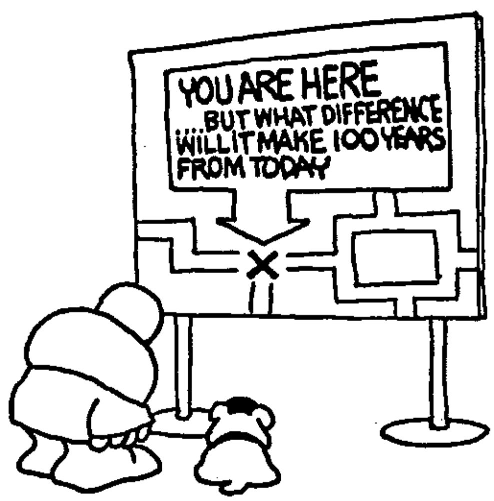
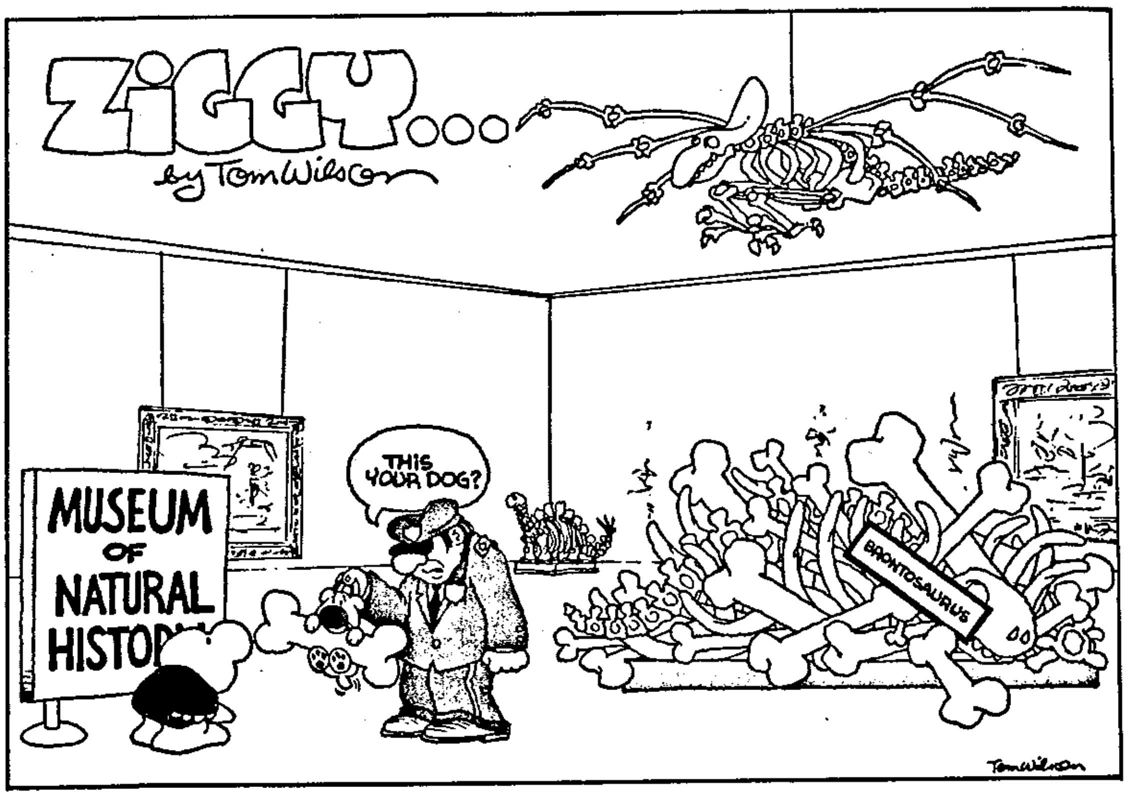
# 6th Grade Accelerated Math Study Guide

Ron Lavine
Friendship Junior High School
Community Consolidated School District #59




Students are issued two copies of this guide to help them throughout the year. One copy should remain at school and one copy should remain at home. Additional help is always available before school, after school, or by telephone.

# LESSONS & CONCEPTS
## ARRANGED IN ORDER OF CLASSROOM PRESENTATIONS



ZIGGY...
by Tom Wilson
THIS YOUR DOG?
MUSEUM OF NATURAL HISTO
BRONTOSAURUS
Tom Wilson

# PLACE VALUE

What is the value of the digit "5" in 35,629 (5000)

# LARGE NUMBER VALUES

Write the number 6,300,025 in words

(six million, three hundred thousand, twenty-five)

Write the standard numeral for: eight billion, five hundred twelve thousand, fifty-six

(8,000,512,056)

# ROUNDING WHOLE NUMBERS

Round 495,827 to the nearest:

| | |
|---|---|
| 10 | (495,830) |
| 100 | (495,800) |
| 1000 | (496,000) |
| 10,000 | (500,000) |
| 100,000 | (500,000) |
| 1,000,000 | (0) |

Notice: In rounding to the nearest 10,000 the "9" becomes "0" and we raise the "4" to "5"

# EXPONENTS

$3^3$ = $3 \times 3 \times 3 \times 1$ (27)
$5^4$ = $5 \times 5 \times 5 \times 5 \times 1$ (625)
$10^3$ = $10 \times 10 \times 10 \times 1$ (1000) Ten to the third power is a "one" with three zeros
$4^0$ = 1

Anything to the zero power equals one

# EXPANDING WHOLE NUMBERS

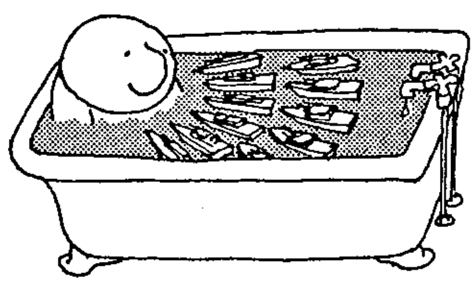
3567 $3000 + 500 + 60 + 7$
$(3 \times 1000) + (5 \times 100) + (6 \times 10) + (7 \times 1)$
$(3 \times 10^3) + (5 \times 10^2) + (6 \times 10^1) + (7 \times 10^0)$

50,206 $50,000 + 200 + 6$
$(5 \times 10,000) + (2 \times 100) + (6 \times 1)$
$(5 \times 10^4) + (2 \times 10^2) + (6 \times 10^0)$

# RENAMING DIVISION

$5 \div 7$ $7\overline{)5}$ $\frac{5}{7}$

$A \div B$ $B\overline{)A}$ $\frac{A}{B}$



# WHOLE NUMBER OPERATIONS

**Addition**

3572
+985
4557

**Subtraction**

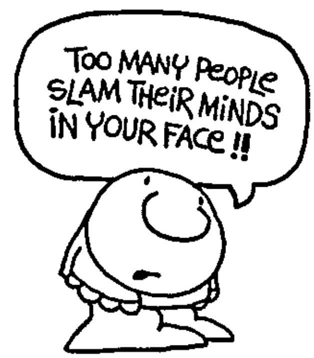
6231
-789
5442

**Multiplication**

573
x58
4584
2865
33,234

**Division**

16
34 | 576
34
236
204
32




**Division With Zero In Quotient**

4
94 | 37826
376
22

40
94 | 37826
376
226

402 $\frac{38}{94} = \frac{19}{47}$
94 | 37826
376
226
188
38

# PRIME AND COMPOSITE NUMBERS

Prime numbers have factors of one and itself. Composite numbers have factors other than one and itself.

Note: By definition "1" is not a prime number. The first prime number is "2."

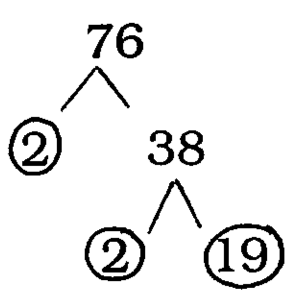
Identify all composite numbers from 10 to 25:

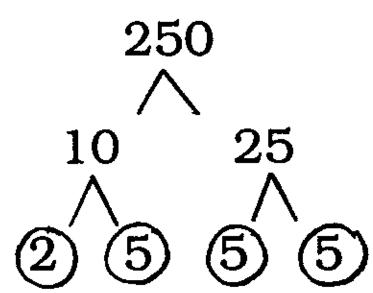
(10, 12, 14, 15, 16, 18, 20, 21, 22, 24, 25)

Identify all prime numbers from 20 to 30:

(23, 29)

# PRIME FACTORIZATION




$76 = 2 \times 2 \times 19$

$76 = 2^2 \times 19$

$250 = 2 \times 5 \times 5 \times 5$

$250 = 2 \times 5^3$

# DIVISIBILITY

2 A number is divisible by "2" if it ends in an even number (0, 2, 4, 6, 8)

5 A number is divisible by "5" if it ends in a 0 or a 5

10 A number is divisible by "10" if it ends in 0

3 A number is divisible by "3" if the sum of its digits is divisible by 3 (example: 381 - sum is 3 + 8 + 1 = 12 and 12 is divisible by 3)

6 A number is divisible by "6" if the original number is "even" and divisible by 3 (example: 18 - sum is 1 + 8 = 9, 9 is divisible by 3, 18 is an even number)

9 A number is divisible by "9" if the sum of its digits is divisible by 9 (example: 360 - sum is 3 + 6 + 0 = 9 and 9 is divisible by 9)

Note: Many students are confused about the rule for "6." The original number must be "even" - not the sum of the digits.

Can you figure out a rule for divisibility by "4"? (A number is divisible by "4" if the last two digits are divisible by 4. Example: 13,528 is divisible by 4 because "28" is divisible by 4.)

# FACTORS

Factors are numbers that divide evenly into an original number.

Identify factors of 36: (1, 2, 3, 4, 6, 9, 12, 18, 36)

# GREATEST COMMON FACTOR (GCF)

The GCF is the largest number that divides evenly into a series of numbers.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| GCF (20, 30) | 20 | 1 | 2 | 4 | 5 | (10) | 20 | | |
| | 30 | 1 | 2 | 3 | 5 | 6 | (10) | 15 | 30 |

# MULTIPLES

Multiples are numbers that the original number divides into evenly.

Identify the first five multiples of 8: (8, 16, 24, 32, 40)

# LEAST COMMON MULTIPLE (LCM)

The LCM is the smallest number that the original numbers divide into evenly.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| LCM (5, 8) | 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | (40) |
| | 8 | 8 | 16 | 24 | 32 | (40) | 48 | | |

# REDUCING FRACTIONS

$\frac{15}{30} = \frac{1}{2}$ divide top and bottom by 15

$\frac{12}{18} = \frac{2}{3}$ divide top and bottom by 6

# RENAMING FRACTIONS

Rename from an improper fraction to a mixed numeral by dividing denominator into numerator and reducing:

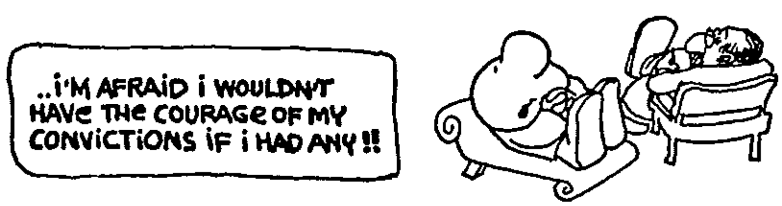
$$\frac{13}{6} = 2\frac{1}{6}$$

$$\frac{14}{8} = 1\frac{6}{8} = 1\frac{3}{4}$$

Rename from a mixed numeral to an improper fraction by multiplying the whole number x the denominator and adding the product to the numerator:

$$3\frac{1}{4} = \frac{13}{4}$$

$$5\frac{2}{3} = \frac{17}{3}$$



# EQUIVALENT FRACTIONS

| | | | |
|---|---|---|---|
| $\frac{2}{3} = \frac{\phantom{00}}{15}$ | multiply by 5<br>multiply by 5 | $\frac{2 \times 5}{3 \times 5}$ | $\frac{(10)}{15}$ |
| $\frac{4}{6} = \frac{6}{\phantom{0}}$ | cross products<br>must be equal | $6 \times 6$<br>$4 \times (9)$ | 36<br>36 |

# COMPARING FRACTIONS

| | | | |
|---|---|---|---|
| $\frac{3}{5} \ \square \ \frac{4}{7}$ | cross multiply "up"<br>and compare | $21 > 20$ | $\frac{3}{5} \times \frac{4}{7}$ |
| $\frac{2}{5} \ \square \ \frac{3}{7}$ | cross multiply "up"<br>and compare | $14 < 15$ | $\frac{2}{5} \times \frac{3}{7}$ |
| $\frac{3}{7} \ \square \ \frac{9}{21}$ | cross multiply "up"<br>and compare | $63 = 63$ | $\frac{3}{7} \times \frac{9}{21}$ |
| $\frac{13}{5} \ \square \ 2\frac{5}{6}$ | change to improper<br>and multiply "up" | $78 < 85$ | $\frac{13}{5} \times \frac{17}{6}$ |

# ADDING FRACTIONS

Adding With
Like Denominators

$$\frac{3}{7} + \frac{2}{7} = \frac{5}{7}$$

Adding With
Different Denominators

$$\frac{2 \times 5}{3 \times 5} = \frac{10}{15}$$

$$+\ \frac{4 \times 3}{5 \times 3} = \frac{12}{15}$$

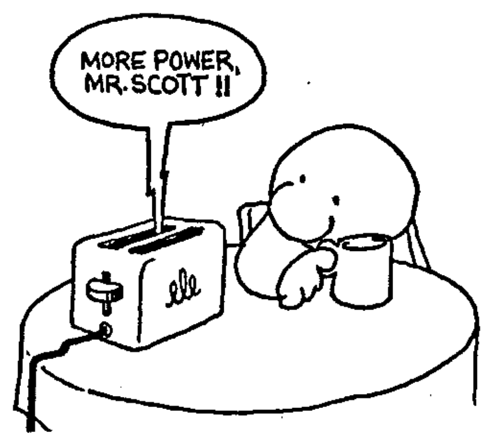
$$\frac{22}{15} = 1\frac{7}{15}$$

Adding With
Mixed Numerals

$$2\frac{1 \times 5}{2 \times 5} \quad 2\frac{5}{10}$$

$$+\ 3\frac{3 \times 2}{5 \times 2} \quad 3\frac{6}{10}$$

$$5\frac{11}{10} = 6\frac{1}{10}$$



# SUBTRACTING FRACTIONS

Subtracting With Like Denominators

$$\begin{array}{r} \frac{6}{7} \\ -\ \frac{2}{7} \\ \hline \frac{4}{7} \end{array}$$

Subtracting With Different Denominators

$$\begin{array}{rcccl} 2\frac{3}{5} & \begin{matrix} x \\ x \end{matrix} & \begin{matrix} 2 \\ 2 \end{matrix} & & 2\frac{6}{10} \\ -\ 1\frac{1}{2} & \begin{matrix} x \\ x \end{matrix} & \begin{matrix} 5 \\ 5 \end{matrix} & & 1\frac{5}{10} \\ \hline & & & & 1\frac{1}{10} \end{array}$$

Subtraction With Borrowing

$$\begin{array}{rccr} 3\frac{2}{5} & \begin{matrix} x \\ x \end{matrix} & \begin{matrix} 4 \\ 4 \end{matrix} & 3\frac{8}{20} \\ -\ 1\frac{3}{4} & \begin{matrix} x \\ x \end{matrix} & \begin{matrix} 5 \\ 5 \end{matrix} & 1\frac{15}{20} \\ \hline \end{array}$$

borrow from "3"
add "20" to "8"

$$\begin{array}{r} 2\frac{28}{20} \\ -\ 1\frac{15}{20} \\ \hline 1\frac{13}{20} \end{array}$$

Borrowing Not Needed:

$$3\frac{2}{3} - 2 = 1\frac{2}{3}$$

Borrowing Needed:

$$4 - 2\frac{2}{5}$$

$$3\frac{5}{5} - 2\frac{2}{5} = 1\frac{3}{5}$$

# MULTIPLYING FRACTIONS

(A) $\frac{2}{3} \times \frac{5}{7} = \frac{10}{21}$

multiply numerators and denominators

(B) $3 \times \frac{3}{5} = \frac{9}{5} = 1\frac{4}{5}$

multiply whole number times numerator

(C) $2\frac{1}{2} \times \frac{2}{3}$

change to improper before multiplying

$\frac{5}{2} \times \frac{2}{3} = \frac{10}{6} = 1\frac{2}{3}$

(D) $3\frac{2}{3} \times 5$

change to improper before multiplying

$\frac{11}{3} \times 5 = \frac{55}{3} = 18\frac{1}{3}$

multiply whole number times numerator

(E) $\frac{35}{60} \times \frac{30}{49}$

$\frac{{}^{5}\cancel{35}}{60} \times \frac{30}{{}^{7}\cancel{49}}$

cross reduce by 7

$\frac{5}{{}^{2}\cancel{60}} \times \frac{{}^{1}\cancel{30}}{7}$

cross reduce by 30

$\frac{5}{2} \times \frac{1}{7} = \frac{5}{14}$

# DIVIDING FRACTIONS

(A) $\frac{3}{5} \div \frac{2}{3}$

take reciprocal of divisor and multiply

$$\frac{3}{5} \times \frac{3}{2} = \frac{9}{10}$$

(B) $\frac{3}{4} \div 2$

notice that reciprocal of 2 is 1/2

$$\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}$$

(C) $6 \div \frac{3}{4}$

do <u>not</u> take reciprocal of whole number <u>dividend</u>

$$^{2}\not{6} \times \frac{4}{^{1}\not{3}} = \frac{8}{1} = 8$$

remember to cross reduce whole number & <u>denominator</u>

(D) $3\frac{2}{3} \div 2\frac{1}{2}$

with mixed numeral in <u>divisor</u> use three steps

$$\frac{11}{3} \div \frac{5}{2}$$

change to improper first

$$\frac{11}{3} \times \frac{2}{5} = \frac{22}{15} = 1\frac{7}{15}$$

then take reciprocal and multiply

# COMPLEX FRACTIONS

Always divide "up" when simplifying a complex fraction. It is also important to rewrite each step as you are solving:

(A)

$$\frac{3}{\frac{2}{3}} = \frac{9}{2} = 4\frac{1}{2}$$


$$3 \div \frac{2}{3}$$

$$3 \times \frac{3}{2} = \frac{9}{2} = 4\frac{1}{2}$$

(B)

$$\frac{\frac{2\frac{1}{2}}{3}}{4} = \frac{\frac{5}{6}}{4} = \frac{5}{24}$$

$$2\frac{1}{2} \div 3 \qquad \frac{5}{6} \div 4$$

$$\frac{5}{2} \times \frac{1}{3} = \frac{5}{6} \qquad \frac{5}{6} \times \frac{1}{4} = \frac{5}{24}$$

# FRACTION WORD PROBLEMS

There are forty tomatoes. Three-fifths of them are ripe. How many tomatoes are ripe?

$40 \times \frac{3}{5} = 24$ tomatoes are ripe

Bill can read one and one-half pages in one minute. How many pages can he read in three and one-half minutes?

$1\frac{1}{2} \times 3\frac{1}{2} = 5\frac{1}{4}$ pages

The parking lot is half full of cars. One-third of the cars are brand new. What fraction of the parking lot is full of new cars?

$\frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ of the parking lot

There was two-thirds of a cake left for dessert. Four people wanted cake. What fraction of the cake did each person receive?

$\frac{2}{3} \div 4 = \frac{1}{6}$ of the cake

It takes Randy two and one-half hours to mow a lawn. How many lawns can he mow in fifteen hours?

$15 \div 2\frac{1}{2} = 6$ lawns

Three-fifths of the work was left to do. Five people shared the responsibility of completing it. What fraction of the work did each person do?

$\frac{3}{5} \div 5 = \frac{3}{25}$ of the work

# READING AND WRITING DECIMALS

Write in words: 2,050,312.053

(two million, fifty thousand, three hundred twelve, and fifty-three thousandths)

Write the standard decimal for: three billion, four hundred eleven thousand, and twelve ten thousandths

(3,000,411,000.0012)

# COMPARING DECIMALS

| | |
|---|---|
| .3 □ .27 | (.30 > .27) |
| 2.56 □ 2.582 | (2.560 < 2.582) |
| 25.38 □ 25.380 | (25.380 = 25.380) |

# DECIMAL VALUE

What is the value of "6" in the numeral 34.065?

(.06) or (6/100) or (six hundredths)

# EXPANDING DECIMALS

Expand 27.354

$20 + 7 + .3 + .05 + .004$

$(2 \times 10) + (7 \times 1) + (3 \times \frac{1}{10}) + (5 \times \frac{1}{100}) + (4 \times \frac{1}{1000})$

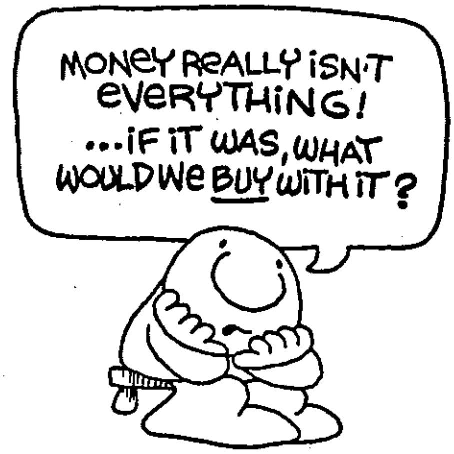
$(2 \times 10^1) + (7 \times 10^0) + (3 \times \frac{1}{10}) + (5 \times \frac{1}{10^2}) + (4 \times \frac{1}{10^3})$

Expand 50,030.005

$50{,}000 + 30 + .005$

$(5 \times 10{,}000) + (3 \times 10) + (5 \times \frac{1}{1000})$

$(5 \times 10^4) + (3 \times 10^1) + (5 \times \frac{1}{10^3})$




You can use the following graphic to help remember how to expand decimals:

$$10^3 \quad 10^2 \quad 10^1 \quad 10^0 \quad \bullet \quad \frac{1}{10} \quad \frac{1}{10^2} \quad \frac{1}{10^3} \quad \frac{1}{10^4}$$

To the left of the decimal point, the exponent is determined by counting digits <u>between</u> the place value and the decimal point.

To the right of the decimal point, the exponent is determined by counting <u>all</u> digits behind the decimal point.

# ROUNDING DECIMALS

Round 354.8392 to the nearest:

| | |
|---|---|
| 100 | 400 |
| 10 | 350 |
| 1 | 355 |
| 1/10 | 354.8 |
| 1/100 | 354.84 |
| 1/1000 | 354.839 |

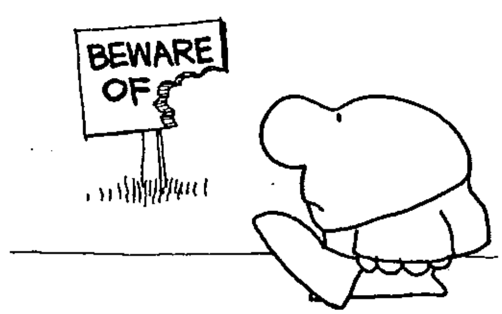
When rounding to a value greater than "0" <u>do</u> <u>not</u> use a decimal point or any digits to the right of the decimal point.

When rounding to a value less than "0" use exactly the number of decimal places required right of the decimal point.

Round 38.995 to the nearest:

| | |
|---|---|
| 10 | 40 |
| 1 | 39 |
| 1/10 | 39.0 |
| $1/10^2$ | 39.00 |

Please note that 39, 39.0, and 39.00 all have the same "value," but they are all different answers to specific rounding questions.



# ADDING & SUBTRACTING DECIMALS

Be sure to line up the decimal points. Whole numbers have a decimal point after the last digit.

35.24 + 3.8 =

```
 35.24
+ 3.80
 39.04
```

5 + 2.65 + .087 =

```
 5.
 2.65
+ .087
 7.737
```

14.6 - 6.84 =

```
 14.60
- 6.84
  7.76
```

# MULTIPLYING DECIMALS

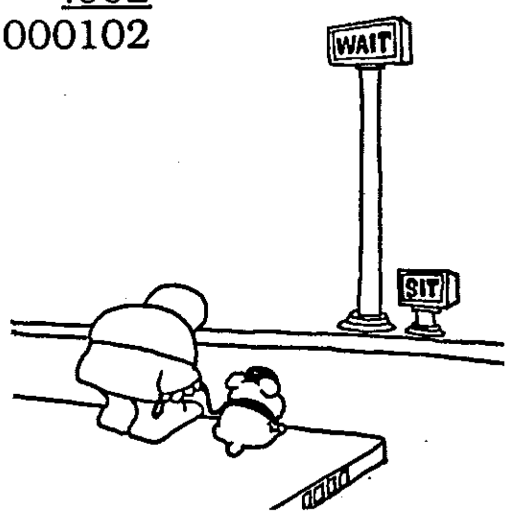
Remember to count all decimal places in the problem to determine the number of decimal places in the product.

3.2 x .6 =

```
  3.2
x  .6
 1.92
```

.051 x .002 =

```
   .051
x  .002
.000102
```



# DIVIDING DECIMALS

The first step in dividing is to eliminate the decimal point from the divisor.

$2 \div .5 = \quad .5\overline{)2} = \quad 5\overline{)20}$

```
      4
  5 ⟌20
     20
      0
```

Add zeros after the decimal point in the dividend to continue a problem.

$3.3 \div .4 = \quad .4\overline{)3.3} = \quad 4\overline{)33.00}$

```
      8.25
  4 ⟌33.00
     32
      1 0
        8
       20
```

If asked to round a quotient, solve one place further and round back to the requested place.

Round to 1/10

$1.5 \div .11 = \quad .11\overline{)1.5} = \quad 11\overline{)150.00}$

```
        13.63 ≈ 13.6
  11 ⟌150.00
       11
        40
        33
         7 0
         6 6
           40
```

If an endless pattern exists in the quotient, use repeating decimal notation.

$20 \div .3 = \quad .3\overline{)20} = \quad 3\overline{)200.00}$

```
      66.66... = 66.6̄
  3 ⟌200.00
     18
      20
      18
       2 0
```

$66.66\ldots = 66.\overline{6}$

# POWERS OF TEN

## MULTIPLICATION

When multiplying by a power of ten, move the decimal place to the right.

$3.65 \times 10 =$ (36.5)

$.46725 \times 10{,}000 =$ (4672.5)

$25.84 \times 10^2 =$ (2584)

$.6 \times 10^3 =$ (600)

## DIVISION

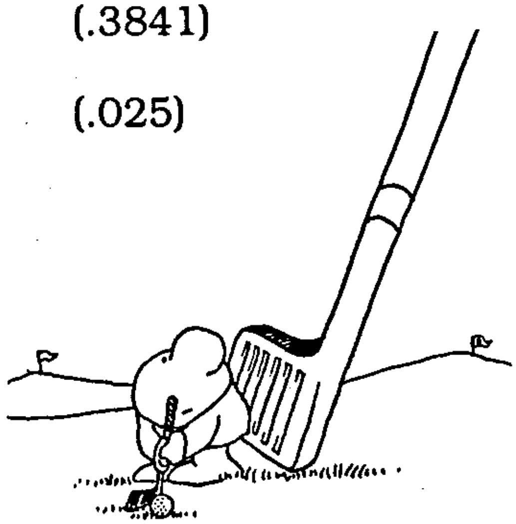
When dividing by a power of ten, move the decimal place to the left.

$3.67 \div 100 =$ (.0367)

$2.63 \div 10^2 =$ (.0263)

$384.1 \div 10^3 =$ (.3841)

$25 \div 10^3 =$ (.025)



# CONVERTING DECIMALS TO FRACTIONS

All decimals have the same value as fractions with a denominator that is a power of ten.

| | |
|---|---|
| .5 = | .5 = 5/10 = (1/2) |
| .34 = | .34 = 34/100 = (17/50) |
| 5.8 = | 5.8 = 5 8/10 = (5 4/5) |

# CONVERTING FRACTIONS TO DECIMALS

Fractions with a denominator that is a power of ten can be easily changed to decimals.

| | |
|---|---|
| 3/10 = | 3/10 = (.3) |
| 4/100 = | 4/100 = (.04) |

Any fraction can be changed into a decimal by dividing the denominator into the numerator.

| | |
|---|---|
| 3/5 = | 3/5 = $3 \div 5$ = (.6) |
| 2/3 = | 2/3 = $2 \div 3$ = $(.\overline{6})$ |

# FRACTION & DECIMAL EQUIVALENCE

The following equivalent values should be memorized. Finding patterns will help make this easier to do.

$\frac{1}{2} = .5$

$\frac{1}{3} = .\overline{3}$

$\frac{2}{3} = .\overline{6}$

$\frac{1}{4} = .25$

$\frac{3}{4} = .75$

$\frac{1}{5} = .2$

$\frac{2}{5} = .4$

$\frac{3}{5} = .6$

$\frac{4}{5} = .8$

$\frac{1}{6} = .1\overline{6}$

$\frac{5}{6} = .8\overline{3}$

$\frac{1}{8} = .125$

$\frac{3}{8} = .375$

$\frac{5}{8} = .625$

$\frac{7}{8} = .875$


$\frac{1}{9} = .\overline{1}$

$\frac{2}{9} = .\overline{2}$

$\frac{3}{9} = .\overline{3}$

$\frac{4}{9} = .\overline{4}$

$\frac{5}{9} = .\overline{5}$

$\frac{6}{9} = .\overline{6}$

$\frac{7}{9} = .\overline{7}$

$\frac{8}{9} = .\overline{8}$

$\frac{1}{11} = .\overline{09}$

$\frac{2}{11} = .\overline{18}$

$\frac{3}{11} = .\overline{27}$

$\frac{4}{11} = .\overline{36}$

$\frac{5}{11} = .\overline{45}$

$\frac{6}{11} = .\overline{54}$

$\frac{7}{11} = .\overline{63}$

$\frac{8}{11} = .\overline{72}$

$\frac{9}{11} = .\overline{81}$

$\frac{10}{11} = .\overline{90}$

# SOLVING PROPORTIONS

A proportion is two ratios set equal to each other. To solve a proportion, set up an equation using the idea that cross products must be equal.

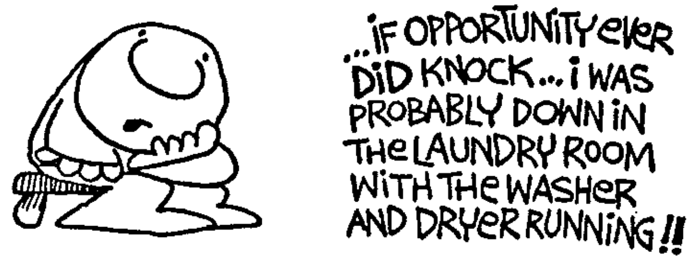
These cross products are called the "means" and the "extremes."

| | |
|---|---|
| $\frac{X}{8} = \frac{9}{18}$ | $\frac{3}{7} = \frac{6}{X}$ |
| $18X = (9)(8)$ | $3X = (6)(7)$ |
| $18X = 72$ | $3X = 42$ |
| $18X\left(\frac{1}{18}\right) = 72\left(\frac{1}{18}\right)$ | $3X\left(\frac{1}{3}\right) = 42\left(\frac{1}{3}\right)$ |
| $X = \frac{72}{18}$ | $X = \frac{42}{3}$ |
| $X = 4$ | $X = 14$ |

To solve the equation, you have to isolate the variable.

Multiply both sides of the equation by the reciprocal of the coefficient.

Then simplify.



# PERCENTAGES

What percent of 12 is 4?

<u>Proportion Method</u>

$$\frac{4}{12} = \frac{X}{100}$$

$X = 33.\overline{3}$

$33.\overline{3}\%$

<u>Decimal Method</u>

Part divided by whole

$4 \div 12 = .33333...$

$33.\overline{3}\%$

What is 5% of 60?

<u>Proportion Method</u>

$$\frac{X}{60} = \frac{5}{100}$$

X = 3

3

<u>Decimal Method</u>

Multiply whole x percent

60 x .05 = 3

3

30 is 25% of what?

<u>Proportion Method</u>

$$\frac{30}{X} = \frac{25}{100}$$

X = 120

120

<u>Decimal Method</u>

Whole divided by percent

$30 \div .25 = 120$

120

# PERCENTAGE WORD PROBLEMS

25 students in the class. 14 are boys. What percent are boys? What percent are girls?

boys / total class $\frac{14}{25} = \frac{X}{100}$

$25X = 1400$
$25X\,(1/25) = 1400\,(1/25)$
$X = 56$

56% boys 44% girls

50 questions on the test. 88% answered correctly. How many questions were answered incorrectly?

incorrect / total questions $\frac{X}{50} = \frac{12}{100}$

$100X = 600$
$100X\,(1/100) = 600\,(1/100)$
$X = 6$

6 incorrect answers

9 items on sale. This is 75% of the total items. How many items in stock?

items on sale / total items $\frac{9}{X} = \frac{75}{100}$

$75X = 900$
$75X\,(1/75) = 900\,(1/75)$
$X = 12$

12 items in stock

# RATE OF DISCOUNT

Rate of discount problems are solved the same way as percentage word problems, but you must be familiar with the vocabulary words specific to discount problems:

| | |
|---|---|
| original price | cost of item before discount |
| regular price | cost of item before discount |
| purchase price | cost of item after discount |
| selling price | cost of item after discount |
| discount | amount of money item reduced by |
| rate of discount | percent of cost reduction |

$9.95 is the original price. There is a 10% discount. What is the amount of discount?

discount / original price

$$\frac{X}{9.95} = \frac{10}{100}$$

100X = 99.5
X = .995 (round money to 1/100)

$1.00

$10.50 is the regular price. There is a 25% discount. What is the selling price?

selling price / regular price

$$\frac{X}{10.50} = \frac{75}{100}$$

100X = 787.5
X = 7.875 (round money to 1/100)

$7.88

# RATE OF DISCOUNT (Continued)

$200 is the original price. $150 is the purchase price. What is the rate of discount?

Note: In this problem, the question asked (rate of discount) does not match the information given (purchase price). It is necessary to find the amount of discount first.

200 - 150 = 50

discount / original price $\frac{50}{200} = \frac{X}{100}$

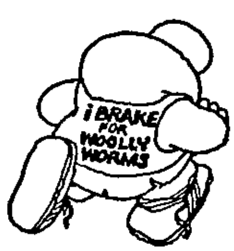
200X = 5000
200X (1/200) = 5000 (1/200)
X = 25

25% discount

$12 discount. 15% discount. What is the purchase price?

Note: In this problem, you are given the purchase price instead of the original price. You must use original price as the denominator in your proportion and subtract the discount to find the purchase price.

discount / original price $\frac{12}{X} = \frac{15}{100}$




15X = 1200
15X (1/15) = 1200 (1/15)
X = 80
80 - 12 = 68

$68

# EQUIVALENCE
## FRACTIONS - DECIMALS - PERCENTS

| FRACTION | DECIMAL | PERCENT |
|---|---|---|
| 1/5 | ? | ? |

Decimal: Divide "up" 1 ÷ 5 = **(.2)**
Percent: Move decimal point two places right .20 = **(20%)**

| FRACTION | DECIMAL | PERCENT |
|---|---|---|
| 2/3 | ? | ? |

Decimal: Divide "up" 2 ÷ 3 = **($.\overline{6}$)**
Percent: Move decimal point two places right .666... = **($66.\overline{6}$%)**
Percent: $66.\overline{6}$% = **(66 2/3 %)**

| FRACTION | DECIMAL | PERCENT |
|---|---|---|
| ? | .4 | ? |

Fraction: .4 = 4/10 = **(2/5)**
Percent: Move decimal point two places right .400 = **(40%)**

| FRACTION | DECIMAL | PERCENT |
|---|---|---|
| ? | 2.5 | ? |

Fraction: 2.5 = 2 5/10 = **(2 1/2)**
Percent: Move decimal point two places right 2.50 = **(250%)**

| FRACTION | DECIMAL | PERCENT |
|---|---|---|
| ? | ? | 7.5% |

Decimal: Move decimal point two places left 07.5 = **(.075)**
Fraction: .075 = 75/1000 = **(3/40)**

| FRACTION | DECIMAL | PERCENT |
|---|---|---|
| ? | ? | .625% |

Decimal: Move decimal point two places left 00.625 = **(.00625)**
Fraction: .00625 = 625/100,000 = **(1/160)**

# REPEATING DECIMALS

## CONVERTING REPEATING DECIMALS TO FRACTIONS

Use these steps to convert a repeating decimal to its equivalent fraction:

1. Establish an initial equation (variable = repeating decimal)
2. Move the decimal point next to the repeating bar if necessary (change the equation)
3. Use this equation as the subtrahend
4. Move the decimal point past one set of repeating digits (establish a new equation for the minuend)
5. Subtract
6. Solve the equation

Change $.\overline{7}$ to a fraction

$$X = .\overline{7}$$

$$\begin{array}{r} 10X = 7.\overline{7} \\ \underline{X = \ \ .\overline{7}} \\ 9X = 7 \end{array}$$

$$X = \mathbf{(7/9)}$$

Change $.\overline{73}$ to a fraction

$$X = .\overline{73}$$

$$\begin{array}{r} 100X = 73.\overline{73} \\ \underline{X = \ \ \ .\overline{73}} \\ 99X = 73 \end{array}$$

$$X = \mathbf{(73/99)}$$

Change $.0\overline{8}$ to a fraction

$$X = .0\overline{8} \longrightarrow 10X = .\overline{8}$$

$$\begin{array}{r} 100X = 8.\overline{8} \\ \underline{10X = \ \ .\overline{8}} \\ 90X = 8 \end{array}$$

$$X = 8/90 = \mathbf{(4/45)}$$

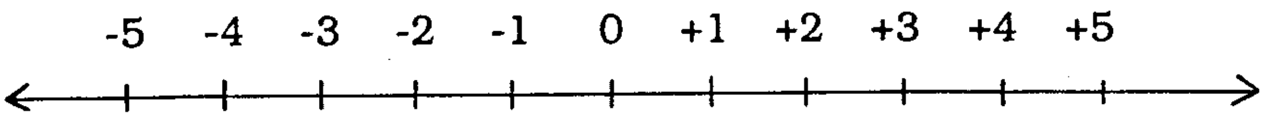
Change $.1\overline{36}$ to a fraction

$$X = .1\overline{36} \longrightarrow 10X = 1.\overline{36}$$

$$\begin{array}{r} 1000X = 136.\overline{36} \\ \underline{10X = \ \ \ 1.\overline{36}} \\ 990X = 135 \end{array}$$

$$X = 135/990 = \mathbf{(3/22)}$$

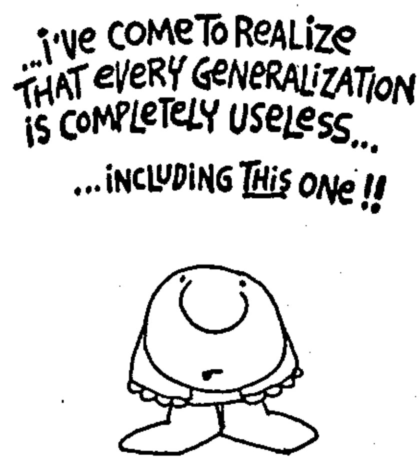
# INTEGER NUMBER LINE




# COMPARING INTEGERS

Integers become greater as you move right on the number line. They become less as you move left on the number line.

| | | | |
|---|---|---|---|
| +6 | ☐ | +3 | (>) |
| +8 | ☐ | -2 | (>) |
| -4 | ☐ | -6 | (>) |
| 0 | ☐ | -2 | (>) |
| -4 | ☐ | -3 | (<) |
| -5 | ☐ | 0 | (<) |
| -3 | ☐ | +2 | (<) |



# ADDING INTEGERS

Some people find it easy to add integers if they think in terms of winning and losing money. A positive integer represents winning and a negative integer represents losing.

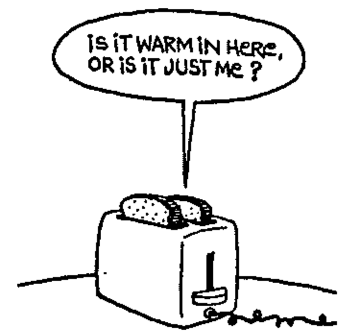
(+3) + (+5) = (+8)

(+6) + (-8) = (-2)

(-12) + (+5) = (-7)

(-6) + (+7) = (+1)

(-9) + (-6) = (-15)




# SUBTRACTING INTEGERS

The preferred method of subtracting integers is to change the subtraction to addition of a negative. (Change the subtraction sign to its opposite. Change the sign of the following integer to its opposite.)

(+6) - (+4) =
(+6) + (-4) = (+2)

(-8) - (-6) =
(-8) + (+6) = (-2)

# MULTIPLYING & DIVIDING INTEGERS

The rules for multiplying and dividing are the same:

1. Ignore the signs and compute (multiply or divide)
2. If the integer signs were the same (both positive or both negative) the answer will be positive.
3. If the integer signs were opposites (one positive and one negative) the answer will be negative.

Multiply and Divide:

| | |
|---|---|
| $(+3) \times (-4) =$ | (-12) |
| $(-6) \div (-2) =$ | (+3) |
| $(-7) \times (-3) =$ | (+21) |
| $(+8) \div (-4) =$ | (-2) |
| $(-6) \times (+6) =$ | (-36) |
| $(-14) \div (+2) =$ | (-7) |

# NUMBER SENTENCES

A number sentence is a statement that shows an expression on the left, an expression on the right, and a comparison sign inbetween.

| | |
|---|---|
| $6 + 8 < 16$ | inequality / closed / true |
| $12 = a + 3$ | equation / open |
| $n + 9 > 10$ | inequality / open |
| $35 > 16 + 74$ | inequality / closed / false |

Note: Since an "open" sentence is open to different possibilities depending upon the value of the variable, it is not possible to identify it as either true or false.

# COMPARISON SIGNS

$=$ equal to

$\neq$ not equal to

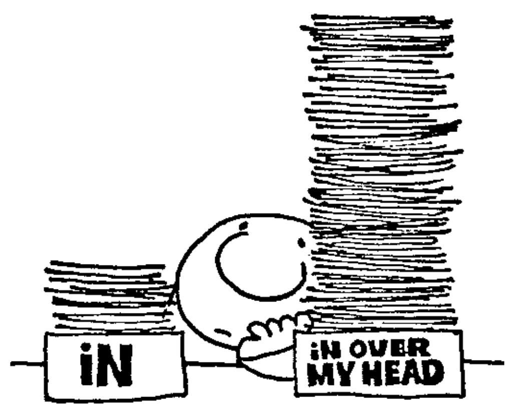
$\approx$ approximately equal to

$<$ less than

$>$ greater than

$\leq$ less than or equal to

$\geq$ greater than or equal to



# ORDER OF OPERATIONS

When determining the value of an expression, the following rules indicate the order in which operations should be performed:

| | |
|---|---|
| 1. Evaluate inner parenthesis | |
| 2. Evaluate outer parenthesis | PARENTHESIS |
| 3. Evaluate powers | EXPONENTS |
| 4. Multiply & divide (left to right) | x ÷ |
| 5. Add & subtract (left to right) | + − |

Note: A scientific calculator will automatically perform order of operations, but some other calculators will not.

1. $4^2 = (16)$ $\quad 4^3 = (64)$

2. $-4^2 = (-16)$ $\quad -4^3 = (-64)$

3. $(-4)^2 = (16)$ $\quad (-4)^3 = (-64)$

4. $-(-5) = (5)$

5. $3 + 4 \times 2 - 1 =$
   $3 + 8 - 1 =$
   $11 - 1 = (10)$

6. $-(3-4) \times (2+5) =$
   $-(-1) \times (7) =$
   $1 \times 7 = (7)$

7. $3 \times (5+3) =$
   $3 \times 8 = (24)$

8. $3 - 5^2 =$
   $3 - 25 = (-22)$

9. $(-4+6)^0 \times (2/3) =$
   $1 \times (2/3) = (2/3)$

10. $4 - \left(\frac{(-3+1)^2}{5^0}\right) \times (-3) =$

    $4 - \left(\frac{(-2)^2}{+1}\right) \times (-3) =$

    $4 - \left(\frac{4}{1}\right) \times (-3) =$

    $4 - \quad 4 \quad \times (-3) =$

    $4 - (-12) = (16)$

# EVALUATING EXPRESSIONS

When substituting values in these problems, always use parenthesis.

Evaluate:

1. h - (-13)    for h = -8

   (-8) - (-13) =

   (-8) + (+13) = (+5)

2. 32/m    for m = -8

   32/(-8) = (-4)

3. xy    for x = -5, y = -8

   (-5)(-8) = (+40)

4. $3a^2b$ - 2a/4    for a = -2, b = -3

   $3(-2)^2(-3)$ - (2)(-2)/(4) =

   3(4)(-3) - (-4)/(4) =

   (-36) - (-1) =

   (-36) + (+1) = (-35)

# AVERAGING ALGEBRAIC EXPRESSIONS

To find the average, combine like terms and divide each term by the number of expressions.

Find the average:

$(3x + 2y) \quad (5x + 4y)$

$3x + 2y + 5x + 4y = 8x + 6y$

$(8x + 6y) \div 2 =$ $4x + 3y$

$(5x + y) \quad (2x + 3y) \quad (4x) \quad (-8y)$

$5x + y + 2x + 3y + 4x - 8y = 11x - 4y$

$(11x - 4y) \div 4 =$ $\frac{11}{4}x - y$

$(a + b) \quad (2a - b) \quad (4b) \quad (5b-1) \quad (-3a + 6)$

$a + b + 2a - b + 4b + 5b - 1 - 3a + 6 = 9b + 5$

$(9b + 5) \div 5 =$ $\frac{9}{5}b + 1$

# SIMPLIFYING EXPRESSIONS

Simplifying an expression maintains the same value but expresses it in simplest form.

## UNDERSTANDING THE DISTRIBUTIVE PROPERTY

$3(4 + 5) = (3 \times 4) + (3 \times 5) = 27$

$-3(2a + b) = (-3 \times 2a) + (-3 \times b) = -6a - 3b$

## SIMPLIFY THESE EXPRESSIONS:

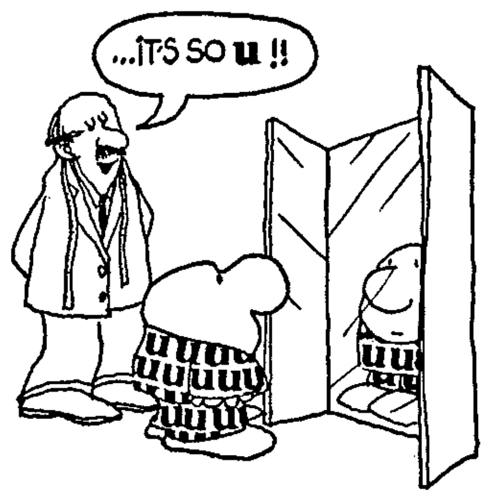
$2x(5x - 3)$

$10x^2 - 6x$

$5(3x - 2) + 8x$

$15x - 10 + 8x$

$23x - 10$




$5x(2x - 1) - 3(x^2 - 2x)$

$10x^2 - 5x - 3x^2 + 6x$

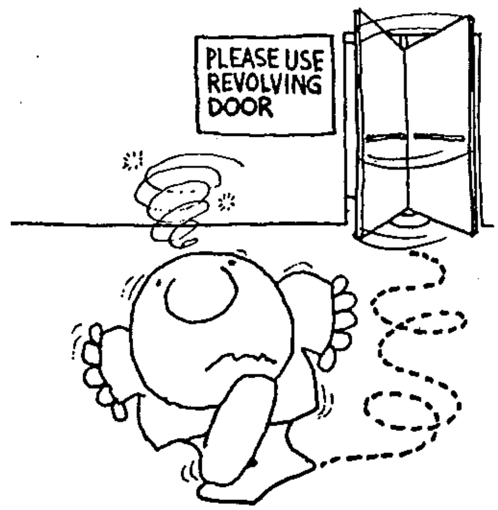
$7x^2 + x$

$14ab + 7m + 21ab$

$35ab + 7m$

# METRIC LENGTH

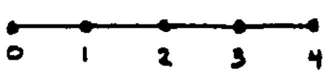
| | | |
|---|---|---|
| kilometer | **km** | 1000m |
| hectometer | hm | 100m |
| dekameter | dam | 10m |
| meter | **m** | |
| decimeter | dm | 1/10m |
| centimeter | **cm** | 1/100m |
| millimeter | **mm** | 1/1000m |



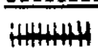

<u>kilometer</u>
5/8 of a mile

<u>meter</u>
slightly more than a yard

<u>centimeter</u>




<u>millimeter</u>



10mm in one centimeter

What unit of measure would be most appropriate for each of the following?

| | |
|---|---|
| height of a flagpole | (m) |
| width of a pencil | (mm) |
| length of a hammer | (cm) |
| dist. between cities | (km) |
| height of a building | (m) |

# METRIC WEIGHT

| | | |
|---|---|---|
| metric ton | **t** | 1000kg |
| kilogram | **kg** | 1000g |
| hectogram | hg | 100g |
| dekagram | dag | 10g |
| gram | **g** | |
| decigram | dg | 1/10g |
| centigram | cg | 1/100g |
| milligram | **mg** | 1/1000g |


<u>metric ton</u>
1000 kg
more than 2000 pounds

<u>kilogram</u>
more than two pounds

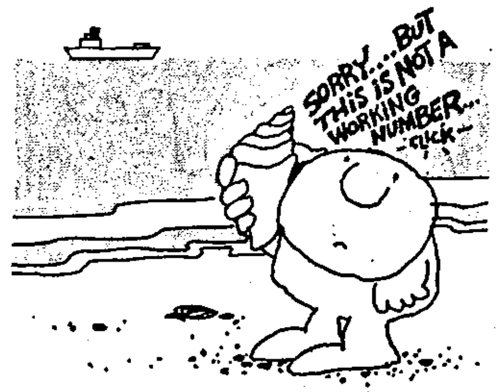
<u>gram</u>
more than 30 g in an ounce
about 500g in a pound

What unit of measure would be most appropriate to weigh each of these?

| | |
|---|---|
| weight of a truck | (t) |
| weight of a pencil | (g) |
| weight of a horse | (kg) |
| weight of a person | (kg) |

# METRIC CAPACITY

| | | |
|---|---|---|
| kiloliter | kℓ | 1000ℓ |
| hectoliter | hℓ | 100ℓ |
| dekaliter | daℓ | 10ℓ |
| liter | ℓ | |
| deciliter | dℓ | 1/10ℓ |
| centiliter | cℓ | 1/100ℓ |
| milliliter | mℓ | 1/1000ℓ |




<u>kiloliter</u>
used for very large units of capacity

<u>liter</u>
a little more than one quart

<u>milliliter</u>
less than a spoonful

What unit of measure would be most appropriate to describe these?

juice for breakfast (mℓ)

paint for the house (ℓ)

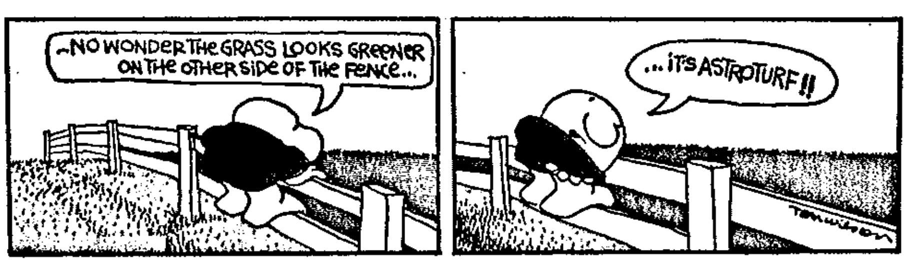
water in a lake (kℓ)

cup of coffee (mℓ)

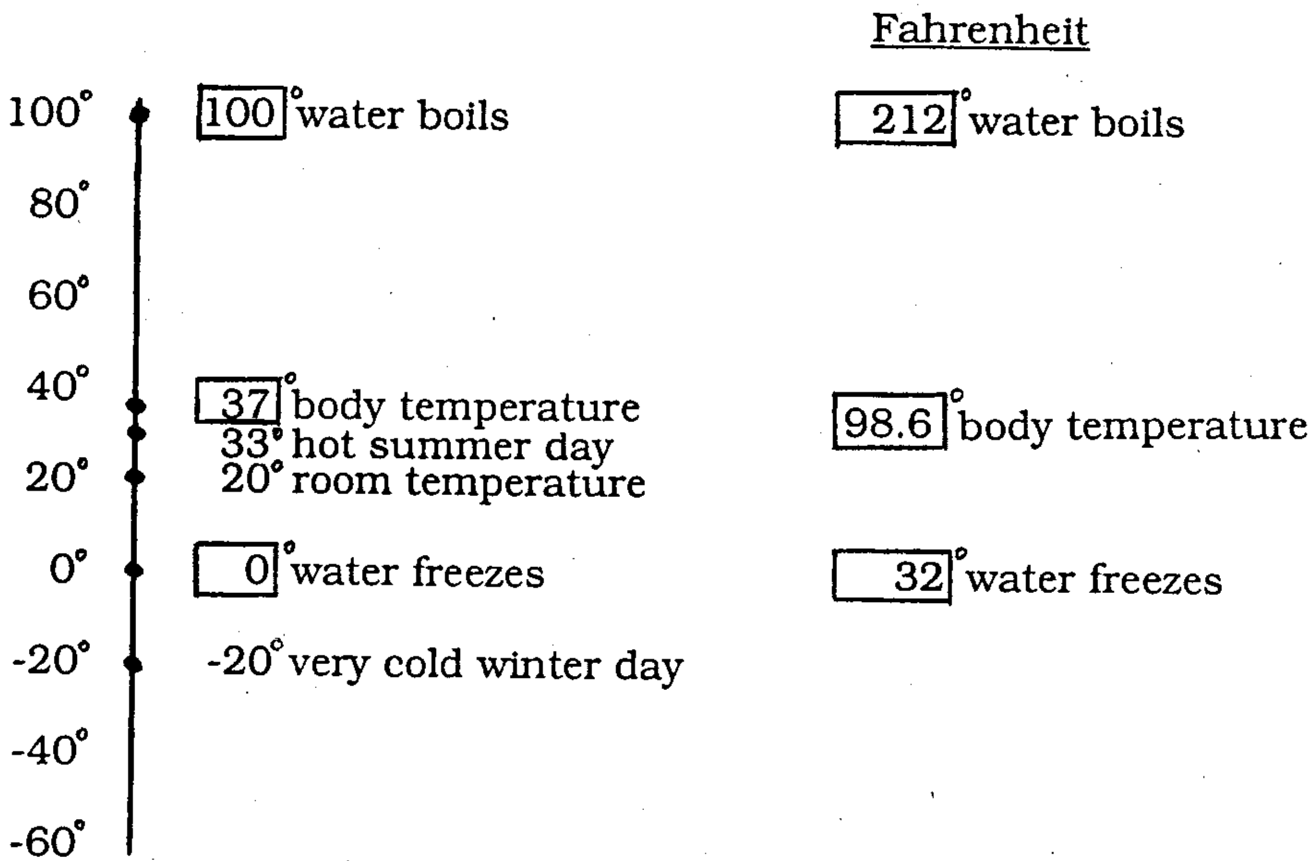
# METRIC CONVERSIONS

To do a metric conversion it is necessary to memorize the positions of the different metric measurements.

| | | | |
|---|---|---|---|
| 5 km | = | _______ m | (5000 m) |
| 2.6 cm | = | _______ km | (.000026 km) |
| .05 kℓ | = | _______ ℓ | (50ℓ) |
| 600 m | = | _______ km | (.6 km) |
| 8 cg | = | _______ g | (.08 g) |
| 50 g | = | _______ mg | (50,000 mg) |
| 2.4 m | = | _______ cm | (240 cm) |
| 3 mg | = | _______ g | (.003 g) |
| 9ℓ | = | _______ kℓ | (.009 kℓ) |
| 38.1 mm | = | _______ cm | (3.81 cm) |



# CELSIUS THERMOMETER




**Converting from Celsius to Fahrenheit**

1) multiply by 1.8
2) add 32

37°C = _____ °F
37 x 1.8 = 66.6
66.6 + 32 = 98.6

**Converting from Fahrenheit to Celsius**

1) subtract 32
2) divide by 1.8

212°F = _____ °C
212 - 32 = 180
180 ÷ 1.8 = 100

# TERMINATING / REPEATING

To determine if a fraction is equal to a terminating or repeating decimal, first reduce to simplest terms. If the prime factors of the denominator are 2's and/or 5's, the decimal will terminate.

| | | |
|---|---|---|
| 3/12 | reduce to 1/4 $4 = 2 \times 2$ | (terminating) |
| 1/12 | $12 = 2 \times 2 \times 3$ | (repeating) |

# COMPARATIVE PURCHASING

To determine the better buy, divide units into price to find the per unit cost.

Which is the better buy?

| | | |
|---|---|---|
| 3m for \$2.00 | $2.00 \div 3 = .\overline{6}$ | \$.67 per meter |
| 5m for \$3.50 | $3.50 \div 5 = .7$ | \$.70 per meter |

The first choice (3m for \$2.00) is the better buy

It may be necessary to make a metric conversion so that both choices use the same unit of metric measure.

Which is the better buy?

4kg for \$6.00
500g for \$0.70 first change 500g to .5kg

| | | |
|---|---|---|
| 4kg for \$6.00 | $6.00 \div 4 = 1.5$ | \$1.50 per kilogram |
| .5kg for \$0.70 | $0.70 \div .5 = 1.4$ | \$1.40 per kilogram |

The second choice (500g for \$.70) is the better buy

# MEASURES OF CENTRAL TENDENCY

| | |
|---|---|
| data | a series of number values to be analyzed |
| mean | average of the data |
| median | middle value |
| mode | most frequently occurring value |
| range | difference between highest and lowest value |

DATA: 3, 5, 7, 3, 4

| | | | |
|---|---|---|---|
| mean | 3 + 5 + 7 + 3 + 4 = 22 | 22 ÷ 5 = | (4 2/5) |
| median | 3 3 **4** 5 7 in order | | (4) |
| mode | **3 3** 4 5 7 | | (3) |
| range | 7 - 3 = 4 | | (4) |

Note: If there is an odd number of items in data, the median is the middle score.

Note: If one value occurs more often than all others, it is the mode.

DATA: 2, 3, 6, 4, 8, 10

| | | | |
|---|---|---|---|
| mean | 2 + 3 + 6 + 4 + 8 + 10 = 33 | 33÷6 | (5 1/2) |
| median | 2 3 **4 6** 8 10 in order | (4+6)÷2 | (5) |
| mode | 2 3 4 6 8 10 | | (none) |
| range | 10 - 2 = 8 | | (8) |

Note: If there is an even number of items in data, the median is the average of the two middle values.

Note: If no item occurs more frequently than the others, there is no mode.

# FREQUENCY TABLE

Constructing a frequency table when there are a large number of items in data helps to simplify the process of making a statistical analysis.

DATA: Spelling Test Scores

| | | | | | |
|---|---|---|---|---|---|
| 85 | 70 | 95 | 80 | 70 | 100 |
| 80 | 75 | 85 | 70 | 95 | 100 |
| 85 | 85 | 95 | 85 | 70 | 100 |
| 70 | 75 | 70 | 80 | 95 | 75 |
| 85 | 80 | 85 | 75 | 80 | 95 |


**Frequency Table**

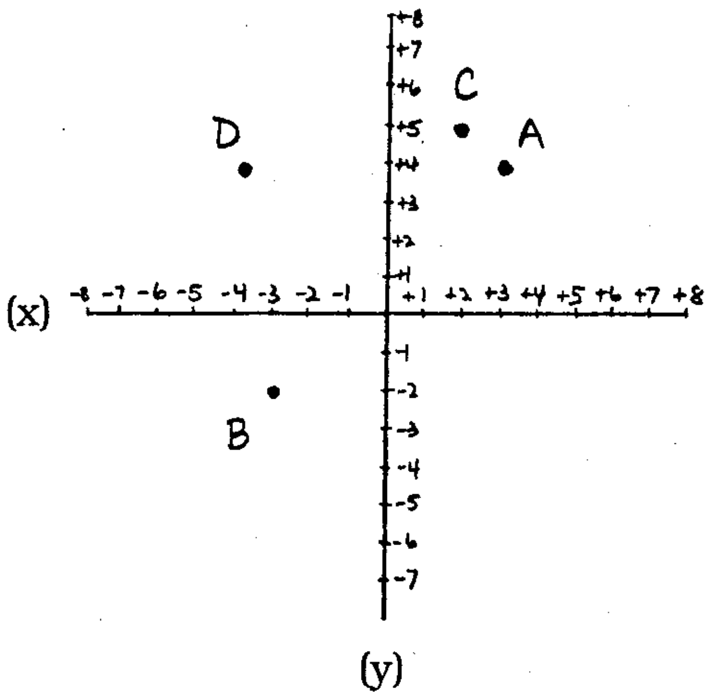
| Value | Frequency | Product | |
|---|---|---|---|
| 100 | \|\|\| | 300 | Mean<br>2490÷30<br>(83) |
| 95 | 卌 | 475 | |
| 90 | | 0 | Median<br>see below<br>(82 1/2) |
| 85 | 卌 \|\| | 595 | |
| 80 | 卌 | 400 | Mode<br>most frequent<br>(85) |
| 75 | \|\|\|\| | 300 | |
| 70 | 卌 \| | 420 | Range<br>100 - 70<br>(30) |
| | 30 | 2490 | |

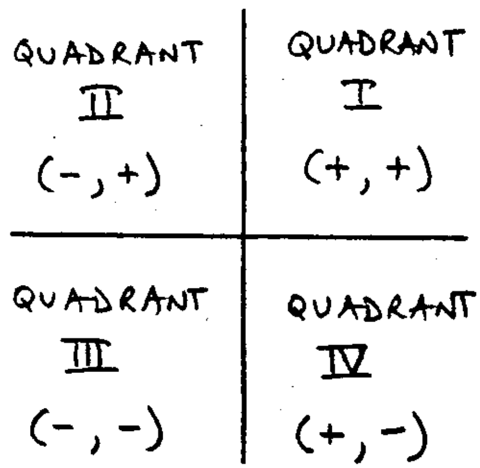
To determine the median, figure out the number of the middle item. If there are 30 items, the middle items are #15 & #16. Count frequency marks up from the bottom (or down from the top). The two middle values are 85 & 80. Average them.

# COORDINATE AXIS

The coordinate axis is divided into four numbered quadrants. We use Roman Numerals to name them.

| | |
|---|---|
| origin | the point of intersection (0,0) |
| x-axis | the horizontal axis |
| y-axis | the vertical axis |
| ordered pair | (horiziontal, vertical) (x,y) |



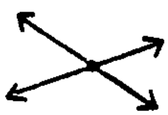
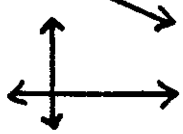
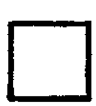

Name the ordered pairs for the indicated points:

| | |
|---|---|
| A | (3,4) |
| B | (-3,-2) |
| C | (2,5) |
| D | (-4,4) |

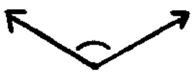
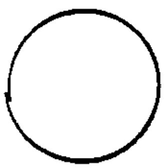
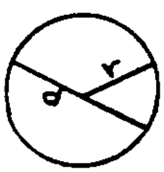
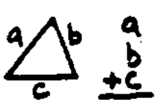
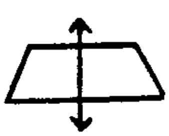
If a coordinate point lies on an axis, it is not considered to be in any of the four quadrants.

**Remember:**
When writing an ordered pair, always name the x-coordinate (horizontal) first.

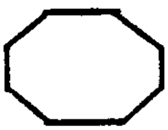
# GEOMETRIC TERMS

| | | |
|---|---|---|
|  | point | location without dimensions |
|  | line | straight set of connecting points extending to infinity in two directions |
|  | line segment | section of a line with a definite starting and ending point |
|  | ray | section of a line that extends to infinity in one direction |
|  | angle | rotation (measured in degrees) |
|  | plane | flat surface in two dimensions extending to infinity (length & width) |
|  | intersection | points in common between geometric figures |
|  | parallel lines | lines in the same plane that never intersect |
|  | perpendicular | lines intersecting to form right angles |
|  | right angle | angle that measures 90 degrees |
|  | curve | set of connecting points in a plane |
|  | closed curve | curve with a common starting and ending point (it can intersect) |
|  | simple cl. curve | closed curve that does not intersect |
|  | polygon | simple closed curve made entirely of line segments |
|  | regular polygon | polygon with all sides and angles congruent |

# GEOMETRIC TERMS (Continued)

| | | |
|---|---|---|
|  | vertex | point where an angle is formed (plural is "vertices") |
|  | acute angle | angle measuring greater than 0 and less than 90 degrees |
|  | obtuse angle | angle measuring greater than 90 and less than 180 degrees |
|  | protractor | instrument used to measure angles |
| $x^{\circ}$ | degree | unit of measure |
|  | circle | simple closed curve with all points an equal distance from the center point |
|   | diameter | distance between two points on a circle passing through the center point |
| | radius | distance from the center point to any point on a circle |
| $(2\pi r)$ | circumference | distance around a circle or partial circle |
| $(3.14)$ | pi | ratio of circumference to diameter in a circle |
|   | perimeter | distance around a polygon |
|  | line of symmetry | line dividing a region into two congruent parts |

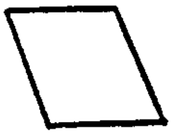
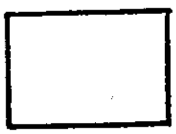
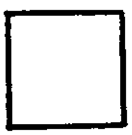
# POLYGONS

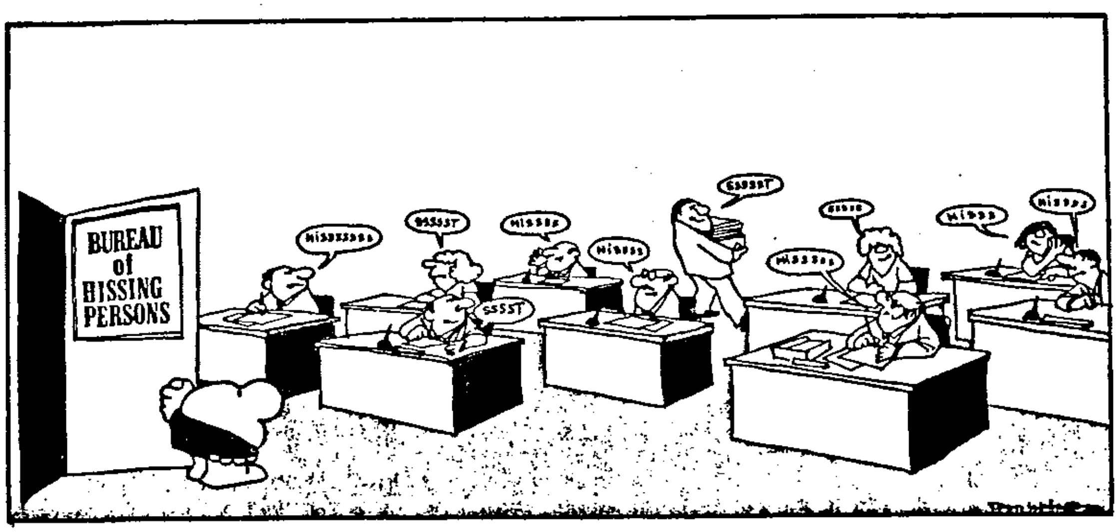
| | | |
|---|---|---|
|  | triangle | three sided polygon |
|  | quadrilateral | four sided polygon |
|  | pentagon | five sided polygon |
|  | hexagon | six sided polygon |
|  | heptagon | seven sided polygon |
|  | octagon | eight sided polygon |

# TRIANGLES

| | | |
|---|---|---|
|  | scalene triangle | triangle with no sides congruent |
|  | isosceles triangle | triangle with two sides congruent |
|  | equilateral triangle | triangle with all sides congruent |
|  | acute triangle | triangle with all acute angles |
|  | obtuse triangle | triangle with one obtuse angle |
|  | right triangle | triangle with one right angle |
|  | equiangular triangle | triangle with all angles congruent |

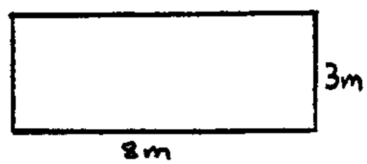
# QUADRILATERALS

| | | |
|---|---|---|
|  | quadrilateral | four sided polygon |
|  | trapezoid | quadrilateral with exactly one pair of opposite sides parallel |
|  | parallelogram | quadrilateral with two pairs of opposite sides parallel |
|  | rhombus | parallelogram with all sides congruent |
|  | rectangle | parallelogram with four right angles |
|  | square | rectangle with all sides congruent |



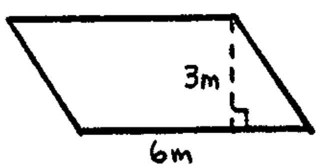
# AREA OF POLYGONS

**RECTANGLE** **A = B x H**



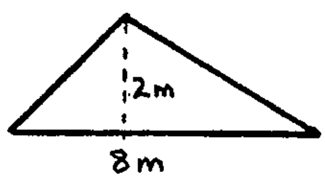

A = 8 x 3 = 24m²

**PARALLELOGRAM** **A = B x H**



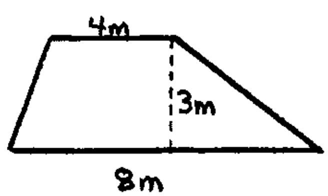

A = 6 x 3 = 18m²

**TRIANGLE** **A = (1/2)(B x H)**




A = (1/2)(8 x 2) = 8m²

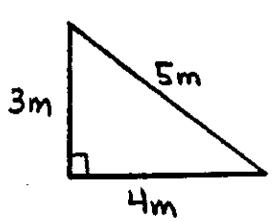
**TRAPEZOID** **A = (AVG OF BASES)(H)**



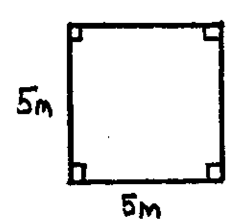

A = (1/2)(8 + 4)(3) = 18m²

# PERIMETER OF POLYGONS

Perimeter is the distance around a polygon. Sum the sides.




P = 3+4+5
P = 12m

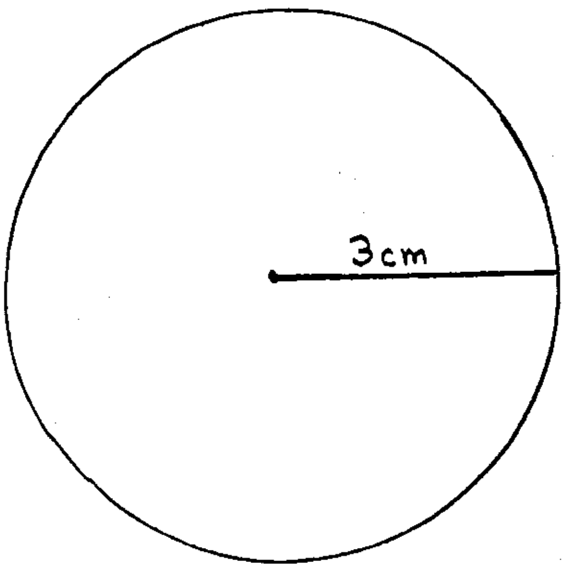



P = 5+5+5+5
P = 20m

# CIRCLES

The formula for area of a circle is $A = \pi r^2$

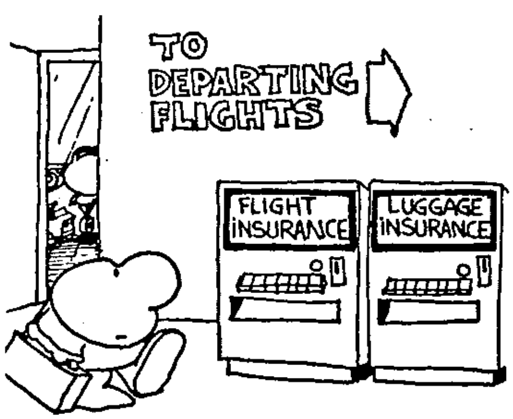
The formula for circumference of a circle is $C = 2\pi r$




**AREA**

$A = \pi r^2$
$A = (3.14)(3)^2$
$A = (3.14)(9)$
$A = 28.26 cm^2$

$A = \pi r^2$
$A = \pi (3)^2$
$A = 9\pi \ cm^2$



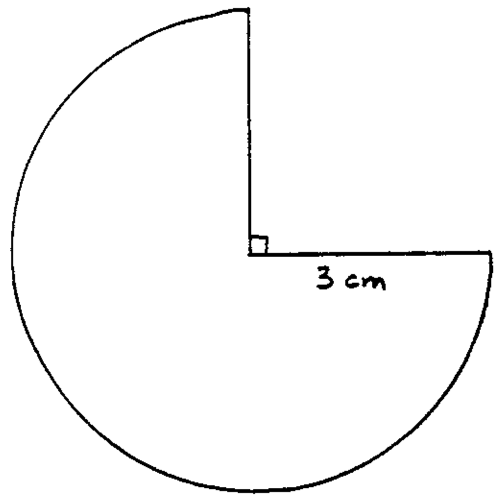

**CIRCUMFERENCE**

$C = 2\pi r$
$C = (2)(3.14)(3)$
$C = 18.84 cm$

$C = 2\pi r$
$C = (2)\pi (3)$
$C = 6\pi \ cm$

# SECTORS
## PARTIAL CIRCLES

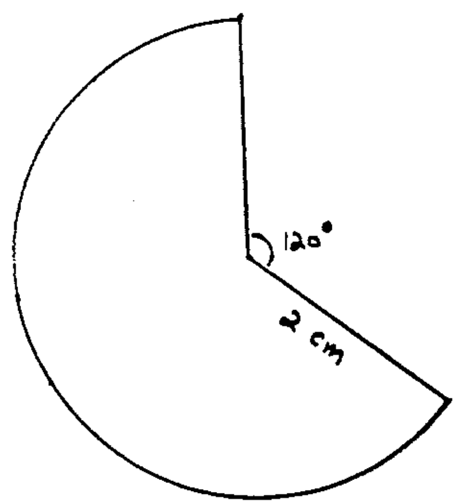
Central Angle = 270 degrees




$A = (\pi r^2)(\text{part of circle})$
$A = (3.14)(9)(270/360)$
$A = (3.14)(9)(.75) = 21.195\text{cm}^2$
$A = (\pi)(9)(.75) = 6.75\pi \text{ cm}^2$

$C = (2\pi r)(\text{part of circle}) + (2r)$
$C = (2)(3.14)(3)(.75) + (2)(3) = 20.13\text{cm}$
$C = (2)(\pi)(3)(.75) + (2)(3) = 4.5\pi + 6\text{cm}$

Central Angle = 240 degrees




$A = (\pi r^2)(\text{part of circle})$
$A = (3.14)(4)(240/360)$
$A = (3.14)(4)(2/3) = 8.37\overline{3}\text{cm}^2$
$A = (\pi)(4)(2/3) = 2.\overline{6}\pi \text{ cm}^2$

$C = (2\pi r)(\text{part of circle}) + (2r)$
$C = (2)(3.14)(2)(2/3) + (2)(2) = 12.37\overline{3}\text{cm}$
$C = (2)(\pi)(2)(2/3) + (2)(3) = 2.\overline{6}\pi + 6\text{cm}$

Note: When calculating the circumference of a sector, you have to add as many radius measures as appear in the diagram. In the above examples, there are only two radius measures in each. In some problems, there will be more than two.

# SOLVING EQUATIONS

## BASIC EQUATION

$a - 7 = 5$

$a + (-7) = 5$

$a + (-7) + 7 = 5 + 7$

$a = 12$

Check:

$a - 7 = 5$

$(12) - 7 = 5$

$5 = 5$

## VARIABLE ON THE RIGHT

$3 = x + 4$

$3 + (-4) = x + 4 + (-4)$

$-1 = x$

$x = -1$

Check:

$3 = x + 4$

$3 = (-1) + 4$

$3 = 3$

## VARIABLE WITH COEFFICIENT

$6n = 14$

$(1/6)(6n) = (1/6)(14)$

$n = 14/6$

$n = 7/3$

Check:

$6n = 14$

$6(7/3) = 14$

$14 = 14$

# SOLVING EQUATIONS (Continued)

### FRACTIONAL COEFFICIENT

$3a/5 = 4$

$(5/3)(3a/5) = (5/3)(4)$

$a = 20/3$

### VARIABLE ON BOTH SIDES

$4x + 3 = 7 - 2x$

$4x + 3 = 7 + (-2x)$

$4x + 3 + 2x = 7 + (-2x) + 2x$

$6x + 3 = 7$

$6x + 3 + (-3) = 7 + (-3)$

$6x = 4$

$(1/6)(6x) = (1/6)(4)$

$x = 4/6$

$x = 2/3$

### DISTRIBUTIVE PROPERTY

$2(5n - 3) = -6n$

$10n - 6 = -6n$

$10n + (-6) = -6n$

$10n + (-6) + 6n = -6n + 6n$

$16n + (-6) = 0$

$16n + (-6) + 6 = 0 + 6$

$16n = 6$

$(1/16)(16n) = (1/16)(6)$

$n = 6/16 = 3/8$

### ELIMINATE FRACTIONS

$6x - 3x/5 = 7$

$5\ (6x - 3x/5 = 7)$

$30x - 3x = 35$

$27x = 35$

$(1/27)(27x) = (1/27)(35)$

$x = 35/27$

# INTEGER PROBLEMS

Find three consecutive integers whose sum is 57:

| | |
|---|---|
| $x$ | $x + (x+1) + (x+2) = 57$ |
| $x+1$ | $3x + 3 = 57$ |
| $x+2$ | $3x = 54$ |

$x = 18 \quad x+1 = 19 \quad x+2 = 20$

Find the largest of four consecutive even integers whose sum is 44:

| | |
|---|---|
| $x$ | $x + (x+2) + (x+4) + (x+6) = 44$ |
| $x+2$ | $4x + 12 = 44$ |
| $x+4$ | $4x = 32$ |
| $x+6$ | $x = 8$ |

$x+6 = 14$

Find the middle integer of three consecutive odd integers if the largest is 7 more than twice the smallest:

| | |
|---|---|
| $x$ | $(x+4) = 2x + 7$ |
| $x+2$ | $-3 = x$ |
| $x+4$ | $x = -3$ |

$x+2 = -1$

Find a number if twice the number exceeds three less than the number by ten:

| | |
|---|---|
| $x$ | $2x - (x-3) = 10$ |
| | $2x - x + 3 = 10$ |
| | $x + 3 = 10$ |

$x = 7$

# SOLVING INEQUALITIES

Use the same procedure to solve inequalities as you use to solve equations with this exception:

The Inequality Sign Changes

When you multiply (or divide) the entire inequality by a negative value (see the third to the last step below)

When you switch the variable from the right to the left (see the last two steps below)

Solve This Inequality:

$4 - (2a/5) < -3a$

$4 + (-2a/5) < -3a$

$5\left[4 + (-2a/5) < -3a\right]$

$20 + (-2a) < -15a$

$20 + (-2a) + 2a < -15a + 2a$

$20 < -13a$

$(-1/13)(20) > (-1/13)(-13a)$

$-20/13 > a$

$a < -20/13$

# ABSOLUTE VALUE

The absolute value of a number is its distance from "0" on the number line. It is always a positive value.

Find the absolute value:

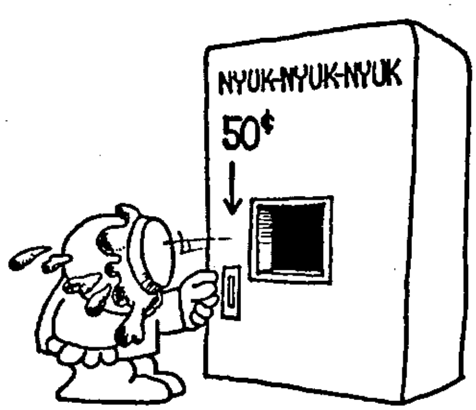
$|+5| =$ (+5)

$|-5| =$ (+5)

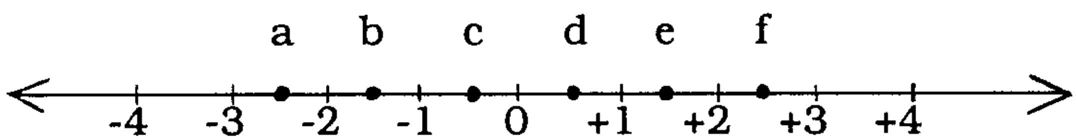
$-|+5| =$ (-5)

$|-5^2| =$ (+25)

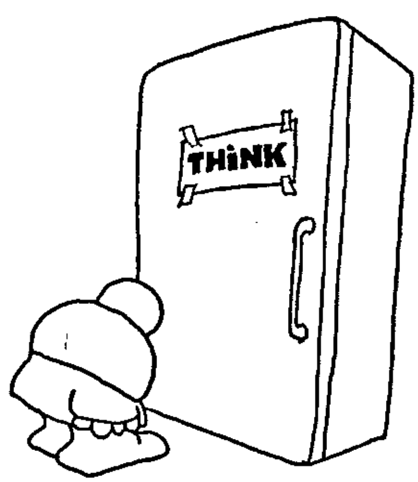
$|-5|^3 =$ (+125)



# NUMBER LINE PROBLEMS




| | | | |
|---|---|---|---|
| e | ☐ | b | (>) |
| d | ☐ | $b^2$ | (<) |
| $c^2$ | ☐ | 1 | (<) |
| d-e | ☐ | $c^2$ | (<) |
| ab | ☐ | $d^3$ | (>) |
| e/a | ☐ | $-b^2$ | (>) |
| a(b-c) | ☐ | $(d-e)^3$ | (>) |



# UNDERSTANDING RADICALS

The square root of 64 (+8 or -8)

Radical 64 ($\sqrt{64}$) = +8 the principal (non-negative) root

Vocabulary Terms

| | |
|---|---|
| radical | the symbol ($\sqrt{\ }$) |
| radicand | the value under the radical |
| index | number indicating root to be taken |
| principal root | non-negative square root |

Note: When the index is "2" it is not written.

Solve:

$$\sqrt{25} = (5)$$

$$\sqrt{\frac{9}{16}} = \frac{\sqrt{9}}{\sqrt{16}} = \frac{3}{4}$$

$$\sqrt{\frac{10}{49}} = \frac{\sqrt{10}}{\sqrt{49}} = \frac{\sqrt{10}}{7}$$


# SIMPLIFYING RADICALS

Simplify:

$$\sqrt{72} = \sqrt{2 \times 2 \times 2 \times 3 \times 3} = 2 \times 3\sqrt{2} = 6\sqrt{2}$$

$$\sqrt{150} = \sqrt{2 \times 3 \times 5 \times 5} = 5\sqrt{2 \times 3} = 5\sqrt{6}$$

$$\frac{\sqrt{32}}{4} = \sqrt{\frac{2 \times 2 \times 2 \times 2 \times 2}{4}} = \frac{2 \times 2\sqrt{2}}{4} = \frac{4\sqrt{2}}{4} = \sqrt{2}$$

$$\frac{\sqrt{80}}{6} = \frac{\sqrt{2 \times 2 \times 2 \times 2 \times 5}}{6} = \frac{2 \times 2\sqrt{5}}{6} = \frac{4\sqrt{5}}{6} = \frac{2\sqrt{5}}{3}$$

# RADICAL OPERATIONS

## ADDING & SUBTRACTING

When adding and subtracting, you can only combine terms that have "like" radicals.

$\sqrt{3} + \sqrt{3} =$ $(2\sqrt{3})$

$5\sqrt{7} + \sqrt{7} + 2\sqrt{7} =$ $(8\sqrt{7})$

$16\sqrt{3} - 5\sqrt{3} =$ $(11\sqrt{3})$

$2\sqrt{3} + 2\sqrt{18} + 3\sqrt{2}$

$2\sqrt{3} + 6\sqrt{2} + 3\sqrt{2} =$ $(2\sqrt{3} + 9\sqrt{2})$

## MULTIPLYING & DIVIDING

When multiplying and dividing, you do not need "like" radicals.

$\sqrt{3} \times \sqrt{3} = \sqrt{9} =$ $(3)$

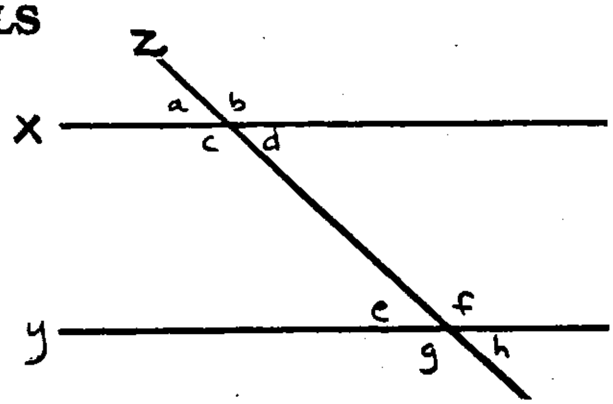
$3\sqrt{5} \times 4\sqrt{7} =$ $(12\sqrt{35})$

$\sqrt{12} \times \sqrt{6} = \sqrt{72} =$ $(6\sqrt{2})$

$3\sqrt{2} \times 5\sqrt{8} = 15\sqrt{16} =$ $(60)$

# ANGLE RELATIONSHIPS

## PARALLELS AND TRANSVERSALS




$x \parallel y$ x and y are parallel

z is a transversal

<u>Vocabulary Terms</u>

| | |
|---|---|
| corresponding angles | angles in the exact same position (corresponding angles are equal) |
| vertical angles | angles on opposite sides of intersecting lines (equal) |
| alternate interior angles | angles between the parallel lines on opposite sides of the transversal (alt. int. angles are equal) |
| supplementary angles | angles that sum to 180 degrees |
| complementary angles | angles that sum to 90 degrees |
| linear pair | two adjacent supplementary angles |

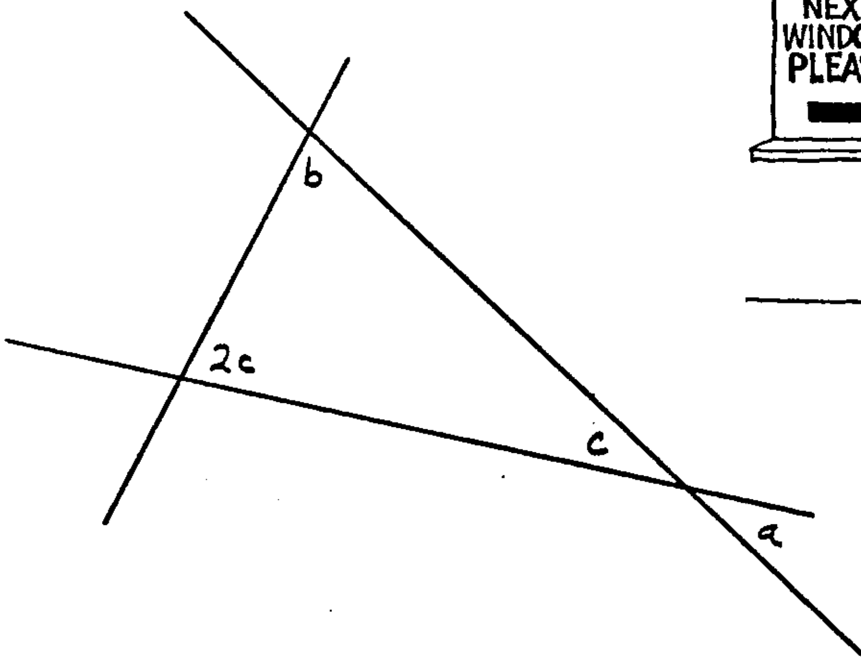
If g = 130 degrees, find d:

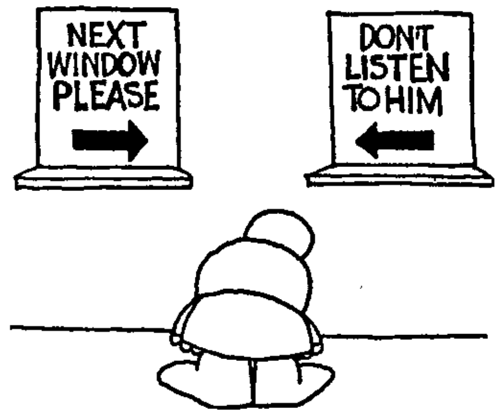
| | |
|---|---|
| $g + h = 180$ | g and h are supplementary |
| $130 + h = 180$ | substitution property of equality |
| $h = 50$ | |
| $h = d$ | corresponding angles are equal |
| $d = 50$ | substitution property of equality |

$d = 50$ degrees

# ANGLE RELATIONSHIPS

## TRIANGLES




If a = 40 degrees, find b:

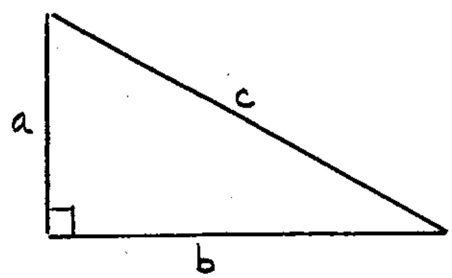
| | |
|---|---|
| $a = c$ | vertical angles are equal |
| $c = 40$ | substitution property of equality |
| $2c = 80$ | |
| $c + 2c + b = 180$ | angles of a triangle sum to 180 |
| $120 + b = 180$ | substitution property of equality |
| $b = 60$ | |

b = 60 degrees

# PYTHAGOREAN THEOREM

In a right triangle, the legs are the sides adjacent to the right angle. The hypotenuse is the side opposite the right angle.

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the legs.




**Pythagorean Theorem**

$a^2 + b^2 = c^2$

Solve these triangles:

a = 4cm
b= 6cm

$4^2 + 6^2 = c^2$
$16 + 36 = c^2$
$52 = c^2$
$c = \sqrt{52}$
$c = \sqrt{2 \times 2 \times 13}$

$c = 2\sqrt{13}$ cm

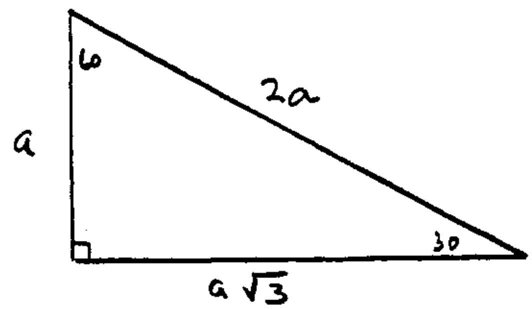
a = 7cm
c = 10 cm

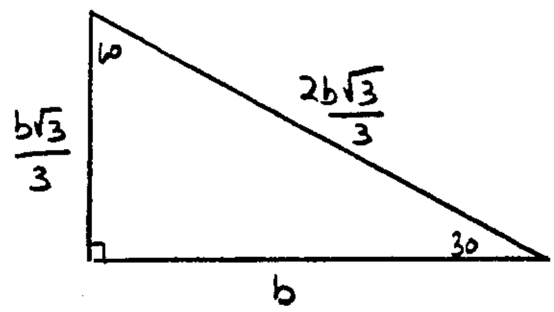
$7^2 + b^2 = 10^2$
$49 + b^2 = 100$
$b^2 = 51$

$b = \sqrt{51}$ cm

# SPECIAL RIGHT TRIANGLES

## 30-60-90 RIGHT TRIANGLE




In a 30-60-90 right triangle, the hypotenuse is exactly twice the length of the short leg. The long leg is $\sqrt{3}$ times as long as the short leg.

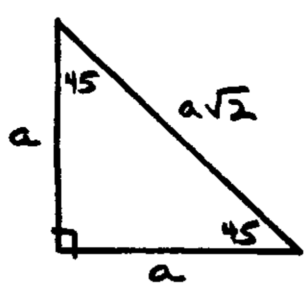
Solve, given a = 6m

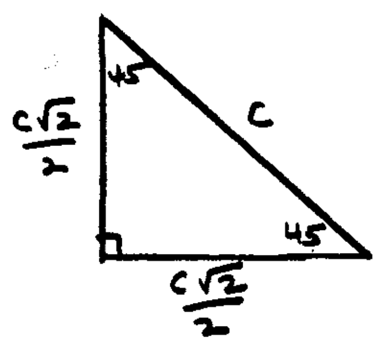
$b = 6\sqrt{3}$ m long leg
2a = 12m hypotenuse

Solve, given b = 8m

$(b\sqrt{3} / 3) = (8\sqrt{3} / 3)$m
$(2b\sqrt{3} / 3) = (16\sqrt{3} / 3)$m

## 45-45-90 ISOSCELES RIGHT TRIANGLE




In a 45-45-90 right triangle, the legs are equal to each other, and the hypotenuse is $\sqrt{2}$ times the length of the legs.

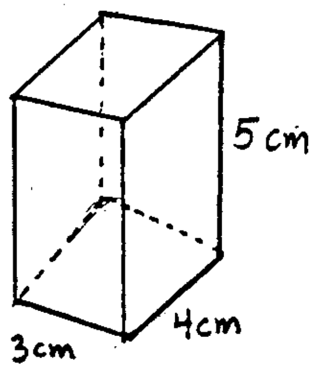
Solve, given a = 4m

$a\sqrt{2} = 4\sqrt{2}$ m

Solve, given c = 8m

$(c\sqrt{2} / 2) = 4\sqrt{2}$ m

# RECTANGULAR PRISM




8 vertices
12 edges
8 base edges
4 lateral edges
2 bases
6 total faces
4 lateral faces

**Determine VOLUME**

Base Area = 4 x 3 = 12cm

Height = 5cm

Volume = Base Area x Height
12 x 5

Volume = $60cm^3$

Note: Volume is measured in <u>cubic</u> units

**Determine SURFACE AREA**

Base Area = 4 x 3 = $12cm^2$
Perimeter of the base = 14cm
Height = 5cm

SA = 2(base area) + (Per x Ht)
SA = (2)(12) + (14)(5)

Surface Area = $94cm^2$

Note: Surface Area is measured in square units

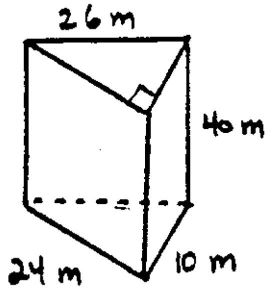
**VOLUME** (cubic units)

Volume = Base Area x Height

**SURFACE AREA** (square units)

SA = 2(Base Area) + (Per x Ht)

# TRIANGULAR PRISM




6 vertices
9 edges
6 base edges
3 lateral edges
2 bases
5 total faces
3 lateral faces

**Determine VOLUME**

Base Area = (1/2)(24)(10)
120$m^2$

Height = 40m

Volume = Base Area x Height
120 x 40

Volume = 4800$m^3$

Note: Volume is measured in cubic units

**Determine SURFACE AREA**

Base Area = 120$m^2$ (see left)
Perimeter of the base = 60m
Height = 40m

SA = 2(Base Area) + (Per x Ht)
SA = (2)(120) + (60)(40)

Surface Area = 2640$m^2$

Note: Surface Area is measured in square units

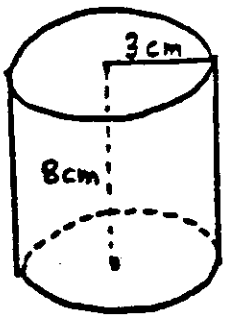
**VOLUME** (cubic units)

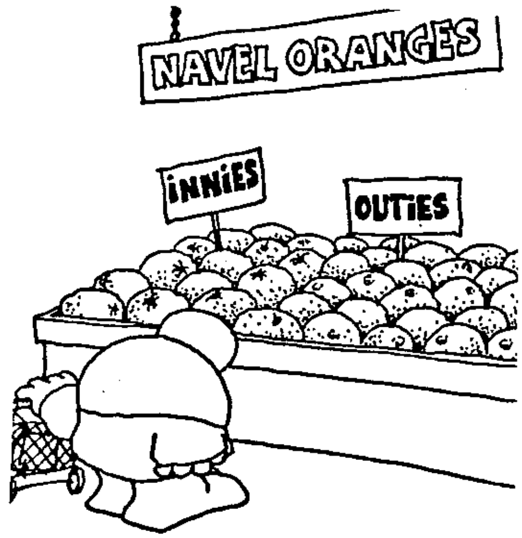
Volume = Base Area x Height

**SURFACE AREA** (square units)

SA = 2(Base Area) + (Per x Ht)

# CYLINDER




**Determine VOLUME**

Base Area = $\pi r^2 = 9\pi$

Volume = Base Area x Height

Volume = $(9\pi)(8)$

Volume = $\boxed{72\pi \text{ cm}^3}$

Note: Volume is measured in cubic units

**Determine SURFACE AREA**

Base Area = $\pi r^2 = 9\pi$
Circumference = $2\pi r = 6\pi$

SA = 2(Base Area) + (Cir x Ht)
SA = $(2)(9\pi) + (6\pi \times 8)$

Surface Area = $\boxed{66\pi \text{ cm}^2}$

Note: Surface Area is measured in square units

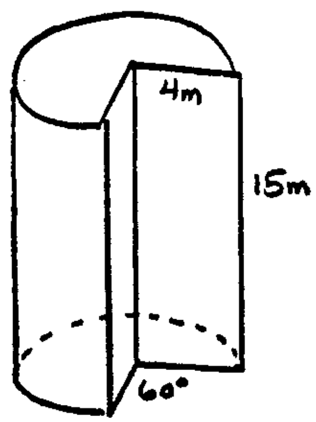
**VOLUME** (cubic units)

Volume = Base Area x Height

**SURFACE AREA** (square units)

SA = 2(Base Area) + (Cir x Ht)

# PARTIAL CYLINDER




Partial Circle Base
Central Angle

$\frac{300}{360} = \frac{5}{6}$

**Determine VOLUME**

Base Area = $(\pi r^2)$(part)
Base Area = $(16\pi)(5/6)$
Base Area = $(40/3)\pi$

Volume = Base Area x Height
Volume = $(40/3)\pi \times (15)$

Volume = $\boxed{200\pi m^3}$

Note: Volume is measured in cubic units and surface area is measured in square units

**Determine SURFACE AREA**

Base Area = $(40/3)\pi$ (see left)

Circumference = $(2\pi r)$(part) + $(2r)$
Circumference = $(8\pi)(5/6) + (8)$
Circumference = $(20/3)\pi + 8$

SA = 2(Base Area) + (Cir + Ht)
SA = $(2)(40/3)(\pi) + (20/3\pi + 8)(15)$
SA = $(80/3)(\pi) + (100\pi) + (120)$

SA = $\boxed{380/3\,\pi + 120m^2}$

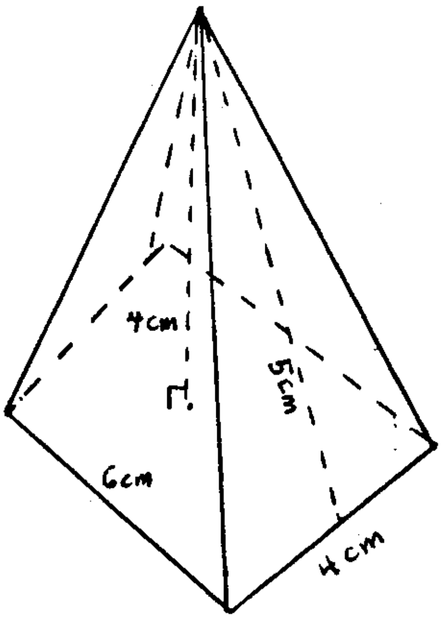
**VOLUME** (cubic units)

Volume = Base Area x Height

**SURFACE AREA** (square units)

SA = 2(Base Area) + (Cir x Ht)

# PYRAMIDS, CONES, & SPHERES




**PYRAMID VOLUME**

V = (1/3)(Base Area)(Height)
V = (1/3)(6 x 4)(4) = $\boxed{32cm^3}$

**PYRAMID SURFACE AREA**

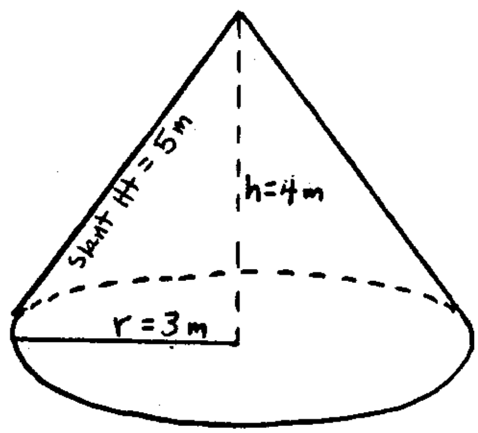
SA = (BA) + (1/2)(Per)(Slant Ht)
SA = (24) + (1/2)(20)(5) = $\boxed{74cm^2}$

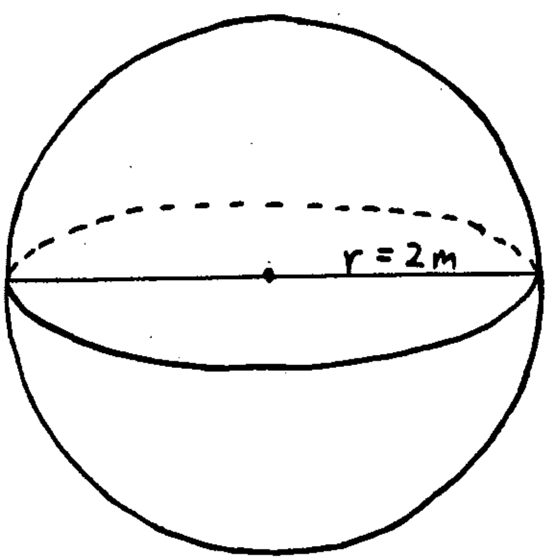
**CONE VOLUME**

V = (1/3)(Base Area)(Height)
V = (1/3)(9π)(4) = $\boxed{12\pi m^3}$

**CONE SURFACE AREA**

SA = (BA) + (1/2)(Cir)(Slant Ht)
SA = (9π) + (1/2)(6π)(5) = $\boxed{24\pi m^2}$




**SPHERE VOLUME**

V = (4/3)($\pi r^3$)
V = (4/3)(π)($2^3$) = $\boxed{32/3\ \pi m^3}$

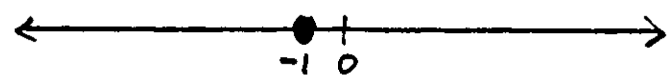
**SPHERE SURFACE AREA**

SA = $4\pi r^2$
SA = (4)(π)($2^2$) = $\boxed{16\pi m^2}$

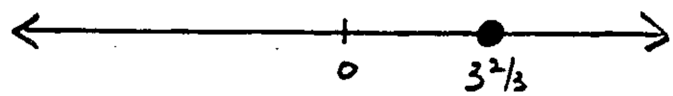
# NUMBER LINE GRAPHING

## EQUATIONS

x - 3 = -4
x = -1

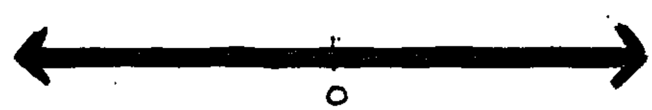



3x - 15 = -4
3x = 11
x = 11/3

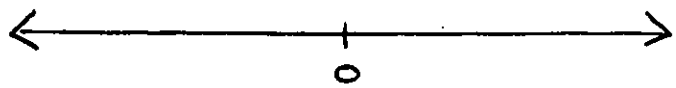



If the variable drops out of the equation entirely, there are two possibilities. If you are left with a true statement once the variable drops out, the equation is an <u>identity</u> and all solutions will work. If you are left with a false statement when the variable drops out, you have a <u>false equation</u> and no solutions will work.

3(2x - 1) = 6x - 3
6x - 3 = 6x - 3
0 = 0
Identity
All Solutions




4x + 3 = 2(1 + 2x)
4x + 3 = 2 + 4x
0 = -1
False Equation
No Solutions

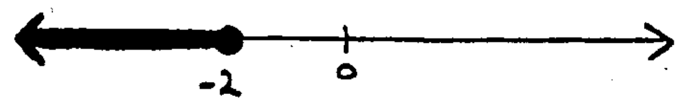


# NUMBER LINE GRAPHING

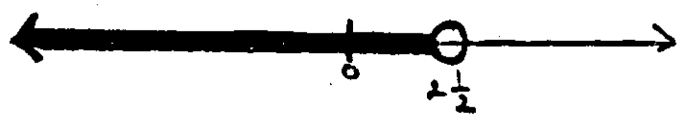
## INEQUALITIES

When graphing an <u>inequality</u> on a number line, a filled-in circle is used with the $\leq$ and $\geq$ signs to indicate that the particular point <u>is</u> part of the solution. An open circle is used with the $<$ and $>$ signs to indicate that the point <u>is</u> <u>not</u> part of the solution.

$-4x \geq 8$
$(-1/4)(-4x) \leq (-1/4)(8)$
$x \leq -2$

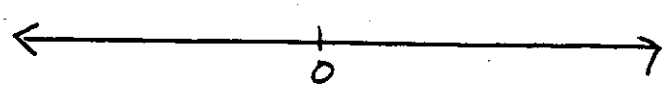



$4 - (2x/5) > 3$
$20 - 2x > 15$
$-2x > -5$
$(-1/2)(-2x) < (-1/2)(-5)$
$x < 5/2$




$3(x - 1) > 3x - 5$
$3x - 3 > 3x - 5$
$0 > -2$
Identity
All Solutions


$2(3x + 2) \leq 1 + 6x$
$6x + 4 \leq 1 + 6x$
$0 \leq -3$
False Inequality
No Solutions



# GRAPHING LINEAR EQUATIONS
## CHART METHOD

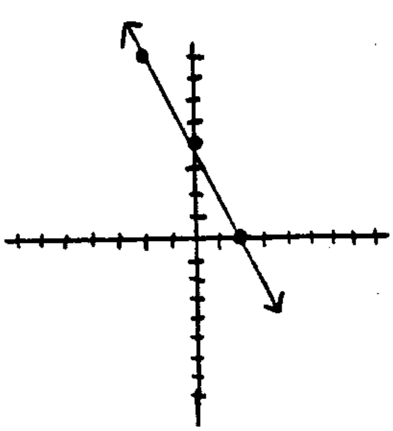
A linear equation is an equation with two variables. There are an infinite number of combinations of values that can satisfy a linear equation. These values can be named as ordered pairs and graphed on a coordinate axis.

The graph of all solutions of a linear equation will be a straight line. This line represents all possible points (ordered pairs) that satisfy the equation.

When graphing by the chart method, change the original equation to the form $y =$ Plug in any value for "x" and find the corresponding value for "y." Find at least three ordered pairs before graphing.

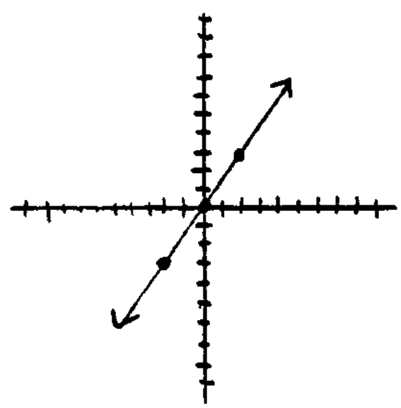
$2x + y = 4$
$y = -2x + 4$

| x | y |
|---|---|
| 2 | 0 |
| 0 | 4 |
| -2 | 8 |



$3x - 2y = 0$
$-2y = -3x$
$y = 3x/2$

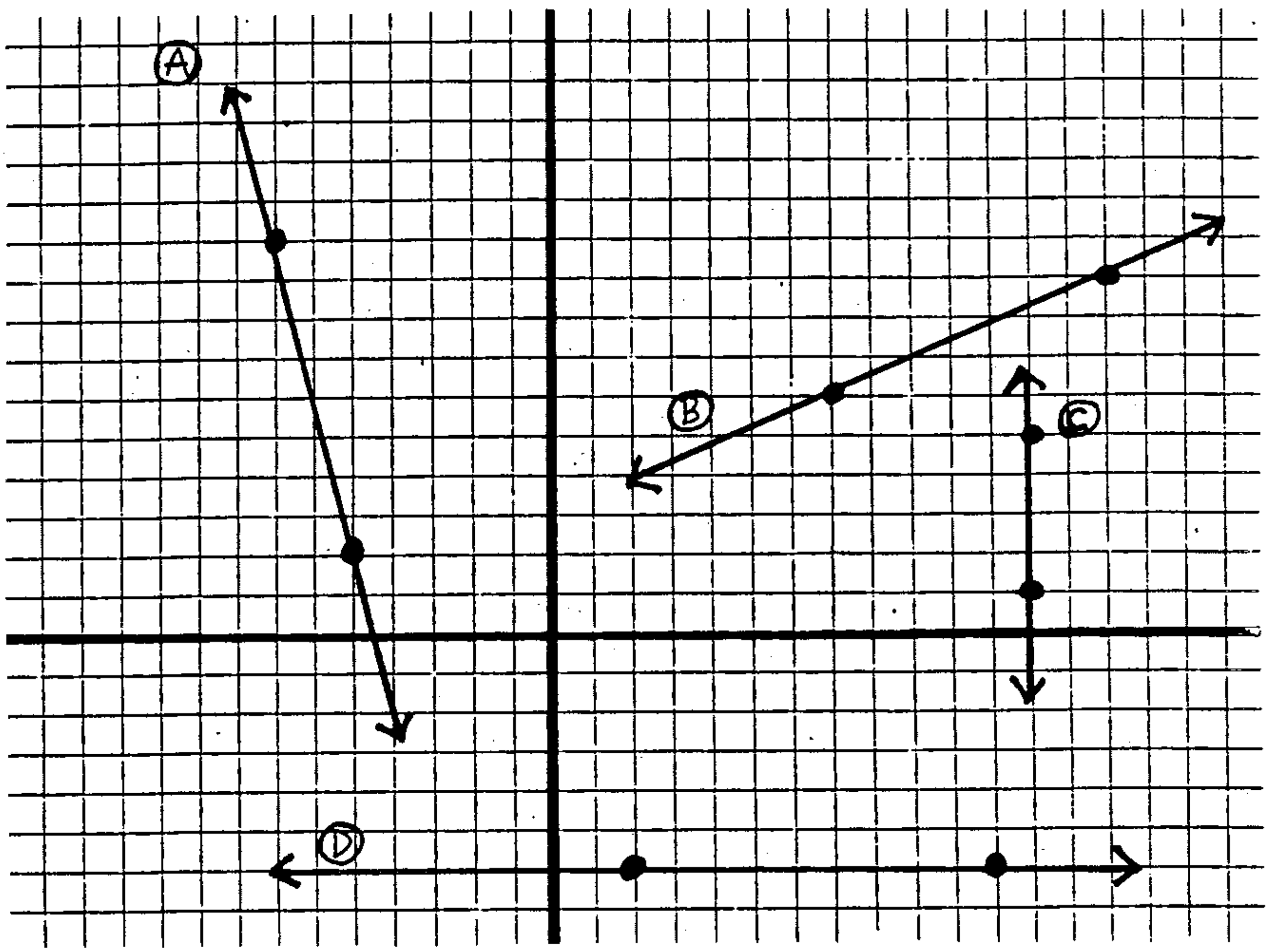
| x | y |
|---|---|
| 2 | 3 |
| 0 | 0 |
| -2 | -3 |



# DETERMINE SLOPE
## FROM A GRAPHED LINE

To determine slope, select any two points on the graph of the line. Count the "rise over run." The slope is a fraction:

| | | | |
|---|---|---|---|
| rise / run | vertical difference / horizontal difference | up (+) / right (+) | down (-) / left (-) |




Slope of (A) -8/2 = -4 — all negative slopes slant down to the right

Slope of (B) 3/7 — all positive slopes slant up to the right

Slope of (C) 4/0 = undefined — all vertical lines have <u>no slope</u> (undefined)

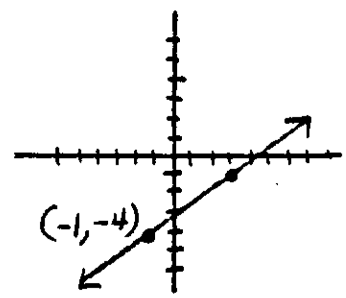
Slope of (D) 0/9 = 0 — all horizontal lines have a "0" slope

# GRAPHING LINEAR EQUATIONS
## SLOPE METHOD

Given a point and a slope, it is possible to graph a linear equation. Start by marking the known point on the graph. Then use the slope to count the rise over run to identify another point. Repeat this process until enough points are marked. (Note: "m" represents slope.)

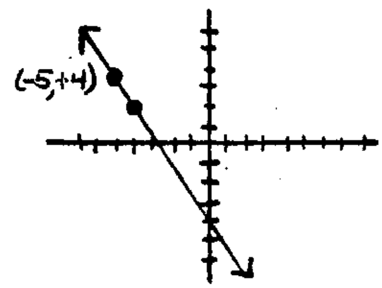
Graph
(-1,-4) m = 3/4

Count 3 up, 4 right



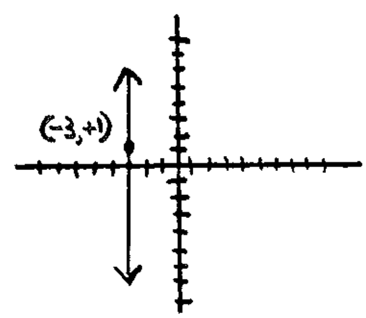

Graph
(-5,+4) m = -2

Count down 2, right 1



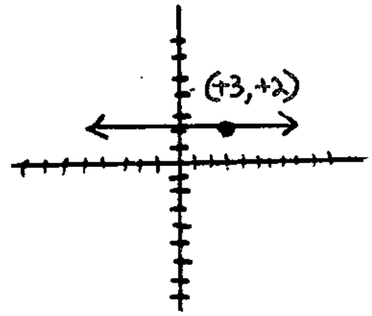

Graph
(-3,+1) m = no slope

All lines with an
undefined slope (no slope)
are vertical




Graph
(+3,+2) m = 0

All lines with a "0" slope
are horizontal



# DETERMINE SLOPE
## FROM TWO POINTS

To determine slope from two points, subtract the coordinates - always remaining consistent in moving from one point to the other. (You may start with either point, but you must subtract in the same direction - numerator and denominator.)

Determine Slope
(-6,+2) (-3,-1)

$$\frac{\text{change in rise (y)}}{\text{change in run (x)}} = \frac{(+2) - (-1)}{(-6) - (-3)} = \frac{3}{-3}$$

$$\boxed{-1}$$

Determine Slope
(-4,+3) (-4,+5)

$$\frac{\text{change in rise (y)}}{\text{change in run (x)}} = \frac{(+3) - (+5)}{(-4) - (-4)} = \frac{-2}{0}$$

$$\boxed{\text{no slope}}$$

Determine Slope
(+6,-1) (+8,-1)

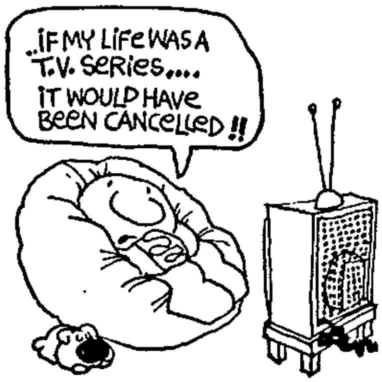
$$\frac{\text{change in rise (y)}}{\text{change in run (x)}} = \frac{(-1) - (-1)}{(+6) - (+8)} = \frac{0}{-2}$$

$$\boxed{0}$$

# SLOPE-INTERCEPT FORM

**y = mx + b**

| | |
|---|---|
| slope | m |
| y-intercept | b |
| x-intercept | -b/m |



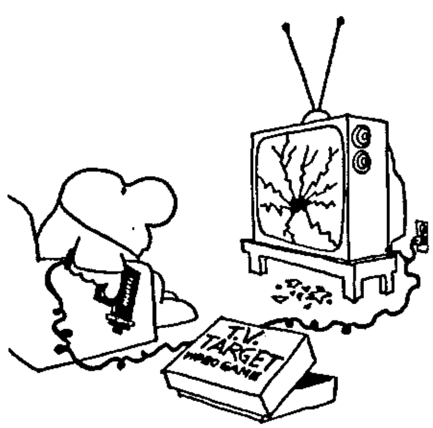

Note: In slope-intercept form, the variable "y" must have a coefficient of "1."

It is permissible to have a fractional constant or a fractional coefficient for "x."

# STANDARD FORM

**Ax + By = C**

| | |
|---|---|
| slope | -A/B |
| y-intercept | C/B |
| x-intercept | C/A |




Note: In standard form, "x" must have a positive whole number coefficient.

It is <u>not</u> permissible to have any fractional coefficients or constants.

# DETERMINE SLOPE & INTERCEPTS

## USING SLOPE-INTERCEPT & STANDARD FORM

On homework and test papers, show all steps when changing the form of the original equation. Also show work when determining complex and simplified fractions for the slope and intercepts.

Determine slope and intercepts for $y = 5x + 2$:

| Slope-Intercept Form | | Standard Form | |
|---|---|---|---|
| $y = 5x + 2$ | | $5x - y = -2$ | |
| slope (m) | 5 | slope (-A/B) | 5 |
| y-intercept (b) | 2 | y-intercept (C/B) | 2 |
| x-intercept (-b/m) | -2/5 | x-intercept (C/A) | -2/5 |

Determine slope and intercepts for $3x - 2y = 3/2$:

| Slope-Intercept Form | | Standard Form | |
|---|---|---|---|
| $y = 3x/2 - 3/4$ | | $6x - 4y = 3$ | |
| slope (m) | 3/2 | slope (-A/B) | 3/2 |
| y-intercept (b) | -3/4 | y-intercept (C/B) | -3/4 |
| x-intercept (-b/m) | 1/2 | x-intercept (C/A) | 1/2 |

Determine slope and intercepts for $x = 4$:

| Slope-Intercept Form | | Standard Form | |
|---|---|---|---|
| none | | $x = 4$ | |
| | | slope (-A/B) | no slope |
| | | y-intercept (C/B) | none |
| | | x-intercept (C/A) | 4 |

# DETERMINE SLOPE & INTERCEPTS
## USING SLOPE-INTERCEPT & STANDARD FORM

**(Continued)**

Determine slope and intercepts for -5y = 4:

| Slope-Intercept Form | | Standard Form | |
|---|---|---|---|
| y = -4/5 | | -5y = 4 | |
| slope (m) | 0 | slope (-A/B) | 0 |
| y-intercept (b) | -4/5 | y-intercept (C/B) | -4/5 |
| x-intercept (-b/m) | none | x-intercept (C/A) | none |

Determine slope and intercepts for 3x = 2y:

| Slope-Intercept Form | | Standard Form | |
|---|---|---|---|
| y = 3x/2 | | 3x - 2y = 0 | |
| slope (m) | 3/2 | slope (-A/B) | 3/2 |
| y-intercept (b) | 0 | y-intercept (C/B) | 0 |
| x-intercept (-b/m) | 0 | x-intercept (C/A) | 0 |

**Be Sure To Recognize These Forms:**

No "y" Term in the Equation
Vertical line, no slope, no y-intercept

No "x" Term in the Equation
Horizontal line, zero (0) slope, no x-intercept

No Constant in the Equation
Graph will pass through the origin, both intercepts are (0,0)

# PROPERTIES OF REAL NUMBERS

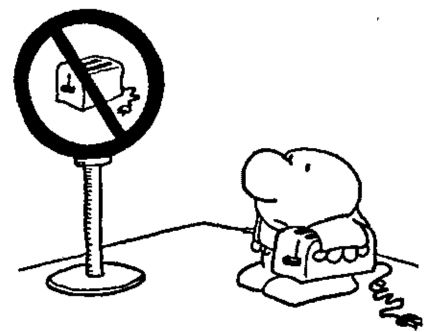
| Property | Addition | Multiplication |
|---|---|---|
| Commutative | $a+b = b+a$ | $ab = ba$ |
| Associative | $a+(b+c) = (a+b)+c$ | $a(bc) = (ab)c$ |
| Identity | $a + 0 = a$<br>additive identity (0) | $a \times 1 = a$<br>multiplicative identity (1) |
| Closure | If a and b are real numbers, a+b is real | If a and b are real numbers, ab is real |
| Zero | | $a \times 0 = 0$ |
| Inverse | $a + (-a) = 0$<br>For any number "a" the additive inverse is "-a" | $a \times (1/a) = 1$<br>For any number "a" the multiplicative inverse is "1/a" (except for a=0) |
| Distributive | $a(b+c) = ab + ac$ | $a(b+c) = ab + ac$ |

Property Names

On tests and quizzes, please refer to the properties of real numbers by using the following:

Commutative Property of Addition (or Multiplication)
Associative Property of Addition (or Multiplication)
Additive Identity / Multiplicative Identity
Closure
Zero Property
Additive Inverse / Multiplicative Inverse
Distributive Property

# PROPERTIES OF EQUALITY



Reflexive Property of Equality

For any number "a"
$a = a$

Example: $3x - 4y/2 = 3x - 4y/2$

Substitution Property of Equality

For any numbers "a" and "b"
If $a = b$, then "a" may be replaced by "b"

Example: $6 + (7^2 - 3) = 6 + (49 - 3)$

Symmetric Property of Equality

For any numbers "a" and "b"
If $a = b$ then $b = a$

Example: If $x - y = 7a$ then $7a = x - y$

Transitive Property of Equality

For any numbers "a" and "b" and "c"
If $a = b$ and $b = c$, then $a = c$

Example: If $2x + 3 = b$ and $b = n - 3$, then $2x + 3 = n - 3$

# TYPES OF NUMBERS

| | |
|---|---|
| NATURAL NUMBERS | Counting numbers: 1, 2, 3 ... |
| WHOLE NUMBERS | Natural numbers and zero: 0, 1, 2, 3 ... |
| INTEGERS | Whole numbers and negative counting numbers ... -3, -2, -1, 0, 1, 2, 3 ... |
| RATIONAL NUMBERS | Whole numbers and fractions (includes repeating and terminating decimals) Examples: -4.7, 2/3, $5.3\overline{4}$ |
| IRRATIONAL NUMBERS | Only infinite decimals (non-terminating, non-repeating) Examples: $\pi$ , $\sqrt{3}$ |
| IMAGINARY NUMBERS | Values that cannot exist in the real world Examples: $\sqrt{-1}$, $\sqrt[4]{-8}$ |

| | |
|---|---|
| <u>Inclusive</u><br>Rational Numbers | Includes integers, wholes, and naturals |
| <u>Exclusive</u><br>Irrational Numbers | Exclusive of all others |
| Imaginary Numbers | Exclusive of all others |
| REAL NUMBERS | All rational and irrational numbers |

Classify the following as rational, irrational, or imaginary:

| | |
|---|---|
| -80 | rational |
| $-3.\overline{5}$ | rational |
| $-\sqrt{8}$ | irrational |
| $\sqrt{-5}$ | imaginary |
| $\sqrt{70}$ | irrational |
| $\sqrt{16}$ | rational |
| $-\sqrt{25}$ | rational |

# RADICALS & CONSECUTIVE INTEGERS

In each of the following examples, place consecutive integers in the open boxes. Note: Be careful to place integers in the correct order when dealing with negative radicals

$\square < \sqrt{164} < \square$

$\sqrt{144} < \sqrt{164} < \sqrt{169}$

$\boxed{12} < \sqrt{164} < \boxed{13}$

$\square < -\sqrt{17} < \square$

$-\sqrt{25} < -\sqrt{17} < -\sqrt{16}$

$\boxed{-5} < -\sqrt{17} < \boxed{-4}$

$\square < -\sqrt{45} < \square$

$-\sqrt{49} < -\sqrt{45} < -\sqrt{36}$

$\boxed{-7} < -\sqrt{45} < \boxed{-6}$

# INTERPOLATING RADICALS

## SQUARES & SQUARE ROOTS

| $N$ | $N^2$ | $\sqrt{N}$ | $N$ | $N^2$ | $\sqrt{N}$ |
|---|---|---|---|---|---|
| **1** | 1 | 1.000 | **51** | 2601 | 7.141 |
| **2** | 4 | 1.414 | **52** | 2704 | 7.211 |
| **3** | 9 | 1.732 | **53** | 2809 | 7.280 |
| **4** | 16 | 2.000 | **54** | 2916 | 7.348 |
| **5** | 25 | 2.236 | **55** | 3025 | 7.416 |
| **6** | 36 | 2.449 | **56** | 3136 | 7.483 |
| **7** | 49 | 2.646 | **57** | 3249 | 7.550 |
| **8** | 64 | 2.828 | **58** | 3364 | 7.616 |
| **9** | 81 | 3.000 | **59** | 3481 | 7.681 |
| **10** | 100 | 3.162 | **60** | 3600 | 7.746 |
| **11** | 121 | 3.317 | **61** | 3721 | 7.810 |
| **12** | 144 | 3.464 | **62** | 3844 | 7.874 |
| **13** | 169 | 3.606 | **63** | 3969 | 7.937 |
| **14** | 196 | 3.742 | **64** | 4096 | 8.000 |
| **15** | 225 | 3.873 | **65** | 4225 | 8.062 |
| **16** | 256 | 4.000 | **66** | 4356 | 8.124 |
| **17** | 289 | 4.123 | **67** | 4489 | 8.185 |
| **18** | 324 | 4.243 | **68** | 4624 | 8.246 |
| **19** | 361 | 4.359 | **69** | 4761 | 8.307 |
| **20** | 400 | 4.472 | **70** | 4900 | 8.367 |
| **21** | 441 | 4.583 | **71** | 5041 | 8.426 |
| **22** | 484 | 4.690 | **72** | 5184 | 8.485 |
| **23** | 529 | 4.796 | **73** | 5329 | 8.544 |
| **24** | 576 | 4.899 | **74** | 5476 | 8.602 |
| **25** | 625 | 5.000 | **75** | 5625 | 8.660 |
| **26** | 676 | 5.099 | **76** | 5776 | 8.718 |
| **27** | 729 | 5.196 | **77** | 5929 | 8.775 |
| **28** | 784 | 5.292 | **78** | 6084 | 8.832 |
| **29** | 841 | 5.385 | **79** | 6241 | 8.888 |
| **30** | 900 | 5.477 | **80** | 6400 | 8.944 |
| **31** | 961 | 5.568 | **81** | 6561 | 9.000 |
| **32** | 1024 | 5.657 | **82** | 6724 | 9.055 |
| **33** | 1089 | 5.745 | **83** | 6889 | 9.110 |
| **34** | 1156 | 5.831 | **84** | 7056 | 9.165 |
| **35** | 1225 | 5.916 | **85** | 7225 | 9.220 |
| **36** | 1296 | 6.000 | **86** | 7396 | 9.274 |
| **37** | 1369 | 6.083 | **87** | 7569 | 9.327 |
| **38** | 1444 | 6.164 | **88** | 7744 | 9.381 |
| **39** | 1521 | 6.245 | **89** | 7921 | 9.434 |
| **40** | 1600 | 6.325 | **90** | 8100 | 9.487 |
| **41** | 1681 | 6.403 | **91** | 8281 | 9.539 |
| **42** | 1764 | 6.481 | **92** | 8464 | 9.592 |
| **43** | 1849 | 6.557 | **93** | 8649 | 9.644 |
| **44** | 1936 | 6.633 | **94** | 8836 | 9.695 |
| **45** | 2025 | 6.708 | **95** | 9025 | 9.747 |
| **46** | 2116 | 6.782 | **96** | 9216 | 9.798 |
| **47** | 2209 | 6.856 | **97** | 9409 | 9.849 |
| **48** | 2304 | 6.928 | **98** | 9604 | 9.899 |
| **49** | 2401 | 7.000 | **99** | 9801 | 9.950 |
| **50** | 2500 | 7.071 | **100** | 10000 | 10.000 |

To find an approximation of a value not included in the table, it is necessary to use a method called interpolation

Find $\sqrt{500}$

$$\underset{22}{\square} < \sqrt{500} < \underset{23}{\square}$$

$$\left.\begin{array}{l} 22 = \left.\begin{array}{l}\sqrt{484} \\ \sqrt{500}\end{array}\right\} 16 \\ 23 = \sqrt{529} \end{array}\right\} 45 \qquad \boxed{22\frac{16}{45}}$$

Find $\sqrt{8000}$

$$\underset{89}{\square} < \sqrt{8000} < \underset{90}{\square}$$

$$\left.\begin{array}{l} 89 = \left.\begin{array}{l}\sqrt{7921} \\ \sqrt{8000}\end{array}\right\} 79 \\ 90 = \sqrt{8100} \end{array}\right\} 179 \qquad \boxed{89\frac{79}{179}}$$

# CALCULATING SQUARE ROOTS

## PROCEDURE FOR CALCULATING SQUARE ROOTS

1. Set up problem in blocks of two digits left and right of the decimal point
2. Find the largest perfect square in the first block, write it under the first block, and put the square root in the answer
3. Subtract, bring down the next block, double the answer with an open box
4. Fill the open box with the largest digit that can double as a units digit and a multiplier
5. Put the boxed digit in the answer, multiply, and repeat steps 3-5 until enough digits exist to round the answer

Calculate $\sqrt{18}$ and round to the nearest 1/100

```
            4. 2  4  2   ≈   4.24
          √18.00 00 00
           16               16 is largest square in 18, put 4 in answer
  8 [2] |   2 00            double answer, subtract, bring down next block
            1 64            82 x 2 = 164, put 2 in answer
 84 [4] |     36 00         double answer, subtract, bring down next block
              33 76         844 x 4 = 3376, put 4 in answer
848 [2] |      2 24 00      double answer, subtract, bring down next block
               1 79 74      8482 x 2 = 17,974, put 2 in answer - round off
```

# SIMPLE INTEREST

When money is invested at simple interest, it earns the same amount of annual interest each year for the life of the investment.

INTEREST = PRINCIPAL x RATE x TIME

| Determine Simple Interest | Principal + Interest |
|---|---|
| $350 @ 6% for 1 year<br>I = $350 x .06 x 1<br>I = $21 | P + I = $371 |
| $400 @ 5% for 6 months<br>I = $400 x .05 x .5<br>I = $10 | P + I = $410 |
| $1000 @ 7.5% for 18 months<br>I = $1000 x .075 x 1.5<br>I = $112.50 | P + I = $1112.50 |
| $800 @ 4.5% for 3 months<br>I = $800 x .045 x .25<br>I = $9 | P + I = $809 |
| $500 @ 6% for 3 years<br>I = $500 x .06 x 3<br>I = $90 | P + I = $590 |

# COMPOUND INTEREST

When money is invested at compound interest, it earns "interest on the interest" that is credited to the account at the end of each compounding period.

$$\text{PRIN} + \text{INT} = \text{PRIN} \times (\text{RATE FOR 1 PERIOD})^{\text{NUMBER OF PERIODS}}$$

<u>Determine Principal + Compound Interest</u>

| | | |
|---|---|---|
| \$500 @ 8% compounded annually for 3 years | $\$500 \times (1.08)^{3}$ | \$629.86 |
| \$1200 @ 9.5% compounded annually for 4 years | $\$1200 \times (1.095)^{4}$ | \$1725.19 |
| \$800 @ 8% compounded semi-annually for 2 years | $\$800 \times (1.04)^{4}$ | \$935.89 |
| \$2000 @ 9% compounded semi-annually for 18 months | $\$2000 \times (1.045)^{3}$ | \$2282.33 |
| \$950 @ 12% compounded quarterly for 1 year | $\$950 \times (1.03)^{4}$ | \$1069.23 |
| \$5000 @ 10% compounded quarterly for 9 months | $\$5000 \times (1.025)^{3}$ | \$5384.45 |
| \$650 @ 8% compounded quarterly for 2.5 years | $\$650 \times (1.02)^{10}$ | \$792.35 |

# VOCABULARY TERMS

Absolute Value
Acute Angle
Acute Triangle
Addend
Additive Identity
Additive Inverse
Adjacent Angles
Alt. Interior Angles
Angle
Arc
Area
Associative Property
Capacity
Celsius
Central Tendency
Central Angle
Circle
Circumference
Closed Curve
Closed Sentence
Closure
Coefficient
Commutative Property
Complementary Angles
Complex Fraction
Composite Number
Compound Interest
Congruent
Constant
Coordinate Axis
Corresponding Angles
Curve
Cylinder
Data
Degree
Denominator
Diameter
Difference
Discount
Distributive Property
Dividend
Divisor
Edge
Equation
Equiangular Triangle
Equilateral Triangle
Equivalent
Evaluating Expressions
Even Number
Exclusive
Exponent (Power)
Expression
Face
Factor
Fahrenheit
False Equation
False Inequality
Frequency Table
Gram
Graphing
Greatest Common Factor
Heptagon
Hexagon
Horizontal
Hypotenuse
Identity
Imaginary Numbers
Improper Fraction
Inclusive
Index
Inequality
Infinite Decimal
Infinity
Integers
Intercept
Interpolating
Intersection
Irrational Numbers
Isosceles Triangle
Lateral Face
Lateral Surface
Least Common Multiple
Legs
Line
Linear Equation
Linear Pair
Line of Symmetry
Line Segment
Liter
Mean
Median
Meter
Minuend
Mixed Numeral
Mode
Multiple
Multiplicative Identity
Multiplicative Inverse
Natural Numbers
Numerator
Obtuse Angle
Obtuse Triangle
Octagon
Odd Number
Open Sentence
Order of Operations
Ordered Pair
Origin
Original Price
Parallel Lines
Parallelogram
Partial Circle
Pentagon
Percent
Perimeter
Perpendicular Lines
Pi
Plane
Point
Polygon
Prime Factorization
Prime Number
Principal
Product
Proportion
Protractor
Purchase Price
Pythagorean Theorem
Pythagorean Triple
Quadrant
Quadrilateral
Quotient
Radical
Radicand
Radius
Range
Rate of Discount
Ratio
Rational Numbers
Ray
Real Numbers
Reciprocal
Rectangle
Rectangular Prism
Reflexive Property
Regular Polygon
Regular Price
Repeating Decimal
Rhombus
Right Angle
Right Triangle
Scalene Triangle
Sector
Selling Price
Semi-circle
Similar Polygons
Simple Closed Curve
Simple Interest
Simplifying Expressions
Slope
Slope-Intercept Form
Square
Standard Form
Straight Angle
Substitution
Subtrahend
Sum
Supplementary Angles
Suface Area
Symmetric Property
Term
Terminating Decimal
Transitive Property
Transversal
Trapezoid
Triangle
Triangular Prism
Undefined Value
Variable
Vertex
Vertical
Vertical Angles
Volume
Whole Numbers
Zero Property

# DEFINITIONS

| | |
|---|---|
| Absolute Value | The positive value of a real number (distance from 0 on a number line) |
| Acute Angle | Angle measuring greater than 0 and less than 90 degrees |
| Acute Triangle | A triangle with three acute angles |
| Addend | A number added to another number |
| Additive Identity | Zero is the additive inverse (a + 0 = a) |
| Additive Inverse | The sum of any number and its additive identity is zero (opposite) |
| Adjacent Angles | Angles next to each other |
| Alt. Interior Angles | Angles between two parallel lines on opposite sides of a transversal |
| Angle | Rotation (measured in degrees) between two rays with a common endpoint |
| Arc | Section of the circumference of a circle |
| Area | The number of square units needed to cover a surface |
| Associative Property | For addition: (a+b)+c=a+(b+c) / For multiplication: (ab)c=a(bc) |
| Capacity | The amount that can be held within a container |
| Celsius | Temperature scale based on water freezing at 0 and boiling at 100 degrees |
| Central Angle | Angle formed by two radii of a circle |
| Central Tendency | Statistical measures (mean, median, mode, range) |
| Circle | Simple closed curve with all points an equal distance from the center point |
| Circumference | The distance around a circle or partial circle |
| Closed Curve | Curve with a common starting and ending point - no loose ends (can intersect) |
| Closed Sentence | Equation or inequality with all terms being constants - no variables |
| Closure | Property indicating that all solutions for an operation are included |
| Coefficient | A value used as a multiplier for a variable |
| Commutative Property | For addition: a + b = b + a / For multiplication: ab = ba |
| Complementary Angles | Angles whose measures sum to 90 degrees |
| Complex Fraction | A fraction containing another fraction in its numerator or denominator |
| Composite Number | A number with factors other than one and itself |
| Compound Interest | Interest calculated on principal and interest already earned |
| Congruent | Equal in all respects - size, shape, etc. |
| Constant | A term within an expression that is numerical (no variable) |
| Coordinate Axis | Perpendicular number lines dividing a plane into four quadrants |
| Corresponding Angles | Angles that relate to each other by position |
| Curve | Set of connected points in a plane |
| Cylinder | Three dimensional figure with two parallel, congruent cirlces as bases |
| Data | Set of values |
| Degree | Unit of measure for angles |
| Denominator | Bottom value in a fraction (represents the whole in a ratio) |
| Diameter | Distance between two points on a circle passing through the center point |
| Difference | Solution to a subtraction problem |
| Discount | Money subtracted from the original price of an item on sale |
| Distributive Property | Distributive Property of Multiplication over Addition: a(b+c)=ab+ac |
| Dividend | Number divided by another number (inside bracket, left of sign, numerator) |
| Divisor | Number that divides into another (outside bracket, right of sign, denominator) |
| Edge | Line segment at the intersection of two faces in a three dimensional figure |
| Equation | A number sentence showing two equal expressions |
| Equiangular Triangle | Triangle with three congruent angles (also equilateral) |
| Equilateral Triangle | Triangle with three congruent sides (also equiangular) |
| Equivalent | Having equal measures |
| Evaluating Expressions | Substituting specified numbers to determine the value of an expression |
| Even Number | Any number divisible evenly by 2 (has a units digit of 0, 2, 4, 6, or 8) |
| Exclusive | Not containing or overlapping anything else |
| Exponent (Power) | Value indicating how many times the base number is used as a factor |
| Expression | An algebraic value including a term or addition/subtraction of terms |
| Face | Flat region in a three dimensional figure |
| Factor | Number that can be divided evenly into another number |

| | |
|---|---|
| Fahrenheit | Temperature scale based on water freezing at 32 and boiling at 212 degrees |
| False Equation | Equation with no solutions (variable drops out leaving false statement) |
| False Inequality | Inequality with no solutions (variable drops out leaving a false statement) |
| Frequency Table | Table constructed to organize data for computing measures of central tendency |
| Gram | Metric unit of measure for weight |
| Graphing | Showing a set of solutions on a number line or coordinate axis |
| Greatest Common Factor | The largest number that divides evenly into two or more given numbers |
| Heptagon | A seven sided polygon |
| Hexagon | A six sided polygon |
| Horizontal | Across (from side to side) |
| Hypotenuse | The side opposite the right angle in a right triangle |
| Identity | Equation or inequality for which all solutions are correct |
| Imaginary Numbers | Numbers that cannot exist in the real world (example: sq root of negative) |
| Improper Fraction | Fraction with numerator larger than denominator |
| Inclusive | Including or overlapping |
| Index | Number to upper left of radical sign indicating root to be taken |
| Inequality | Number sentence showing two expressions separated by an inequality sign |
| Infinite Decimal | A non-repeating, non-terminating decimal (example: pi, sq root of 2) |
| Infinity | Concept of boundlessness in time, space, quantity |
| Integers | Positive and negative counting numbers and zero |
| Intercept | Point of intersection between a graphed solution and one of the coordinate axis |
| Interpolating | Determining an approximate value not included in a table |
| Intersection | Point or points in common between geometric figures |
| Irrational Numbers | Non-repeating, non-terminating real numbers (set of all infinite decimals) |
| Isosceles Triangle | Triangle with two congruent sides |
| Lateral Face | Plane region of a three dimensional figure (not one of the bases) |
| Lateral Surface | All of the regions of a three dimensional figure that are not bases |
| Least Common Multiple | The smallest number that the original numbers can divide into evenly |
| Legs | Sides adjacent to the right angle in a right triangle |
| Line | Straight set of connecting points extending to infinity in two directions |
| Linear Equation | An equation in two variables for which the solution graph is a straight line |
| Linear Pair | Two adjacent supplementary angles |
| Line of Symmetry | A line dividing a region into two congruent parts |
| Line Segment | Section of a line with definite starting and ending points |
| Liter | Metric unit of measure for capacity |
| Mean | Average of the data (sum divided by number of items in data) |
| Median | Middle value in data (avg of two middle values if even number of items) |
| Meter | Metric unit of measure for length |
| Minuend | Number from which another is subtracted (top number in subtraction problem) |
| Mixed Numeral | Value expressed by a whole number and a fraction |
| Mode | Item occurring most frequently in data |
| Multiple | Number divisible evenly by the original number |
| Multiplicative Identity | One (1) is the multiplicative identity such that a x 1 = a |
| Multiplicative Inverse | Reciprocal of the original value, produces product of (1) when multiplied |
| Natural Numbers | Positive integers (counting numbers starting with 1) |
| Numerator | Top value in a fraction (represents part of a whole in a ratio) |
| Obtuse Angle | An angle measuring greater than 90 and less than 180 degrees |
| Obtuse Triangle | Triangle with one obtuse angle |
| Octagon | Eight sided polygon |
| Odd Number | Every other number starting with 1 (has units digit of 1, 3, 5, 7, or 9) |
| Open Sentence | Equation or inequality containing at least one variable |
| Order of Operations | Rules that govern order in which calculations are to be done |
| Ordered Pair | Two values specifying the horizontal and vertical coordinates (x,y) |
| Origin | The point of intersection (0,0) between the two coordinate axis |
| Original Price | The beginning price of an item before a discount is subtracted |
| Parallel Lines | Lines in the same plane that never intersect |
| Parallelogram | Quadrilateral with two sets of parallel sides |
| Partial Circle | Sector of a circle bordered by radii and arcs (part of 360 degrees) |
| Pentagon | Five sided polygon |

| | |
|---|---|
| Percent | Ratio with 100 as the bottom term (part out of 100) |
| Perimeter | Distance around a polygon or simple closed curve |
| Perpendicular Lines | Lines intersecting to form right angles |
| Pi | Ratio of the circumference of a circle to its diameter (approx. 3.14) |
| Plane | Flat surface extending to infinity in two dimensions |
| Point | Location without dimensions |
| Polygon | A simple closed curve made entirely of line segments |
| Prime Factorization | Product of prime numbers (in ascending order) producing the original value |
| Prime Number | A whole number greater than 1 with factors of only 1 and itself |
| Principal | Amount of money invested |
| Product | Solution to a multiplcation problem |
| Proportion | Comparison of two ratios |
| Protractor | Instrument used for measuring angles |
| Purchase Price | Price of an item after the discount has been subtracted |
| Pythagorean Theorem | In a right triangle, sum of the legs squared equals the hypotenuse squared |
| Pythagorean Triples | Sets of three whole numbers that can serve as sides of a right triangle |
| Quadrant | One of the four regions formed by the coordinate axis |
| Quadrilateral | Four sided polygon |
| Quotient | Solution to a division problem |
| Radical | Symbol for square (or other specified) root - indicates principal root |
| Radicand | The value under the radical sign |
| Radius | The distance from the center point to any point on a circle (half the diameter) |
| Range | The difference between the highest and lowest values in data |
| Rate of Discount | Percent of the original price deducted to determine the selling price |
| Ratio | Indicates part of a whole - fractional value |
| Rational Numbers | Set of all numbers expressed by terminating or repeating values |
| Ray | Section of a line with a definite starting point |
| Real Numbers | Set of all rational and irrational numbers |
| Reciprocal | Value which multiplied by the original gives a product of 1 (mult. inverse) |
| Rectangle | Parallelogram with four right angles |
| Rectangular Prism | Prism with parallel, congruent rectangles for bases |
| Reflexive Property | Property of equality for any number "a" such that a=a |
| Regular Polygon | Polygon with all sides and angles congruent |
| Regular Price | Price of an item before discount is deducted (original price) |
| Repeating Decimal | Decimal that does not terminate and repeats a pattern of digits to infinity |
| Rhombus | Parallelogram with all sides congruent |
| Right Angle | Angle measuring 90 degrees formed by perpendicular lines or segments |
| Right Triangle | Triangle that includes one right angle |
| Scalene Triangle | Triangle with no congruent sides |
| Sector | Section of a circle bounded by two radii and an arc |
| Selling Price | Price of an item after discount is subtracted (purchase price) |
| Semi-Circle | Exactly half of a circle |
| Similar Polygons | Polygons with all measures in direct proportion |
| Simple Closed Curve | Closed curve that does not intersect itself |
| Simple Interest | Interest calculated on original principal only for a period of time |
| Simplifying Expressions | Combining like terms in an algebraic expression |
| Slope | Rise over run, ratio of change in "y" to change in "x" in a linear equation |
| Slope-Intercept Form | Form for a linear equation: y=mx+b (m = slope, b = y-int, -b/m = x-int) |
| Square | Rectangle with all sides congruent |
| Standard Form | Form for a linear equation: Ax+By=C (-A/B = slope, C/B = y-int, C/A = x-int) |
| Straight Angle | Angle measuring 180 degrees |
| Substitution | Property of equality in which equal terms can replace each other |
| Subtrahend | A number subtracted from another number (bottom number in subtraction) |
| Sum | Solution to an addition problem |
| Supplementary Angles | Angles whose measures sum to 180 degrees |
| Surface Area | Sum of the areas of the faces of a three dimensional geometric figure |
| Symmetric Property | Property of equality for "a" and "b": if a=b then b=a |
| Term | Single value or product of coefficients and variables |
| Terminating Decimal | Decimal value with a definite number of digits |

| | |
|---|---|
| Transitive Property | Property of equality: if a=b and b=c then a=c |
| Transversal | Line or section of a line intersecting a set of parallel lines |
| Trapezoid | Quadrilateral with exactly one set of parallel sides |
| Triangle | Three sided polygon |
| Triangular Prism | Prism with two congruent, parallel triangular bases |
| Undefined Value | Any value that includes a division by zero |
| Variable | Letters or symbols representing values in an expression |
| Vertex | Point where an angle is formed (plural is vertices) |
| Vetical | Up and down, from top to bottom |
| Vertical Angles | Equal angles formed on opposite sides of intersecting lines |
| Volume | Measure of the capacity of a three dimensional figure (in cubic units) |
| Whole Numbers | Set of all positive counting numbers and zero |
| Zero Property | The product of zero (0) and any value is zero (0) |