:

① $(x+4)(x+7)$

② $(4n-3)(3n+5)$

Use special product patterns to multiply:

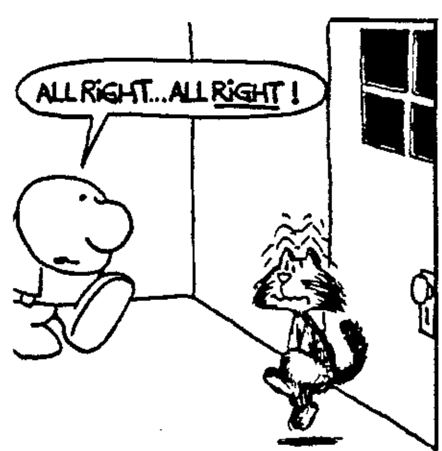
③ $(3n+7)(3n-7)$

④ $(3x+4)^2$

Divide:

⑤ $(5x^2-13x-6) \div (x-3)$

⑥ $(6n^3+n^2-17n-13) \div (3n+2)$



UNITS 9-14

Cumulative Review

REVIEW & PRACTICE

Integers, exponents, and order of operations:

① $(-8)-(-10)$

② $(-3)+(-5)-(-6)-(+7)+(+8)$

③ $(-2)^3$

④ $(-6)-(-3)(+4)+(-2)$

⑤ $-3^2-(-2)^2$

⑥ $(-1)(-2)(+3)(-1)(-2)$

Evaluating expressions:
$a=-1 \quad b=-2 \quad c=-3$

⑦ $2a-b$

⑧ $3c-2a^2$

⑨ $abc-a^2b^2$

⑩ $-2(a+b)^2$

Simplifying expressions:

⑪ $5a-2b-3a+4b$

⑫ $a^2+a^2b-2ab^2-3a^2b-4a^2$

⑬ $3(2x-y)-2(x+3y)$

⑭ $x(x-3y)-2(xy+3x^2)$

⑮ $a(2a-b)+b(3a-b)$

Simplify each radical:

⑯ $\sqrt{120}$

⑰ $3\sqrt{72}$

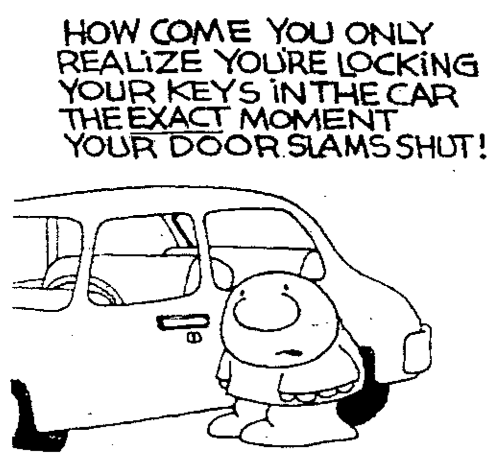
Multiply, combine, simplify:

⑱ $(\sqrt{5})^2$

⑲ $(2\sqrt{3})(5\sqrt{6})$

⑳ $(\sqrt{8})(\sqrt{12})$

㉑ $\sqrt{28}-2\sqrt{7}+3\sqrt{63}$

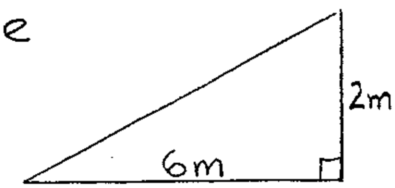


UNITS 9-14

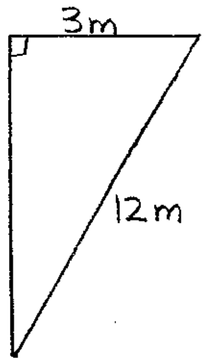
Cumulative Review

Pythagorean Theorem:

(22) Find the missing side.



(23) Find the missing side.



Solve each equation or inequality:

(24) $3(n-4) = n-6$

(25) $4(x-3) - 2(3x+5) = 2x-2$

(26) $3(a+2) \geq 24$

(27) $x-3 < 3(x-2)+5$

Solve and graph on a number line:

(28) $2n+5 > 5n-13$

(29) $2(x+3) \leq 4(x-1)$

Problem solving:

(30) Three more than five times a number decreased by two less than twice the number is three less than the number. Find the number.

(31) Four times a number decreased by six less than twice the number is one more than the number. Find the number.

Multiply, divide, simplify (monomials):

(32) $(3ab)(2a^2b^3)$

(33) $(-2xy^2)^2(-x^2y)^3$

(34) $(-2a)^3(2a)^2$

(35) $\dfrac{12x^2yz^3}{8xz}$

(36) $\dfrac{-3a^2b^{-3}c}{6a^{-4}b^2c^{-3}}$

(37) $\dfrac{-15x^{-2}y^{-3}z}{10x^{-5}yz^{-1}}$

UNITS 9-14

Cumulative Review

38. $-3xy(-xy^2)^{-2}$

39. $ab(3ab)^{-2}$

Use FOIL to multiply these binomials:

40. $(x+5)(x-4)$

41. $(a+b)(a-2b)$

42. $(3x+y)(2x-y)$

43. $(4n-3)(2n+1)$

Use special product patterns to multiply:

44. $(a+4)^2$

45. $(2n-m)^2$

46. $(x+3)(x-3)$

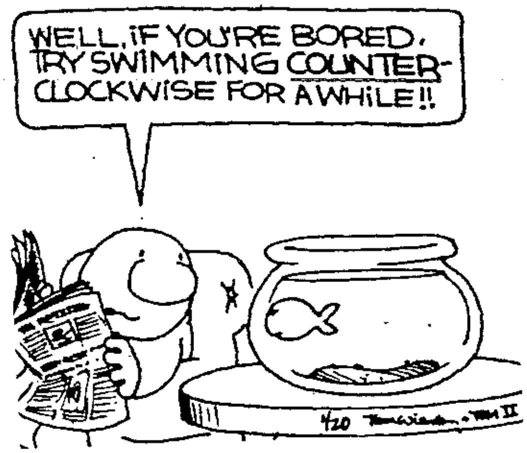
47. $(3a-2b)(3a+2b)$

Polynomial division:

48. $(6x^3+5x^2+2x+15)\div(2x+3)$

49. $(5n^3+11n^2-4n-6)\div(n+2)$

50. $(3a^3-11a^2-7a-7)\div(3a+1)$



PRACTICE TEST

Order of operations:

1. $(-1)^3-(-2)(-3)+(-5)$

Evaluating expressions:
$x=2 \quad y=-2 \quad z=-1$

2. $3x-5y$

3. $3xz^2-y^3$

Simplifying expressions:

4. $3a-4ab+5a-b^2+2ab$

5. $2(x-3y)-3(2x+3y)$

Simplify the radical:

6. $\sqrt{108}$

UNITS 9-14

Cumulative Review

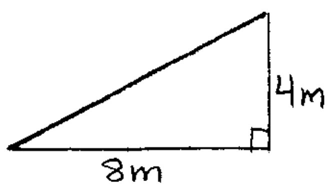
Multiply, combine, simplify:

⑦ $(3\sqrt{2})^2$

⑧ $\sqrt{12} - 4\sqrt{3} + \sqrt{27}$

Pythagorean Theorem:

⑨ Find the missing side.



Solve each equation or inequality:

⑩ $3(n-5) = n+7$

⑪ $2x-3 \leq 4(x-2)+7$

Solve and graph on a number line:

⑫ $3(n-2) > 7n+14$

Problem solving:

⑬ Four less than three times a number decreased by four more than the number is negative two. Find the number.

Simplifying monomials:

⑭ $(-2xy^2)^2(-xy)^3$

⑮ $\dfrac{-12a^{-2}bc^3}{8ab^{-3}c^2}$

Use FOIL to multiply:

⑯ $(n-3)(n+7)$

⑰ $(2a+b)(3a-2b)$

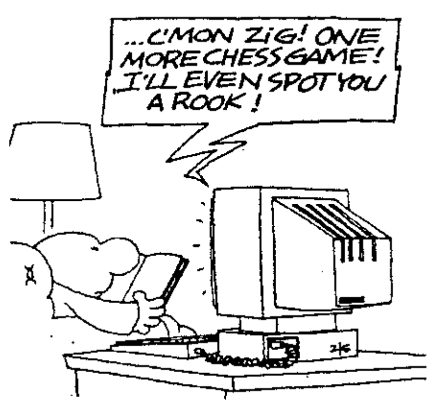
Use special product patterns to multiply:

⑱ $(x-2y)^2$

⑲ $(3a+2)(3a-2)$

Polynomial division:

⑳ $(3n^3+10n^2-3n+14) \div (n+4)$



UNIT 15

Linear Equations

1. UNDERSTANDING SLOPE

A linear equation has two variables, and all ordered pairs (x, y) that satisfy the equation can be graphed as a straight line on the coordinate axis.

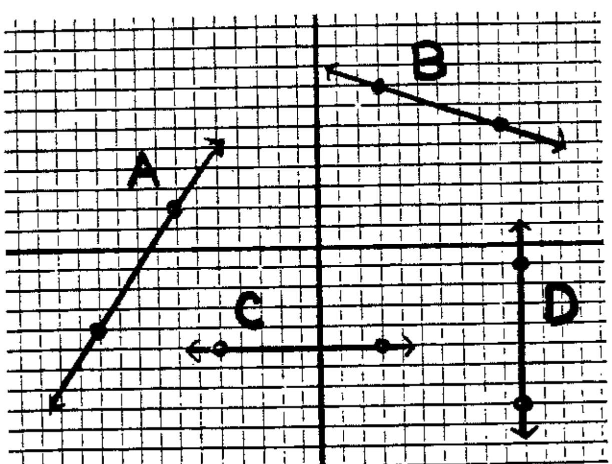
The <u>slope</u> of a linear equation is a numerical value that indicates the direction of the line as it moves through the coordinate plane.

The slope of a line is often referred to as "rise over run."

$\frac{\text{rise}}{\text{run}}$ $\frac{\text{change in } y \text{ (vertical)}}{\text{change in } x \text{ (horizontal)}}$

Demonstration

A) Identify the slope of each line:



Line A
slope = $3/2$
$\frac{\text{rise}}{\text{run}}$ $\frac{+6}{+4}$

Line B
slope = $-1/3$
$\frac{\text{rise}}{\text{run}}$ $\frac{-2}{+6}$

Line C
slope = 0
$\frac{\text{rise}}{\text{run}}$ $\frac{0}{8}$
all horizontal lines have a slope of 0

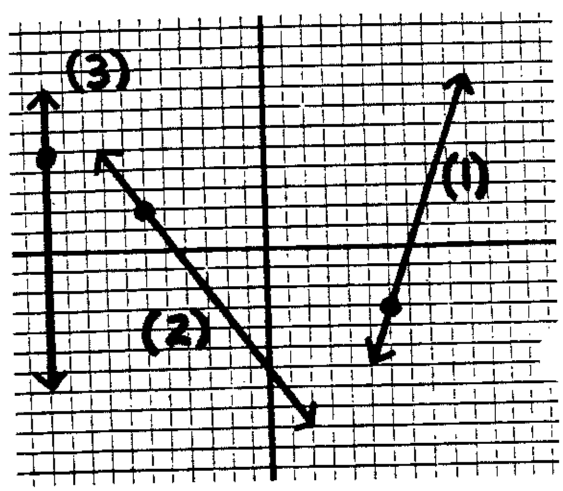
Line D
slope = und.
$\frac{\text{rise}}{\text{run}}$ $\frac{7}{0}$
all vertical lines have an undefined slope

B) Draw a line through a given point with a given slope:

(1) Through (6, −3) slope = 3

(2) Through (−6, 2) slope = $-4/3$

(3) Through (−11, 5) slope = und.

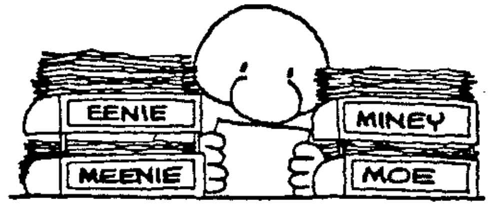


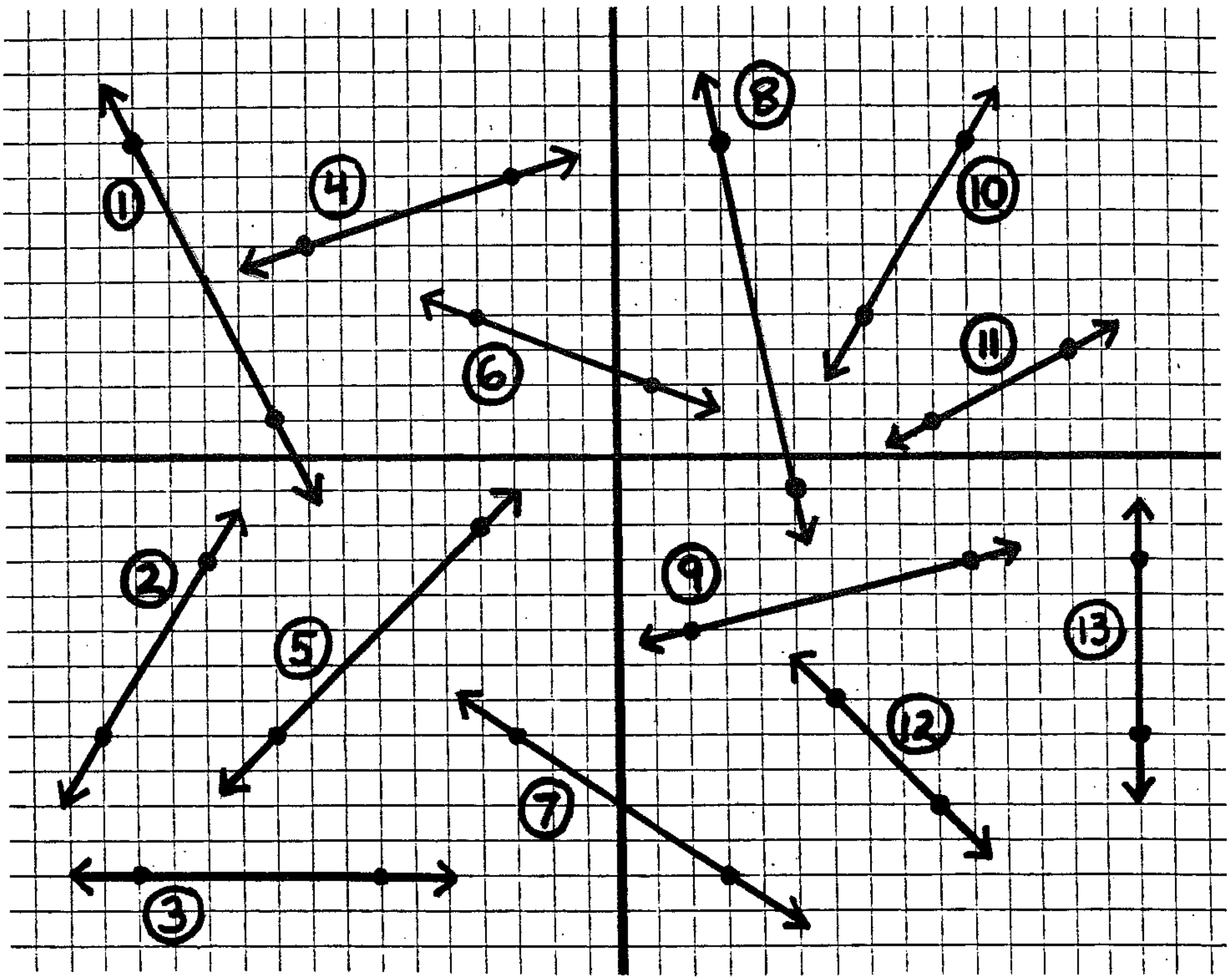
UNIT 15

Linear Equations

Problem Set 15.1
Understanding Slope

Identify the slope of each line shown below:





On a few sheets of graph paper, draw lines as indicated:

14 Through (4, 8) slope = 2/3

15 Through (−6, 0) slope = 4

16 Through (−2, −6) slope = −2/5

17 Through (0, 4) slope = 0

UNIT 15

Linear Equations

(18) Through (0,0) slope = -5

(19) Through $(-3,-3)$ slope = $\frac{1}{2}$

(20) Through (2,0) slope = $\frac{7}{4}$

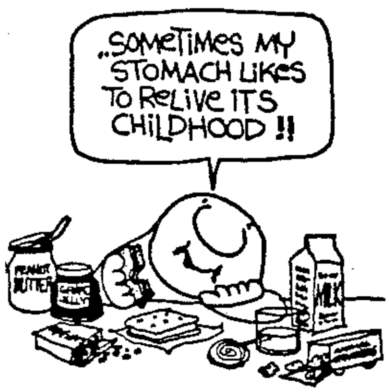
(21) Through (-5,11) slope = undefined

(22) Through $(6,-3)$ slope = -3

(23) Through $(-6,-8)$ slope = $-\frac{3}{4}$

(24) Through (0,10) slope = 4

(25) Through (8,2) slope = $\frac{1}{4}$



2. SLOPE-INTERCEPT FORM

A linear equation can be put into a special form that provides important information. It also makes it easier to graph the equation when it is in this form.

Slope - Intercept Form

$$y = mx + b$$

- slope = m
- y-intercept = b
- x-intercept = $-b/m$

An intercept is the point where a line intersects with the x or y axis.

Once you have identified the two intercepts, you can immediately graph the line by connecting them.

Demonstration

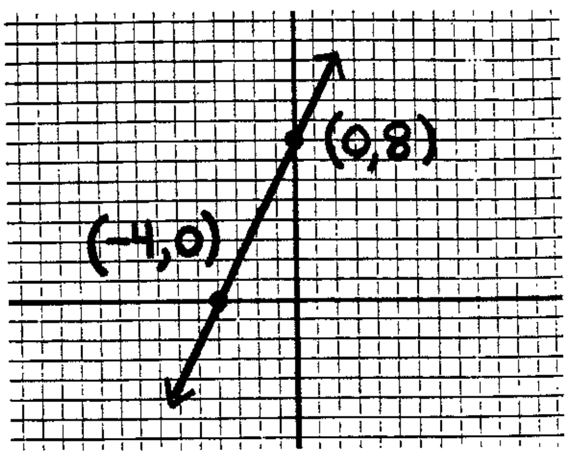
From an equation in slope-intercept form, identify the slope, both intercepts, and draw the graph:

A) $y = 2x + 8$

$m = 2$	slope (m)	=	2
$b = 8$	y-int (b)	=	8
	x-int $(-b/m)$	=	-4

To graph the line, identify and connect the intercepts.

Linear Equations



Note: In this problem, the equation was already in slope-intercept form. In the next problem, you have to change the form of the equation using these steps:

- Isolate the y-term
- multiply both sides of the equation by the reciprocal of the co-efficient of y

Demonstration

Change the form of the equation to slope-intercept form. Then identify slope and both intercepts. Draw the graph.

B) $x - 2y = 12$

$$x^{-x} - 2y = 12^{-x}$$

$$-2y = -x + 12$$

$$\left(-\tfrac{1}{2}\right)(-2y) = \left(-\tfrac{1}{2}\right)(-x + 12)$$

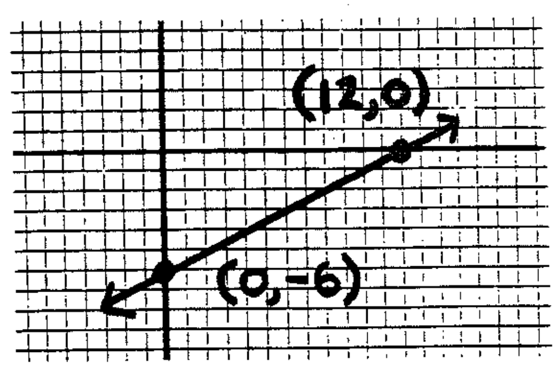
$$y = \tfrac{1}{2}x - 6$$

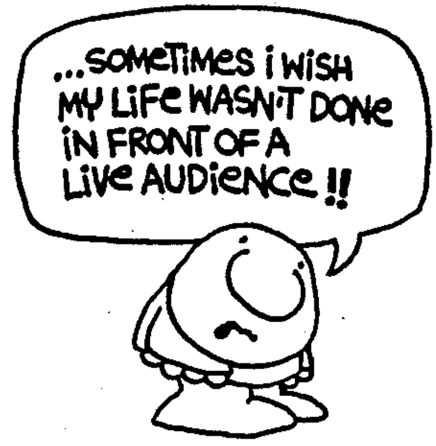
Slope - Intercept Form

$y = \frac{1}{2}x - 6$

$m = \frac{1}{2}$	slope (m)	=	$\frac{1}{2}$
$b = -6$	y-int (b)	=	-6
	x-int ($-b/m$)	=	12

$$\frac{-b}{m} = \frac{-(-6)}{(\frac{1}{2})} = \frac{6}{\frac{1}{2}} = 6 \times \frac{2}{1} = 12$$





UNIT 15

Linear Equations

c) $x + 4y = 16$

$$\overset{-x}{x} + 4y = \overset{-x}{16}$$

$$4y = -x + 16$$

$$\left(\tfrac{1}{4}\right)(4y) = \left(\tfrac{1}{4}\right)(-x + 16)$$

$$y = \tfrac{-1}{4}x + 4$$

Slope-Intercept Form

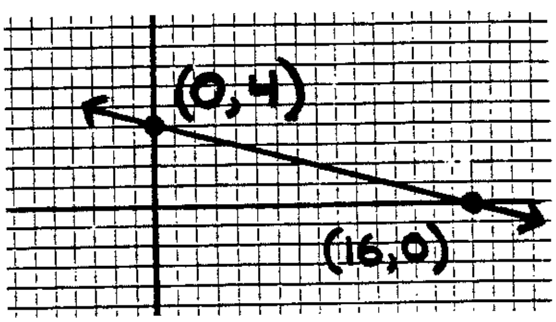
$$y = \tfrac{-1}{4}x + 4$$

$m = \frac{-1}{4}$ slope (m) $= {}^{-}1/4$

$b = 4$ y-int (b) $= 4$

x-int $\left(\frac{-b}{m}\right) = 16$

$$\frac{-b}{m} = \frac{-(4)}{(-1/4)} = \frac{4}{\frac{1}{4}} = 4 \times \frac{4}{1} = 16$$



Note: When determining the value of the x-intercept, you may have to simplify a complex fraction. Also, be careful about (+) and (-) signs.

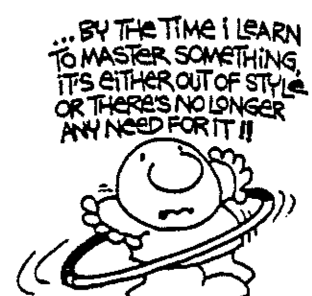
Problem Set 15.2

Slope-Intercept Form

PART I

For each equation in slope-intercept form, identify the slope, both intercepts, and graph the equation.

1. $y = 3x + 12$
2. $y = x + 5$
3. $y = -2x + 10$
4. $y = -5x - 10$
5. $y = -x - 7$
6. $y = 4x + 8$
7. $y = -3x + 15$
8. $y = 2x + 14$
9. $y = x - 9$
10. $y = -4x - 16$



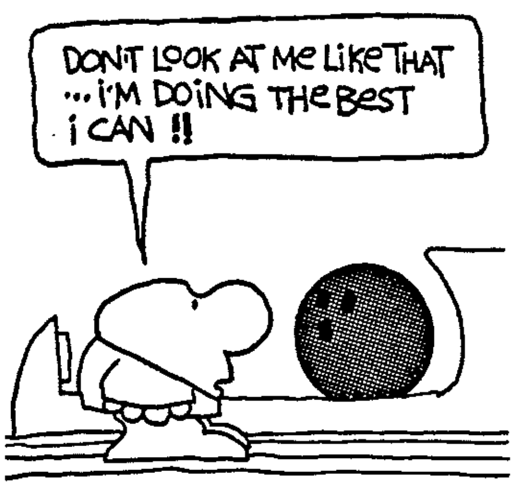
UNIT 15

Linear Equations

PART II

Change each equation to slope-intercept form. Then identify the slope and both intercepts. Graph each equation using the intercepts.

(11) $2x + y = 4$	(16) $x + 3y = 6$
(12) $x + y = -5$	(17) $2x + 3y = 12$
(13) $x - 3y = -12$	(18) $3x - 4y = 12$
(14) $x - 2y = 8$	(19) $2x + 5y = -10$
(15) $x + 4y = 8$	(20) $3x - 2y = 6$



3. STANDARD FORM

In addition to slope-intercept form, a linear equation can be put into another form that makes it easy to determine the slope and both intercepts.

Standard Form

$$Ax + By = C$$

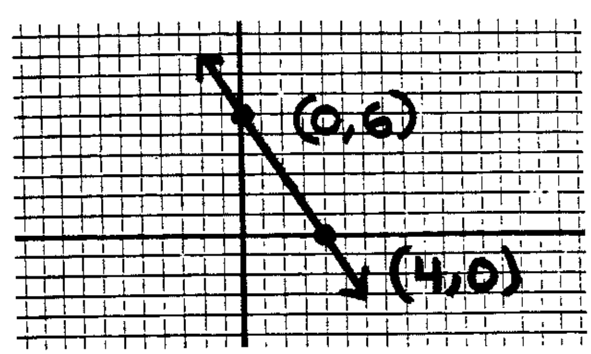
- slope $= -A/B$
- y-intercept $= C/B$
- x-intercept $= C/A$

Demonstration

From an equation in standard form, identify the slope, both intercepts, and draw the graph:

A) $3x + 2y = 12$

$A = 3$	slope $(-A/B) = -3/2$
$B = 2$	y-int $(C/B) = 6$
$C = 12$	x-int $(C/A) = 4$

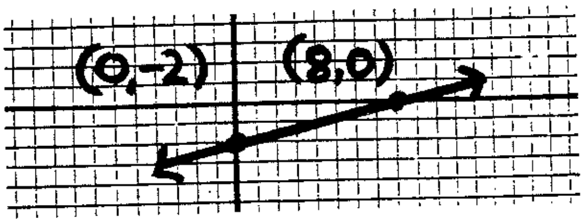


UNIT 15

Linear Equations

B) $x - 4y = 8$

$A = 1$ slope $(-A/B) = 1/4$
$B = -4$ y-int $(C/B) = -2$
$C = 8$ x-int $(C/A) = 8$



Demonstration

In the following problems, you first have to put the equation in standard form. Then find the slope, both intercepts, and draw the graph.

Be aware of the following conditions that apply to all equations in standard form:

- "A" (coefficient of x) must be positive
- No fractions
- Divide by common factor (if there is one)

See if you can tell which condition applies to each example below:

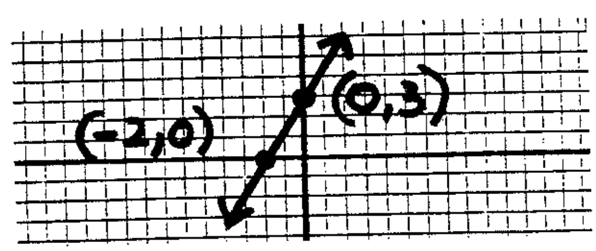
Incorrect: $-2x + 3y = 8$
Change to: $2x - 3y = -8$

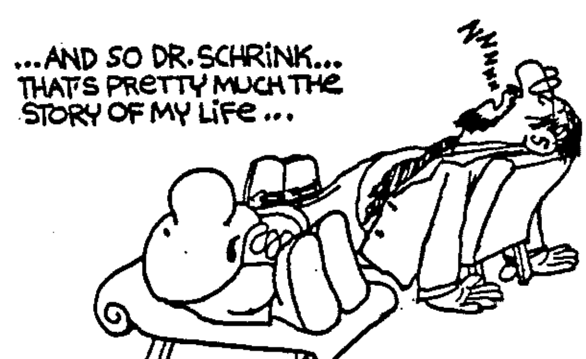
Incorrect: $x + y = 2/3$
change to: $3x + 3y = 2$

Incorrect: $4x - 6y = 10$
change to: $2x - 3y = 5$

c) $\overset{-3x}{2y} = \overset{-3x}{3x} + 6$
$-3x + 2y = 6$ $\rightarrow$ "A" must be
$3x - 2y = -6$

$A = 3$ Slope $(-A/B) = 3/2$
$B = -2$ y-int $(C/B) = 3$
$C = -6$ x-int $(C/A) = -2$





UNIT 15

Linear Equations

D) $\overset{+\frac{2}{3}x}{2y} = \overset{+\frac{2}{3}x}{-\frac{2}{3}x} + 4$

$\frac{2}{3}x + 2y = 4$ → no fractions

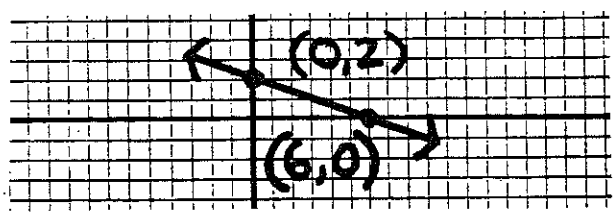
$3\left(\frac{2}{3}x + 2y = 4\right)$

$2x + 6y = 12$ → divide by 2 (common factor)

$\frac{1}{2}(2x + 6y = 12)$

$x + 3y = 6$

$A = 1$	Slope $(-A/B) = -1/3$
$B = 3$	y-int $(C/B) = 2$
$C = 6$	x-int $(C/A) = 6$



Problem Set 15.3

Standard Form

PART I

From an equation in standard form, identify the slope, both intercepts, and draw the graph:

① $3x + y = 6$

② $2x - y = 4$

③ $x + y = 5$

④ $x - y = 3$

⑤ $2x - 3y = -6$

⑥ $3x + 4y = -12$

⑦ $x - 2y = 6$

⑧ $5x + 4y = 20$

⑨ $3x - y = 6$

⑩ $x + 4y = 8$

PART II

Change the equation to standard form. (Be aware of all conditions). Identify slope, both intercepts, and draw the graph:

⑪ $2y = -x + 4$

⑫ $5y = 2x - 10$

⑬ $8y = -4x + 16$

⑭ $y = -\frac{1}{4}x - 2$

⑮ $3y = 3x + 9$

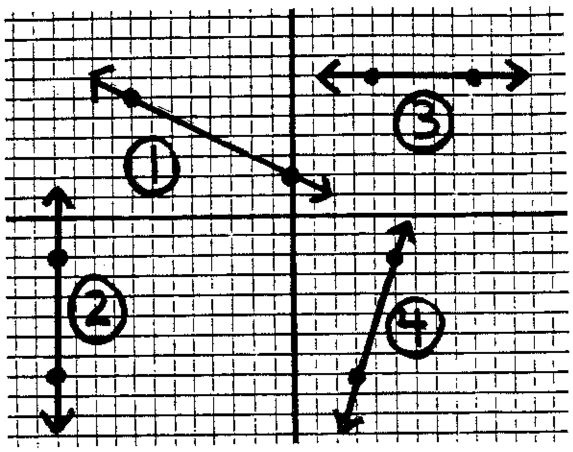
⑯ $y = -\frac{2}{3}x + 4$

UNIT 15

Linear Equations

REVIEW & PRACTICE

Identify the slope of each line indicated below:



On graph paper, graph each line with the indicated slope through the given point:

5. Through $(-4, 2)$ slope -3
6. Through $(5, -3)$ slope $2/5$
7. Through $(4, 6)$ slope 0
8. Through $(-3, -5)$ slope $-3/5$

For each equation in slope-intercept form, determine the slope, identify the intercepts, and draw the graph:

9. $y = 4x + 8$
10. $y = -2x - 6$
11. $y = -2/5\, x + 4$
12. $y = \frac{1}{2} x - 3$

Change each equation to slope-intercept form. Then determine slope, identify intercepts, and draw the graph:

13. $3x - y = -6$
14. $x + 2y = 8$
15. $2x + 3y = 18$
16. $3x - 4y = 12$

continued

UNIT 15

Linear Equations

For each equation in standard form, determine the slope, identify the intercepts, and graph the line:

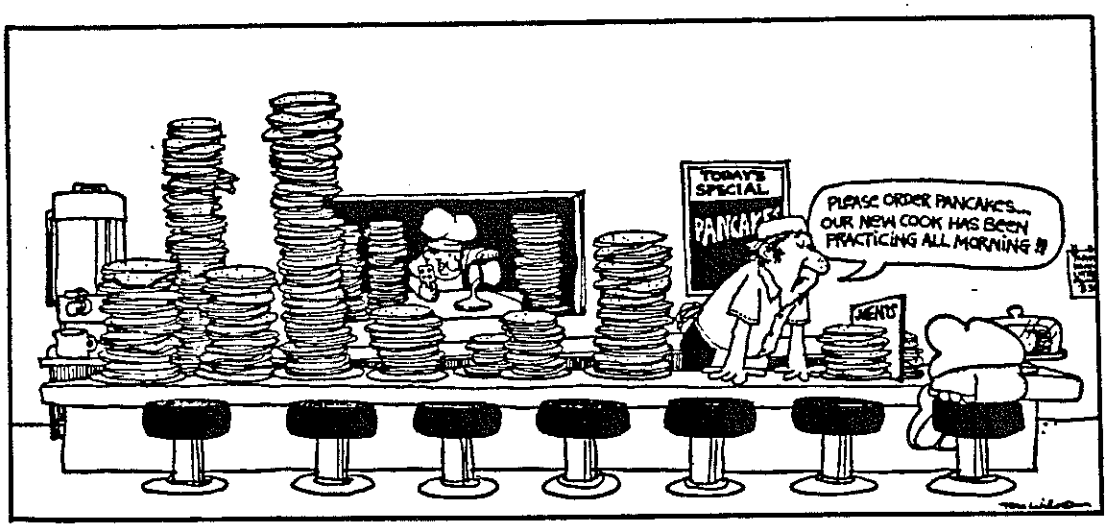
(17) $2x + y = 6$ (19) $x - y = -5$

(18) $3x + 2y = -6$ (20) $x - 2y = 8$

Change to standard form. Then determine the slope, identify the intercepts, and draw the graph. (Be aware of conditions)

(21) $y = 2x - 4$ (23) $2x - \frac{2}{3}y = -4$

(22) $y = \frac{-2}{3}x + 3$ (24) $2y = \frac{1}{2}x + 4$



UNIT 15

Linear Equations

PRACTICE TEST #1

1. Graph a line through the point $(-4, 2)$ with a slope of $-3/5$

2. Determine the slope and both intercepts for this equation in slope-intercept form:

$$y = 3x - 9$$

3. Graph this equation using the intercepts:

$$y = -2x + 10$$

4. Determine the slope and both intercepts for this equation in standard form:

$$3x + 2y = -18$$

5. Graph this equation using the intercepts:

$$x - 4y = -8$$

6. Change this equation to slope-intercept form:

$$2x - 3y = -15$$

PRACTICE TEST #2

1. Graph a line through the point $(6, -3)$ with an undefined slope

2. Determine the slope and both intercepts for this equation in slope-intercept form:

$$y = -2x - 14$$

3. Graph this equation using the intercepts:

$$y = 5x - 10$$

4. Determine the slope and both intercepts for this equation in standard form:

$$4x - 5y = 20$$

5. Graph this equation using the intercepts:

$$2x + y = -8$$

6. Change this equation to slope-intercept form:

$$x - 2y = -7$$

UNIT 16

Linear Systems

1. GRAPHING SYSTEMS

A system consists of two equations, each with two variables.

The solution to a system consists of an ordered pair (x, y) that satisfies both equations.

System:	Solution:
$y = x - 5$ $y = -2x + 4$	$(3, -2)$

Notice how the values $x = 3$ and $y = -2$ make both equations true.

There are several ways to solve a system. In this unit, we will learn three methods:

1. Graphing
2. Substitution
3. Elimination

This first lesson explores graphing and requires you to graph each equation in the system as a linear equation. Use intercepts to graph.

Demonstration

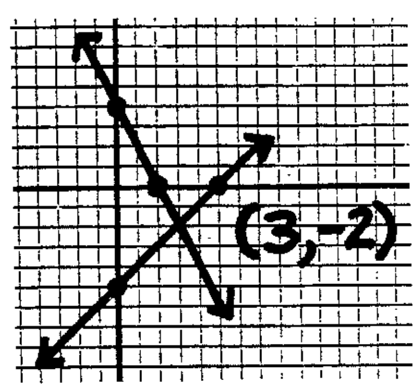
The point of intersection between the two lines is the solution to the system.

A) Slope-Intercept Form

$y = x - 5$
$y = -2x + 4$

Both equations are in slope-intercept form. Determine the intercepts, graph the lines, and identify the solution. Check the solution to see if it satisfies both equations.

$y = x - 5$	$y = -2x + 4$
y-int $(0, -5)$	y-int $(0, 4)$
x-int $(5, 0)$	x-int $(2, 0)$



Be very careful to be accurate when graphing systems

$y = x - 5$	$y = -2x + 4$
$(-2) = (3) - 5$	$(-2) = -2(3) + 4$

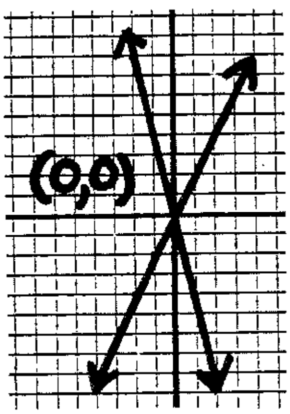
UNIT 16

Linear Systems

B) Using Slope To Graph

$y = 2x$
$y = -4x$

Because there are no constants in the equations, you must use the origin and slope.



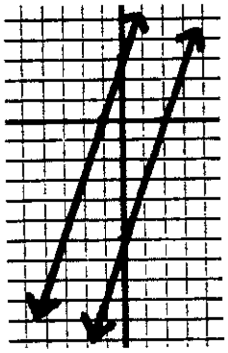
$y = 2x$
slope = 2
rise (2), run (1)

$y = -4x$
slope = −4
rise (−4), run (1)

C) Parallel Lines

$y = 3x + 3$
$y = 3x - 6$

Because both lines have the same slope, they are parallel.



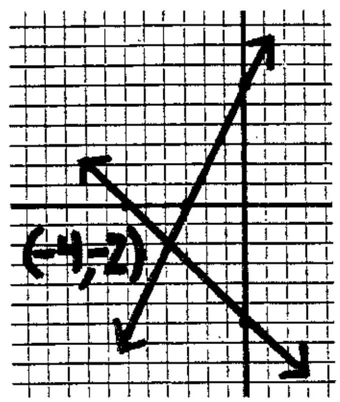
No Solution

There are no points that satisfy both equations

D) Using Standard Form

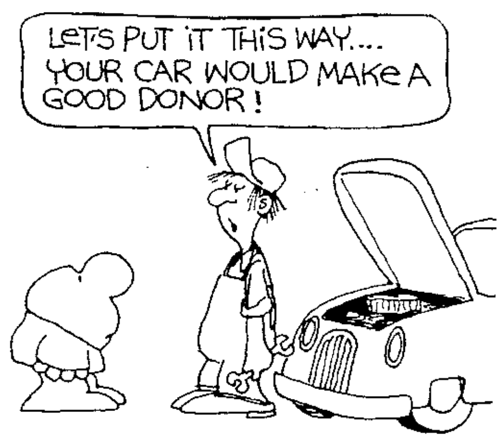
$2x - y = -6$
$x + y = -6$

Use the standard form to determine intercepts when graphing this system.



$2x - y = -6$
$2(-4) - (-2) = -6$
$-6 = -6$

$x + y = -6$
$(-4) + (-2) = -6$
$-6 = -6$



Review:

$y = mx + b$	$Ax + By = C$
y-int = $(0, b)$	y-int = (0, [illegible]
x-int = $(\frac{-b}{m}, 0)$	x-int = $(C/A, 0)$

UNIT 16

Linear Systems

Problem Set 16.1

Graphing Systems

Graph each system to determine the solution. Check the solution in both equations.

① $y=2x-2$
$y=-x+10$

② $y=3x+9$
$y=-x+1$

③ $y=2x-4$
$y=-4x-4$

④ $y=3x+6$
$y=-x-6$

⑤ $y=3x$
$y=-2x$

⑥ $y=2x+6$
$y=2x-4$

⑦ $y=2x+6$
$y=-4x$

⑧ $3x-2y=6$
$x+2y=10$

⑨ $2x-y=-6$
$x+3y=-3$

⑩ $x+3y=6$
$-x-3y=3$

⑪ $x-3y=3$
$2x+3y=-12$

⑫ $x+y=7$
$5x-y=5$

⑬ $x+4y=-4$
$x-2y=8$

⑭ $2x+y=-6$
$-3x+6y=-6$

Be sure to check your solutions.

2. SUBSTITUTION

An algebraic method called "Substitution" can be used to solve a system without graphing.

It involves substituting the value for a variable from one equation into the other.

Demonstration

Solve each system by substitution.

A) $y=5$
$3x+2y=1$

Since the first equation has only one variable, you can substitute $y=5$ into the second equation to solve for x.

$3x+2y=1$
$3x+2(5)=1$
$3x+10^{-10}=1^{-10}$
$3x=-9$
$(\frac{1}{3})(3x)=(\frac{1}{3})(-9)$
$x=-3$

Always check your solution in the system

$(-3,5)$

UNIT 16

Linear Systems

B) $3x - 2y = -6$
$4x - 3 = 5$

Because the second equation has only one variable (x), solve that equation first. Then substitute.

$4x - \overset{+3}{3} = \overset{+3}{5}$

$4x = 8$

$(\frac{1}{4})(4x) = (\frac{1}{4})(8)$

$x = 2$

Now substitute $x=2$ into the first equation to solve for y.

$3x - 2y = -6$

$3(2) - 2y = -6$

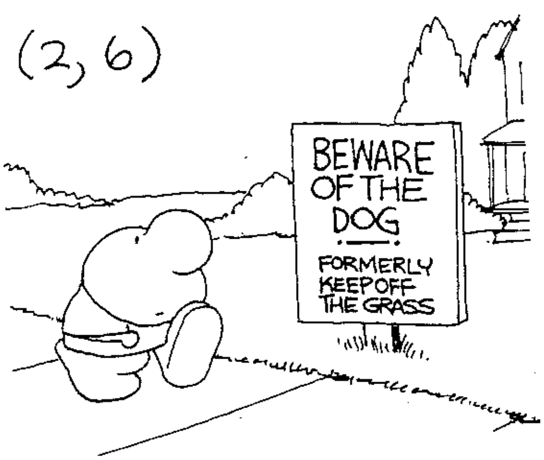
$\overset{-6}{6} - 2y = \overset{-6}{-6}$

$-2y = -12$

$(-\frac{1}{2})(-2y) = (-\frac{1}{2})(-12)$

$y = 6$

$(2, 6)$



C) $x = y + 4$
$3x - 2y = 5$

Notice that the first equation gives you a value of (x) in terms of (y). You can substitute this value into the second equation.

$3x - 2y = 5$

$3(y+4) - 2y = 5$

$3y + 12 - 2y = 5$

$y + \overset{-12}{12} = \overset{-12}{5}$

$y = -7$

Now that you have a value for (y), substitute back into the first equation to determine the value of (x).

$x = y + 4$

$x = (-7) + 4$

$x = -3$

$(-3, -7)$

This type of "double" substitution must be used when both equations have two variables.

Linear Systems

D) $5x - 4y = -2$
$4x - y = 5$

Because both equations have two variables, look for an equation with a variable having a coefficient of (1) or (-1). Solve for that variable (y) in terms of (x).

$$4x \overset{-4x}{-} y = 5^{\,-4x}$$
$$-y = 5 - 4x$$
$$y = -5 + 4x$$

Now substitute back into the first equation. Be careful to distribute the subtraction sign into the quantity.

$$5x - 4y = -2$$
$$5x - 4(-5 + 4x) = -2$$
$$5x + 20 - 16x = -2$$
$$-11x + 20^{\,-20} = -2^{\,-20}$$
$$-11x = -22$$
$$\left(\tfrac{-1}{11}\right)(-11x) = \left(\tfrac{-1}{11}\right)(-22)$$
$$x = 2$$

Now solve for (y).

$$y = -5 + 4x$$
$$y = -5 + 4(2)$$
$$y = 3 \qquad (2, 3)$$

Problem Set 16.2
Substitution

Solve each system using substitution.

① $y = 2$
$3x + 2y = 16$

② $x = -3$
$2x + y = -2$

③ $3x - 4y = -3$
$2y + 4 = -2$

④ $2x - y = -8$
$3x - 1 = -4$

⑤ $2x - 5 = 3$
$5x + 4y = 4$

⑥ $3y + 7 = 1$
$4x - 7y = 2$

⑦ $y = x + 8$
$2x + y = -1$

⑧ $x = 3y - 3$
$3x - 4y = 6$

⑨ $3x - 2y = 5$
$y = 2x - 2$

⑩ $2x - 3y = -5$
$x = 2y - 6$

UNIT 16

Linear Systems

Challenge Problems

(11) $2x + y = 8$
$3x - 5y = -1$

(12) $x - 2y = 7$
$4x - 2y = -2$

(13) $2x - 3y = 6$
$3x - y = -5$

(14) $3x - 2y = 7$
$4x - y = 1$

(15) $3x - y = 1$
$5x - 2y = 4$

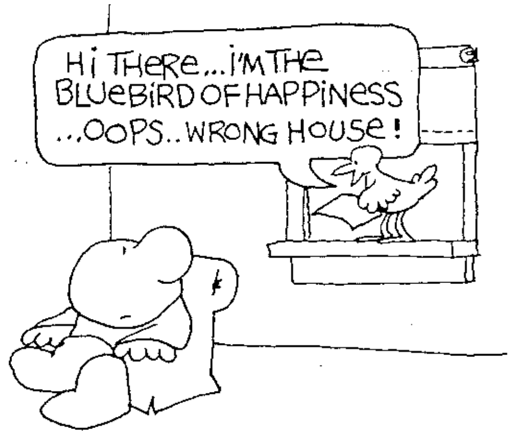
(16) $3x - y = 3$
$2x - 3y = -12$

(17) $2x - y = -10$
$3x + 5y = -2$

(18) $3x - y = 15$
$3x + 2y = 6$

(19) $4y - x = -2$
$2y - 3x = 14$

(20) $2y - x = 7$
$-3y - 4x = 6$



3. ELIMINATION

Even though substitution will work for any system, it is sometimes easier to use a different algebraic method when there are no coefficients of (1) or (-1).

This method is called elimination. It allows you to eliminate a variable by adding equations together.

Demonstration

A) $3x + 2y = -1$
$5x - 2y = -23$

Because the coefficients of (y) are opposite, adding the equations will eliminate one of the variables.

$$\begin{array}{l} 3x + 2y = -1 \\ \underline{5x - 2y = -23} \\ 8x \qquad = -24 \\ (1/8)(8x) = (1/8)(-24) \\ x = -3 \end{array}$$

Eliminate (y) and solve for (x)

Now, substitute for (x) to solve for (y)

$$\begin{array}{l} 3x + 2y = -1 \\ 3(-3) + 2y = -1 \\ -9^{+9} + 2y = -1^{+9} \\ 2y = 8 \\ (1/2)(2y) = (1/2)(8) \\ y = 4 \end{array}$$

(-3, 4)

UNIT 16

Linear Systems

B) $3x - 4y = -14$
$2x + 2y = -14$ mult. by 2

To eliminate the (y) variable, you can multiply the second equation by (2) before adding.

$$\begin{array}{l} 3x - 4y = -14 \\ \underline{4x + 4y = -28} \\ 7x \qquad = -42 \end{array}$$

$(\tfrac{1}{7})(7x) = (\tfrac{1}{7})(-42)$
$x = -6$

Solve for (x) by eliminating (y)

$3x - 4y = -14$
$3(-6) - 4y = -14$
$-18^{+18} - 4y = -14^{+18}$
$-4y = 4$
$(-\tfrac{1}{4})(-4y) = (-\tfrac{1}{4})(4)$
$y = -1$

substitute to solve for (y)

$(-6, -1)$

C) $3x + 5y = -3$ mult. by 2
$2x + 3y = -3$ mult. by -3

To eliminate the (x) variable, you must multiply both equations.

$$\begin{array}{l} 6x + 10y = -6 \\ \underline{-6x - 9y = 9} \\ \qquad y = 3 \end{array}$$

$3x + 5y = -3$
$3x + 5(3) = -3$
$3x + 15^{-15} = -3^{-15}$
$3x = -18$
$(\tfrac{1}{3})(3x) = (\tfrac{1}{3})(-18)$
$x = -6$

$(-6, 3)$

D) $3x + 3y = -9$ div. by 3
$8x - 8y = 56$ div. by 8

You can also divide an equation before adding.

$$\begin{array}{l} x + y = -3 \\ \underline{x - y = 7} \\ 2x \qquad = 4 \end{array}$$

$(\tfrac{1}{2})(2x) = (\tfrac{1}{2})(4)$
$x = 2$

$x + y = -3$
$(2) + y = -3$
$y = -5$

$(2, -5)$

UNIT 16

Linear Systems

Problem Set 16.3
Elimination

Solve using elimination.

1. $2x+3y=13$
   $3x-3y=-3$

2. $3x-2y=2$
   $5x+2y=30$

3. $4x+3y=12$
   $-4x-2y=-4$

4. $2x-3y=33$
   $-2x-5y=7$

5. $5x-2y=-14$
   $2x+y=-11$

6. $3x+4y=-7$
   $2x-y=-12$

7. $5x+6y=22$
   $3x-2y=30$

8. $4x-3y=-1$
   $-2x+4y=-12$

9. $3x+2y=8$
   $5x+3y=11$

10. $3x+4y=4$
    $2x+5y=19$

11. $3x-5y=12$
    $4x-2y=16$

12. $2x-3y=3$
    $5x-4y=-10$

13. $5x-7y=-13$
    $2x-5y=-3$

14. $4x-3y=-9$
    $-5x+2y=-1$

15. $2x-2y=-18$
    $4x+4y=12$

16. $3x+3y=6$
    $7x-7y=42$

4. PROBLEM SOLVING

The word problems in this unit are based on the formula:

$\boxed{\text{Rate} \times \text{Time} = \text{Distance}}$

Demonstration

A) A boat travels upstream 24 miles in 3 hours. Later in the day, the boat travels downstream and covers twice the distance in 4 hours. Determine the constant speed of the boat in still water and the rate of the current.

Note: In a river, the current pushes a boat faster going downstream and slows down a boat travelling upstream.

UNIT 16

Linear Systems

In the chart below, (r) represents the speed of the boat and (c) represents the speed of the current.

	Rate	× Time	= Dist.
Upstream	$r-c$	3	24
Downstream	$r+c$	4	48

$3r-3c=24$ div. by 3
$4r+4c=48$ div. by 4

$$\begin{aligned} r-c &= 8 \\ r+c &= 12 \\ \hline 2r &= 20 \\ r &= 10 \end{aligned}$$

The system is based on $R \times T = D$

$$\begin{aligned} r+c &= 12 \\ (10)+c &= 12 \\ c &= 2 \end{aligned}$$

Solve for r and substitute to determine the value of c

Boat: 10 mph
Current: 2 mph

B) While flying with the wind, a plane flies 300 miles in 40 minutes. It returns against the wind in 45 minutes. Find the speed of the plane in still air and the rate of the wind.

Note: Wind pushes behind and against a plane much like the current of a river effects a boat.

Note: Because rate is measured in miles per hour, time must also be measured in hours.

40 minutes = $2/3$ hours
45 minutes = $3/4$ hours

	Rate	× Time	= Dist.
With Wind	$r+w$	$2/3$	300
Against Wind	$r-w$	$3/4$	300

$\frac{2}{3}r+\frac{2}{3}w=300$ mult. by $3/2$
$\frac{3}{4}r-\frac{3}{4}w=300$ mult. by $4/3$

$$\begin{aligned} r+w &= 450 \\ r-w &= 400 \\ \hline 2r &= 850 \\ r &= 425 \end{aligned}$$

$$\begin{aligned} r+w &= 450 \\ (425)+w &= 450 \\ w &= 25 \end{aligned}$$

Plane: 425 mph
Wind: 25 mph

UNIT 16

Linear Systems

Problem Set 16.4
Problem Solving

Set up a chart and create a system to solve:

① A boat travels 48 miles downstream in 4 hours. The return trip upstream takes 6 hours. Find the speed of the boat in still water and the rate of the current.

② Travelling downstream, a boat goes 60 miles in 3 hours. Against the current, the boat travels the same distance in twice the time. What is the rate of the boat and the rate of the current?

③ A boat travels 30 miles upstream in 5 hours. With the current behind it, the boat can travel 2 miles less in only 2 hours. Determine the rate of the boat and the current.

④ A plane flies 600 miles in 2 hours with a tailwind behind it. Flying against the wind, the plane can travel 80 miles less in the same amount of time. Determine the rate of the plane and the wind.

⑤ A plane flies 150 miles in 20 minutes with a tailwind behind it. Against the wind, the plane can fly 25 miles farther in a half hour. Determine the rate of the plane and the rate of the wind.

⑥ A plane travels 1800 miles in 3 hours flying with a tailwind. Travelling in the opposite direction, it takes 4 hours for the plane to cover 2000 mile. Determine the rate of

Linear Systems

the plane and the rate of the wind.

⑦ Flying with the wind, a plane travels 300 miles in 45 minutes. Against the wind, the plane can cover half that distance in 30 minutes. Determine the rate of the plane and the rate of the wind.

⑧ With the wind behind it, a plane travels 600 miles in 1½ hours. Against the wind, the plane covers the same distance in 2 hours. Determine the rate of the plane and the rate of the wind.

REVIEW & PRACTICE

Graph each system to determine the solution. Check your solution.

① $y=2x-2$
$y=-2x-10$

② $y=x+8$
$y=-2x-4$

③ $4x-y=4$
$2x+y=8$

④ $3x+2y=6$
$x-3y=-9$

Use substitution to solve:

⑤ $y=5$
$2x-3y=-21$

⑥ $x=3$
$3x+2y=-5$

⑦ $2x+7=-1$
$3x+4y=12$

⑧ $3y-11=16$
$3x-2y=-3$

⑨ $3x-y=3$
$-4x+3y=1$

⑩ $3x-7y=-5$
$x-5y=1$

⑪ $7x-2y=-8$
$3x-y=-5$

⑫ $-4x-y=6$
$3x+2y=3$

Use elimination to solve each system.

⑬ $2x+3y=7$
$-3x-4y=-8$

⑭ $3x-2y=-10$
$7x-3y=-10$

⑮ $2x-2y=-2$
$3x+3y=33$

⑯ $5x+5y=15$
$3x-3y=-33$

Linear Systems

(17) $4x-3y=-12$
$5x-2y=-1$

(18) $6x-3y=6$
$-3x+7y=41$

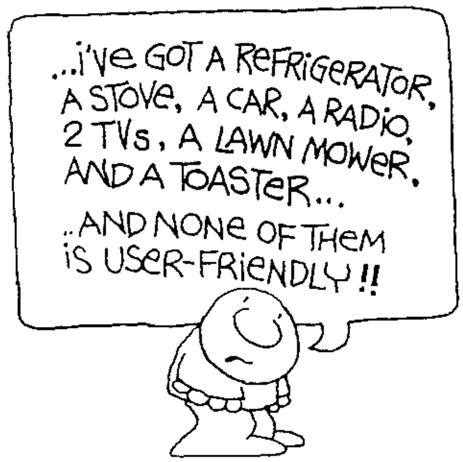
Make a chart and create a system of equations to solve each problem.

(19) Travelling downstream, a boat can go 36 miles in 4 hours. When coming upstream, the boat can go only half the distance in 6 hours. Determine the rate of the boat and the rate of the current.

(20) It took 11 hours for the scouts to bring supplies 55 miles upstream by canoe. Returning to base camp took only 5 hours. Determine the rate of the canoe and the rate of the current.

(21) A plane makes a 280 mile trip in 40 minutes with a tailwind. The same plane travels 225 miles in 45 minutes against the wind. Determine the rate of the plane and the rate of the wind.

(22) Flying against the wind, a plane travels 960 miles in 3 hours. With the wind behind it, the plane makes the return trip in half the time. Determine the rate of the plane and the rate of the wind.



Methods For Solving A System of Equations:

1. Graphing
2. Substitution
3. Elimination

UNIT 16

Linear Systems

PRACTICE TEST #1

Solve by graphing:

1. $y = 4x + 8$
   $2x + y = 2$

Solve by substitution:

2. $2x + 5 = -7$
   $3x + 4y = -2$

3. $2x - y = 5$
   $3x + 2y = 4$

Solve by elimination:

4. $3x + 2y = 6$
   $4x - 2y = 22$

5. $4x - 2y = -6$
   $3x - 3y = -15$

Solve:

6. A boat travels downstream 9 miles in a half hour and makes the return trip upstream in 90 minutes. Determine the rate of the boat and the rate of the current.

PRACTICE TEST #2

Solve by graphing

1. $y = 5x + 5$
   $x + y = -7$

Solve by substitution:

2. $2x - 5y = -5$
   $3y - 4 = 5$

3. $3x - y = 8$
   $5x + 4y = 2$

Solve by elimination:

4. $2x - 3y = -9$
   $-2x + 5y = 19$

5. $5x + 3y = 3$
   $2x - 4y = 22$

Solve:

6. Against the wind, a plane flies 320 miles in 40 minutes. With the wind, the plane travels 145 miles <u>farther</u> in 45 minutes. Determine the rates of the plane and the wind.

UNIT 17

Factoring

1. GREATEST COMMON FACTOR

The greatest common factor of a series of terms is the product of all common prime factors to the highest common power.

Demonstration

Identify the GCF:

A) $6xy$, $12x^2$, $9xy^2$

$3x$

B) $12a^2b$, $24a^3bc$, $40ab^2c^2$

$4ab$

C) $5x^3$, $8yz^2$, $9xy$

1

When you <u>factor</u> an expression, you change its form but not its value.

The first step in factoring an expression is to factor out the GCF. This step is a process that could be considered the "reverse of the Distributive Property."

Demonstration

If possible, factor out the GCF:

D) $10y^2 + 15y$

$5y(2y+3)$

E) $21ab^2 - 33a^2bc$

$3ab(7b - 11ac)$

F) $12a^5b + 8a^3 - 24a^3c$

$4a^3(3a^2b + 2 - 6c)$

G) $6x^3y^2 + 14x^2y + 2x^2$

$2x^2(3xy^2 + 7y + 1)$

UNIT 17

Factoring

H) $25x^2y - 32xz^2 + 18y^3$

not factorable

Problem Set 17.1

Greatest Common Factor

Identify the GCF:

① $6a^2,\ 18b^2,\ 9b^3$

② $12x^2y,\ 9xy^2,\ 15x^3y^2$

③ $8a^2b^2,\ 12a^3c^2,\ 14abc$

④ $8b^4,\ 5c,\ 3a^2b$

⑤ $18a^2b^2,\ 6b,\ 42a^2b^3$

⑥ $15a^2b^3c,\ 25ab^2c^2,\ 30a^2b^3$

⑦ $32a^3b,\ 40a^2bc,\ 48a^2b^2c$

⑧ $16x^3y^3,\ 24x^2z^2,\ 36x^2yz^3$

If possible, factor out the GCF from each expression:

⑨ $3x^2y + 9y^2 + 6$

⑩ $5a^2 + 10ab - 15b^2$

⑪ $2a^3b^2 - 16a^2b^3 + 8ab$

⑫ $3x^3y + 9xy^2 + 36xy$

⑬ $ab^2 - bc^3 + a^2c$

⑭ $xy^2z + x^3y^2 - x^2y^3z^2$

⑮ $24x^2y^2 + 12xy + x$

⑯ $28a^2b^2c + 21a^2bc^2 - 14abc$

⑰ $12ax + 20bx + 32cx$

⑱ $a + a^2b + a^3b^3$

⑲ $ax^3 + 5bx^3 + 9cx^3$

⑳ $14a^3x + 19a^3y + 11a^3z$

㉑ $6x^2 - 9xy + 24x^2y^2$

㉒ $20a^2b - 30a^3b^3 + 40a^2b^2$

㉓ $5x^2y^2 - 10x^3y^3 - 15x^4y^4$

㉔ $a + ab + abc + abcd$

UNIT 17

Factoring

2. DIFF. OF PERFECT SQUARES

A basic pattern for factoring the difference of two perfect square terms is shown below:

$$\boxed{a^2-b^2=(a+b)(a-b)}$$

Notice: This pattern is the reverse of the "Product of a Sum and Difference" (Lesson 14.2)

Demonstration

State whether each binomial is the difference of perfect squares:

A) y^2-25 Yes

$4b^2-c^2$ Yes

x^2+y^2 No

$8a^2-b^2$ No

x^2-1 Yes

Factor each of these binomials (if possible).

B) a^2-c^2
$(a+c)(a-c)$

C) $4n^2-1$
$(2n+1)(2n-1)$

D) $16x^2+9y^2$
not factorable

Remember that the first step in factoring any expression is to factor out the GCF.

E) $3x^2-27y^2$
$3(x^2-9y^2)$
$3(x+3y)(x-3y)$

F) x^3-16xy^2
$x(x^2-16y^2)$
$x(x+4y)(x-4y)$

When factoring any expression, be sure to continue factoring until all quantities are completely factored.

G) $5n^5-5n$
$5n(n^4-1)$
$5n(n^2+1)(n^2-1)$
$5n(n^2+1)(n+1)(n-1)$

UNIT 17

Factoring

H) $48a^6b^2 - 3a^2b^2$

$3a^2b^2(16a^4-1)$

$3a^2b^2(4a^2+1)(4a^2-1)$

$3a^2b^2(4a^2+1)(2a+1)(2a-1)$

Note: In Problem Set 17.2 you should employ all methods of factoring from Lessons 17.1 and 17.2.

Problem Set 17.2

Diff. of Perfect Squares

State whether each binomial is the difference of perfect squares.

① x^2-y^2

② a^2+b^2

③ $9x^2-10$

④ b^2-49

⑤ $36x^4-1$

⑥ $25a^2b^2-100$

⑦ $64x^4+y^2$

⑧ $4a-16b$

Write each expression and show all steps as you factor it completely (if possible).

⑨ x^2-y^2

⑩ n^2-16

⑪ a^2-1

⑫ a^4-1

⑬ n^2-4m^2

⑭ a^2+b^2

⑮ x^4-y^4

⑯ $25x^2-4y^2$

⑰ $3a^3-12ab^2$

⑱ $6m^2-24n^2$

⑲ $16ax^4-a^5$

⑳ $15n^3-60m^2n$

㉑ $8x^2+4y^2$

㉒ $2xy^4-162x$

㉓ $8x^3y-98xy^3$

㉔ $5n^3-10nm^2$

㉕ $12x^2y^3+8xy^4-16xy^3$

㉖ $9ab^2-27a^2b^3+18a^2b^2$

㉗ $6x^2-9xy^3+12x^2y$

㉘ $a^2b^3c-a^3b^2c^2+a^2b^2c^2$

㉙ a^4b^4-1

㉚ $16x^5y^3-4xy$

㉛ $2a^2b^3-72a^2b$

UNIT 17

Factoring

3. FACTORING TRINOMIALS

When factoring a trinomial with a lead term co-efficient of (+1), look for factors of the constant term that sum to the middle term coefficient.

$\boxed{\begin{array}{l} x^2 + bx + c \\ \text{Look for factors of } c \\ \text{that sum to } b \end{array}}$

Example:

$x^2 + 10x + 21$

Sum $3+7$

factors $3 \cdot 7$

$(x+3)(x+7)$

Note: Start with 21. Find all possible pairs of factors until you find a pair that sums to 10:

$(1)\cdot(21)$ * $(3)\cdot(7)$

$(-1)\cdot(-21)$ $(-3)\cdot(-7)$

You can check your factored expression by FOIL:

$(x+3)(x+7)$

$x^2+10x+21$

Demonstration

Factor each trinomial:

A) $y^2 + 8y + 12$

Factors:

$(1)\cdot(12)$ * $(2)\cdot(6)$ $(3)\cdot(4)$

$(-1)\cdot(-12)$ $(-2)\cdot(-6)$ $(-3)\cdot(-4)$

$(x+2)(x+6)$

B) $a^2 - 9a + 18$ * $(-3)\cdot(-6)$

$(a-3)(a-6)$

c) $y^2+5y-14$ * $(7)\cdot(-2)$

$(a+7)(a-2)$

D) x^2+8x+8

not factorable

When a second variable is involved, use the last term coefficient in place of the constant.

$a^2 - 3ab - 28b^2$

$(-7)\cdot(4)$

$(-7)+(4)$

$(a-7b)(a+4b)$ ← don't forget to inclu the "b"

UNIT 17

Factoring

E) $x^2-3xy-40y^2$ $\quad *(-8)\cdot(5)$

$(x-8y)(x+5y)$

F) $a^2-10ab+24b^2$ $\quad *(-6)\cdot(-4)$

$(a-6b)(a-4b)$

Remember: The first step in factoring an expression is look for a GCF.

G) $4a^3-12a^2b-40ab^2$

$4a(a^2-3ab-10b^2)$

$4a(a-5b)(a+2b)$

H) $2x^2y+2xy^2-40y^3$

$2y(x^2+xy-20y^2)$

$2y(x+5y)(x-4y)$

Problem Set 17.3
Factoring Trinomials

Factor each trinomial:

1. $y^2+12y+27$
2. $x^2+9x+20$
3. $m^2-12m+27$
4. $b^2-11b+28$
5. c^2+3c+6
6. $r^2-12r+20$
7. $a^2+22a+21$
8. $c^2+10c+20$
9. $a^2+5a-50$
10. $b^2+2b-48$
11. $x^2-10x+39$
12. $c^2-2cd-8d^2$
13. $a^2+2ab-3b^2$
14. $a^2-4ab-32b^2$
15. $m^2-mn-6n^2$
16. $x^2-4xy-5y^2$

UNIT 17

Factoring

Challenge Problems

Factor each expression and show all steps:

⑰ $3x^2 + 15x - 108$

⑱ $5n^2 - 15n - 90$

⑲ $4a^2 + 8ab - 12b^2$

⑳ $5x^2 - 20xy + 20y^2$

㉑ $3x^3 - 3x^2y - 18xy^2$

㉒ $4ab^2 - 8abc + 4ac^2$

㉓ $3x^3 - 3xy^2$

㉔ $12ab^2 - 3a^3$

㉕ $2x^5y - 2xy$

㉖ $16a^5b^2 - 81ab^6$

4. GROUPING TERMS

When factoring a trinomial with a lead term coefficient other than (+1), start by multiplying the lead term coefficient and the constant.

$2x^2 + 7x + 6 \qquad (2)\cdot(6) = 12$

Next, find factors that sum to the middle term coefficient.

factors of 12 $\qquad (4)\cdot(3)$

$\qquad (4)+(3) = 7$

Next, split the middle term into two parts using these factors.

$2x^2 + 7x + 6$

$2x^2 + 4x + 3x + 6$

Next, factor by grouping the first two terms and the last two terms.

$2x^2 + 4x + 3x + 6$

$2x(x+2) + 3(x+2)$

Next, factor out the GCF of $(x+2)$.

$2x(x+2) + 3(x+2)$

$(x+2)(2x+3)$

UNIT 17
Factoring

Demonstration

Factor each expression:

A) $3x^2 + 11x + 6$

$(3) \cdot (6) = 18$

$(9) + (2) = 11$

$3x^2 + 9x + 2x + 6$

$3x(x+3) + 2(x+3)$

$(x+3)(3x+2)$

B) $5x^2 - 17x + 14$

$(5) \cdot (14) = 70$

$(-7) + (-10) = -17$

$5x^2 - 7x - 10x + 14$

$x(5x-7) - 2(5x-7)$

$(5x-7)(x-2)$

Note: Be sure to have a (−) sign here.

Note: When you split the middle term, you can put the factors in either order. Try one of the first two demonstration problems again with the middle terms reversed.

C) $8x^2 - 6xy - 9y^2$

$(8) \cdot (-9) = -72$

$(6) + (-12) = -6$

$8x^2 + 6xy - 12xy - 9y^2$

$2x(4x+3y) - 3y(4x+3y)$

$(4x+3y)(2x-3y)$

In the following problem, remember to start by factoring out the GCF.

D) $18a^2 + 33ab - 30b^2$

$3(6a^2 + 11ab - 10b^2)$

$(6) \cdot (-10) = -60$

$(15) + (-4) = 11$

$3[6a^2 + 15ab - 4ab - 10b^2]$

$3[3a(2a+5b) - 2b(2a+5b)]$

$3(2a+5)(3a-2b)$

Demonstration (continues)

UNIT 17

Factoring

E) $2x^2 - 7x - 4$

$(2)\cdot(-4) = -8$

$(-8)+(1) = -7$

$2x^2 - 8x + x - 4$

$2x(x-4) + 1(x-4)$

$(x-4)(2x+1)$

Note: You can factor (1) out of any quantity

Problem Set 17.4
Grouping Terms

Factor each expression:

① $4b^2 + 5b - 6$

② $4y^2 - 17y - 15$

③ $2x^2 - x - 6$

④ $3a^2 - 4a - 15$

⑤ $5b^2 - 13b - 10$

⑥ $4y^2 - 16y + 7$

⑦ $6x^2 + 7x + 2$

⑧ $6n^2 - 11n + 4$

Be alert! The following expressions have two variables:

⑨ $2a^2 + 5ab - 3b^2$

⑩ $2x^2 - 5xy - 3y^2$

⑪ $15x^2 - 13xy + 2y^2$

⑫ $4a^2 - 8ab + 3b^2$

Challenge Problems

Be sure to factor out the GCF:

⑬ $9a^2 + 24ab + 12b^2$

⑭ $8x^2 + 8xy - 30y^2$

⑮ $18x^2 - 21xy + 6y^2$

UNIT 17

Factoring

(16) $24a^3b - 44a^2b + 12ab$

(17) $12n^4 - 2n^3 - 2n^2$

(18) $4x^2y^2 + 5xy^2 - 6y^2$

(19) $6a^3 + 15a^2b - 9ab^2$

Review Problems

(20) $x^4 - 1$

(21) $x^2 - 3x - 18$

(22) $n^2 - 10n + 16$

(23) $16a^2 - 9b^2$

(24) $x^2 - 7x + 12$

(25) $a^4 - b^4$

REVIEW & PRACTICE

Factor out the GCF:

(1) $4xy^2 - 6x^2y + 2xy$

(2) $36a^2b^2 - 12ab$

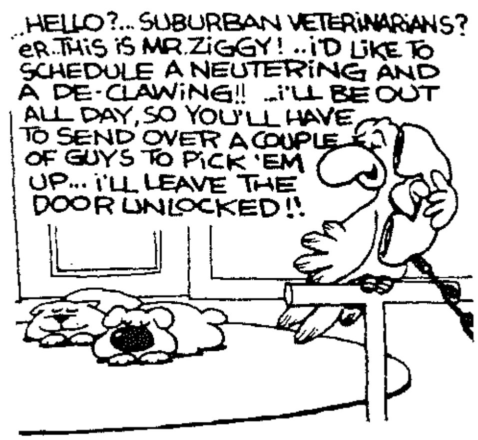
(3) $3x^5y - 3x + 6xy$

Difference of perfect squares:

(4) $9x^2 - 16y^2$

(5) $a^2b^2 - 1$

(6) $x^4 - 81y^4$



Factor trinomials with a lead coefficient of (1):

(7) $a^2 + 17a + 72$

(8) $x^2 - 15xy + 36y^2$

(9) $n^2 - 5nm - 14m^2$

Factor trinomials with lead coefficients $\neq$ (1):

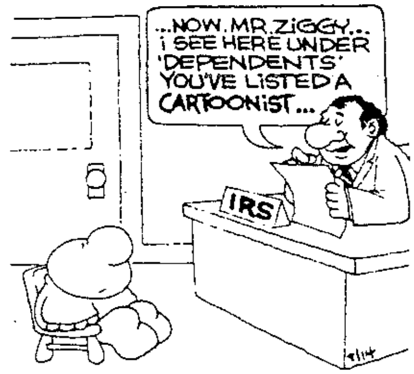
(10) $3a^2 - 10ab - 8b^2$

(11) $4x^2 - 4xy - 3y^2$

UNIT 17

Factoring

⑫ $6a^2 - 19a + 10$



Factor each expression completely. Use all methods. Remember to factor out the GCF first:

⑬ $2a^2 - 8b^2$

⑭ $3n^2 + 9n - 12$

⑮ $4x^2y - 6xy^2 + 10xy^3$

⑯ $x^2 + 6x + 8$

⑰ $16x^4 - 1$

⑱ $2x^2 + 7x + 3$

⑲ $12a^3 - 24a^2b$

⑳ $3a^2 + 5ab + 2b^2$

㉑ $a^2 - 2ab - 3b^2$

㉒ $x^4 - x^2y^2$

㉓ $6abc - 3ab + 9a^2b$

㉔ $2x^2 + 7x + 5$

㉕ $36a^2 - 25$

㉖ $2a^2 + 10a + 12$

㉗ $6a^2 + 11a + 3$

㉘ $x^2y^2z^2 - xy^3 + x^2yz$

㉙ $8a^2 + 12ab + 4b^2$

㉚ $81n^4 - 1$

㉛ $3n^2 - 9n + 6$

㉜ $x^2 + x - 2$

㉝ $6a^2 + 5a - 6$

㉞ $18x^2 - 24x^3$

㉟ $a^3 - 9a$

㊱ $x^2 - 3xy - 10xy$

㊲ $2a^4 + 7a^3 + 6a^2$

㊳ $a^4b^3 - a^3b^4$

㊴ $64a^2 - 49b^2$

UNIT 17

Factoring

PRACTICE TEST #1

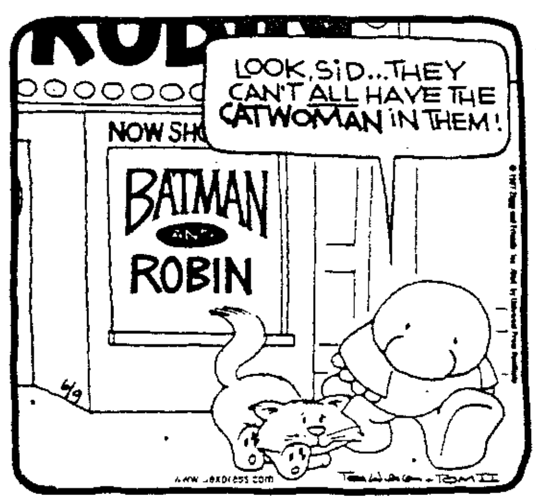
Factor each expression completely. Show all steps.

1. $6x^2y^2 - 9xy^3z$
2. $16a^2 - b^2$
3. $2n^2 - 14n + 24$
4. $6x^4 - 6$
5. $2a^2 - 5a - 12$
6. $12x^2 + 14xy - 6y^2$

PRACTICE TEST #2

Factor each expression completely. Show all steps.

1. $4a^3b - 8a^2bc + 12ab$
2. $a^2 - 5a - 14$
3. $98x^2 - 2$
4. $81a^4 - 16b^4$
5. $4x^2 - 18xy + 18y^2$
6. $8n^2 - 16n + 6$



UNIT 18

Quadratic Equations

1. FACTORING

The Zero Product Property can be used to solve equations that have already been factored.

Zero Product Property
If the product is zero, one of the factors must be zero.

$(x-3)(x+7) = 0$

To solve: $(x-3)=0$ -or- $(x+7)=0$

Therefore: $x=3$ -or- $x=-7$

In some quantities, it is a little tougher to determine the value of the variable that makes the quantity equal to zero.

Determining Zero Values
The quantity equals zero if the variable equals the opposite of the constant divided by the coefficient.

$(2x-3)(3x+7)=0$

If $(2x-3)=0$, $x = 3/2$
If $(3x+7)=0$, $x = {-7}/3$

Demonstration

Solve each equation:

A) $(n-5)(n+4)=0$
$n = 5$ or -4

B) $(2n-5)(3n+2)=0$
$n = 5/2$ or ${-2}/3$

c) $(x-7)(3x+4)=0$
$x = 7$ or ${-4}/3$

To solve the following equations, you must first factor the quadratic expression.

Demonstration

Solve each equation:

D) $x^2-7x+12=0$
$(x-4)(x-3)=0$
$x = 4$ or 3

UNIT 18

Quadratic Equations

E) $2x^2+7x-15=0$

$(2)(-15)=-30$

$(10)+(-3)=7$

$2x^2+10x-3x-15=0$

$2x(x+5)-3(x+5)=0$

$(x+5)(2x-3)=0$

$x=-5$ or $3/2$

When solving the following equation, you have to subtract to use the Zero Product Property.

F) $2n^2+7n+2=6$

$2n^2+7n+2\,{}^{-6}=6\,{}^{-6}$

$2n^2+7n-4=0$

$(2)(-4)=-8$

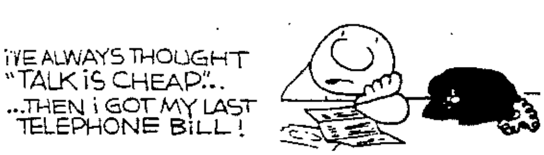
$(8)+(-1)=7$

$2n^2+8n-n-4=0$

$2n(n+4)-1(n+4)=0$

$(n+4)(2n-1)=0$

$n=-4$ or $1/2$



Problem Set 18.1

Factoring

Solve each equation:

1. $(x-5)(x+7)=0$
2. $(n+4)(n+9)=0$
3. $(a-3)(2a-5)=0$
4. $(3a-1)(4a+3)=0$
5. $(5x+4)(2x-7)=0$
6. $(3x+8)(2x-1)=0$
7. $n^2-8n+15=0$
8. $x^2+2x-24=0$
9. $x^2-9=0$
10. $4n^2-1=0$
11. $a^2-6a+8=0$
12. $x^2+5x-6=0$
13. $3x^2-13x-10=0$
14. $2n^2+5n-12=0$
15. $25a^2-16=0$

UNIT 18

Quadratic Equations

Challenge Problems

(16) $2x^2 - x - 16 = 5$

(17) $6n^2 - 7n + 5 = 8$

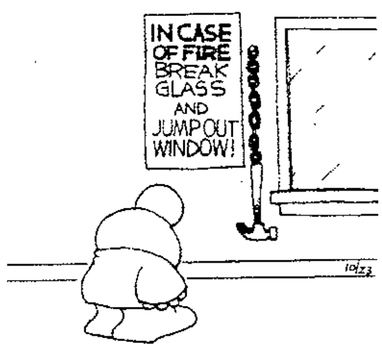
(18) $9x^2 + 6 = 7$

(19) $5a^2 + 11a - 3 = -5$

(20) $(x+3)(x+4) = 6$

(21) $(n-5)(n+3) = 9$

(22) $(2n-1)(n+3) = 4$



2. PROBLEM SOLVING

The key to problem solving involves careful reading when translating a problem into an equation. It is also important to check solutions.

Demonstration

Define a key, set up an equation, and solve:

A) Determine two consecutive negative odd integers whose product is fifteen.

n	$\boxed{-5}$	3
$n+2$	$\boxed{-3}$	5

$(n)(n+2) = 15$

$n^2 + 2n \overset{-15}{=} 15^{-15}$

$n^2 + 2n - 15 = 0$

$(n+5)(n-3) = 0$

$n = -5$ or 3

Note: Read carefully. The problem asks for negative integers.

Remember:

Consecutive	$n, n+1$
Consecutive even	$n, n+2$
Consecutive odd	$n, n+2$

B) An integer is added to the square of the next consecutive integer. Find the integers if the sum is forty-one.

n

$n+1$

$n + (n+1)^2 = 41$

$n + n^2 + 2n + 1 = 41$

$n^2 + 3n + 1 \overset{-41}{=} 41^{-41}$

continued

Quadratic Equations

$n^2 + 3n - 40 = 0$
$(n+8)(n-5) = 0$
$n = -8$ or 5

n	-8	5
$n+1$	-7	6

Note: Two pairs of integers are possible in this problem.

c) The difference of two integers is nine and their product is negative fourteen. Determine the integers.

n	7	2
$n-9$	-2	-7

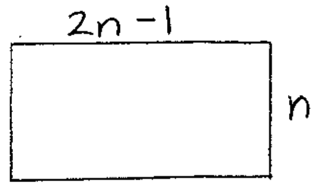
$(n)(n-9) = -14$
$n^2 - 9n \;{}^{+14} = -14 \;{}^{+14}$
$n^2 - 9n + 14 = 0$
$(n-7)(n-2) = 0$
$n = 7$ or 2

Note: Check out how the key handles the difference of two integers.

Question: Would a key with n and $\underline{n+9}$ work for this problem?

D) The length of a rectangle is one inch less than twice the width. The area is twenty-eight square inches. What are the dimensions?

Note: Combine your key with a diagram.

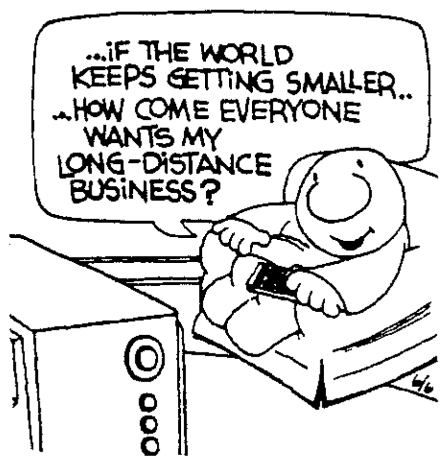


$(n)(2n-1) = 28$
$2n^2 - n \;{}^{-28} = 28 \;{}^{-28}$
$2n^2 - n - 28 = 0$
$(2)(-28) = -56$
$(-8) + (7) = -1$
$2n^2 - 8n + 7n - 28 = 0$
$2n(n-4) + 7(n-4) = 0$
$(n-4)(2n+7) = 0$
$n = 4$ or $\cancel{-\frac{7}{2}}$

Note: A dimension must be positive.

n	4
$2n-1$	7

$\boxed{\text{4 by 7 in.}}$



UNIT 18

Quadratic Equations

Problem Set 18.2
Problem Solving

Define a Key, write an equation, and solve:

① Find two consecutive even integers whose product is twenty-four.

② Find two consecutive negative integers whose product is twenty.

③ Find two consecutive odd integers whose product is fifteen.

④ An integer is added to the square of the next consecutive integer. Find the integers if the sum is twenty-nine.

⑤ An integer is added to the square of the next consecutive integer. Find the integers if the sum is five.

⑥ The square of a positive odd integer is added to the square of the next consecutive odd integer. Find the integers if the sum is thirty-four.

⑦ The square of a negative even integer is added to the square of the next consecutive even integer. Find the integers if the sum is fifty-two.

⑧ An integer is added to the square of the next consecutive integer and the sum is eleven. Find the integers.

⑨ The difference of two integers is two and their product is thirty-five. Find the integers.

⑩ The difference of two integers is three and their product is eighteen. Both integers are positive. Find the integers.

Quadratic Equations

(11) The difference of two integers is six and their product is negative five. Find the integers.

(12) The difference of two negative integers is three and their product is ten. Find the integers.

(13) The difference of two integers is six and their product is negative eight. Find the integers.

(14) The length of a rectangle is one more than twice its width. Find the dimensions if the area is twenty-one square inches.

(15) The length of a rectangle is two more than its width and the area is twenty-four square inches. Find the dimensions.

(16) The length of a rectangle is one less than three times its width. The area is ten square inches. Find the dimensions of the rectangle.

(17) The length of a rectangle is two more than twice its width. Find the dimensions if the area is twenty-four square inches.

(18) The length of a rectangle is two less than twice its width. The area is forty square inches. Find the dimensions.

3. COMPLETING THE SQUARE

So far, we have learned only one method for solving quadratic equations. That one method is factoring.

Factoring does not work on all equations. A method that does work all of

Quadratic Equations

the time is called Completing the Square.

Demonstration

Solve by completing the square:

$x^2 - 2x - 15 = 0$

First, clear the constant from the left side (add 15 to both sides):

$x^2 - 2x \quad = 15$

Then, take half the middle term coefficient, square it, and add it to both sides:

half of -2 is -1
$(-1)^2 = 1$

$x^2 - 2x + 1 = 15 + 1$

The trinomial on the left is now a perfect square trinomial:

$(x - 1)^2 = 16$
↑ This is $\frac{1}{2}$ of the middle term coefficient

Now, take the square root of both sides. Be sure to use a $\pm$ (plus or minus sign) on the right side of the equation:

$x - 1 = \pm\sqrt{16}$
$x - 1^{+1} = \pm 4^{+1}$
$x = 5$ or -3

Demonstration

A) $x^2 - 6x + 8 = 0$

$x^2 - 6x \quad = -8$

half of -6 is -3
$(-3)^2 = 9$

$x^2 - 6x + 9 = -8 + 9$

$(x - 3)^2 = 1$

$x - 3 = \pm\sqrt{1}$

$x - 3 = \pm 1$

$x = 3 \pm 1$
$x = 4$ or 2

Quadratic Equations

B) $x^2 + 4x - 6 = 0$

$x^2 + 4x = 6$

half of 4 is 2

$(2)^2 = 4$

$x^2 + 4x + 4 = 6 + 4$

$(x+2)^2 = 10$

$x + 2 = \pm\sqrt{10}$

$x = -2 \pm \sqrt{10}$

If the equation has a lead term coefficient other than (1), you must divide by the lead term coefficient.

C) $2x^2 - 8x - 6 = 0$

$2x^2 - 8x = 6$

Divide by the lead term co-efficient:

$x^2 - 4x = 3$

half of -4 is -2

$(-2)^2 = 4$

$x^2 - 4x + 4 = 3 + 4$

$(x-2)^2 = 7$

$x - 2 = \pm\sqrt{7}$

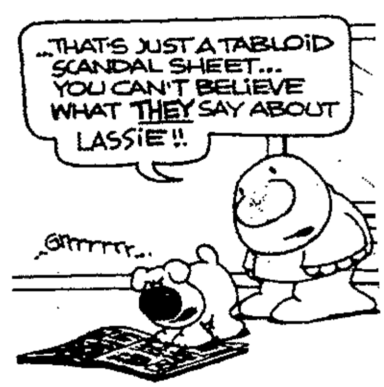
$x = 2 \pm \sqrt{7}$

Problem Set 18.3

Completing The Square

Solve by completing the square:

1. $x^2 + 4x - 5 = 0$
2. $x^2 - 10x + 21 = 0$
3. $x^2 - 6x + 4 = 0$
4. $x^2 + 4x - 2 = 0$
5. $x^2 + 6x + 2 = 0$
6. $x^2 + 10x + 23 = 0$
7. $2x^2 - 16x + 26 = 0$
8. $3x^2 + 12x + 6 = 0$
9. $2x^2 + 20x + 40 = 0$
10. $3x^2 + 6x - 30 = 0$



UNIT 18

Quadratic Equations

Review & Challenge

Solve by factoring:

(11) $x^2 - x - 6 = 0$

(12) $n^2 - 6n + 8 = 0$

(13) $2x^2 - 7x - 4 = 0$

(14) $6n^2 + 7n - 3 = 0$

(15) $(a+2)(a+3) = 20$

(16) $(x-1)(x+4) = 14$

(17) $(2n+3)(n+3) = -1$

(18) $(x+1)(3x-4) = 6$

Define a key, set up an equation, and solve:

(19) An even integer is added to the square of the next consecutive even integer and the sum is zero. Find the integers.

(20) The difference of two integers is seven and their product is negative twelve. Find the integers.

(21) The length of a rectangle is four inches less than three times its width. The area of the rectangle is fifteen square inches. Find the dimensions.

(22) The sum of the squares of two consecutive negative integers is twenty-five. Determine the integers.

(23) The difference of two positive integers is four and their product is twenty-one. Find the integers.

(24) The length of a rectangle is one more than three times the width. The area of the rectangle is fourteen square inches. Determine the rectangle's dimensions.

Quadratic Equations

4. QUADRATIC FORMULA

Another method that can be used to solve a quadratic equation is the quadratic formula.

Values of a, b, and c from any quadratic equation can be substituted into the formula.

Quadratic Equation:

$$ax^2 + bx + c = 0$$

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Demonstration

Solve using the fomula:

A) $2x^2 + 7x - 4 = 0$

$a = 2 \quad b = 7 \quad c = -4$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(7) \pm \sqrt{(7)^2 - 4(2)(-4)}}{2(2)}$$

$$x = \frac{-7 \pm \sqrt{81}}{4} = \frac{-7 \pm 9}{4}$$

$$x = \frac{-16}{4} \text{ or } \frac{2}{4} = -4 \text{ or } \frac{1}{2}$$

B) $x^2 = 25$

In order to use the formula, set the quadratic expression equal to zero.

$$x^2 - 25 = 0$$

$a = 1 \quad b = 0 \quad c = -25$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-0 \pm \sqrt{(0)^2 - 4(1)(-25)}}{2(1)}$$

$$x = \frac{0 \pm \sqrt{100}}{2} = \frac{0 \pm 10}{2}$$

$$x = \frac{10}{2} \text{ or } \frac{-10}{2} \qquad x = 5 \text{ or } -5$$

Quadratic Equations

c) $-n^2 - 6n = -3$

$-n^2 - 6n + 3 = 0$

$a = -1 \quad b = -6 \quad c = 3$

$$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$n = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(-1)(3)}}{2(-1)}$$

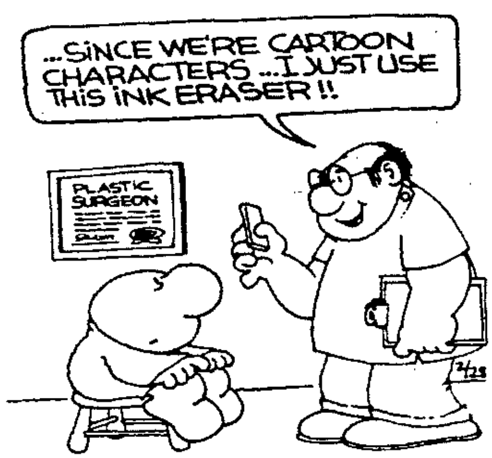
$$n = \frac{6 \pm \sqrt{48}}{-2} = \frac{6 \pm \sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}}{-2}$$

$$n = \frac{6 \pm 4\sqrt{3}}{-2}$$

Remember to simplify all radicals.

$$n = -3 \pm 2\sqrt{3}$$

Also be sure to simplify all fractions.



Problem Set 18.4

The Quadratic Formula

Solve using the formula. To help you memorize the formula, you should write it out as one of the steps in each problem.

1. $x^2 + 4x + 3 = 0$
2. $2n^2 + n - 15 = 0$
3. $2x^2 + 7x = -6$
4. $3n^2 - 10n = 8$
5. $x^2 - 6x + 1 = 0$
6. $x^2 - 4x + 1 = 0$
7. $n^2 + 6n - 11 = 0$
8. $n^2 - 10x + 7 = 0$
9. $x^2 - 3 = 0$
10. $x^2 - 18 = 0$
11. $2n^2 - 5n = -3$
12. $2x^2 + 7x = -3$
13. $x^2 - 8x - 11 = 0$
14. $n^2 + 4n - 4 = 0$

UNIT 18

Quadratic Equations

Review Problems

Solve by factoring:

(15) $x^2 - 5x - 24 = 0$

(16) $2x^2 - 7x + 6 = 0$

(17) $4x^2 + 4x - 3 = 0$

Solve by completing the square:

(18) $x^2 + 8x + 12 = 0$

(19) $x^2 - 4x - 1 = 0$

(20) $x^2 + 6x + 1 = 0$

Solve by any method:

(21) The sum of an even integer and the square of the next consecutive even integer is forty. Find the integers.

(22) The difference of two negative integers is four and their product is thirty-two. Find the integers

REVIEW & PRACTICE

Solve by factoring:

(1) $n^2 - 16 = 0$

(2) $4x^2 - 9 = 0$

(3) $2a^2 - 2 = 0$

(4) $3n^2 - 12 = 0$

(5) $a^2 - 2a - 15 = 0$

(6) $x^2 + 9x + 20 = 0$

(7) $3n^2 - 5n - 2 = 0$

(8) $2x^2 + 5x + 3 = 0$

(9) $3x^2 - 10x - 8 = 0$

(10) $2a^2 + 9a + 10 = 0$

Solve by completing the square:

(11) $x^2 - 2x - 24 = 0$

(12) $a^2 - 6a + 3 = 0$

(13) $x^2 + 4x - 1 = 0$

(14) $n^2 + 2n - 1 = 0$

UNIT 18
Quadratic Equations

(15) $a^2 - 8a + 13 = 0$

(16) $x^2 + 6x + 4 = 0$

(17) $2x^2 + 8x + 4 = 0$

(18) $3n^2 - 6n - 18 = 0$

Solve with the quadratic formula:

(19) $x^2 + 3x - 18 = 0$

(20) $n^2 - 3n - 4 = 0$

(21) $a^2 + 4a - 1 = 0$

(22) $x^2 - 6x + 6 = 0$

(23) $2n^2 - 5n - 12 = 0$

(24) $3x^2 + 10x + 3 = 0$

(25) $a^2 + 4a - 4 = 0$

(26) $x^2 - 6x - 9 = 0$

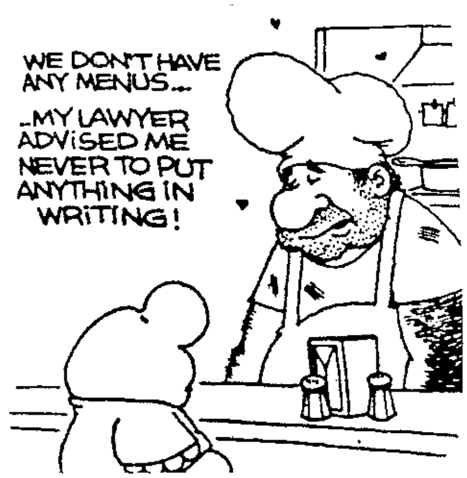
Define a key, write an equation, and solve using any method:

(27) Find two consecutive odd integers whose product is sixty-three.

(28) Find two consecutive negative integers whose product is forty-two.

(29) A negative even integer is added to the square of the next even integer and the sum is zero. Find the integers.

(30) The square of an integer is added to the square of the next consecutive integer and the sum is forty-one. Find the integers.



(31) The difference of two integers is eight and their product is forty-eight. Find the integers.

(32) The difference of two

UNIT 18

Quadratic Equations

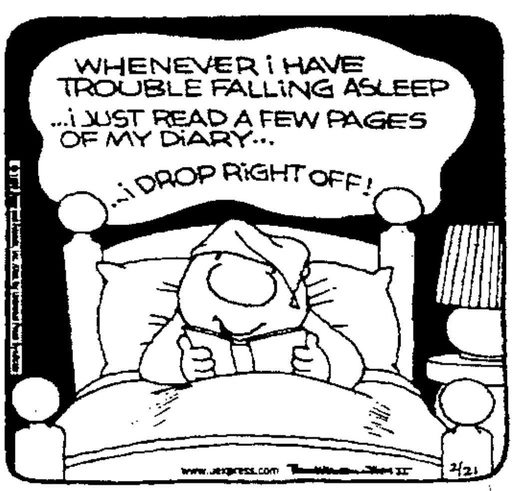
negative integers is three and their product is forty. Find the integers.

(33) The length of a rectangle is three more than the width and the area is twenty-eight square inches. Find the dimensions.

(34) The length of a rectangle is one more than three times its width and the area is thirty square inches. Find the dimensions.

(35) The length of a rectangle is three less than twice its width and the area is thirty-five square inches. Find the dimensions.

(36) The length of a rectangle is two more than twice its width and the area is forty square inches. Find the dimensions.



UNIT 18

Quadratic Equations

PRACTICE TEST #1

Solve by factoring:

① $x^2+6x-16=0$

Solve by completing the square:

② $x^2+4x+1=0$

③ $3x^2+6x-18=0$

Solve by using the quadratic formula. Be sure to simplify fractions and radicals:

④ $2x^2+2x-3=0$

Define a variable, write an equation, and solve using any method:

⑤ The difference of two integers is nine and their product is negative fourteen. Find the integers.

⑥ The length of a rectangle is four less than twice its width and the area is forty-eight square inches. Find the dimensions.

PRACTICE TEST #2

Solve by factoring:

① $4x^2-9=0$

Solve by completing the square:

② $x^2-6x+4=0$

③ $2x^2+8x-6=0$

Solve by using the quadratic formula. Be sure to simplify fractions and radicals:

④ $2x^2-4x-3=0$

Define a variable, write an equation, and solve using any method:

⑤ A negative even integer is added to the square of the next even integer. Find the integers if the sum is ten.

⑥ The length of a rectangle is one less than twice its width and the area is forty-five square inches. Find the dimensions.

UNITS 9-18

Cumulative Review

REVIEW & PRACTICE

Order of operations:

1. $(-3)+(-8)\div(-2)-(-3)$
2. $(-4)^2-(-2)^3$
3. $(-3)(-2)(+1)(-2)(+2)$

Evaluating expressions:
$a=-1 \quad b=-3 \quad c=-5$

4. $a-2b+c$
5. a^3-2b^2
6. $3(b+a)^2$

Simplifying expressions:

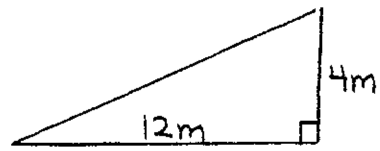
7. $3(2a-b)-2(a+4b)$
8. $2a(a-b)+3ab-4a^2$
9. $a(a-3b)-2a(3a+b)$

Multiply, combine, simplify radicals:

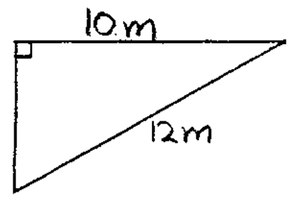
10. $(\sqrt{3})(2\sqrt{3})^2$
11. $2\sqrt{20}-3\sqrt{45}$
12. $\sqrt{32}-4\sqrt{50}+\sqrt{18}$

Pythagorean Theorem:

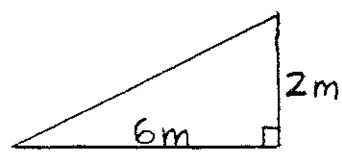
13. Find the missing side.



14. Find the missing side



15. Find the missing side.



Solve and graph on a number line:

16. $4(2x-3)=x-5$
17. $3(2n-4)<4(2n-5)$
18. $2(x-3)\geq 3(2x-1)+5$

Problem solving:

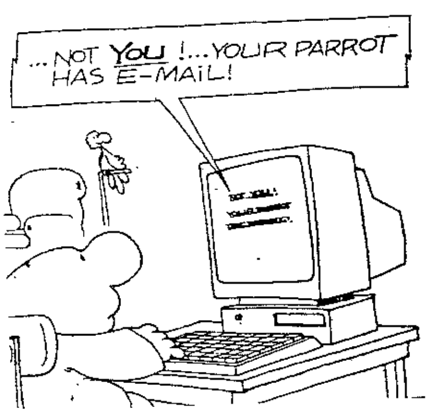
19. Two less than three times a number decreased by four more than the number is

Cumulative Review

equal to five times the number. Find the number.

(20) Twice a number decreased by four less than three times the number is ten less than the number. Find the number.

(21) Three less than four times a number decreased by one more than twice the number is equal to the number. Find it.



Multiply, divide, and simplify monomials:

(22) $(2a^2b)(-ab^3)^2$

(23) $\dfrac{-8x^{-2}y^{-1}z}{6xy^{-3}z^2}$

(24) $\dfrac{-20a^{-3}b^2c^3}{8ab^{-5}c^{-1}}$

Use FOIL and special product patterns to multiply:

(25) $(2x-3)(x-5)$

(26) $(3n-2)^2$

(27) $(4a-b)(4a+b)$

Polynomial division:

(28) $(2n^3+5n^2-n-6)\div(n+1)$

(29) $(2n^3-13n^2+19n-9)\div(2n-3)$

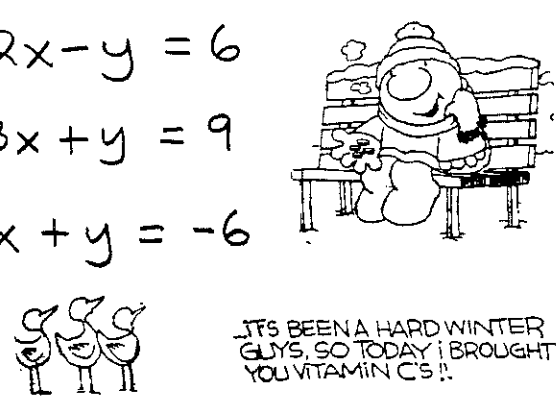
(30) $(4x^3+12x^2+11x+1)\div(x+2)$

Change each equation to slope-intercept form. Determine the slope and both intercepts. Graph the line:

(31) $2x-y=6$

(32) $3x+y=9$

(33) $x+y=-6$



Cumulative Review

Use the standard form to determine the slope and both intercepts. Do not graph:

(34) $3x + 2y = 12$

(35) $x - 4y = -8$

(36) $2x + y = 10$

Use elimination or substitution to solve each system:

(37) $2x - 3y = -1$
$x - 4y = -8$

(38) $4x + 3y = 6$
$2x - 5y = 16$

(39) $x - 5y = 14$
$2x - y = 1$

Make a chart and create a system of equations to solve each problem:

(40) Travelling downstream, a boat can travel 30 miles in 3 hours. Going upstream, it takes 5 hours to cover the same distance. Determine the speed of the boat and the rate of the current.

(41) Travelling upstream, two girls in a canoe can make the 24 miles from base camp to the waterfall in 4 hours. The return trip downstream takes only half the time. Determine the rate of the canoe and the rate of the current

(42) Flying against the wind, a plane can cover 900 miles in 5 hours. With the wind, the plane can travel 600 miles farther in the same amount of time. Determine the speed of the plane and the rate of the wind.

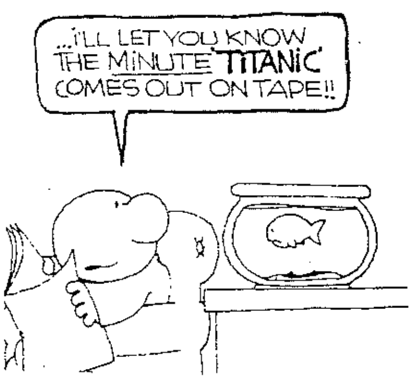
Factor each of the polynomial expressions:

(43) $2x^2 - 2y^2$

UNITS 9-18

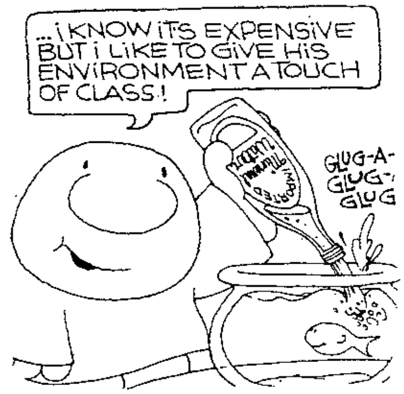
Cumulative Review

44. $5a^2 - 5b^2$
45. $3n^2 - 27$
46. $x^2 + x - 12$
47. $a^2 + 5a + 6$
48. $n^2 + 8n + 15$
49. $2x^2 + 5x - 12$
50. $12a^2 + 5ab - 2b^2$
51. $3x^2 + xy - 2y^2$



Solve each equation by factoring:

52. $x^2 + 2x - 8 = 0$
53. $2n^2 + 11n + 5 = 0$
54. $3x^2 + 5x - 12 = 0$



Solve using completing the square:

55. $x^2 - 6x + 7 = 0$
56. $x^2 + 4x - 2 = 0$
57. $x^2 - 2x - 11 = 0$

Solve using the quadratic formula:

58. $x^2 - 8x + 13 = 0$
59. $2x^2 + 13x + 15 = 0$
60. $3x^2 + 7x + 2 = 0$