Friendship Junior High School
Accelerated Math Program
Mr. Lavine (Room 102A)

A.T.I.M.

Advanced Topics In Mathematics

Ongoing Review Packet

UNIT 1: A B C
Polynomials

UNIT 2: A B
Determinants & Cramer's Rule

UNIT 3: A B
Graphing Equations & Inequalities

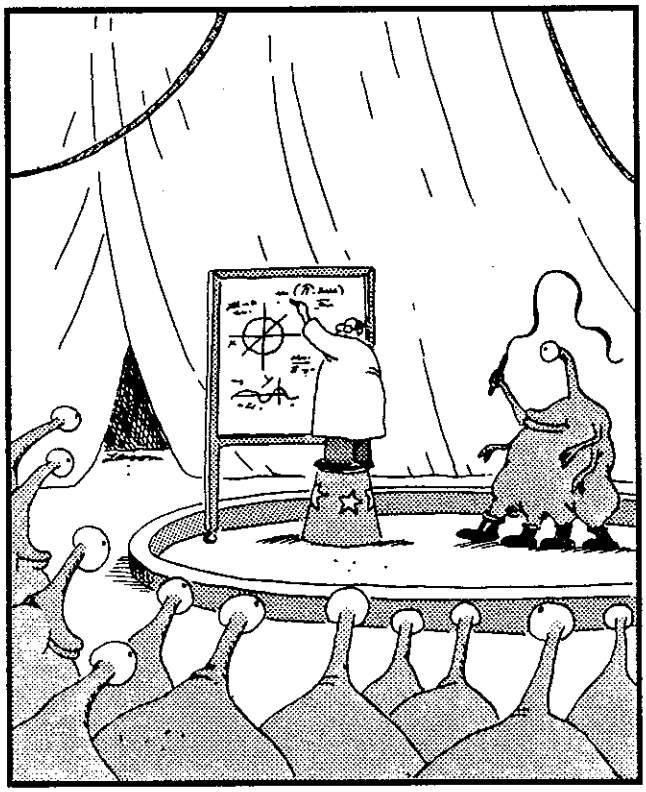
UNIT 4: A B
Systems & Linear Programming

UNIT 5: A B
Radicals & Radical Equations

UNIT 6: A B
Rational & Radical Form
Imaginary & Complex Numbers

UNIT 7: A B C
Quadratics

UNIT 8: A B
Rational Expressions & Equations



Abducted by an alien circus company, Professor Doyle is forced to solve advanced algebra problems in the center ring

Factoring

DEMONSTRATION (Review Unit 1A)

① PERFECT SQUARE TRINOMIAL

$2x^2-12xy+18y^2$
$2(x^2-6xy+9y^2)$
$2(x-3y)^2$

② TRINOMIAL: ax^2+bx+c

$3x^2+10xy+8y^2$
$3x^2+6xy+4xy+8y^2$
$3x(x+2y)+4y(x+2y)$
$(x+2y)(3x+4y)$

③ QUANTITIES

$(4a+3b)^2-(a-b)^2$
$[(4a+3b)+(a-b)][(4a+3b)-(a-b)]$
$(5a+2b)(3a+4b)$

④ PERFECT CUBE: SUM

$27a^3+8b^6$
$(3a+2b^2)(9a^2-6ab^2+4b^4)$

⑤ PERFECT CUBE: DIFFERENCE

$2a^6-128b^9$
$2(a^6-64b^9)$
$2(a^2-4b^3)(a^4+4a^2b^3+16b^6)$

⑥ GROUPING

$x^6-x^4y^2+y^6-x^2y^4$
$x^4(x^2-y^2)+y^4(y^2-x^2)$
$x^4(x^2-y^2)-y^4(x^2-y^2)$
$(x^2-y^2)(x^4-y^4)$
$(x+y)(x-y)(x^2+y^2)(x+y)(x-y)$

⑦ GROUPING: 3-1

$a^2+4ab-9x^2+4b^2$
$(a^2+4ab+4b^2)-9x^2$
$(a+2b)^2-9x^2$
$(a+2b+3x)(a+2b-3x)$

⑧ FACTOR COMPLETELY

$m^3-3m^2a+3ma^2-a^3$
$m^3-a^3-3m^2a+3ma^2$
$(m-a)(m^2+ma+a^2)-3ma(m-a)$
$(m-a)(m^2+ma+a^2-3ma)$
$(m-a)(m^2-2ma+a^2)$
$(m-a)(m-a)^2$
$(m-a)^3$

⑨ QUADRATIC FORM

n^4-10n^2+9
$(n^2)^2-10n^2+9$
$(n^2-9)(n^2-1)$
$(n+3)(n-3)(n+1)(n-1)$

Additional Practice Problems in:

1.2 and 1.3

Factoring

PROBLEM SET (Review Unit 1A)

Factor each expression:

① $(3n-m)^2-(n+5m)^2$

② $(2a+3b)^2-(a-b)^2$

③ $16x^3+2y^6$

④ $27x^3y^3+z^9$

⑤ $x^6-x^3y^3-y^6+x^3y^3$

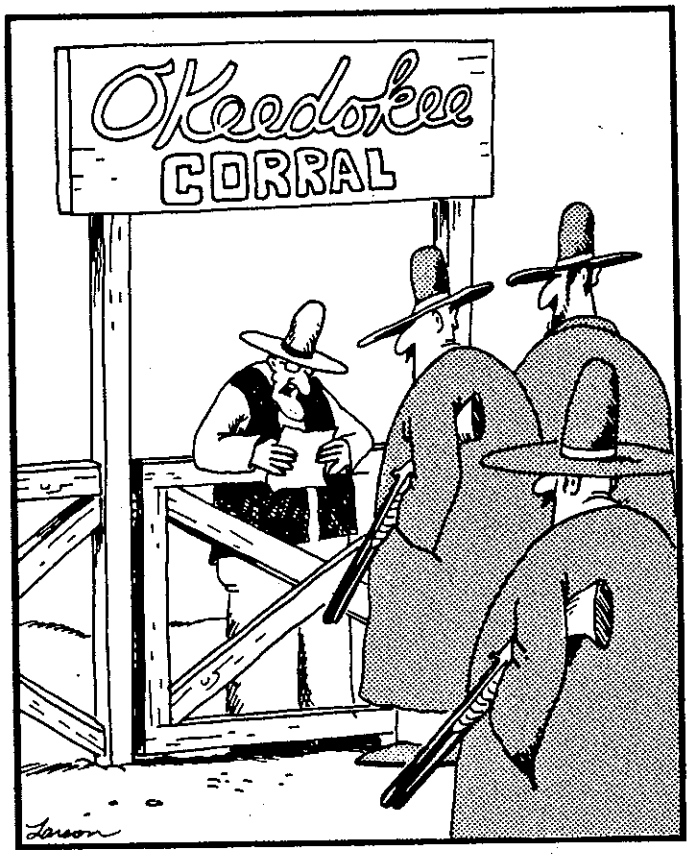
⑥ $n^5-n^3m^2-m^5+n^2m^3$

⑦ $3x^2-3y^2-3z^2-6yz$

⑧ $2a^2-18+4ab+2b^2$

⑨ $a^6-2a^3b^3+b^6$

⑩ $x^6+2x^3y^3-3y^6$



"Let's see here. ... Oh! Close, but no cigar. You want the place up the road—same as I told those other fellahs."

Ed and Barbara are visited by the insects of the Amazon Basin.

Factoring

ANSWER KEY (Review Unit 1A)

① $(3n-m)^2-(n+5m)^2$
$[(3n-m)+(n+5m)][(3n-m)-(n+5m)]$
$(4n+4m)(2n-6m)$
$4(n+m)2(n-3m)$
$8(n+m)(n-3m)$

② $(2a+3b)^2-(a-b)^2$
$[(2a+3b)+(a-b)][(2a+3b)-(a-b)]$
$(3a+2b)(a+4b)$

③ $16x^3+2y^6$
$2(8x^3+y^6)$
$2(2x+y^2)(4x^2-2xy^2+y^4)$

④ $27x^3y^3+z^9$
$(3xy+z^3)(9x^2y^2-3xyz^3+z^6)$

⑤ $x^6-x^3y^3-y^6+x^3y^3$
$x^3(x^3-y^3)-y^3(y^3-x^3)$
$x^3(x^3-y^3)+y^3(x^3-y^3)$
$(x^3-y^3)(x^3+y^3)$
$(x-y)(x^2+xy+y^2)(x+y)(x^2-xy+y^2)$

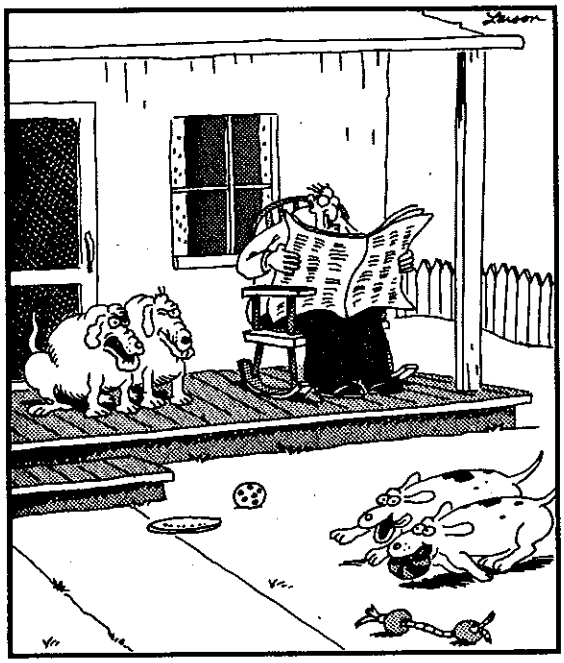
⑥ $n^5-n^3m^2-m^5+n^2m^3$
$n^3(n^2-m^2)-m^3(m^2-n^2)$
$n^3(n^2-m^2)+m^3(n^2-m^2)$
$(n^2-m^2)(n^3+m^3)$
$(n+m)(n-m)(n+m)(n^2-nm+m^2)$
$(n+m)^2(n-m)(n^2-nm+m^2)$

⑦ $3x^2-3y^2-3z^2-6yz$
$3[x^2-(y^2+2yz+z^2)]$
$3[x^2-(y+z)^2]$
$3(x+y+z)(x-y-z)$

⑧ $2a^2-18+4ab+2b^2$
$2[(a^2+2ab+b^2)-9]$
$2[(a+b)^2-9]$
$2(a+b+3)(a+b-3)$

⑨ $a^6-2a^3b^3+b^6$
$(a^3)^2-2a^3b^3+(b^3)^2$
$(a^3-b^3)^2$
$(a-b)^2(a^2+ab+b^2)^2$

⑩ $x^6+2x^3y^3-3y^6$
$(x^3)^2+2x^3y^3-3(y^3)^2$
$(x^3+3y^3)(x^3-y^3)$
$(x^3+3y^3)(x-y)(x^2+xy+y^2)$



"Man, these pups today with all their fancy balls and whatnot. ... Why back in our day, we had to play with a plain old cat's head."

Simplifying Expressions

DEMONSTRATION (Review Unit 1B)

① NEGATIVE EXPONENTS

$$\frac{(2ab)^2(a^3c)^{-2}}{(2b^2c)^{-3}}$$

$$\frac{(4a^2b^2)(a^{-6}c^{-2})}{2^{-3}b^{-6}c^{-3}}$$

$$\frac{4a^{-4}b^2c^{-2}}{2^{-3}b^{-6}c^{-3}} = \frac{32b^8c}{a^4}$$

② NEGATIVE EXPONENTS

$$\left(\frac{-3a^2b^{-3}}{a^4b^{-1}}\right)^{-3}$$

$$\left(\frac{-3}{a^2b^2}\right)^{-3} = \left(\frac{a^2b^2}{-3}\right)^3 = \frac{-a^6b^6}{27}$$

③ VARIABLE EXPONENTS

$$\frac{4x^3y^{2n-1}}{-2xy^{n+3}}$$

$$-2x^2y^{(2n-1)-(n+3)} = -2x^2y^{n-4}$$

④ SQUARING A BINOMIAL

$$(2x^{3n} + 3y^{n+2})^2$$

$$4x^{6n} + 12x^{3n}y^{n+2} + 9y^{2n+4}$$

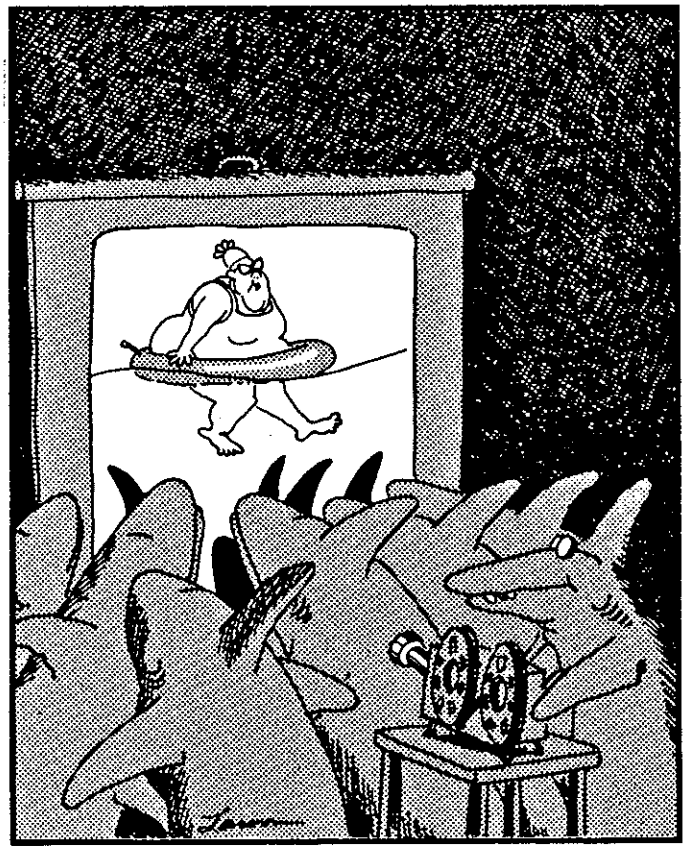
⑤ PRODUCT OF A SUM AND DIFFERENCE

$$(a^{3x+1} + 3b^{x-2})(a^{3x+1} - 3b^{x-2})$$

$$a^{6x+2} - 9b^{2x-4}$$

Additional Practice Problems in:

1.1



Shark nerds always ran the projector.

Simplifying Expressions

PROBLEM SET (Review Unit 1B)

Simplify each polynomial expression:

① $\dfrac{(3xy)^2(xy)^{-3}}{(3xz)^{-1}}$

② $\dfrac{(-2ab^2)^{-3}(-2a^{-1}b)^{-2}}{(2a^2b^{-3})^{-2}}$

③ $\left(\dfrac{-3x^2y^{-3}z^{-1}}{2x^{-2}y}\right)^{-2}$

④ $\left(\dfrac{-2n^3m^{-2}}{3n^{-4}m^{-5}}\right)^{-3}$

⑤ $\dfrac{12a^4b^{x-3}}{-6a^{-2}b^{3x-1}}$

⑥ $\dfrac{8^{-2}x^{-3}y^{a+3b}}{4^{-3}x^{-1}y^{a-b}}$

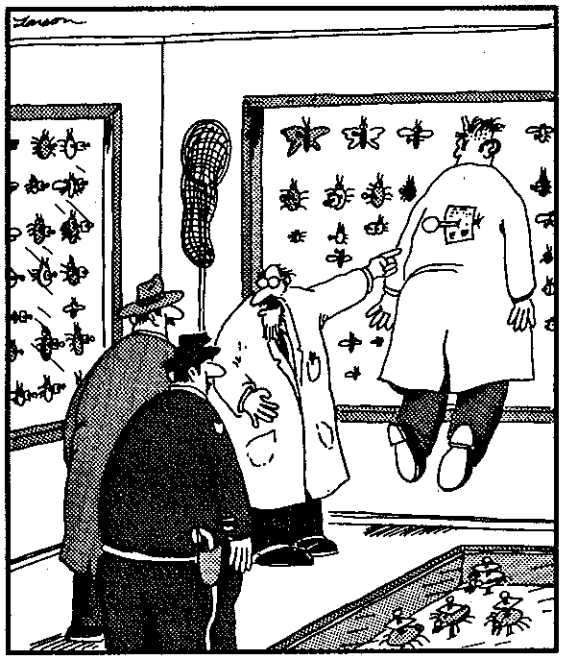
⑦ $(3x^{2n}-2y^{n+1})^2$

⑧ $(5a^{x-1}+4b^{x+y})^2$

⑨ $(x^{2n+m}+y^{n-m})(x^{2n+m}-y^{n-m})$

⑩ $(3a^x-2b^{x+3})(3a^x+2b^{x+3})$

Octopus obedience school



"Professor LaVonne had many enemies in the entomological world, detective, but if you examine that data label, you'll find exactly when and where he was—shall we say—'collected.'"

Simplifying Expressions

ANSWER KEY (Review Unit 1B)

① $\dfrac{(3xy)^2(xy)^{-3}}{(3xz)^{-1}}$

$\dfrac{(9x^2y^2)(x^{-3}y^{-3})}{3^{-1}x^{-1}z^{-1}} = \dfrac{9x^{-1}y^{-1}}{3^{-1}x^{-1}z^{-1}} = \dfrac{27z}{y}$

② $\dfrac{(-2ab^2)^{-3}(-2a^{-1}b)^{-2}}{(2a^2b^{-3})^{-2}}$

$\dfrac{[(-2)^{-3}a^{-3}b^{-6}][(-2)^{-2}a^2b^{-2}]}{2^{-2}a^{-4}b^6}$

$\dfrac{(-2)^{-5}a^{-1}b^{-8}}{2^{-2}a^{-4}b^6} = \dfrac{2^2a^3}{(-2)^5b^{14}} = \dfrac{-4a^3}{32b^{14}} = \dfrac{-a^3}{8b^{14}}$

③ $\left(\dfrac{-3x^2y^{-3}z^{-1}}{2x^{-2}y}\right)^{-2}$

$\left(\dfrac{-3x^4}{2y^4z}\right)^{-2} = \left(\dfrac{2y^4z}{-3x^4}\right)^2 = \dfrac{4y^8z^2}{9x^8}$

④ $\left(\dfrac{-2n^3m^{-2}}{3n^{-4}m^{-5}}\right)^{-3}$

$\left(\dfrac{-2n^7m^3}{3}\right)^{-3} = \left(\dfrac{3}{-2n^7m^3}\right)^3 = \dfrac{-27}{8n^{21}m^9}$

⑤ $\dfrac{12a^4b^{x-3}}{-6a^{-2}b^{3x-1}}$

$-2a^6b^{(x-3)-(3x-1)} = -2a^6b^{-2x-2}$

⑥ $\dfrac{8^{-2}x^{-3}y^{a+3b}}{4^{-3}x^{-1}y^{a-b}}$

$\dfrac{y^{(a+3b)-(a-b)}}{x^2} = \dfrac{y^{4b}}{x^2}$

⑦ $(3x^{2n} - 2y^{n+1})^2$

$9x^{4n} - 12x^{2n}y^{n+1} + 4y^{2n+2}$

⑧ $(5a^{x-1} + 4b^{x+y})^2$

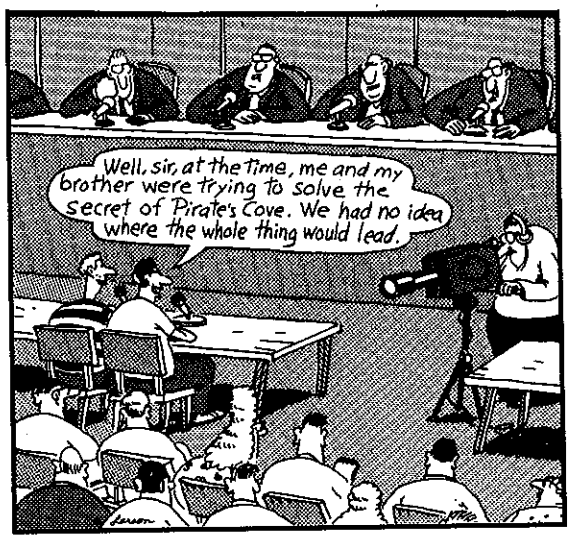
$25a^{2x-2} + 40a^{x-1}b^{x+y} + 16b^{2x+2y}$

⑨ $(x^{2n+m} + y^{n-m})(x^{2n+m} - y^{n-m})$

$x^{4n+2m} - y^{2n-2m}$

⑩ $(3a^x - 2b^{x+3})(3a^x + 2b^{x+3})$

$9a^{2x} - 4b^{2x+6}$



Testifying before a Senate subcommittee, the Hardy boys crack the Iran-contra scandal.

Synthetic Division

DEMONSTRATION (Review Unit 1C)

① POLYNOMIAL DIVISION

$(n^3-3) \div (n+2)$

$$n+2 \;\overline{)\; n^3 \qquad -3}$$

$$\begin{array}{r} n^2-2n+4-\frac{11}{n+2} \\ n+2 \;\overline{)\; n^3 \qquad\qquad -3} \\ \underline{n^3+2n^2} \\ -2n^2 \\ \underline{-2n^2-4n} \\ 4n-3 \\ \underline{4n+8} \\ -11 \end{array}$$

② SYNTHETIC DIVISION

$(2x^4-5x^3-10x+8)(x-3)^{-1}$

3	2	-5	0	-10	8
		6	3	9	-3
	2	1	3	-1	5

$2x^3+x^2+3x-1+\frac{5}{x-3}$

③ SYNTHETIC DIVISION

$(12n^4-2n^3-3) \div (2n+1)$

$(6n^4-n^3-\frac{3}{2}) \div (n+\frac{1}{2})$

$-\frac{1}{2}$	6	-1	0	0	$-\frac{3}{2}$
		-3	2	-1	$\frac{1}{2}$
	6	-4	2	-1	-1 → (-1)×2

$6n^3-4n^2+2n-1-\frac{2}{2n+1}$

SYNTHETIC DIVISION PROCEDURES
(Problem #3 : Explanation)

a) Since the lead coefficient of the divisor is not "1" - divide (by 2)

b) Take the opposite of the constant in the divisor $-\frac{1}{2}$

c) Write coefficients in descending order of n : 6 -1 0 0 $-\frac{3}{2}$

d) Bring down the first number (6)

e) Multiply $-\frac{1}{2} \times (6) = -3$ and put in 2nd column, Add (-1 + -3 = -4)

f) Multiply $-\frac{1}{2} \times (-4) = 2$ and put in 3rd column. Add (0+2=2)

g) Continue to last column (remainder). Multiply to reverse the division in step a). (-1) x2 = 2

h) Use the result to frame the quotient.

Additional Practice Problems in :

1.4

Synthetic Division

PROBLEM SET (Review Unit 1C)

Use polynomial division:

① $(3n^3-4) \div (n+2)$

② $(16x^4-3) \div (2x+1)$

Use synthetic division:

③ $(x^3+3x^2-4x+1) \div (x+3)$

④ $(5n^3-3n^2+2n-5) \div (n+1)$

⑤ $(x^3-11x+10)(x+3)^{-1}$

⑥ $(3y^3-5y+10)(y-2)^{-1}$

⑦ $(6n^4-7n^3+4n^2-9n-8)(2n-1)^{-1}$

⑧ $(10x^4+21x^3+11x^2+11x+16)(2x+3)^{-1}$

⑨ $(12x^4+11x^3-7x^2)(3x+2)^{-1}$

⑩ $(20n^4+22n^3+n^2+7n+10)(5n+3)^{-1}$

Show the synthetic division grid on your worksheet.

The grid will be required on the review quiz.

It was a tough frontier town; but later, after the arrival of the Earp brothers, things calmed down, and the town's name was shortened to simply Dodge City.

More facts of nature: All forest animals, to this very day, remember exactly where they were and what they were doing when they heard that Bambi's mother had been shot.

Synthetic Division

ANSWER KEY (Review Unit 1C)

① $3n^2-6n+12-\frac{28}{n+2}$

$n+2 \overline{)\,3n^3 \qquad -4}$

$3n^3+6n^2$

$-6n^2$

$-6n^2-12n$

$12n-4$

$12n+24$

-28

② $8x^3-4x^2+2x-1-\frac{2}{2x+1}$

$2x+1 \overline{)\,16x^4 \qquad -3}$

$16x^4+8x^3$

$-8x^3$

$-8x^3-4x^2$

$4x^2$

$4x^2+2x$

$-2x-3$

$-2x-1$

-2

③ $(x^3+3x^2-4x+1) \div (x+3)$

−3	1	3	−4	1
		−3	0	12
	1	0	−4	13

$x^2-4+\frac{13}{x+3}$

④ $(5n^3-3n^2+2n-5) \div (n+1)$

−1	5	−3	2	−5
		−5	8	−10
	5	−8	10	−15

$5n^2-8n+10-\frac{15}{n+1}$

⑤ $(x^3-11x+10)(x+3)^{-1}$

−3	1	0	−11	10
		−3	9	6
	1	−3	−2	16

$x^2-3x-2+\frac{16}{x+3}$

⑥ $(3y^3-5y+10)(y-2)^{-1}$

2	3	0	−5	10
		6	12	14
	3	6	7	24

$3y^2+6y+7+\frac{24}{y-2}$

⑦ $(6n^4-7n^3+4n^2-9n-8)(2n-1)^{-1}$

$(3n^4-\frac{7}{2}n^3+2n^2-\frac{9}{2}n-4)(n-\frac{1}{2})^{-1}$

1/2	3	−7/2	2	−9/2	−4
		3/2	−1	1/2	−2
	3	−2	1	−4	−6 → ×2

$3n^3-2n^2+n-4-\frac{12}{2n-1}$

⑧ $(10x^4+21x^3+11x^2+11x+16)(2x+3)^{-1}$

$(5x^4+2\frac{1}{2}x^3+11/2x^2+11/2x+8)(x+3/2)^{-1}$

−3/2	5	2 1/2	11/2	11/2	8
		−15/2	−9/2	−3/2	−6
	5	3	1	4	2 ×2

$5x^3+3x^2+x+4+\frac{4}{2x+3}$

Synthetic Division

ANSWER KEY (Review Unit 1C)

(9) $(12x^4+11x^3-7x^2)(3x+2)^{-1}$

$(4x^4+11/3x^3-7/3x^2)(x+2/3)^{-1}$

$-2/3$	4	$11/3$	$-7/3$	0	0
		$-8/3$	$-2/3$	2	$-4/3$
	4	1	-3	2	$-4/3$

$\times 3$

$4x^3+x^2-3x+2-4/3x+2$

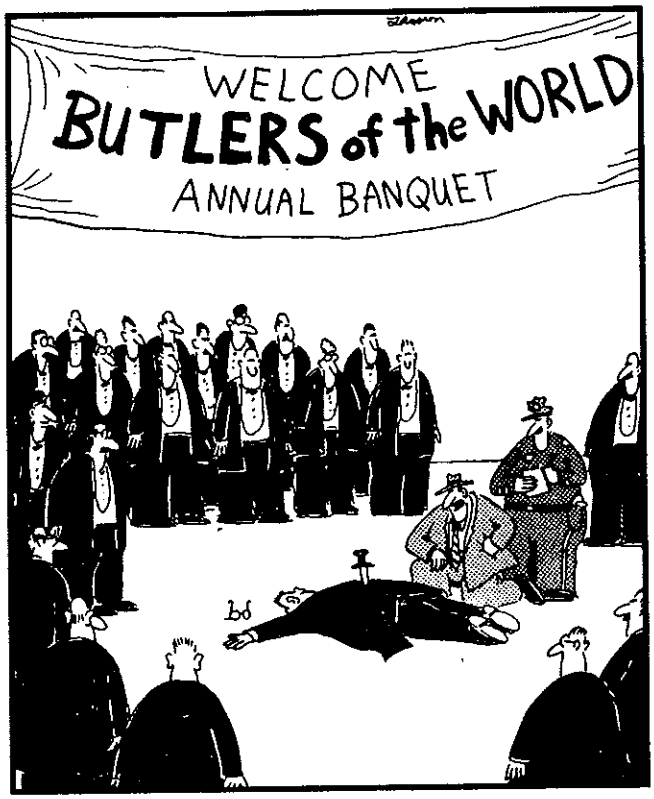
(10) $(20n^4+22n^3+n^2+7n+10)(5n+3)^{-1}$

$(4n^4+22/5n^3+1/5n^2+7/5n+2)(n+3/5)^{-1}$

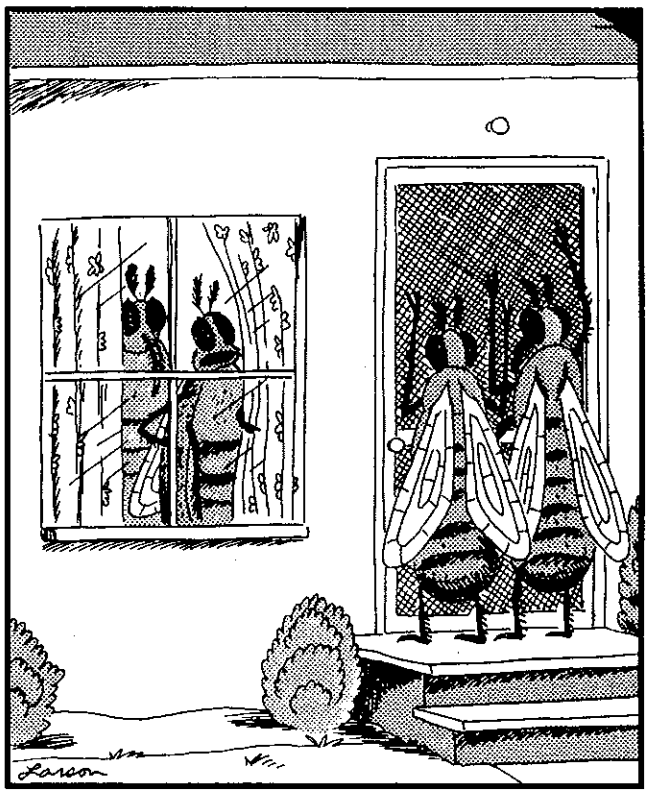
$-3/5$	4	$22/5$	$1/5$	$7/5$	2
		$-12/5$	$-6/5$	$3/5$	$-6/5$
	4	2	-1	2	$4/5$

$\times 5$

$4n^3+2n^2-n+2+4/5n+3$



"God, Collings, I hate to start a Monday with a case like this."



"Man, the Kellermans are bold! ... If it wasn't for our screens, they'd probably walk right in!"

Determinants

DEMONSTRATION (Review Unit 2A)

① 2ND ORDER DETERMINANT

$$\begin{vmatrix} 6 & 3 \\ -8 & -2 \end{vmatrix}$$

$(6)(-2) - (-8)(3)$

$(-12) - (-24)$

$(-12) + (24) = 12$

"Oh, look, Roger! Nerds! ... And some little nerdlings!"

② EXPANSION OF MINORS

$$\begin{vmatrix} 2 & 3 & 4 \\ 6 & 5 & 7 \\ 1 & 2 & 8 \end{vmatrix}$$

$$2\begin{vmatrix} 5 & 7 \\ 2 & 8 \end{vmatrix} - 3\begin{vmatrix} 6 & 7 \\ 1 & 8 \end{vmatrix} + 4\begin{vmatrix} 6 & 5 \\ 1 & 2 \end{vmatrix}$$

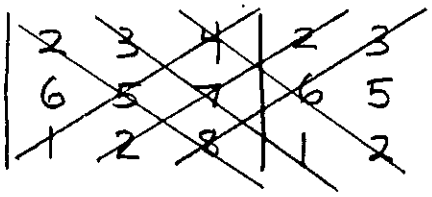
Signed minors

$$\begin{matrix} + & - & + \\ - & + & - \\ + & - & + \end{matrix}$$

$(2)(40-14) - (3)(48-7) + (4)(12-5)$

$(2)(26) - (3)(41) + (4)(7) = -43$

③ DIAGONALS



$(80) + (21) + (48) - (20) - (28) - (144)$

$(80) + (21) + (48) + (-20) + (-28) + (-144) = -43$

Additional Practice Problems In:

2.2 and 2.3

Determinants

PROBLEM SET (Review Unit 2A)

Find the value of each 2nd order determinant:

① $\begin{vmatrix} 6 & -2 \\ -3 & 0 \end{vmatrix}$ ② $\begin{vmatrix} -5 & -9 \\ -4 & 5 \end{vmatrix}$

Use expansion of minors to determine the value:

③ $\begin{vmatrix} 4 & 3 & 5 \\ 2 & 0 & 2 \\ 6 & 1 & 8 \end{vmatrix}$ ④ $\begin{vmatrix} 6 & 0 & 4 \\ -3 & 5 & -1 \\ 2 & 9 & 3 \end{vmatrix}$

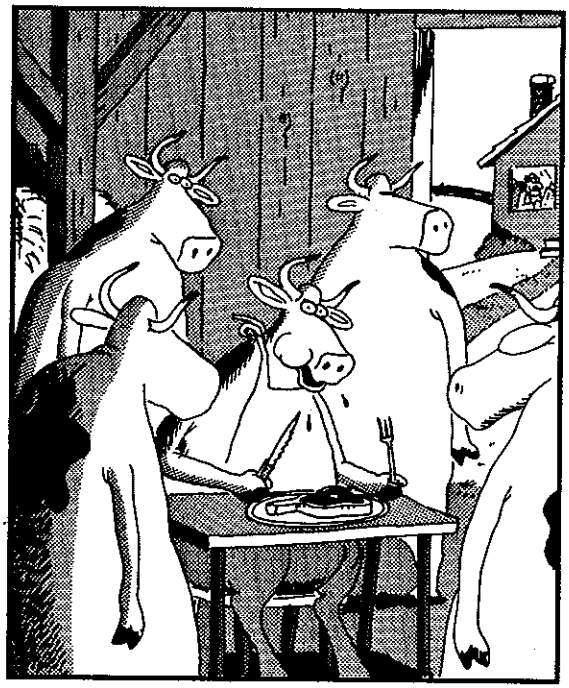
⑤ $\begin{vmatrix} -2 & -3 & -1 \\ -4 & 2 & 5 \\ 0 & 6 & 11 \end{vmatrix}$ ⑥ $\begin{vmatrix} 12 & -2 & 3 \\ 0 & -1 & -2 \\ -3 & 4 & -5 \end{vmatrix}$

Use diagonals to determine the value:

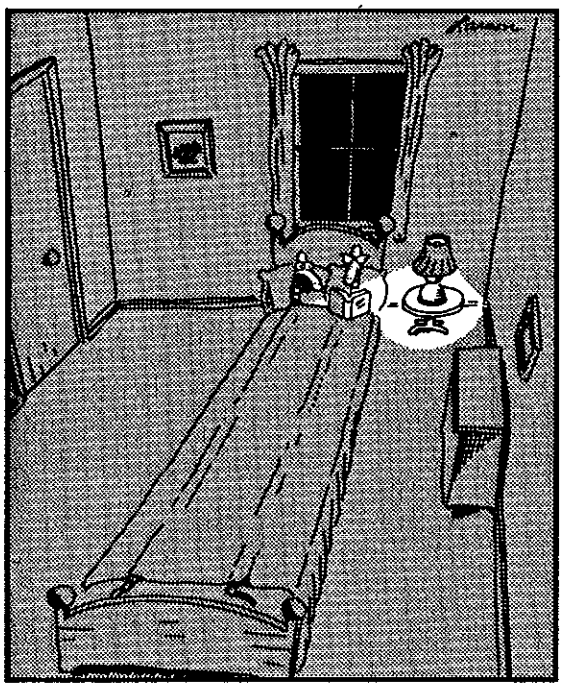
⑦ $\begin{vmatrix} 4 & 1 & -2 \\ 3 & 6 & 3 \\ 0 & 8 & 2 \end{vmatrix}$ ⑧ $\begin{vmatrix} 4 & 1 & -3 \\ 8 & 1 & 2 \\ 3 & -4 & 6 \end{vmatrix}$

⑨ $\begin{vmatrix} 7 & 3 & -2 \\ 9 & 0 & -6 \\ -5 & -1 & 2 \end{vmatrix}$ ⑩ $\begin{vmatrix} -1 & -4 & 6 \\ 8 & -3 & 4 \\ 0 & -1 & -2 \end{vmatrix}$

On the quiz you may wish to use both methods to double check an answer



"Mmmmmm ... interesting ... interesting. ... I'd say we taste a little like chicken."



"Oh, man. There you go again with another one of those hiss-and-tell books."

Determinants

ANSWER KEY (Review Unit 2A)

① $\begin{vmatrix} 6 & -2 \\ -3 & 0 \end{vmatrix}$ $(6)(0)-(-3)(-2)$
$(0)-(6) = -6$

② $\begin{vmatrix} -5 & -9 \\ -4 & 5 \end{vmatrix}$ $(-5)(5)-(-4)(-9)$
$(-25)-(36) = -61$

③ $\begin{vmatrix} 4 & 3 & 5 \\ 2 & 0 & 2 \\ 6 & 1 & 8 \end{vmatrix} = -10$

$4\begin{vmatrix} 0 & 2 \\ 1 & 8 \end{vmatrix} - 3\begin{vmatrix} 2 & 2 \\ 6 & 8 \end{vmatrix} + 5\begin{vmatrix} 2 & 0 \\ 6 & 1 \end{vmatrix}$

$(4)(-2)-(3)(4)+(5)(2)$
$(-8)+(-12)+(10) = -10$

④ $\begin{vmatrix} 6 & 0 & 4 \\ -3 & 5 & -1 \\ 2 & 9 & 3 \end{vmatrix} = -4$

$6\begin{vmatrix} 5 & -1 \\ 9 & 3 \end{vmatrix} - 0\begin{vmatrix} -3 & -1 \\ 2 & 3 \end{vmatrix} + 4\begin{vmatrix} -3 & 5 \\ 2 & 9 \end{vmatrix}$

$(6)(24)-(0)(-7)+(4)(-37)$
$(144)+(0)+(-148) = -4$

⑤ $\begin{vmatrix} -2 & -3 & -1 \\ -4 & 2 & 5 \\ 0 & 6 & 11 \end{vmatrix} = -92$

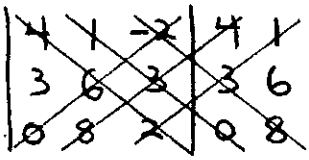
$-2\begin{vmatrix} 2 & 5 \\ 6 & 11 \end{vmatrix} - (-3)\begin{vmatrix} -4 & 5 \\ 0 & 11 \end{vmatrix} + (-1)\begin{vmatrix} -4 & 2 \\ 0 & 6 \end{vmatrix}$

$(-2)(-8)-(-3)(-44)+(-1)(-24)$
$(16)-(132)+(24)$
$(16)+(-132)+(24) = -92$

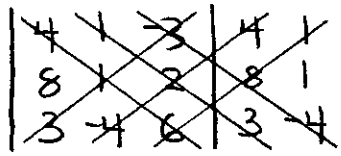
⑥ $\begin{vmatrix} 12 & -2 & 3 \\ 0 & -1 & -2 \\ -3 & 4 & -5 \end{vmatrix} = 135$

$12\begin{vmatrix} -1 & -2 \\ 4 & -5 \end{vmatrix} - (-2)\begin{vmatrix} 0 & -2 \\ -3 & -5 \end{vmatrix} + 3\begin{vmatrix} 0 & -1 \\ -3 & 4 \end{vmatrix}$

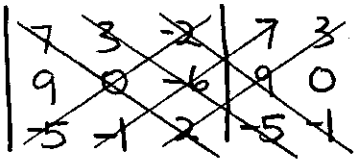
$(12)(13)-(-2)(-6)+(3)(-3)$
$(156)+(-12)+(-9) = 135$

⑦  $= -102$

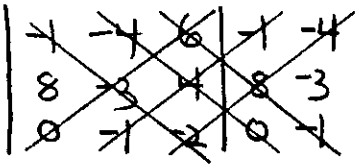
$(48)+(0)+(-48)-(0)-(96)-(6)$

⑧  $= 119$

$(24)+(6)+(96)-(-9)-(-32)-(48)$

⑨  $= 12$

$(0)+(90)+(18)-(0)-(42)-(54)$

⑩  $= -122$

$(-6)+(0)+(-48)-(0)-(4)-(64)$

Cramer's Rule

DEMONSTRATION (Review Unit 2B)

Use Cramer's Rule to solve a 2nd order system:

① $3x - 5y = -7$
$x + 2y = 16$ $\quad (6, 5)$

$$x = \frac{\begin{vmatrix} -7 & -5 \\ 16 & 2 \end{vmatrix}}{\begin{vmatrix} 3 & -5 \\ 1 & 2 \end{vmatrix}} = \frac{66}{11} \qquad y = \frac{\begin{vmatrix} 3 & -7 \\ 1 & 16 \end{vmatrix}}{11} = \frac{55}{11}$$

Use expansion of minors to solve for x:

② $4x - 3y + z = -1$
$2x + 9y + 5z = 2$
$2x - 6y - 3z = 0$

$$x = \frac{\begin{vmatrix} -1 & -3 & 1 \\ 2 & 9 & 5 \\ 0 & -6 & -3 \end{vmatrix}}{\begin{vmatrix} 4 & -3 & 1 \\ 2 & 9 & 5 \\ 2 & -6 & -3 \end{vmatrix}} = \frac{-33}{-66} = \frac{1}{2}$$

$$(-1)\begin{vmatrix} 9 & 5 \\ -6 & -3 \end{vmatrix} - (-3)\begin{vmatrix} 2 & 5 \\ 0 & -3 \end{vmatrix} + (1)\begin{vmatrix} 2 & 9 \\ 0 & -6 \end{vmatrix}$$

$(-1)(3) - (-3)(-6) + (1)(-12) = -33$

$$(4)\begin{vmatrix} 9 & 5 \\ -6 & -3 \end{vmatrix} - (-3)\begin{vmatrix} 2 & 5 \\ 2 & -3 \end{vmatrix} + (1)\begin{vmatrix} 2 & 9 \\ 2 & -6 \end{vmatrix}$$

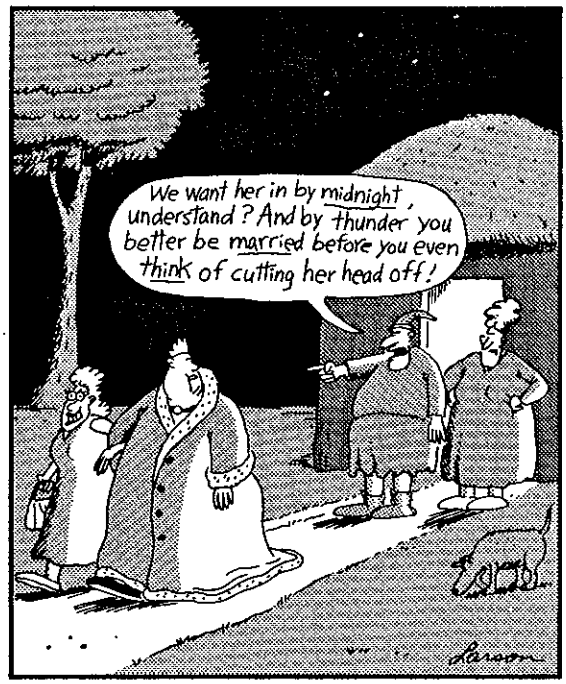
$(4)(3) - (-3)(-16) + (1)(-30) = -66$

Use diagonals to solve for y (same system as problem #2):

③
$$y = \frac{\begin{vmatrix} 4 & 1 & 4 & 1 \\ 2 & 5 & 2 & 2 \\ 2 & 0 & 2 & 0 \end{vmatrix}}{\begin{vmatrix} 4 & -3 & 4 & -3 \\ 2 & 9 & 2 & 9 \\ 2 & -6 & 2 & -6 \end{vmatrix}} = \frac{-44}{-66} = \frac{2}{3}$$

$(-24) + (-10) + (0) - (4) - (0) - (6) = -44$
$(-108) + (-30) + (-12) - (18) - (-120) - (18) = -66$

Additional Practice Problems in: 2.2 and 2.4



Henry VIII on the dating scene

Cramer's Rule

PROBLEM SET (Review Unit 2B)

Use Cramer's Rule to solve a 2nd order system:

① $3x+5y=-2$
$6x-15y=11$

② $2x+3y=-8$
$5x-6y=-11$

Use expansion of minors:

③ Solve for y

$3x-y+z=8$
$2x+3y-4z=-1$
$x-2y-z=9$

④ Solve for x

$6x-y+2z=-3$
$3x+4y-z=2$
$9x-3y-4z=25$

Use diagonals:

⑤ Solve for x

$3x+8y-6z=-1$
$x+12y-3z=4$
$2x+4y-3z=-3$

⑥ Solve for y

$3x-2y+z=4$
$x+4y-3z=2$
$x-y+z=0$

Use elimination and substitution to solve the system:

⑦ $2x+3y-2z=10$
$4x+y-z=14$
$x+y+z=1$

⑧ $2x+y+z=-2$
$x-2y+z=3$
$3x+y+2z=-1$

For extra practice using expansion of minors, diagonals, or elimination and substitution:

Use the ordered triples included at the end of the answer key for problems 3–6.

Later, Edna was forced to sell her brussels sprout house.

Cramer's Rule

ANSWER KEY (Review Unit 2B)

① $3x+5y=-2$, $6x-15y=11$ $\quad \left(\frac{1}{3}, \frac{-3}{5}\right)$

$$x=\frac{\begin{vmatrix}-2 & 5\\ 11 & -15\end{vmatrix}}{\begin{vmatrix}3 & 5\\ 6 & -15\end{vmatrix}}=\frac{-25}{-75} \qquad y=\frac{\begin{vmatrix}3 & -2\\ 6 & 11\end{vmatrix}}{-75}=\frac{45}{-75}$$

② $2x+3y=-8$, $5x-6y=-11$ $\quad (-3, -\frac{2}{3})$

$$x=\frac{\begin{vmatrix}-8 & 3\\ -11 & -6\end{vmatrix}}{\begin{vmatrix}2 & 3\\ 5 & -6\end{vmatrix}}=\frac{81}{-27} \qquad y=\frac{\begin{vmatrix}2 & -8\\ 5 & -11\end{vmatrix}}{-27}=\frac{18}{-27}$$

③ $3x-y+z=8$, $2x+3y-4z=-1$, $x-2y-z=9$

$$y=\frac{\begin{vmatrix}3 & 8 & 1\\ 2 & -1 & -4\\ 1 & 9 & -1\end{vmatrix}}{\begin{vmatrix}3 & -1 & 1\\ 2 & 3 & -4\\ 1 & -2 & -1\end{vmatrix}}=\frac{114}{-38}=-3$$

$(3)\begin{vmatrix}-1 & -4\\ 9 & -1\end{vmatrix}-(8)\begin{vmatrix}2 & -4\\ 1 & -1\end{vmatrix}+(1)\begin{vmatrix}2 & -1\\ 1 & 9\end{vmatrix}$

$(3)(37)-(8)(2)+(1)(19)=114$

$(3)\begin{vmatrix}3 & -4\\ -2 & -1\end{vmatrix}-(-1)\begin{vmatrix}2 & -4\\ 1 & -1\end{vmatrix}+(1)\begin{vmatrix}2 & 3\\ 1 & -2\end{vmatrix}$

$(3)(-11)-(-1)(2)+(1)(-7)=-38$

④ $6x-y+2z=-3$, $3x+4y-z=2$, $9x-3y-4z=25$

$$x=\frac{\begin{vmatrix}-3 & -1 & 2\\ 2 & 4 & -1\\ 25 & -3 & -4\end{vmatrix}}{\begin{vmatrix}6 & -1 & 2\\ 3 & 4 & -1\\ 9 & -3 & -4\end{vmatrix}}=\frac{-138}{-207}=\frac{2}{3}$$

$(-3)\begin{vmatrix}4 & -1\\ -3 & -4\end{vmatrix}-(-1)\begin{vmatrix}2 & -1\\ 25 & -4\end{vmatrix}+(2)\begin{vmatrix}2 & 4\\ 25 & -3\end{vmatrix}$

$(-3)(-19)-(-1)(17)+(2)(-106)=-138$

$(6)\begin{vmatrix}4 & -1\\ -3 & -4\end{vmatrix}-(-1)\begin{vmatrix}3 & -1\\ 9 & -4\end{vmatrix}+(2)\begin{vmatrix}3 & 4\\ 9 & -3\end{vmatrix}$

$(6)(-19)-(-1)(-3)+(2)(-45)=-207$

⑤ $3x+8y-6z=-1$, $x+12y-3z=4$, $2x+4y-3z=-3$

$$x=\frac{\begin{vmatrix}3 & 8 & -6\\ 1 & 12 & -3\\ 2 & 4 & -3\end{vmatrix}}{\begin{vmatrix}3 & 8 & -6\\ 1 & 12 & -3\\ 2 & 4 & -3\end{vmatrix}}=\frac{-120}{24}=-5$$

$(36)+(72)+(-96)-(216)-(12)-(-96)$

$(36)+(72)+(-96)+(-216)+(-12)+(96)$

$(-108)+(-48)+(-24)-(-144)-(-36)-(-24)$

$(-108)+(-48)+(-24)+(144)+(36)+(24)$

Cramer's Rule

ANSWER KEY (Review Unit 2B)

⑥ $3x-2y+z=4$
$x+4y-3z=2$
$x-y+z=0$

$$y=\frac{\begin{vmatrix}3&4&1\\1&2&-3\\1&0&1\end{vmatrix}}{\begin{vmatrix}3&-2&1\\1&4&-3\\1&-1&1\end{vmatrix}}=\frac{-12}{6}=-2$$

$(6)+(-12)+(0)-(2)-(0)-(4)$
$(6)+(-12)+(0)+(-2)+(0)+(-4)$

$(12)+(6)+(-1)-(4)-(9)-(-2)$
$(12)+(6)+(-1)+(-4)+(-9)+(2)$

⑦ a) $2x+3y-2z=10$
b) $4x+y-z=14$
c) $x+y+z=1$

a) $2x+3y-2z=10$
c) $-2x-2y-2z=-2$
$y-4z=8$

b) $4x+y-z=14$
c) $-4x-4y-4z=-4$
$-3y-5z=10$

$(y-4z=8)\times 3$ $\quad 3y-12z=24$
$(-3y-5z=10)$ $\quad -3y-5z=10$
$-17z=34$
$z=-2$

$y-4z=8 \rightarrow y-4(-2)=8$
$y=0$

$x+y+z=1$
$x+(0)+(-2)=1$
$x=3$ $\quad (3,0,-2)$

⑧ a) $2x+y+z=-2$
b) $x-2y+z=3$
c) $3x+y+2z=-1$

a) $2x+y+z=-2$
b) $-2x+4y-2z=-6$
$5y-z=-8$

b) $-3x+6y-3z=-9$
c) $3x+y+2z=-1$
$7y-z=-10$

$5y-z=-8$
$-7y+z=10$
$-2y=2$
$y=-1$

$5y-z=-8$
$5(-1)-z=-8$
$-z=-3$
$z=3$

$x-2y+z=3$
$x-2(-1)+(3)=3$
$x=-2$ $\quad (-2,-1,3)$

PROBLEMS 3-6
COMPLETE ANSWERS
FOR EXTRA PRACTICE

③ $(2,-3,-1)$ ⑤ $(-5,1/4,-2)$

④ $(\frac{2}{3},-1,-4)$ ⑥ $(1,-2,-3)$

Inequalities & Number Line Graphing

DEMONSTRATION (Review Unit 3A)

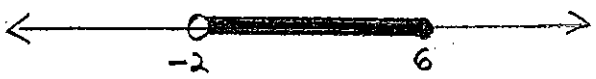
① COMPOUND INEQUALITY

$3 < x+5 \leq 11$

$3 < x+5$ and $x+5 \leq 11$

$-2 < x$ and $x \leq 6$

$\boxed{-2 < x \leq 6}$



② ABSOLUTE VALUE: INTERSECTION (<)

$|3n-2|+1 \leq 5$

$|3n-2| \leq 4$

$3n-2 \leq 4$ and $3n-2 \geq -4$

$3n \leq 6$ and $3n \geq -2$

$n \leq 2$ and $n \geq -2/3$

$\boxed{-2/3 \leq n \leq 2}$

③ ABSOLUTE VALUE: UNION (>)

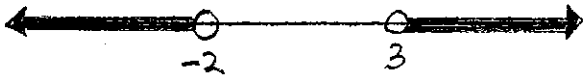
$|4x-2| > 10$

$4x-2 > 10$ or $4x-2 < -10$

$4x > 12$ or $4x < -8$

$x > 3$ or $x < -2$

$\boxed{x > 3 \text{ or } x < -2}$

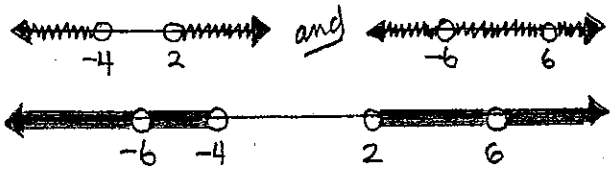


④ INTERSECTION OF ABSOLUTE VALUE SYSTEMS

$|n+1| > 3$ and $|n| \neq 6$

$(n+1 > 3$ or $n+1 < -3)$ and $(n \neq 6$ and $n \neq -6)$

$\boxed{(n > 2 \text{ or } n < -4) \text{ and } (n \neq 6 \text{ and } n \neq -6)}$



⑤ INTERSECTION OF ABSOLUTE VALUE SYSTEMS

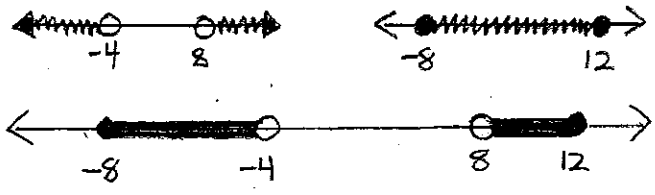
$6 < |x-2| \leq 10$

$|x-2| > 6$ and $|x-2| \leq 10$

$(x-2 > 6$ or $x-2 < -6)$ and $(x-2 \leq 10$ and $x-2 \geq -10)$

$(x > 8$ or $x < -4)$ and $(x \leq 12$ and $x \geq -8)$

$\boxed{(x > 8 \text{ or } x < -4) \text{ and } (-8 \leq x \leq 12)}$



Additional Practice Problems in:

4.1

Inequalities & Number Line Graphing

PROBLEM SET (Review Unit 3A)

Solve each inequality and graph the solution on a number line:

① $-2 \leq x+6 < 12$

② $|x-5|-3 \leq 10$

③ $|2x-1| > 7$

④ $|3x+6| \geq 6$

⑤ $|2x-3|+3 < 12$

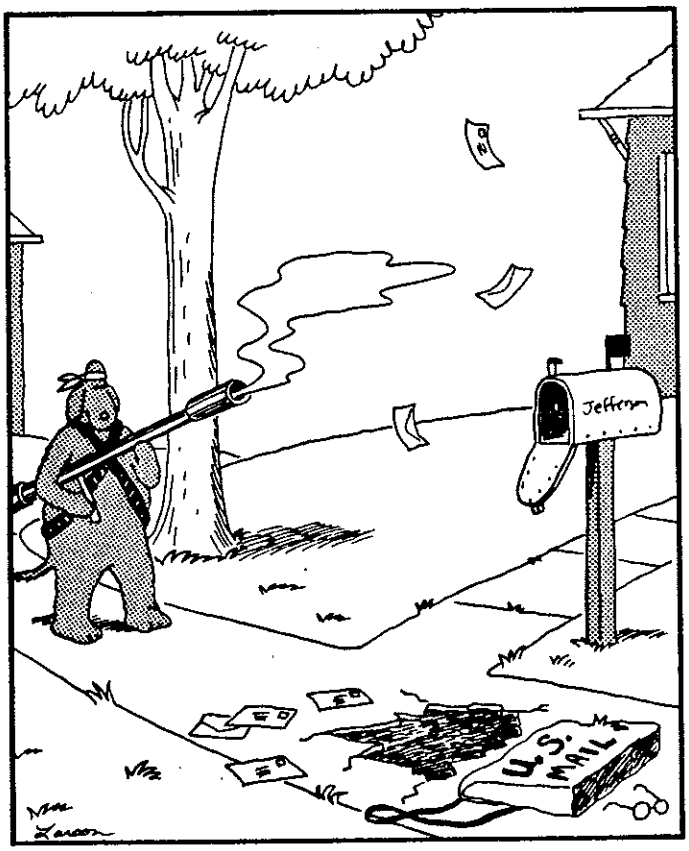
⑥ $-4 < 2x+4 \leq 10$

⑦ $|2x+1| \geq 9$ and $|x-1| < 7$

⑧ $-2 \leq |n-6|-4 < 6$

⑨ $2 < |2n-4| \leq 12$

⑩ $|x+5| > 6$ and $|x+3| \neq 10$



Neither rain nor snow nor sleet nor hail, they said, could stop the mail....But they didn't figure on Rexbo.

"Buffalo breath? *Buffalo* breath?...Shall we discuss your incessant little *grunting* noises?"

Inequalities & Number Line Graphing

ANSWER KEY (Review Unit 3A)

① $-2 \leq x+6 < 12$

$-2 \leq x+6$ and $x+6 < 12$

$-8 \leq x$ and $x < 6$

$\boxed{-8 \leq x < 6}$

② $|x-5|-3 \leq 10$

$|x-5| \leq 13$

$x-5 \leq 13$ and $x-5 \geq -13$

$x \leq 18$ and $x \geq -8$

$\boxed{-8 \leq x \leq 18}$

③ $|2x-1| > 7$

$2x-1 > 7$ or $2x-1 < -7$

$2x > 8$ or $2x < -6$

$\boxed{x > 4 \text{ or } x < -3}$

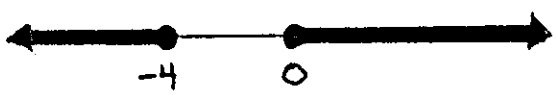
④ $|3x+6| \geq 6$

$3x+6 \geq 6$ or $3x+6 \leq -6$

$3x \geq 0$ or $3x \leq -12$

$x \geq 0$ or $x \leq -4$

$\boxed{x \geq 0 \text{ or } x \leq -4}$



⑤ $|2x-3|+3 < 12$

$|2x-3| < 9$

$2x-3 < 9$ and $2x-3 > -9$

$2x < 12$ and $2x > -6$

$x < 6$ and $x > -3$

$\boxed{-3 < x < 6}$

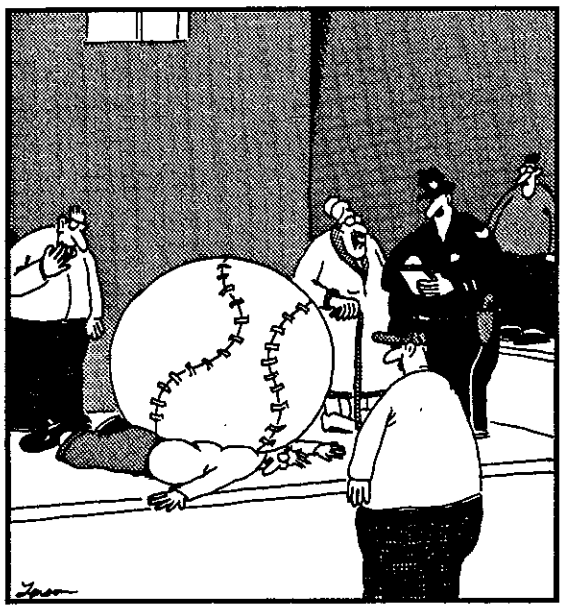
⑥ $-4 < 2x+4 \leq 10$

$-4 < 2x+4$ and $2x+4 \leq 10$

$-8 < 2x$ and $2x \leq 6$

$-4 < x$ and $x \leq 3$

$\boxed{-4 < x \leq 3}$



"And then wham!
This thing just came right out of left field."

Inequalities & Number Line Graphing

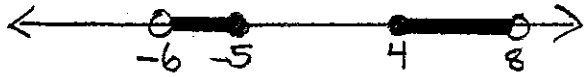
ANSWER KEY (Review Unit 3A)

⑦ $|2x+1| \geq 9$ and $|x-1| < 7$

$(2x+1 \geq 9$ or $2x+1 \leq -9)$ and $(x-1 < 7$ and $x-1 > -7)$

$(2x \geq 8$ or $2x \leq -10)$ and $(x < 8$ and $x > -6)$

$\boxed{(x \geq 4 \text{ or } x \leq -5) \text{ and } (-6 < x < 8)}$



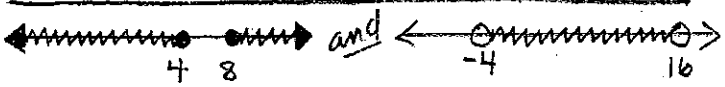
⑧ $-2 \leq |n-6| - 4 < 6$

$|n-6| - 4 \geq -2$ and $|n-6| - 4 < 6$

$|n-6| \geq 2$ and $|n-6| < 10$

$(n-6 \geq 2$ or $n-6 \leq -2)$ and $(n-6 < 10$ and $n-6 > -10)$

$\boxed{(n \geq 8 \text{ or } n \leq 4) \text{ and } (-4 < n < 16)}$



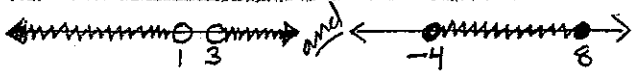
⑨ $2 < |2n-4| \leq 12$

$|2n-4| > 2$ and $|2n-4| \leq 12$

$(2n-4 > 2$ or $2n-4 < -2)$ and $(2n-4 \leq 12$ and $2n-4 \geq -12)$

$(2n > 6$ or $2n < 2)$ and $(2n \leq 16$ and $2n \geq -8)$

$\boxed{(n > 3 \text{ or } n < 1) \text{ and } (-4 \leq n \leq 8)}$

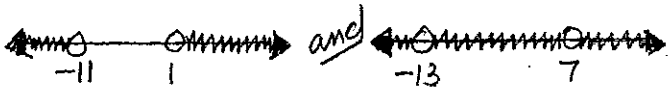


⑩ $|x+5| > 6$ and $|x+3| \neq 10$

$(x+5 > 6$ or $x+5 < -6)$ and $(x+3 \neq 10$ and $x+3 \neq -10)$

$(x > 1$ or $x < -11)$ and $(x \neq 7$ and $x \neq -13)$

$\boxed{(x > 1 \text{ or } x < -11) \text{ and } (x \neq 7 \text{ and } x \neq -13)}$

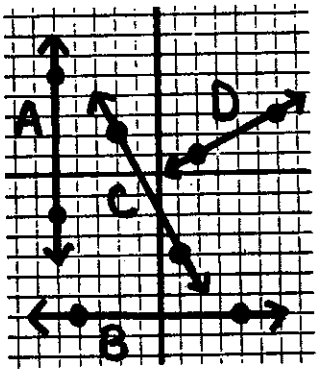


Linear Equations

DEMONSTRATION (Review Unit 3B)

Review information about linear equations:

① SLOPE



A) undef.

B) 0

C) -2

D) ½

slope is "rise over run"

A line containing (-6,2) and (-4,10) has a slope of:

$$\frac{y_1 - y_2}{x_1 - x_2} \quad \frac{(2)-(10)}{(-6)-(-4)} = \frac{-8}{-2} = 4$$

② SLOPE-INTERCEPT FORM

$\boxed{y = mx + b}$

slope = m
y-int = b
x-int = $-b/m$

$y = \frac{1}{2}x + 3$

$m = \frac{1}{2}$

$b = 3$ (0,3) $\quad \frac{-b}{m} = -6$ (-6,0)

③ STANDARD FORM

$\boxed{Ax + By = C}$ $\quad x - 2y = -6$

slope = $-A/B$ $\quad -A/B = \frac{1}{2}$
y-int = C/B $\quad C/B = 3$ (0,3)
x-int = C/A $\quad C/A = -6$ (-6,0)

Standard form: No fractions, the coeffficient of x must be positive, GCF must be factored

④ POINT-SLOPE FORM

$\boxed{y - y_1 = m(x - x_1)}$

slope = m
(x_1, y_1) is on the line

$y - \frac{1}{3} = 2(x - 4)$
slope = 2, $(4, \frac{1}{3})$ is on the line

Change the form to slope-intercept to determine intercepts.

⑤

Parallel lines: equal slope
Perpendicular lines: neg. recip. slope
Horizontal line: no x term
Vertical line: no y term
Through origin: no constant

Additional Practice: 4.2

Linear Equations

PROBLEM SET (Review Unit 3B)

Write an equation for a line that:

① Goes through $(-8,2)$ $(-5,1)$ in slope-intercept form

② Goes through $(-5,0)(6,-3)$ in standard form

③ Goes through $(-4,-2)(2,6)$ in point-slope form

④ Is parallel to $2x-y=6$ through $(-4,6)$ in point-slope form

⑤ Is parallel to $y=\frac{-1}{2}x+4$ through $(-4,-8)$ in standard form

⑥ Is perpendicular to $y=3x-1$ in standard form through $(-6,-2)$

⑦ Is perpendicular to $3x-2y=12$ in slope-int form through $(8,-2)$

Indicate the slope, both intercepts, and draw the graph for each:

⑧ $y+2=\frac{2}{3}(x-8)$

⑨ $y=\frac{-3}{4}x-6$

⑩ $2x-y=-10$

"Hey! You're not lookin' to buy anything, are you? I think you best just keep movin', buddy."

Linear Equations

ANSWER KEY (Review Unit 3B)

① $(-8,2)(-5,1)$

$\frac{(2)-(1)}{(-8)-(-5)} = \frac{-1}{3}$

$y = \frac{-1}{3}x + b$

$(2) = \frac{-1}{3}(-8) + b$

$(2) = \frac{8}{3} + b$

$\frac{-2}{3} = b$

$y = \frac{-1}{3}x - \frac{2}{3}$

② $(-5,0)(6,-3)$

$\frac{(0)-(-3)}{(-5)-(6)} = \frac{-3}{11}$

$\frac{-A}{B} = \frac{-3}{11}$ $A=3$ $B=11$

$3x+11y = C$

$3(-5)+11(0) = C$

$-15 = C$

$3x+11y = -15$

③ $(-4,-2)\ (2,6)$

$\frac{(-2)-(6)}{(-4)-(2)} = \frac{-8}{-6} = \frac{4}{3}$

$y - y_1 = \frac{4}{3}(x - x_1)$

continued

$y+2 = \frac{4}{3}(x+4)$

or

$y-6 = \frac{4}{3}(x-2)$

④ $2x-y=6$ $(-4,6)$

Slope $(-A/B) = 2$

$y - y_1 = 2(x - x_1)$

$y-6 = 2(x+4)$

⑤ $y = \frac{-1}{2}x + 4$ $(-4,-8)$

slope $= \frac{-1}{2}$

$\frac{-A}{B} = \frac{-1}{2}$ $A=1$ $B=2$

$x+2y = C$

$(-4)+2(-8) = C$

$-20 = C$

$x+2y = -20$

⑥ $y = 3x-1$ $(-6,-2)$

perp. slope $= \frac{-1}{3}$

$\frac{-A}{B} = \frac{-1}{3}$ $A=1$ $B=3$

$x+3y = C$

$(-6)+3(-2) = C$

$-12 = C$

$x+3y = -12$

⑦ $3x-2y=12$ $(8,-2)$

perp. slope $= \frac{-2}{3}$

$y = \frac{-2}{3}x + b$

$(-2) = \frac{-2}{3}(8) + b$

$(-2) = \frac{-16}{3} + b$

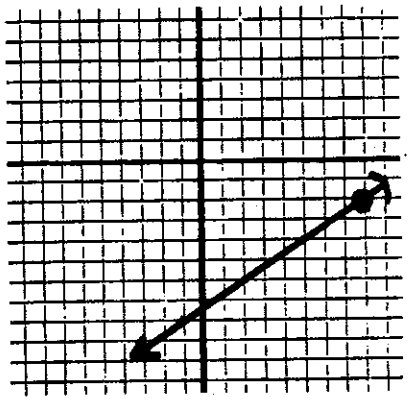
$\frac{10}{3} = b$

$y = \frac{-2}{3}x + \frac{10}{3}$

⑧ $y+2 = \frac{2}{3}(x-8)$

slope $= \frac{2}{3}$

point $(8,-2)$



change form:

$y+2 = \frac{2}{3}(x-8)$

$y = \frac{2}{3}x - \frac{22}{3}$

y-int $= \frac{-22}{3}$ $(0, -22/3)$

x-int $= 11$ $(11,0)$

Linear Equations

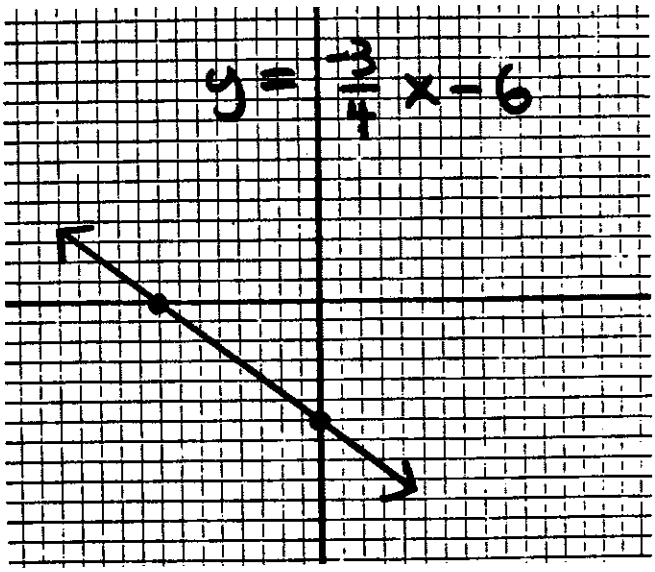
ANSWER KEY (Review Unit 3B)

⑨ $y = -\frac{3}{4}x - 6$

slope $= m = -\frac{3}{4}$

y-int $= b = -6$ $(0, -6)$

x-int $= \frac{-b}{m} = -8$ $(-8, 0)$

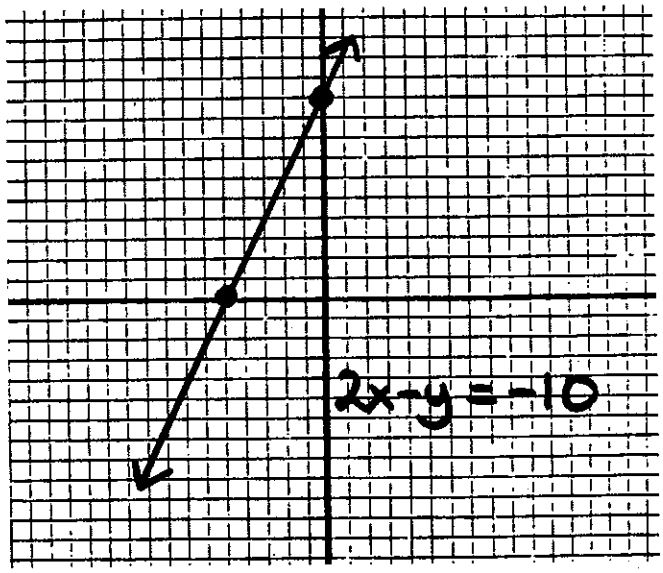


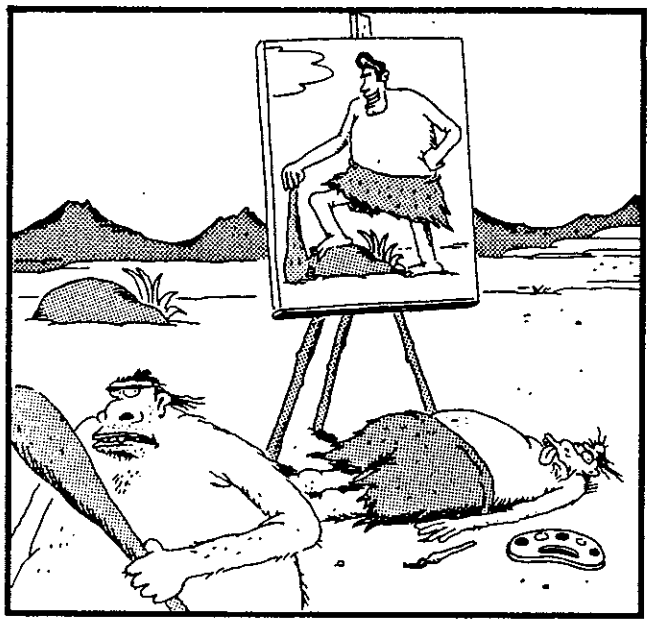
⑯ $2x - y = -10$

slope $= -A/B = 2$

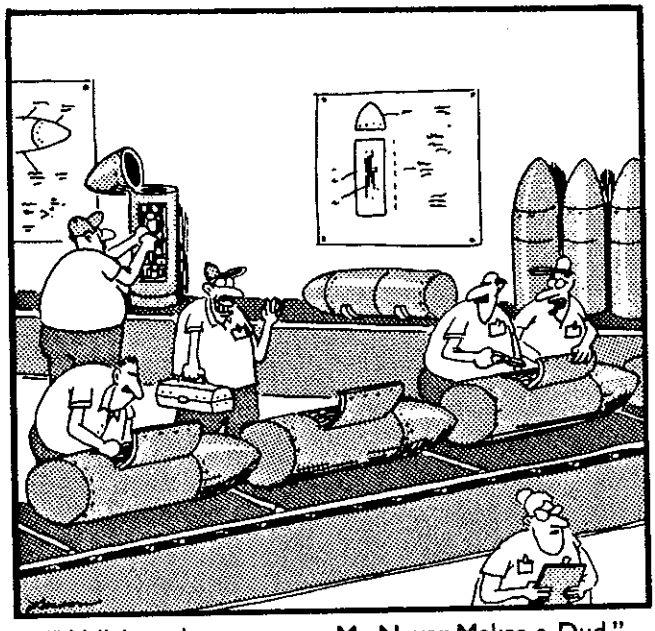
y-int $= C/B = 10$ $(0, 10)$

x-int $= C/A = -5$ $(-5, 0)$





Modern art critic



"Well, here he comes ... Mr. Never-Makes-a-Dud."

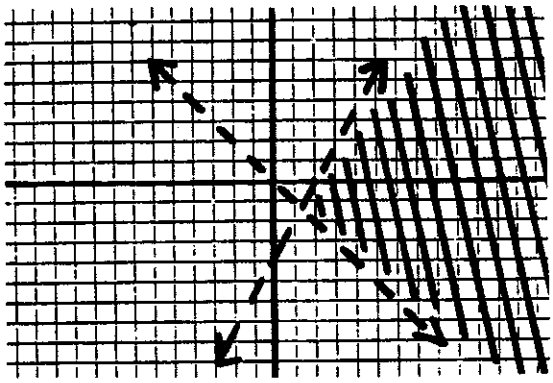
Graphing Systems of Inequalities

DEMONSTRATION (Review Unit 4A)

① SYSTEM OF INEQUALITIES

$2x-y>4 \rightarrow y<2x-4$

$y>-x$



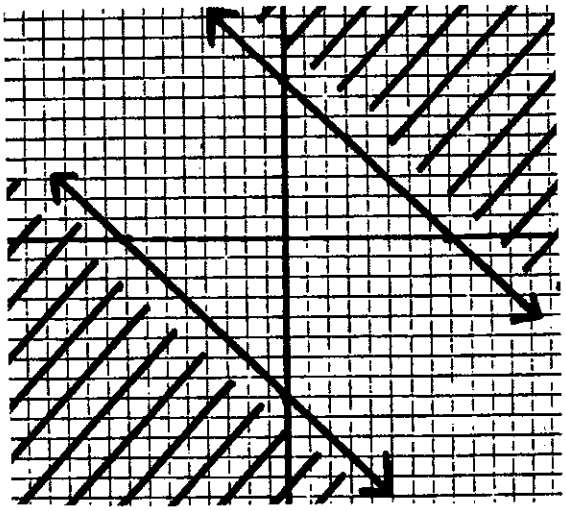
Always change inequalities to slope-intercept form when graphing

② ABSOLUTE VALUE: Union

$|x+y| \geq 8$

$x+y \geq 8$ or $x+y \leq -8$

$y \geq -x+8$ or $y \leq -x-8$



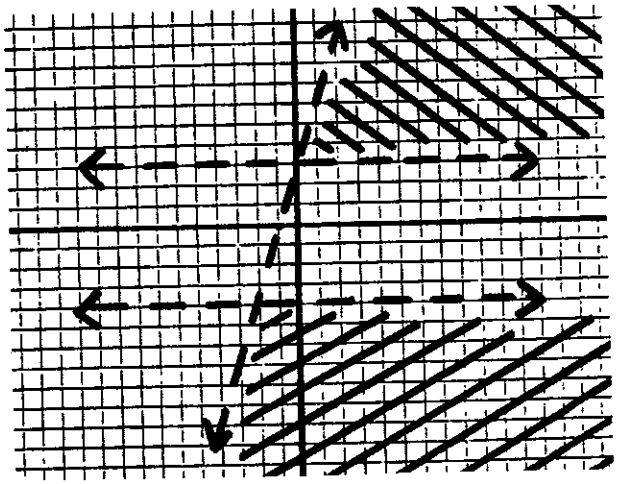
③ ABSOLUTE VALUE: Union and Intersection

$|2y+1|>7$ and $y<3x+3$

$2y+1>7$ or $2y+1<-7$

$2y>6$ or $2y<-8$

$(y>3$ or $y<-4)$ and $(y<3x+3)$



Additional Practice: 4.3

At the Insurance Agents Wax Museum

Graphing Systems of Inequalities

PROBLEM SET (Review Unit 4A)

Graph each system of inequalities:

① $y > x+2$
$y < x-6$

② $y \geq x-5$
$y \geq -x-2$

③ $|x-y| \leq 3$

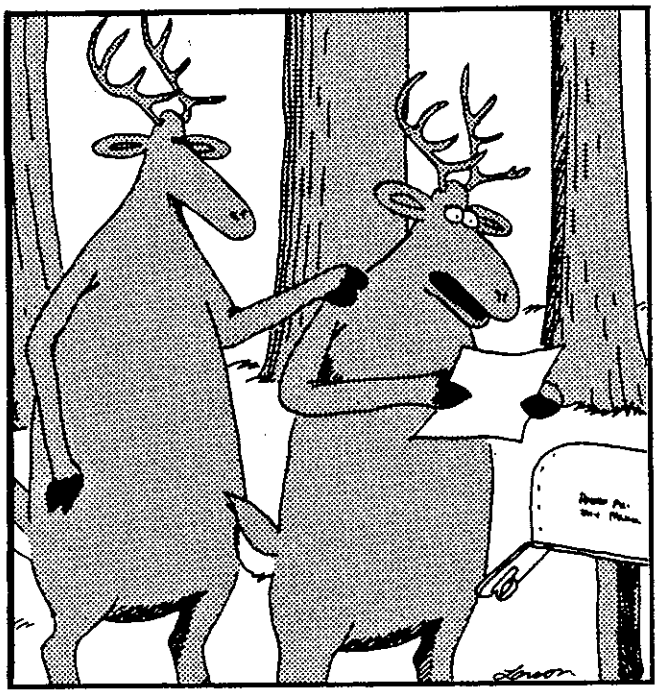
④ $|x-3| > y$

⑤ $|x| \leq 4$
$x+y<3$

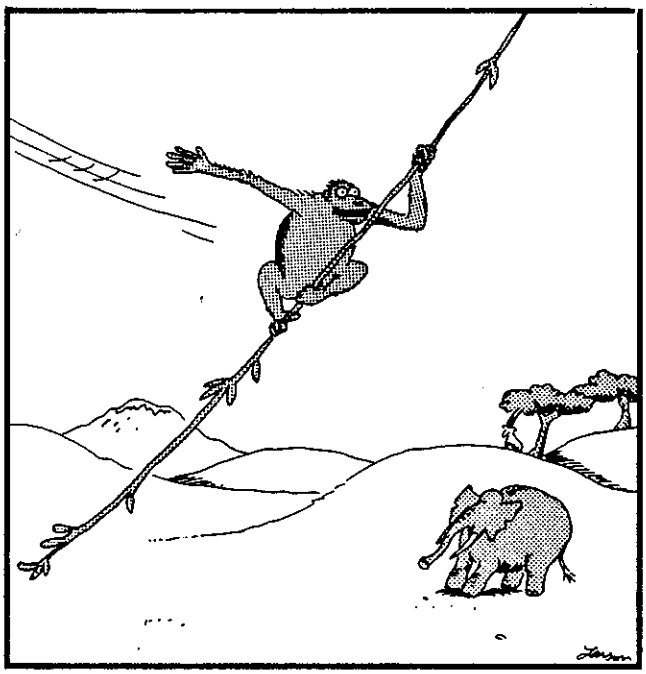
⑥ $|y|<5$
$3x-y<9$

⑦ $|y| \geq 4$
$2x+y<8$

⑧ $|x-y|>4$
$|y|<8$



"Oh my God! It's from Connie! She's written me a 'John deer' letter!"

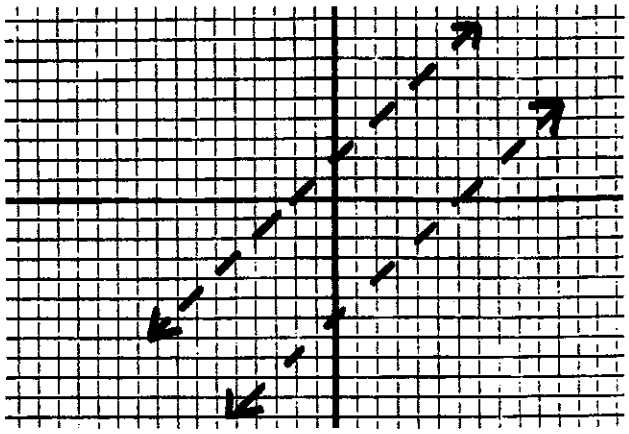


He had seen Tanzania, and most of Mozambique was already behind him. There was no mistake. Chippy had done what most chimps only dream about: He had caught the Perfect Vine.

Graphing Systems of Inequalities

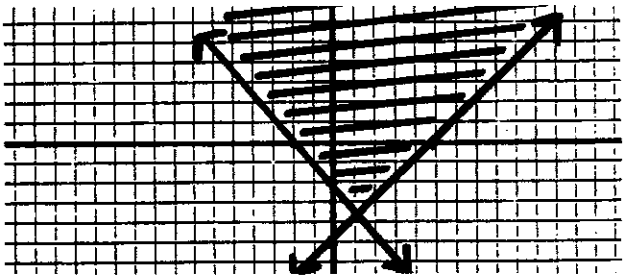
ANSWER KEY (Review Unit 4A)

① $y > x+2$ $y < x-6$



No solution

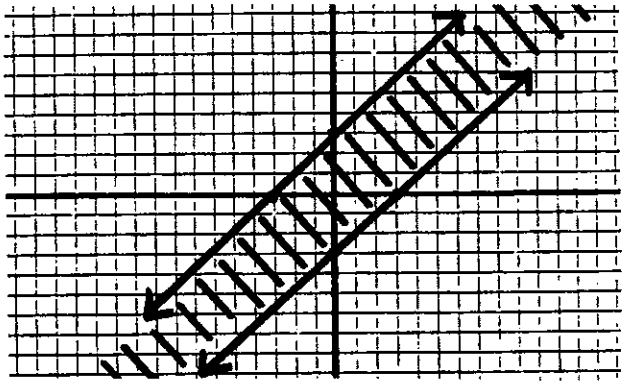
② $y \geq x-5$ $y \geq -x-2$



③ $|x-y| \leq 3$

$x-y \leq 3$ and $x-y \geq -3$

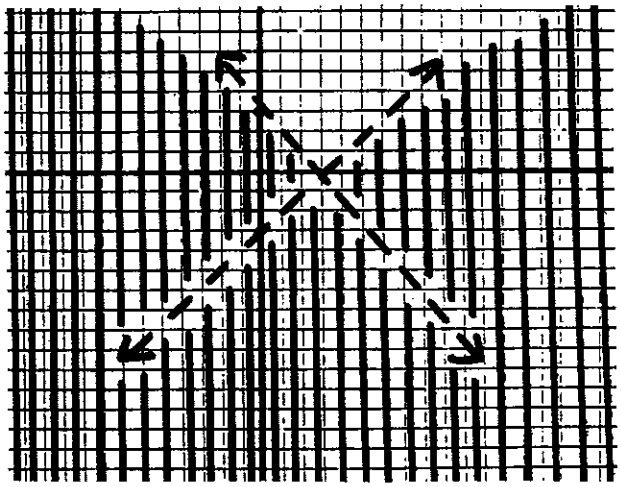
$x-3 \leq y \leq x+3$



④ $|x-3| > y$

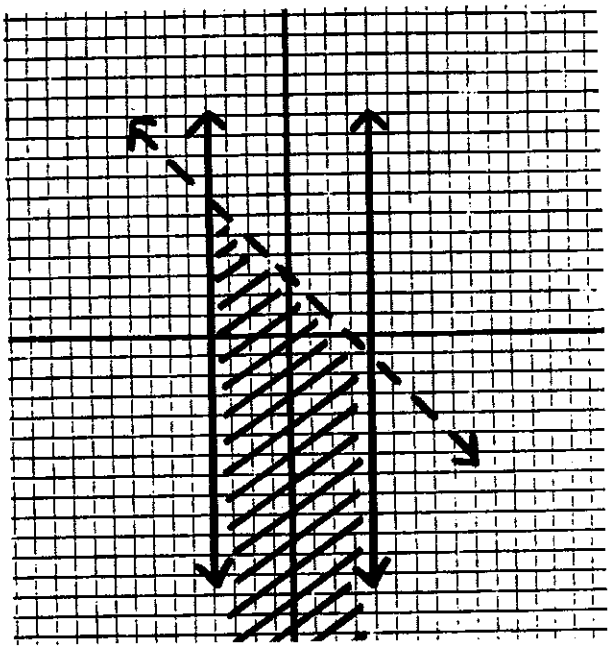
$x-3 > y$ or $x-3 < -y$.

$y < x-3$ or $y < -x+3$



⑤ $|x| \leq 4$ $x+y < 3$

$(-4 \leq x \leq 4)$ and $y < -x+3$

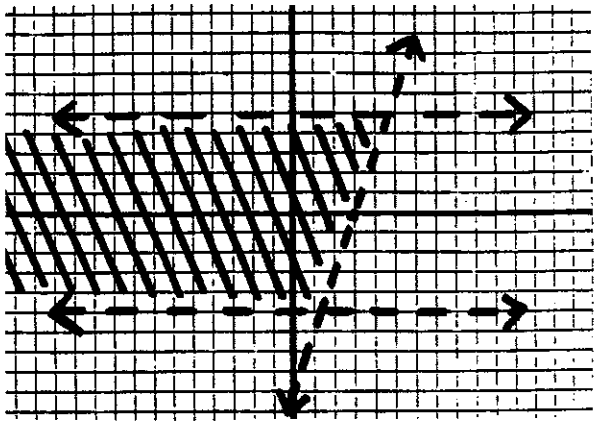


Graphing Systems of Inequalities

ANSWER KEY (Review Unit 4A)

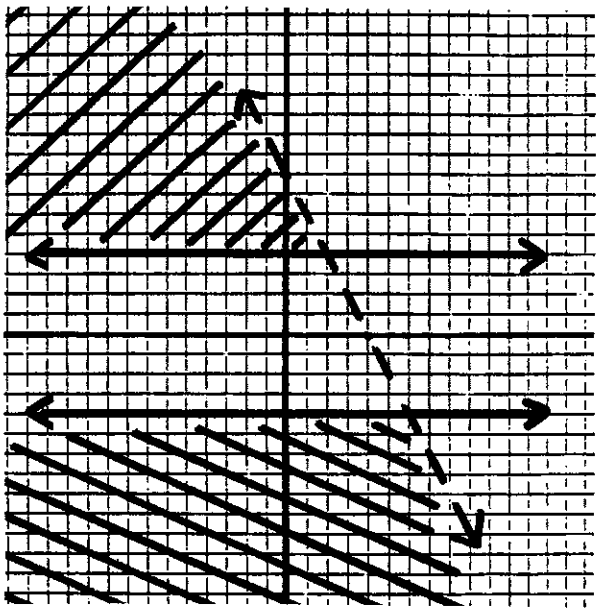
⑥ $|y| < 5 \quad 3x - y < 9$

$(-5 < y < 5)$ and $(y > 3x - 9)$



⑦ $|y| \geq 4 \quad 2x + y < 8$

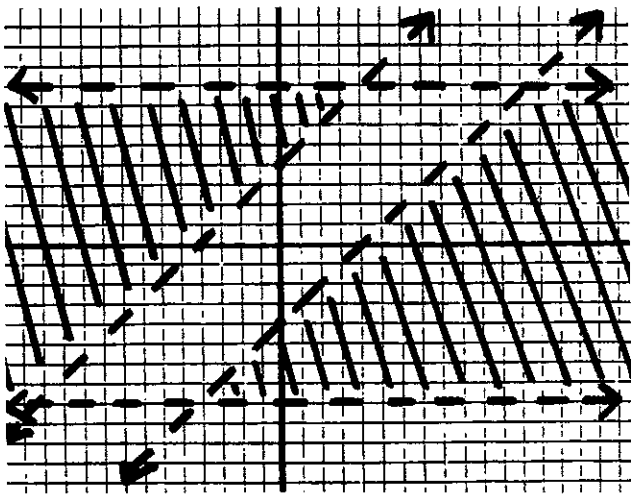
$(y \geq 4$ or $y \leq -4)$ and $(y < -2x + 8)$

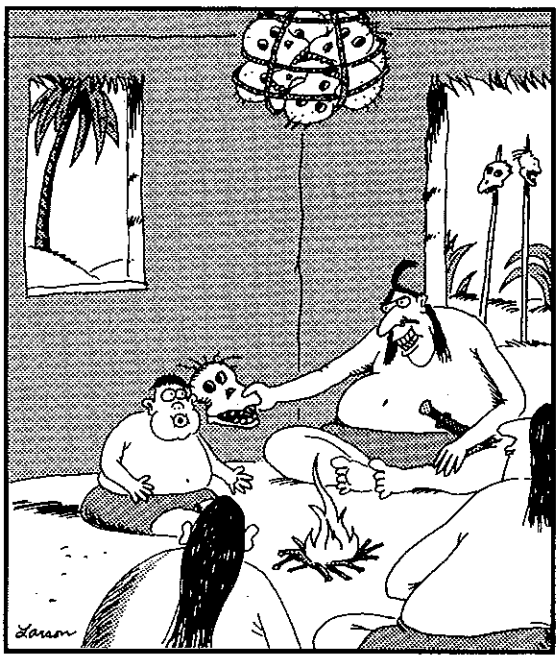


⑧ $|x - y| > 4 \quad |y| < 8$

$(x - y > 4$ or $x - y < -4)$ and $(y < 8$ and $y > -8)$

$(y < x - 4$ or $y > x + 4)$ and $(-8 < y < 8)$





The whole family always enjoyed the way Uncle Numanga could reach over and "find" a skull in little Tooby's ear.

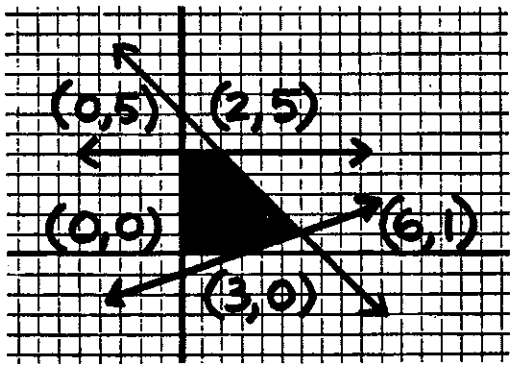
Linear Programming

DEMONSTRATION (Review Unit 4B)

① Function:
$f(x,y) = 5x - 3y$

Conditions:
$x \geq 0$
$x + y \leq 7 \rightarrow y \leq -x + 7$
$0 \leq y \leq 5$
$3y \geq x - 3 \rightarrow y \geq \frac{1}{3}x - 1$

Find the maximum and minimum values for the function subject to the stated conditions.



(x,y)	$5x - 3y$	
(0,5)	$5(0) - 3(5) =$ **-15**	min
(0,0)	$5(0) - 3(0) =$ 0	
(3,0)	$5(3) - 3(0) =$ 15	
(6,1)	$5(6) - 3(1) =$ **27**	max
(2,5)	$5(2) - 3(5) =$ -5	

Suggestion: Use elimination or substitution to find vertices as needed.

"Hey, bucko...I'm through begging."

② The Blair Company makes two kinds of pianos: spinets and consoles. The equipment in the factory allows for making at most 450 spinets and 200 consoles in one month. During the month of June, the company can spend $360,000 to make these pianos (see chart). To maximize profit, how many of each type should the company make? How much profit?

Linear Programming

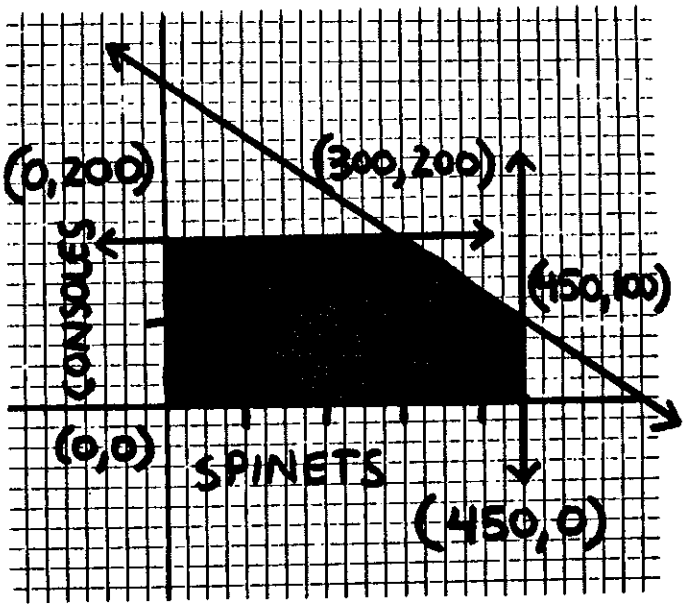
DEMONSTRATION (Review Unit 4B)

	Unit Cost	Unit Profit
Spinet	$600	$125
Console	$900	$200

x = Spinets
y = Consoles

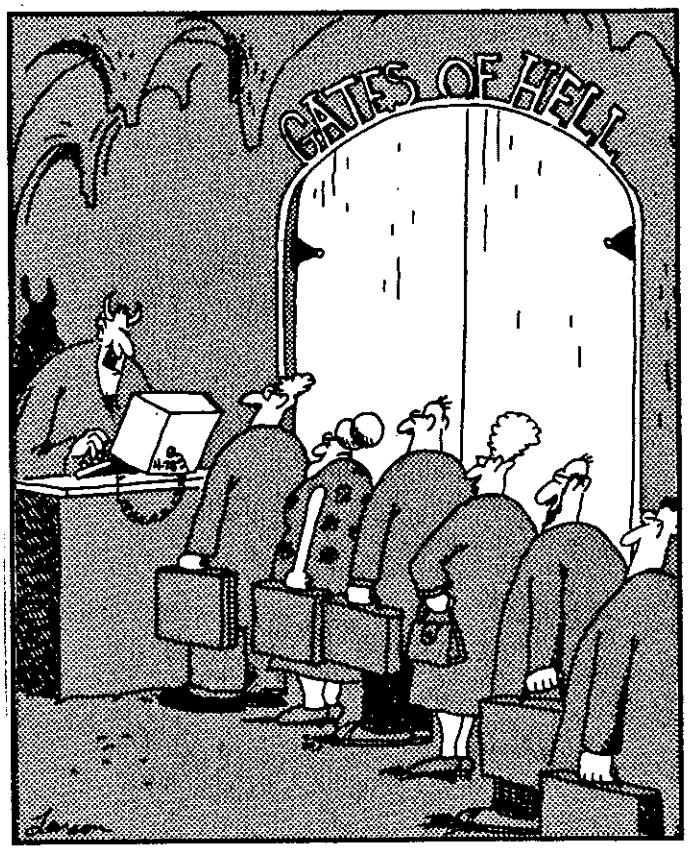
Function:
$f(x,y) = 125x + 200y$

Conditions:
$x \leq 450$
$y \leq 200$
$600x + 900y \leq 360{,}000$
$\hookrightarrow y \leq -\frac{2}{3}x + 400$



Numbers on the axis should be consistent (x and y) and practical (allowing the graph to be reasonable in size).

Additional Practice Problems in: 5.2



"OK, sir, would you like inferno or non-inferno? ... Ha! Just kidding. It's all inferno, of course — I just get a kick out of saying that."

(x, y)	125x + 200y
(0, 200)	125(0) + 200(200) = 40,000
(300, 200)	125(300) + 200(200) = 77,500
(450, 100)	125(450) + 200(100) = 76,250
(450, 0)	125(450) + 200(0) = 56,250
(0, 0)	125(0) + 200(0) = 0

300 Spinets
200 Consoles

$77,500

Linear Programming

PROBLEM SET (Review Unit 4B)

Use linear programming to determine the maximum and minimum values for each function according to the stated conditions:

① Function:
$f(x,y) = 5x + 3y$

Conditions:
$x + y \leq 6$
$x - y \leq 4$
$x \geq 1$
$y \geq -2$

② Function:
$f(x,y) = 3x - 5y$

Conditions:
$2x + y \geq 7$
$x + y \geq 5$
$1 \leq y \leq 5$
$2 \leq x \leq 7$

Use linear programming to solve:

③ Winfield Motors has enough sales personnel to sell up to 100 cars per month. To meet manufacturer's quotas, the company must sell at least 30 large size cars but no more than 60 each month. They must also sell at least 20 compact cars but no more than 60 each month. They make $950 profit on large cars. Due to a factory recall, they lose $125 on each compact car. How many of each should be sold to maximize profit?

④ Lyons Paint and Wallpaper advertised a special sale on two colors of paint: purple and green. To mix a gallon of each color, 3 different dyes are used:

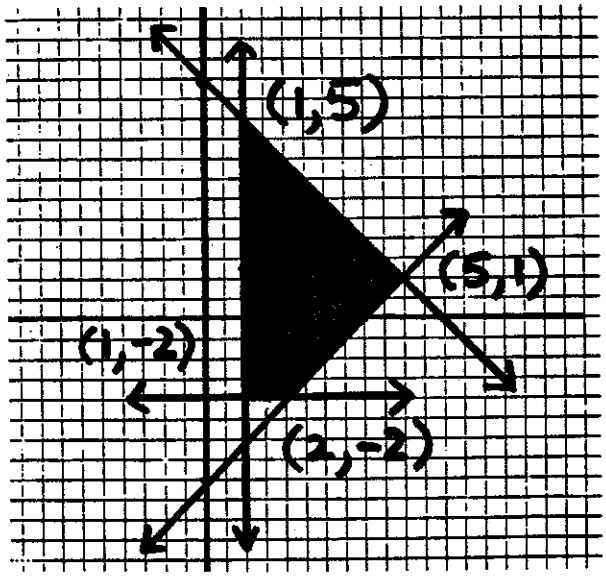
	Dye A	Dye B	Dye C
purple	4 units	1 unit	3 units
green	2 units	8 units	4 units

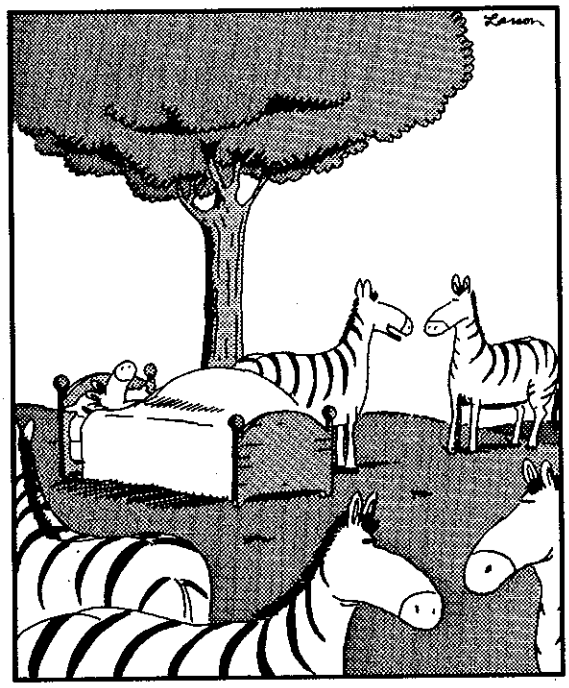
To mix the paint, the store used at least 44 units of A and 56 units of B. They used at most 68 units of C. Because of the sale, they lose $.30 per gallon of purple and $.08 per gallon of green. Selling how much of each color will minimize losses? Maximize losses?

Linear Programming

ANSWER KEY (Review Unit 4B)

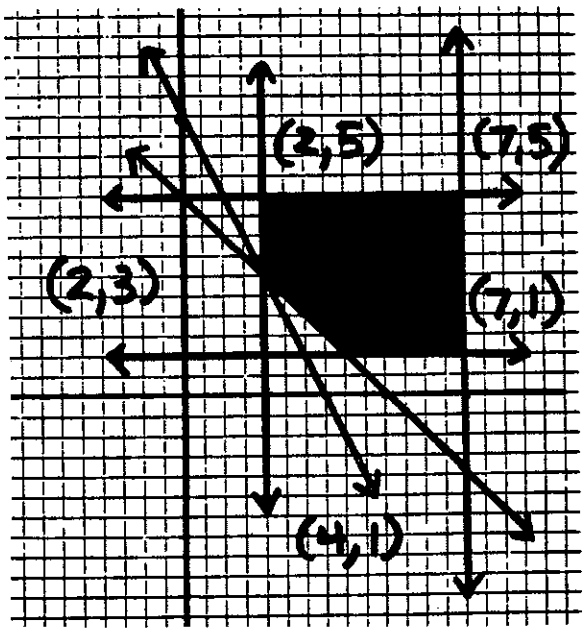
① $x+y \leq 6 \rightarrow y \leq -x+6$
$x-y \leq 4 \rightarrow y \geq x-4$
$x \geq 1$
$y \geq -2$





"Could you come back later?
He's catching a few Y's right now."

(x,y)	$5x+3y$		
$(1,5)$	$5(1)+3(5)$	$= 20$	
$(1,-2)$	$5(1)+3(-2)$	$= \boxed{-1}$	min
$(5,1)$	$5(5)+3(1)$	$= \boxed{28}$	max
$(2,-2)$	$5(2)+3(-2)$	$= 4$	



② $2x+y \geq 7 \rightarrow y \geq -2x+7$
$x+y \geq 5 \quad\quad y \geq -x+5$
$1 \leq y \leq 5$
$2 \leq x \leq 7$

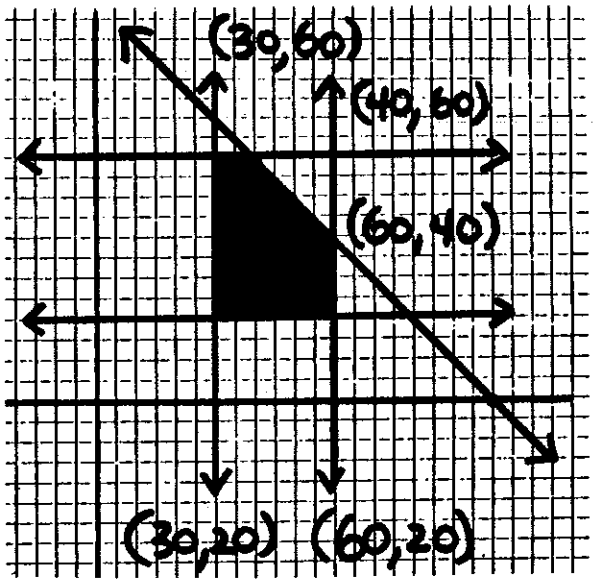
(x,y)	$3x-5y$		
$(2,3)$	$3(2)-5(3)$	$= -9$	
$(2,5)$	$3(2)-5(5)$	$= \boxed{-19}$	min
$(4,1)$	$3(4)-5(1)$	$= 7$	
$(7,5)$	$3(7)-5(5)$	$= -4$	
$(7,1)$	$3(7)-5(1)$	$= \boxed{16}$	max

Linear Programming

ANSWER KEY (Review Unit 4B)

③ x = large size cars
y = compact cars

$x + y \leq 100 \rightarrow y \leq -x + 100$
$20 \leq y \leq 60$
$30 \leq x \leq 60$



(x, y)	$950x - 125y$
(30,60)	$950(30) - 125(60) = 21{,}000$
(40,60)	$950(40) - 125(60) = 30{,}500$
(60,40)	$950(60) - 125(40) = 52{,}000$
(60,20)	$950(60) - 125(20) = \boxed{54{,}500}$
(30,20)	$950(30) - 125(20) = 26{,}000$

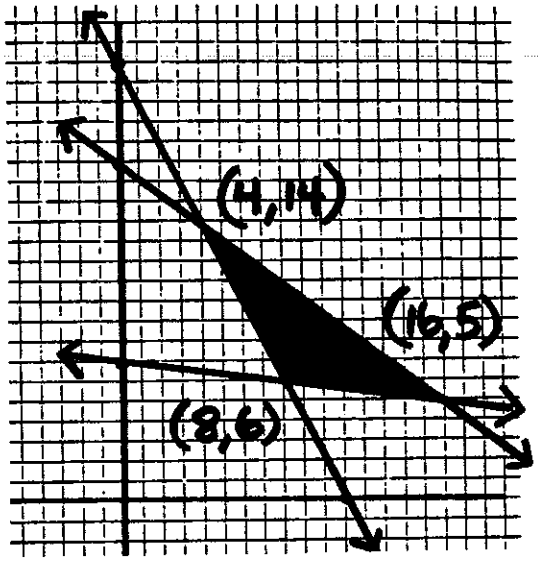
60 large size cars
20 compact cars

$54,500 profit

④ x = gallons of purple
y = gallons of green

Dye

A	$4x + 2y \geq 44$	$y \geq -2x + 22$
B	$x + 8y \geq 56$	$y \geq -\frac{1}{8}x + 7$
C	$3x + 4y \leq 68$	$y \leq -\frac{3}{4}x + 17$



(x, y)	$-.3x - .08y$
(8,6)	$-.3(8) - .08(6) = -2.88$
(4,14)	$-.3(4) - .08(14) = -2.32$
(16,5)	$-.3(16) - .08(5) = -5.20$

To minimize losses:
4 gallons of purple
14 gallons of green $-$2.32

Maximum losses:
16 gallons of purple
5 gallons of green $-$5.20

Radicals

DEMONSTRATION (Review Unit 5A)

① SIMPLIFY RADICALS

a) $\sqrt{32x^2} = 4|x|\sqrt{2}$

b) $\sqrt[3]{-24a^3b^7} = -2ab^2\sqrt[3]{3b}$

c) $\sqrt[4]{a^5b^9c^8} = |a|b^2c^2\sqrt[4]{ab}$

② RADICAL OPERATIONS

a) $\sqrt[4]{16x^2y} \cdot \sqrt[4]{96x^4y^5}$

$\sqrt[4]{2^4x^2y} \cdot \sqrt[4]{2^5 \cdot 3 \cdot x^4y^5}$

$\sqrt[4]{2^9 \cdot 3x^6y^6} = 4|x|y\sqrt[4]{6x^2y^2}$

b) $5\sqrt{27} + 2\sqrt{3} - 7\sqrt{48}$

$5\sqrt{3^3} + 2\sqrt{3} - 7\sqrt{2^4 \cdot 3}$

$15\sqrt{3} + 2\sqrt{3} - 28\sqrt{3} = -11\sqrt{3}$

c) $(6+\sqrt{2})(\sqrt{10}+\sqrt{5})$

$6\sqrt{10} + 6\sqrt{5} + \sqrt{20} + \sqrt{10}$

$6\sqrt{10} + 6\sqrt{5} + 2\sqrt{5} + \sqrt{10}$

$7\sqrt{10} + 8\sqrt{5}$

d) $(\sqrt{6}+\sqrt{3})(\sqrt{6}-\sqrt{3})$

$(\sqrt{6})^2 - (\sqrt{3})^2$

$6 - 3 = 3$ (conjugates)

③ RATIONALIZING

a) $\frac{3}{2\sqrt{5}} = \frac{3}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{10}$

b) $\sqrt[3]{\frac{5}{3b}} = \frac{\sqrt[3]{5}}{\sqrt[3]{3b}} \cdot \frac{\sqrt[3]{3^2b^2}}{\sqrt[3]{3^2b^2}} = \frac{\sqrt[3]{45b^2}}{3b}$

c) $\frac{3-\sqrt{3}}{6+2\sqrt{3}} = \frac{3-\sqrt{3}}{6+2\sqrt{3}} \cdot \frac{6-2\sqrt{3}}{6-2\sqrt{3}}$

$\frac{18-6\sqrt{3}-6\sqrt{3}+6}{36-12} = \frac{2-\sqrt{3}}{2}$

④ RADICAL EXPRESSIONS

$\sqrt{\frac{1}{5}} + \sqrt{20} + \sqrt{75}$

$\frac{\sqrt{1}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}}$

$\frac{\sqrt{5}}{5} + \frac{10\sqrt{5}}{5} + 5\sqrt{3}$

$\frac{11\sqrt{5}}{5} + 5\sqrt{3}$

Additional Practice Problems in:

6.1, 6.2, and 6.3

Radicals

PROBLEM SET (Review Unit 5A)

Simplify:

① $\sqrt{96a^4b^6c^7}$

② $\sqrt{54x^3y^7z^4}$

③ $\sqrt[3]{-81x^7y^9}$

④ $\sqrt[3]{-32a^4b^5}$

⑤ $\sqrt[4]{32x^5y^7z^4}$

⑥ $\sqrt[4]{81a^9b^7c^6}$

⑰ $\dfrac{2-\sqrt{5}}{3+\sqrt{5}}$

⑱ $\dfrac{3-2\sqrt{3}}{6-\sqrt{3}}$

Radical Operations:

⑦ $\sqrt[3]{4x^2yz^2} \cdot \sqrt[3]{16xy^4}$

⑧ $\sqrt[3]{9a^4bc^7} \cdot \sqrt[3]{6ac^2}$

⑨ $\sqrt[4]{8x^2y^5z^7} \cdot \sqrt[4]{4x^3y^7z}$

⑩ $\sqrt[4]{12a^5b^2c^6} \cdot \sqrt[4]{8ab^3c^4}$

⑪ $3\sqrt{12} + \sqrt{\frac{1}{3}} - 3\sqrt{27}$

⑫ $-3\sqrt{45} - \sqrt{\frac{1}{5}} + 4\sqrt{20}$

⑬ $(4-\sqrt{3})(4+\sqrt{3})$

⑭ $(2\sqrt{2}+\sqrt{3})(2\sqrt{2}-\sqrt{3})$

Helen paused. With an audible "wumph," Muffy's familiar yipping had ended, and only the sounds of Ed's football game now emanated from the living room.

Rationalize and Simplify:

⑮ $\sqrt[3]{\frac{3}{2a}}$

⑯ $\sqrt[3]{\frac{3}{32x^2}}$

Radicals

ANSWER KEY (Review Unit 5A)

① $\sqrt{96a^4b^6c^7} = 4a^2|b^3|c^3\sqrt{6c}$

② $\sqrt{54x^3y^7z^4} = 3xy^3z^2\sqrt{6xy}$

③ $\sqrt[3]{-81x^7y^9} = -3x^2y^3\sqrt[3]{3x}$

④ $\sqrt[3]{-32a^4b^5} = -2ab\sqrt[3]{4ab^2}$

⑤ $\sqrt[4]{32x^5y^7z^4} = 2xy|z|\sqrt[4]{2xy^3}$

⑥ $\sqrt[4]{81a^9b^7c^6} = 3a^2|bc|\sqrt[4]{ab^3c^2}$

⑦ $\sqrt[3]{4x^2yz^2} \cdot \sqrt[3]{16xy^4}$

$\sqrt[3]{2^6x^3y^5z^2} = 4xy\sqrt[3]{y^2z^2}$

⑧ $\sqrt[3]{9a^4bc^7} \cdot \sqrt[3]{6ac^2}$

$\sqrt[3]{2\cdot3^3\cdot a^5bc^9} = 3ac^3\sqrt[3]{2a^2b}$

⑨ $\sqrt[4]{8x^2y^5z^7} \cdot \sqrt[4]{4x^3y^7z}$

$\sqrt[4]{2^5x^5y^{12}z^8} = 2x|y^3|z^2\sqrt[4]{2x}$

⑩ $\sqrt[4]{12a^5b^2c^6} \cdot \sqrt[4]{8ab^3c^4}$

$\sqrt[4]{2^5\cdot3a^6b^5c^{10}} = 2abc^2\sqrt[4]{6a^2bc^2}$

⑪ $3\sqrt{12} + \sqrt{\frac{1}{3}} - 3\sqrt{27}$

$6\sqrt{3} + \frac{\sqrt{3}}{3} - 9\sqrt{3}$

$\frac{18\sqrt{3}}{3} + \frac{\sqrt{3}}{3} - \frac{27\sqrt{3}}{3} = \frac{-8\sqrt{3}}{3}$

⑫ $-3\sqrt{45} - \sqrt{\frac{1}{5}} + 4\sqrt{20}$

$-9\sqrt{5} - \frac{\sqrt{5}}{5} + 8\sqrt{5}$

$\frac{-45\sqrt{5}}{5} - \frac{\sqrt{5}}{5} + \frac{40\sqrt{5}}{5} = \frac{-6\sqrt{5}}{5}$

⑬ $(4-\sqrt{3})(4+\sqrt{3}) = 16-3 = 13$

⑭ $(2\sqrt{2}+\sqrt{3})(2\sqrt{2}-\sqrt{3}) = 8-3 = 5$

⑮ $\sqrt[3]{\frac{3}{2a}} = \frac{\sqrt[3]{3}}{\sqrt[3]{2a}} \cdot \frac{\sqrt[3]{4a^2}}{\sqrt[3]{4a^2}} = \frac{\sqrt[3]{12a^2}}{2a}$

⑯ $\sqrt[3]{\frac{3}{32x^2}} = \frac{\sqrt[3]{3}}{\sqrt[3]{32x^2}} \cdot \frac{\sqrt[3]{2x}}{\sqrt[3]{2x}} = \frac{\sqrt[3]{6x}}{4x}$

⑰ $\frac{2-\sqrt{5}}{3+\sqrt{5}} \cdot \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{6-2\sqrt{5}-3\sqrt{5}+5}{9-5}$

$\frac{11-5\sqrt{5}}{4}$

⑱ $\frac{3-2\sqrt{3}}{6-\sqrt{3}} \cdot \frac{6+\sqrt{3}}{6+\sqrt{3}} = \frac{18+3\sqrt{3}-12\sqrt{3}-6}{36-3}$

$\frac{12-9\sqrt{3}}{33} = \frac{4-3\sqrt{3}}{11}$

Radical Equations

DEMONSTRATION (Reivew Unit 5B)

Note: If a variable appears in the radicand of a radical equation, you must check all possible solutions.

① RADICAL EQUATION

$\sqrt{x+16} + x = 14$

$\sqrt{x+16} = 14 - x$

$x + 16 = 196 - 28x + x^2$

$x^2 - 29x + 180 = 0$

$(x-9)(x-20) = 0$

$\boxed{x = 9}$ or ~~20~~

② CUBED ROOT

$\sqrt[3]{3n-1} - 2 = 0$

$\sqrt[3]{3n-1} = 2$

$3n - 1 = 8$

$3n - 9 = 0$

$3(n-3) = 0$

$\boxed{n = 3}$

Note: You must isolate the radical before squaring or cubing both sides of the equation.

③ TWO RADICALS

$\sqrt{y+12} + 1 = \sqrt{y+21}$

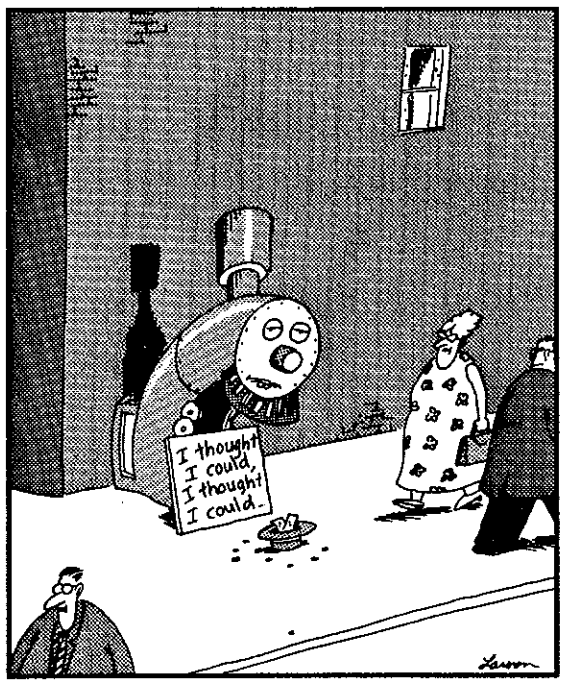
$y + 12 + 2\sqrt{y+12} + 1 = y + 21$

$2\sqrt{y+12} = 8$

$\sqrt{y+12} = 4$

$y + 12 = 16$ $\quad \boxed{y = 4}$

Additional Practice: 6.4



Radical Equations

PROBLEM SET (Reivew Unit 5B)

Solve each radical equation:

① $\sqrt{2x+1} - 1 = 2$

② $\sqrt{3x+1} - 5 = -1$

③ $\sqrt{3x+7} - x = 3$

④ $\sqrt{2n+10} - n = 5$

⑤ $\sqrt{x-3} + x = 5$

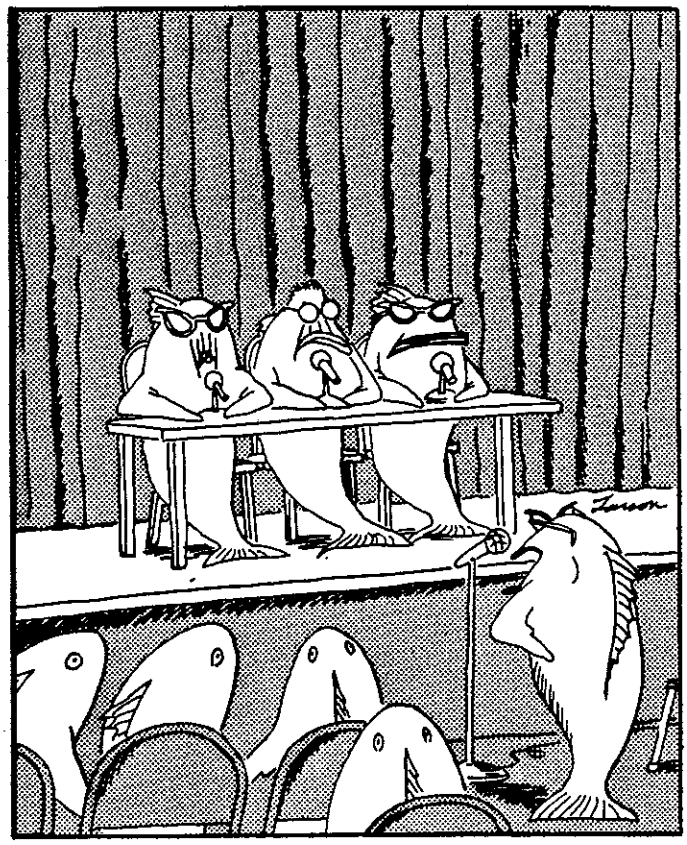
⑥ $\sqrt{10+3x} - x = 4$

⑦ $\sqrt[3]{3n+2} - 3 = -1$

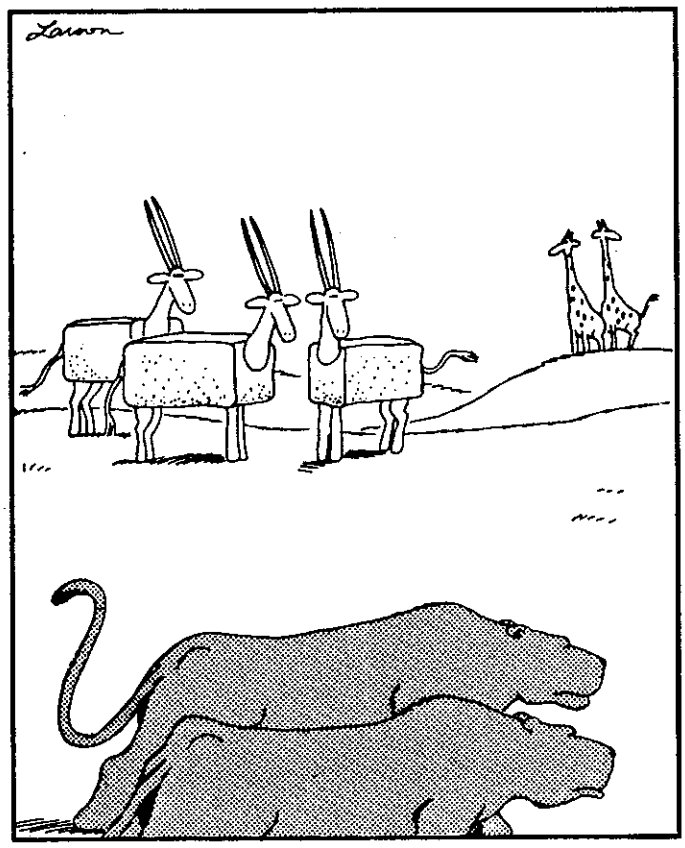
⑧ $\sqrt[3]{4x+7} - 5 = -2$

⑨ $\sqrt{7-x} - 1 = \sqrt{x-2}$

⑩ $\sqrt{x+3} - 1 = \sqrt{x-2}$



The committee to decide whether spawning should be taught in school



Knowing the lions' preference for red meat, the spamalopes remained calm but wary.

Radical Equations

ANSWER KEY (Reivew Unit 5B)

① $\sqrt{2x+1} - 1 = 2$

$\sqrt{2x+1} = 3$

$2x+1 = 9$

$2x = 8$

$x = 4$

② $\sqrt{3x+1} - 5 = -1$

$\sqrt{3x+1} = 4$

$3x+1 = 16$

$3x = 15$

$x = 5$

③ $\sqrt{3x+7} - x = 3$

$\sqrt{3x+7} = 3+x$

$3x+7 = 9 + 6x + x^2$

$x^2 + 3x + 2 = 0$

$(x+2)(x+1) = 0$

$x = -2$ or -1

④ $\sqrt{2n+10} - n = 5$

$\sqrt{2n+10} = 5+n$

$2n+10 = 25 + 10n + n^2$

$n^2 + 8n + 15 = 0$

$(n+5)(n+3) = 0$

$n = -3$ or -5

⑤ $\sqrt{x-3} + x = 5$

$\sqrt{x-3} = 5-x$

$x-3 = 25 - 10x + x^2$

$x^2 - 11x + 28 = 0$

$(x-7)(x-4) = 0$

$x = 4$ or ~~7~~

⑥ $\sqrt{10+3x} - x = 4$

$\sqrt{10+3x} = 4+x$

$10+3x = 16 + 8x + x^2$

$x^2 + 5x + 6 = 0$

$(x+3)(x+2) = 0$

$x = -3$ or -2

⑦ $\sqrt[3]{3n+2} - 3 = -1$

$\sqrt[3]{3n+2} = 2$

$3n+2 = 8$

$3n = 6$

$n = 2$

⑧ $\sqrt[3]{4x+7} - 5 = -2$

$\sqrt[3]{4x+7} = 3$

$4x+7 = 27$

$4x = 20$

$x = 5$

⑨ $\sqrt{7-x} - 1 = \sqrt{x-2}$

$7 - x - 2\sqrt{7-x} + 1 = x-2$

$-2\sqrt{7-x} = 2x - 10$

$\sqrt{7-x} = 5-x$

$7-x = 25 - 10x + x^2$

$x^2 - 9x + 18 = 0$

$(x-6)(x-3) = 0$

$x = 3$ or ~~6~~

⑩ $\sqrt{x+3} - 1 = \sqrt{x-2}$

$x+3 - 2\sqrt{x+3} + 1 = x-2$

$-2\sqrt{x+3} = -6$

$\sqrt{x+3} = 3$

$x+3 = 9$

$x = 6$

Rational Form & Radical Form

DEMONSTRATION (Review Unit 6A)

① FRACTIONAL EXPONENTS

$5^{\frac{2}{3}}$ ← exponent of radicand
← index
↑ radicand

Rational form: $5^{2/3}$

Radical form: $\sqrt[3]{5^2}$ or $\sqrt[3]{25}$

② CONDITIONS FOR SIMPLIFIED EXPRESSIONS

No negative exponents
No fractional exponents in the denominator
No complex fractions
Smallest index possible

③ RATIONAL FORM

a) $27^{\frac{2}{3}} = (3^3)^{\frac{2}{3}} = 3^2 = 9$

b) $8^{\frac{3}{2}} = (2^3)^{\frac{3}{2}} = 2^{\frac{9}{2}} = 16 \cdot 2^{\frac{1}{2}}$

c) $\sqrt[5]{(32x)^2} = \sqrt[5]{2^{10}x^2}$

$2^{\frac{10}{5}} x^{\frac{2}{5}} = 2^2 x^{\frac{2}{5}} = 4x^{2/5}$

④ RADICAL FORM

a) $\sqrt[6]{25} = \sqrt[6]{5^2} = 5^{2/6} = 5^{1/3} = \sqrt[3]{5}$

b) $\sqrt[4]{(64y^2z^2)^3} = \sqrt[4]{2^{18}y^6z^6}$

$2^{9/2} y^{3/2} z^{3/2} = \sqrt{2^9 y^3 z^3} = 16yz\sqrt{2yz}$

c) $4^{2/3} a^{1/6} b^{1/2} = 4^{4/6} a^{1/6} b^{3/6}$

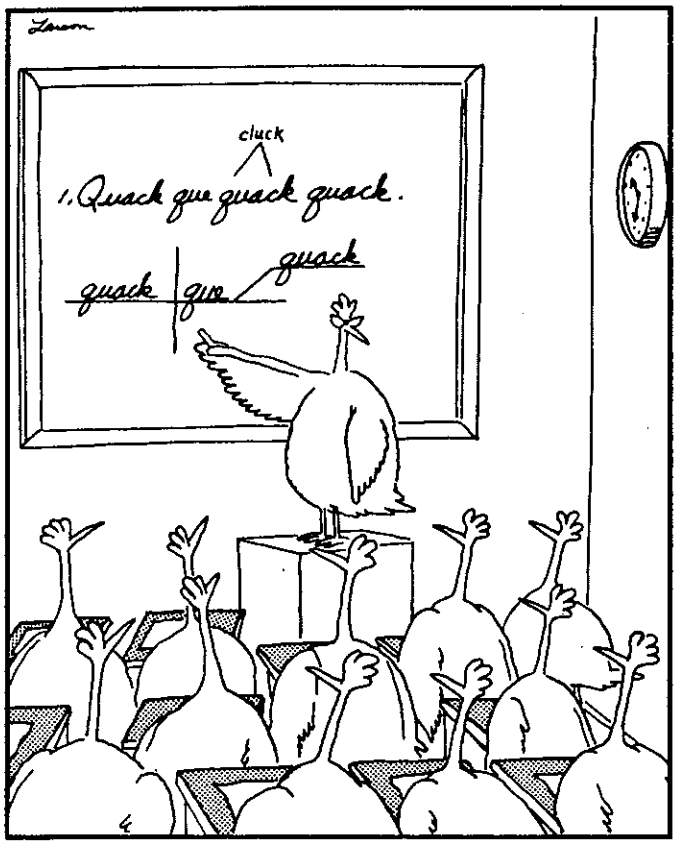
$\sqrt[6]{4^4 a b^3} = \sqrt[6]{2^8 a b^3} = 2\sqrt[6]{4ab^3}$

⑤ NEGATIVE FRACTIONAL EXPONENTS

$5^{-2/3} = \frac{1}{5^{2/3}} = \frac{1}{5^{2/3}} \cdot \frac{5^{1/3}}{5^{1/3}} = \frac{5^{1/3}}{5}$

Rational form: $5^{1/3}/5$

Radical form: $\sqrt[3]{5}/5$



Beginning duck

Rational Form & Radical Form

DEMONSTRATION (Review Unit 6A)

⑥ MULTIPLYING / DIVIDING

a) $n^{1/2}n^{2/3} = n^{3/6}n^{4/6} = n^{7/6}$

Rational form : $n \cdot n^{1/6}$

Radical form : $n\sqrt[6]{n}$

b) $\dfrac{3^{1/2}}{9^{-2/3}} = (3^{1/2})(9^{2/3}) = (3^{1/2})(3^2)^{2/3}$

$(3^{1/2})(3^{4/3}) = (3^{3/6})(3^{8/6}) = 3^{11/6}$

Rational form : $3^{11/6}$

Radical form : $\sqrt[6]{3^{11}} = 3\sqrt[6]{3^5}$

⑦ RATIONALIZE

a) $\dfrac{a^{1/2}b}{a^{2/3}-a^{-1/3}}$

$\dfrac{a^{1/2}b}{a^{2/3}-a^{-1/3}} \cdot \dfrac{a^{1/3}}{a^{1/3}} = \dfrac{a^{5/6}b}{a^1-a^0}$

$\dfrac{a^{5/6}b}{a-1}$

b) $\dfrac{a^{1/2}-b^{1/2}}{a^{1/2}+b^{1/2}}$

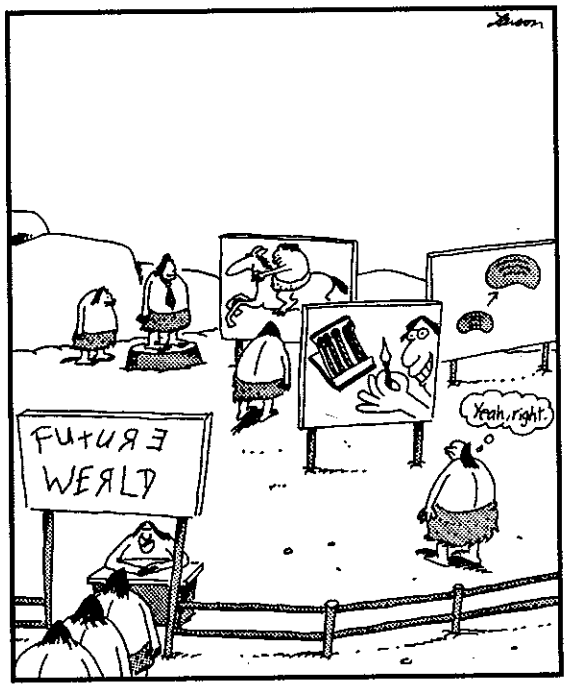
$\dfrac{a^{1/2}-b^{1/2}}{a^{1/2}+b^{1/2}} \cdot \dfrac{a^{1/2}-b^{1/2}}{a^{1/2}-b^{1/2}} = \dfrac{(a^{1/2}-b^{1/2})^2}{(a^{1/2})^2-(b^{1/2})^2} = \dfrac{a-2a^{1/2}b^{1/2}+b}{a-b}$

⑧ SIMPLIFY

$\left(\dfrac{a^{-1/2}}{3^{-2}a^{-3/2}}\right)^{1/2} = \dfrac{a^{-1/4}}{3^{-1}a^{-3/4}}$

$(3)(a^{-1/4})(a^{3/4}) = 3a^{1/2}$

Additional Practice Problems in : 7.1 and 7.2



Primitive theme parks

Rational Form & Radical Form

PROBLEM SET (Review Unit 6A)

Express in simplified rational form:

① $\sqrt[4]{32x^3y^2}$

② $\sqrt[6]{81a^8b^6c}$

③ $16^{2/3}$

④ $125^{2/3}$

Express in simplified radical form:

⑤ $\sqrt[6]{(9a^3b)^4}$

⑥ $\sqrt[4]{(3^3x^4y^5)^2}$

⑦ $9^{2/3}x^{1/2}y^{5/6}$

⑧ $16^{1/3}a^{5/4}b^{3/2}$

Express in <u>both</u> forms:

⑨ $4^{-1/3}$

⑩ $8^{-1/4}$

⑪ $(a^{3/2})(a^{2/3})$

⑫ $(n^{1/2})(n^{4/5})$

Rationalize and simplify:

⑬ $\dfrac{x^{2/3}y}{x^{-1/3}-x^{5/3}}$

⑭ $\dfrac{a^{1/2}b^{1/3}}{a^{-1/4}+a^{3/4}}$

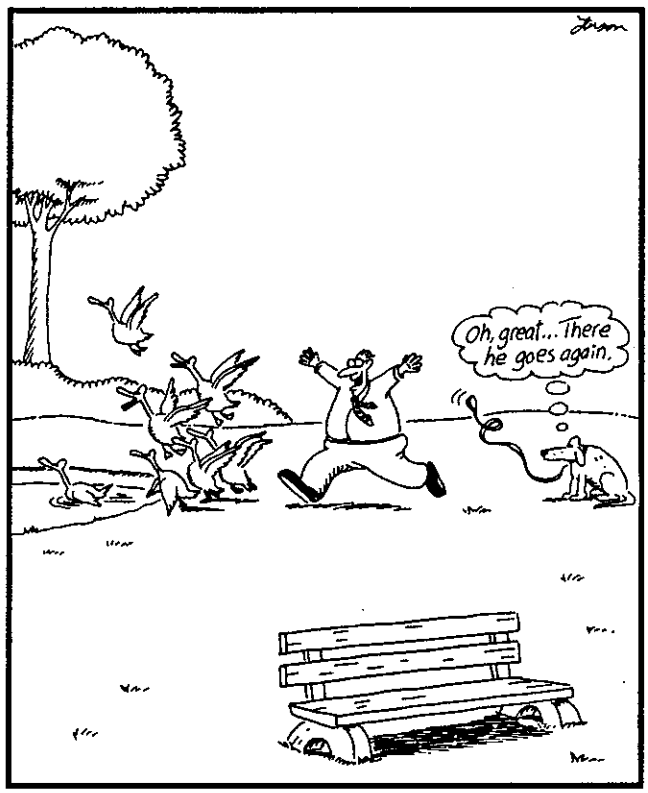
⑮ $\dfrac{a^{1/2}+b}{a^{1/2}-b}$

⑯ $\dfrac{x^2-y^{1/2}}{x^2+y^{1/2}}$

Simplify (rationalize if needed):

⑰ $\left(\dfrac{x^{2/3}}{2^2xy^{-1/3}}\right)^{-3}$

⑱ $\left(\dfrac{a^{-1/2}b^3}{4ab^{-2}c}\right)^{-1/2}$



Rational Form & Radical Form

ANSWER KEY (Review Unit 6A)

① $\sqrt[4]{32x^3y^2} = 2^{5/4}x^{3/4}y^{2/4}$

$2^{5/4}x^{3/4}y^{1/2}$

② $\sqrt[6]{81a^8b^6c} = 3^{4/6}a^{8/6}b^{6/6}c^{1/6}$

$3^{2/3}a^{4/3}bc^{1/6}$

③ $16^{2/3} = (2^4)^{2/3} = 2^{8/3} = 4\cdot 2^{2/3}$

④ $125^{2/3} = (5^3)^{2/3} = 5^2 = 25$

⑤ $\sqrt[6]{(9a^3b)^4} = \sqrt[6]{3^8a^{12}b^4}$

$3^{8/6}a^{12/6}b^{4/6} = 3^{4/3}a^2b^{2/3}$

$3a^2\sqrt[3]{3b^2}$

⑥ $\sqrt[4]{(3^3x^4y^5)^2} = \sqrt[4]{3^6x^8y^{10}}$

$3^{6/4}x^{8/4}y^{10/4} = 3^{3/2}x^2y^{5/2}$

$3x^2y^2\sqrt{3y}$

⑦ $9^{2/3}x^{1/2}y^{5/6} = (3^2)^{2/3}x^{1/2}y^{5/6}$

$3^{8/6}x^{3/6}y^{5/6} = 3\sqrt[6]{3^2x^3y^5}$

$3\sqrt[6]{9x^3y^5}$

⑧ $16^{1/3}a^{5/4}b^{3/2} = (2^4)^{1/3}a^{5/4}b^{3/2}$

$2^{4/3}a^{5/4}b^{3/2} = 2^{16/12}a^{15/12}b^{18/12}$

$2ab\sqrt[12]{2^4a^3b^6} = 2ab\sqrt[12]{16a^3b^6}$

⑨ $4^{-1/3} = (2^2)^{-1/3} = 2^{-2/3} = 1/2^{2/3}$

$\frac{1}{2^{2/3}}\cdot\frac{2^{1/3}}{2^{1/3}} = \frac{2^{1/3}}{2}$

Rational form: $2^{1/3}/2$

Radical form: $\sqrt[3]{2}/2$

⑩ $8^{-1/4} = (2^3)^{-1/4} = 2^{-3/4} = 1/2^{3/4}$

$\frac{1}{2^{3/4}}\cdot\frac{2^{1/4}}{2^{1/4}} = \frac{2^{1/4}}{2}$

Rational form: $2^{1/4}/2$

Radical form: $\sqrt[4]{2}/2$

⑪ $(a^{3/2})(a^{2/3}) = a^{\frac{3}{2}+\frac{2}{3}}$

$a^{\frac{9}{6}+\frac{4}{6}} = a^{13/6}$

Rational form: $a^2\cdot a^{1/6}$

Radical form: $a^2\sqrt[6]{a}$

⑫ $(n^{1/2})(n^{4/5}) = n^{1/2+4/5}$

$n^{5/10+8/10} = n^{13/10}$

Rational form: $n\cdot n^{3/10}$

Radical form: $n\sqrt[10]{n^3}$

⑬ $\frac{x^{2/3}y}{x^{-1/3}-x^{5/3}}\cdot\frac{x^{1/3}}{x^{1/3}} = \frac{x^{3/3}y}{x^0-x^{6/3}}$

$\frac{xy}{1-x^2}$

Rational Form & Radical Form

ANSWER KEY (Review Unit 6A)

(14) $\dfrac{a^{1/2}b^{1/3}}{a^{-1/4}+a^{3/4}} \cdot \dfrac{a^{1/4}}{a^{1/4}} = \dfrac{a^{3/4}b^{1/3}}{1+a}$

(15) $\dfrac{a^{1/2}+b}{a^{1/2}-b} \cdot \dfrac{a^{1/2}+b}{a^{1/2}+b} = \dfrac{(a^{1/2}+b)^2}{(a^{1/2})^2-b^2}$

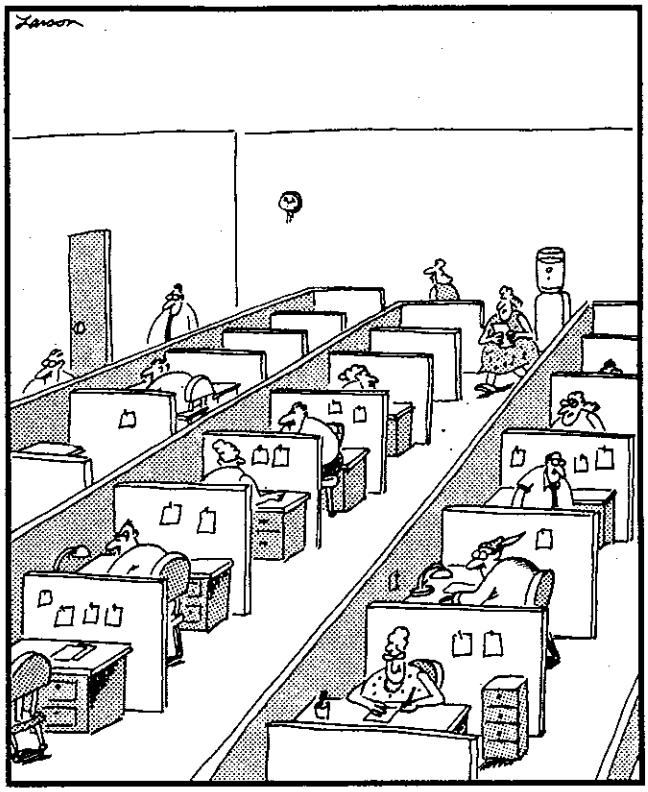
$\dfrac{a+2a^{1/2}b+b^2}{a-b^2}$

(16) $\dfrac{x^2-y^{1/2}}{x^2+y^{1/2}} \cdot \dfrac{x^2-y^{1/2}}{x^2-y^{1/2}} = \dfrac{(x^2-y^{1/2})^2}{(x^2)^2-(y^{1/2})^2}$

$\dfrac{x^4-2x^2y^{1/2}+y}{x^4-y}$

(17) $\left(\dfrac{x^{2/3}}{2^2 x\, y^{-1/3}}\right)^{-3} = \dfrac{x^{-2}}{2^{-6}x^{-3}y}$

$\dfrac{2^6 x}{y} = \dfrac{64x}{y}$



Thirty years had passed, and although he had no real regrets about marrying Wendy, buying a home, and having two kids, Peter found his thoughts often going back to his life in Never-Never-Land.

(18) $\left(\dfrac{a^{-1/2}b^3}{4ab^{-2}c}\right)^{-1/2} = \dfrac{a^{1/4}b^{-3/2}}{4^{-1/2}a^{-1/2}bc^{-1/2}}$

$\dfrac{4^{1/2}a^{1/4}a^{1/2}c^{1/2}}{b\,b^{3/2}} = \dfrac{2a^{3/4}c^{1/2}}{b^{5/2}}$ ← Remember: No fractional exponents in the denominator of a simplified expression. you must rationalize in this problem.

$\dfrac{2a^{3/4}c^{1/2}}{b^{5/2}} \cdot \dfrac{b^{1/2}}{b^{1/2}} = \dfrac{2a^{3/4}b^{1/2}c^{1/2}}{b^3}$

Imaginary & Complex Numbers

DEMONSTRATION (Review Unit 6B)

① THE IMAGINARY UNIT

a) $(3i)(5i) = 15i^2 = -15$

b) $\sqrt{-16} = i\sqrt{16} = 4i$

c) $(\sqrt{-3})(\sqrt{72}) = (i\sqrt{3})(2i\sqrt{3})$

$2i^2\sqrt{9} = 6i^2 = -6$

Note: You must remove the negative from under the radical before multiplying radicands.

Note: The i unit is only used to replace negative radicands if the index is even.

Simplifying powers of i:

$i^1 = i,\ i^2 = -1,\ i^3 = -i,\ i^4 = 1$

② IMAGINARY SOLUTIONS

$4x^2 + 45 = 0$

$4x^2 = -45$

$x^2 = -45/4$

$x = \pm\sqrt{-45}/2$

$x = \dfrac{\pm 3i\sqrt{5}}{2}$

③ COMPLEX NUMBERS

a) $(6-5i)-(3-2i)$

$6-5i-3+2i = 3-3i$

b) $(6-7i)(4+3i)$

$24+18i-28i-21i^2$

$24-10i-21i^2 = 45-10i$

c) Multiplying Conjugates

$(3+5i)(3-5i)$

$(3)^2-(5i)^2 = 9-25i^2 = 34$

④ SIMPLIFY COMPLEX NUMBERS

$\dfrac{3+7i}{2i}$ Note: i unit cannot be in the denominator

$\dfrac{3+7i}{2i} \cdot \dfrac{i}{i} = \dfrac{3i+7i^2}{2i^2} = \dfrac{-7+3i}{-2}$

$\dfrac{7-3i}{2}$ Note: avoid negative denominator

Additional Practice Problems in:

7.3 and 7.4

Imaginary & Complex Numbers

PROBLEM SET (Review Unit 6B)

Compute and simplify:

① $(4i^2)(5i)(2i^3)$

② $(3i^3)(i^4)(2i)$

③ $(\sqrt{-6})(\sqrt{-8})$

④ $(\sqrt{-2})(\sqrt{-6})(\sqrt{-8})$

Solve:

⑤ $3n^2+42=-12$

⑥ $25n^2+27=0$

Compute and Simplify:

⑦ $(2+3i)(5-4i)$

⑧ $(6-6i)(2-8i)$

Multiply conjugates:

⑨ $(4-6i)(4+6i)$

⑩ $(3+7i)(3-7i)$

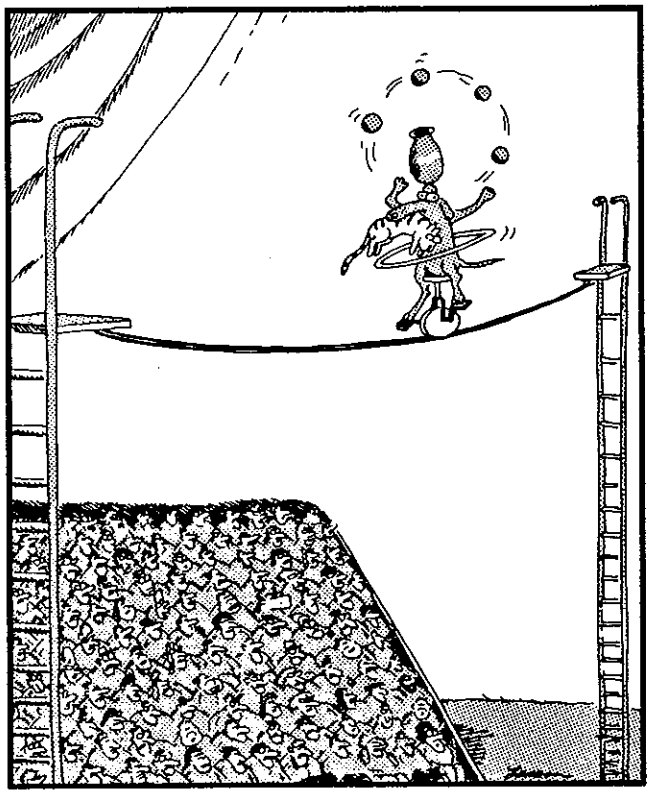
Simplify:

⑪ $\frac{4-2i}{3i}$

⑫ $\frac{2+3i}{5i}$

⑬ $\frac{3-2i}{2+i}$

⑭ $\frac{5+3i}{5-3i}$



High above the hushed crowd, Rex tried to remain focused. Still, he couldn't shake one nagging thought: He was an old dog and this was a new trick.

Imaginary & Complex Numbers

ANSWER KEY (Review Unit 6B)

① $(4i^2)(5i)(2i^3)$
$40i^6 = 40i^2 = -40$

② $(3i^3)(i^4)(2i)$
$6i^8 = 6i^4 = 6$

③ $(\sqrt{-6})(\sqrt{-8}) = (i\sqrt{6})(2i\sqrt{2})$
$2i^2\sqrt{12} = 4i^2\sqrt{3} = -4\sqrt{3}$

④ $(\sqrt{-2})(\sqrt{-6})(\sqrt{-8}) = (i\sqrt{2})(i\sqrt{6})(2i\sqrt{2})$
$2i^3\sqrt{24} = 4i^3\sqrt{6} = -4i\sqrt{6}$

⑤ $3n^2 + 42 = -12$
$3n^2 = -54$
$n^2 = -18$
$n = \pm\sqrt{-18}$
$n = \pm 3i\sqrt{2}$

⑥ $25n^2 + 27 = 0$
$25n^2 = -27$
$n^2 = -27/25$
$n = \pm\frac{\sqrt{-27}}{5}$
$n = \pm\frac{3i\sqrt{3}}{5}$

⑦ $(2+3i)(5-4i)$
$10 - 8i + 15i - 12i^2$
$10 + 7i - 12i^2 = 22 + 7i$

⑧ $(6-6i)(2-8i)$
$12 - 48i - 12i + 48i^2$
$12 - 60i + 48i^2 = -36 - 60i$

⑨ $(4-6i)(4+6i)$
$(4)^2 - (6i)^2 = 16 - 36i^2 = 52$

⑩ $(3+7i)(3-7i)$
$(3)^2 - (7i)^2 = 9 - 49i^2 = 58$

⑪ $\frac{4-2i}{3i} \cdot \frac{i}{i} = \frac{4i-2i^2}{3i^2} = \frac{2+4i}{-3} = \frac{-2-4i}{3}$

⑫ $\frac{2+3i}{5i} \cdot \frac{i}{i} = \frac{2i+3i^2}{5i^2} = \frac{-3+2i}{-5} = \frac{3-2i}{5}$

⑬ $\frac{3-2i}{2+i} \cdot \frac{2-i}{2-i} = \frac{6-3i-4i+2i^2}{(2)^2-(i)^2}$

$\frac{6-7i+2i^2}{4-i^2} = \frac{4-7i}{5}$

⑭ $\frac{5+3i}{5-3i} \cdot \frac{5+3i}{5+3i} = \frac{25+30i+9i^2}{(5)^2-(3i)^2}$

$\frac{16+30i}{25-9i^2} = \frac{16+30i}{34} = \frac{8+15i}{17}$

Quadratic Equations

DEMONSTRATION (Review Unit 7A)

① FACTORING

$3x^2 = 10x - 3$

$3x^2 - 10x + 3 = 0$

$3x^2 - 9x - x + 3 = 0$

$3x(x-3) - 1(x-3) = 0$

$(x-3)(3x-1) = 0$

$x = 3$ or $1/3$

② COMPLETING THE SQUARE

$2m^2 - 8m + 3 = 0$

$m^2 - 4m = -3/2$

$m^2 - 4m + 4 = -3/2 + 4$

$(m-2)^2 = 5/2$

$m - 2 = \pm\sqrt{5}/\sqrt{2}$

$m - 2 = \pm\sqrt{10}/2$

$m = \dfrac{4 \pm \sqrt{10}}{2}$

③ QUADRATIC FORMULA

$3x^2 - 5x + 9 = 0$ $\quad a = 3$, $b = -5$, $c = 9$

$x = \dfrac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(9)}}{2(3)}$

$x = \dfrac{5 \pm \sqrt{-83}}{6}$

$x = \dfrac{5 \pm i\sqrt{83}}{6}$

④ QUADRATIC FORMULA
Three Solutions

$x^3 = 27$

$x^3 - 27 = 0$

$(x-3)(x^2 + 3x + 9) = 0$

$x = \dfrac{-(3) \pm \sqrt{(3)^2 - 4(1)(9)}}{2(1)}$

$x = \dfrac{-3 \pm \sqrt{-27}}{2} = \dfrac{-3 \pm 3i\sqrt{3}}{2}$

$x = 3, \dfrac{-3 \pm 3i\sqrt{3}}{2}$

⑤ SPECIAL COEFFICIENTS

$ax^2 + abx + ab^2 = 0$

$x^2 + bx = -b^2$

$x^2 + bx + \dfrac{b^2}{4} = -b^2 + \dfrac{b^2}{4}$

$(x + b/2)^2 = \dfrac{b^2 - 4b^2}{4} = \dfrac{-3b^2}{4}$

$x + b/2 = \pm\dfrac{\sqrt{-3b^2}}{2} = \pm\dfrac{bi\sqrt{3}}{2}$

$x = \dfrac{-b \pm bi\sqrt{3}}{2}$

Additional Practice Problems in:

8.1

Quadratic Equations

PROBLEM SET (Review Unit 7A)

Solve by factoring:

① $2x^2 - 11x + 12 = 0$

② $12n^2 - n - 1 = 0$

Imaginary answers are acceptable for all of the following

Solve by completing the square:

③ $4x^2 + 8x - 6 = 0$

④ $3x^2 + 12x - 8 = 0$

Solve by using the quadratic formula:

⑤ $2x^2 - 6x + 7 = 0$

⑥ $5x^2 - 4x + 8 = 0$

⑦ $n^3 = 8$

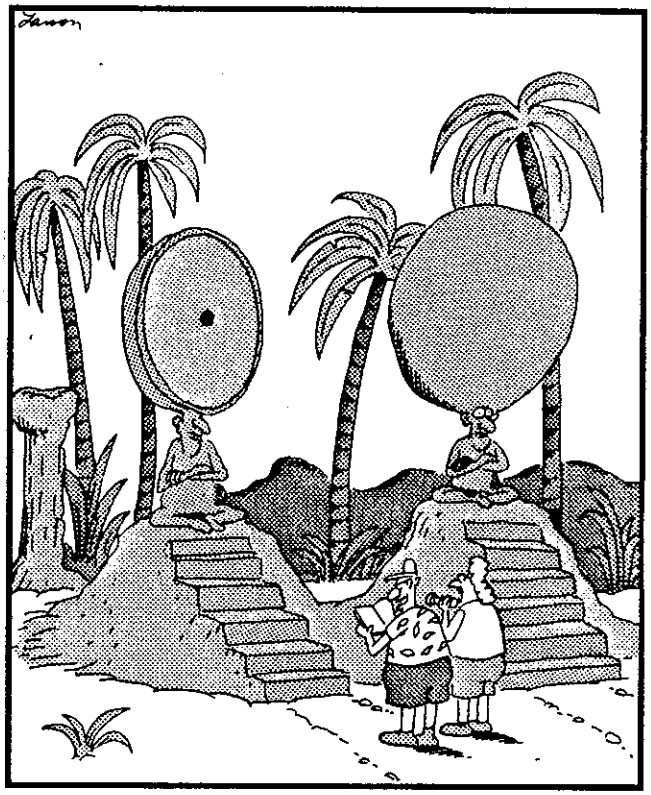
⑧ $x^3 = 1$

Problems #7 and #8 have 3 solutions

Solve by completing the square:

⑨ $ax^2 + b^2x - c = 0$

⑩ $ax^2 + a^2x + a^3 = 0$



"Well, this guidebook is worthless! It just says these people worshipped two gods: one who was all-knowing and one who was all-seeing—but they don't tell you which is which, for crying out loud!"

Quadratic Equations

ANSWER KEY (Review Unit 7A)

① $2x^2-11x+12=0$

$2x^2-8x-3x+12=0$

$2x(x-4)-3(x-4)=0$

$(x-4)(2x-3)=0$

$x=4$ or $3/2$

② $12n^2-n-1=0$

$12n^2-4n+3n-1=0$

$4n(3n-1)+1(3n-1)=0$

$(3n-1)(4n+1)=0$

$n=1/3$ or $-1/4$

③ $4x^2+8x-6=0$

$x^2+2x = 3/2$

$x^2+2x+1 = 3/2+1$

$(x+1)^2 = 5/2$

$x+1 = \frac{\pm\sqrt{5}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}} = \frac{\pm\sqrt{10}}{2}$

$x=\frac{-2\pm\sqrt{10}}{2}$

④ $3x^2+12x-8=0$

$x^2+4x = 8/3$

$x^2+4x+4 = 8/3+4$

$(x+2)^2 = 20/3$

$x+2 = \frac{\pm2\sqrt{5}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} = \frac{\pm2\sqrt{15}}{3}$

$x=\frac{-6\pm2\sqrt{15}}{3}$

⑤ $2x^2-6x+7=0$

$x=\frac{-(-6)\pm\sqrt{(6)^2-4(2)(7)}}{2(2)}$

$x=\frac{6\pm\sqrt{-20}}{4}=\frac{6\pm2i\sqrt{5}}{4}=\frac{3\pm i\sqrt{5}}{2}$

⑥ $5x^2-4x+8=0$

$x=\frac{-(-4)\pm\sqrt{(-4)^2-4(5)(8)}}{2(5)}$

$x=\frac{4\pm\sqrt{-144}}{10}=\frac{4\pm12i}{10}=\frac{2\pm6i}{5}$

⑦ $n^3=8$

$n^3-8=0$

$(n-2)(n^2+2n+4)=0$

$n=\frac{-2\pm\sqrt{(2)^2-4(1)(4)}}{2(1)}$

$n=\frac{-2\pm\sqrt{-12}}{2}=\frac{-2\pm2i\sqrt{3}}{2}$

$n=2, -1\pm i\sqrt{3}$

⑧ $x^3=1$

$x^3-1=0$

$(x-1)(x^2+x+1)=0$

$x=\frac{-1\pm\sqrt{(1)^2-4(1)(1)}}{2(1)}$

$x=\frac{-1\pm\sqrt{-3}}{2}=\frac{-1\pm i\sqrt{3}}{2}$

$x=1, \frac{-1\pm i\sqrt{3}}{2}$

Quadratic Equations

ANSWER KEY (Review Unit 7A)

⑨ $ax^2 + b^2x - c = 0$

$x^2 + \frac{b^2}{a}x = \frac{c}{a}$

$x^2 + \frac{b^2}{a}x + \frac{b^4}{4a^2} = \frac{c}{a} + \frac{b^4}{4a^2}$

$\left(x + \frac{b^2}{2a}\right)^2 = \frac{b^4 + 4ac}{4a^2}$

$x + \frac{b^2}{2a} = \frac{\pm\sqrt{b^4 + 4ac}}{2a}$

$x = \frac{-b^2 \pm \sqrt{b^4 + 4ac}}{2a}$

⑩ $ax^2 + a^2x + a^3 = 0$

$x^2 + ax = -a^2$

$x^2 + ax + \frac{a^2}{4} = -a^2 + \frac{a^2}{4}$

$\left(x + \frac{a}{2}\right)^2 = \frac{a^2 - 4a^2}{4} = \frac{-3a^2}{4}$

$x + \frac{a}{2} = \frac{\pm\sqrt{-3a^2}}{2} = \frac{\pm ai\sqrt{3}}{2}$

$x = \frac{-a \pm ai\sqrt{3}}{2}$

Douglas is ejected from the spoon band.

Humboys

The Discriminant, Roots, & Quadratic Form

DEMONSTRATION (Review Unit 7B)

① DISCRIMINANT VALUE

Value	Roots
0	1 rational
+ (per sq)	2 rational
+ (non per sq)	2 irrational
−	2 imaginary

a) $4x^2+20x+25=0$
$b^2-4ac=0$
1 rational root

b) $a^2+a-12=0$
$b^2-4ac=49$
2 rational roots

c) $x^2+5x-3=0$
$b^2-4ac=37$
2 irrational roots

d) $3y^2+4y+5=0$
$b^2-4ac=-44$
2 imaginary roots

② SUM AND PRODUCT OF ROOTS

ax^2+bx+c
sum $=-b/a$ product $=c/a$

Find a quadratic equation with the given roots:

Note: No fractions

a) $-2/3$, 6 sum: $16/3$ prod: -4
$x^2-\frac{16}{3}x-4=0$
$3x^2-16x-12=0$

b) $5\pm 2i$ sum: 10 prod: 29
$x^2-10x+29=0$

c) $\sqrt{5}$, $-\frac{3}{2}\sqrt{5}$ sum: $-\frac{1}{2}\sqrt{5}$ prod: $\frac{-15}{2}$
$x^2+\frac{1}{2}x\sqrt{5}-\frac{15}{2}=0$
$2x^2+x\sqrt{5}-15=0$

③ THE QUADRATIC FORM

* a) $x-7\sqrt{x}-8=0$
$(\sqrt{x})^2-7\sqrt{x}-8=0$
$(\sqrt{x}-8)(\sqrt{x}+1)=0$
$\sqrt{x}=8$ $\sqrt{x}=-1$
$x=64$ $x=\not{x}$ ← does not check

* b) $x^{\frac{2}{3}}-16=0$
$(x^{1/3})^2-16=0$
$(x^{1/3}+4)(x^{1/3}-4)=0$
$x^{1/3}=-4$ $x^{1/3}=4$
$x=-64$ $x=64$

Both answers check. (Fractional exponents are like radicals – check variables)

The Discriminant, Roots, & Quadratic Form

DEMONSTRATION (Review Unit 7B)

c) $x^4 - 13x^2 + 36 = 0$

$(x^2)^2 - 13x^2 + 36 = 0$

$(x^2 - 9)(x^2 - 4) = 0$

$(x+3)(x-3)(x+2)(x-2) = 0$

$x = \pm 3, \pm 2$

Additional Practice Problems in:

8.2 and 8.3

d) $x^4 - 49 = 0$

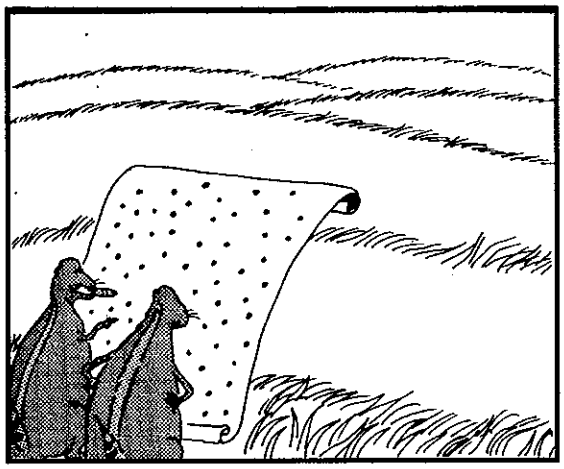
$(x^2)^2 - 49 = 0$

$(x^2+7)(x^2-7) = 0$

$x^2 = -7 \quad x^2 = 7$

$x = \pm i\sqrt{7} \quad x = \pm\sqrt{7}$

$x = \pm i\sqrt{7}, \pm\sqrt{7}$



Prairie dog developers

e) $n^6 - 9n^3 + 8 = 0$

$(n^3)^2 - 9n^3 + 8 = 0$

$(n^3 - 8)(n^3 - 1) = 0$

$(n-2)(n^2+2n+4)(n-1)(n^2+n+1) = 0$

$\frac{-2 \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)}$ $\qquad$ $\frac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$

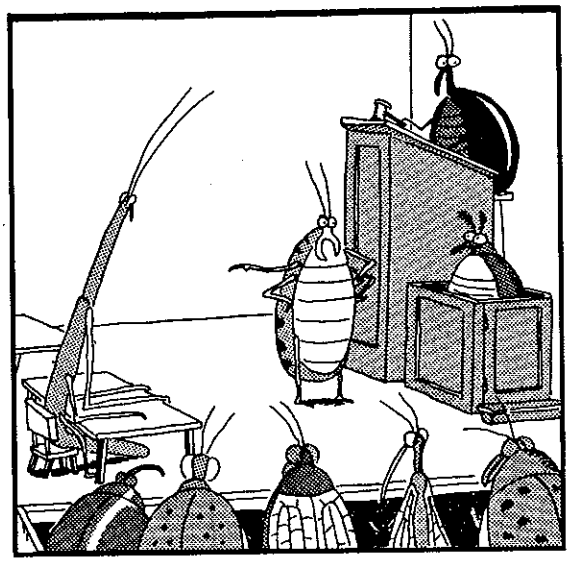
$\frac{-2 \pm \sqrt{-12}}{2}$ $\qquad$ $\frac{-1 \pm \sqrt{-3}}{2}$

$-1 \pm i\sqrt{3}$ $\qquad$ $\frac{-1 \pm i\sqrt{3}}{2}$

2 $\qquad$ 1

$n = 2, -1 \pm i\sqrt{3}, 1, -1 \pm \frac{i\sqrt{3}}{2}$

* Answers in (a) and (b) must be checked



"Most interesting, ma'am—you've identified the defendant as the one you saw running from the scene. I take it, then, that you're unaware that my client is a *walking* stick?"

The Discriminant, Roots, & Quadratic Form

PROBLEM SET (Review Unit 7B)

Use the discriminant to determine the nature of the roots:

① $2x^2+3x+5=0$

② $3x^2-4x+1=0$

③ $4x^2-12x+9=0$

④ $3x^2-2x-3=0$

⑬ $x^4-5x^2+4=0$

⑭ $x^4-10x^2+9=0$

⑮ $x^4-25=0$

⑯ $x^4-9=0$

⑰ $x^6-28x^3+27=0$

⑱ $x^6-64=0$

Determine the equation for a quadratic with the given roots:

⑤ $-\frac{1}{3}, 4$

⑥ $\frac{2}{5}, \frac{1}{4}$

⑦ $3\pm4i$

⑧ $\sqrt{3}, -\frac{2}{3}\sqrt{3}$

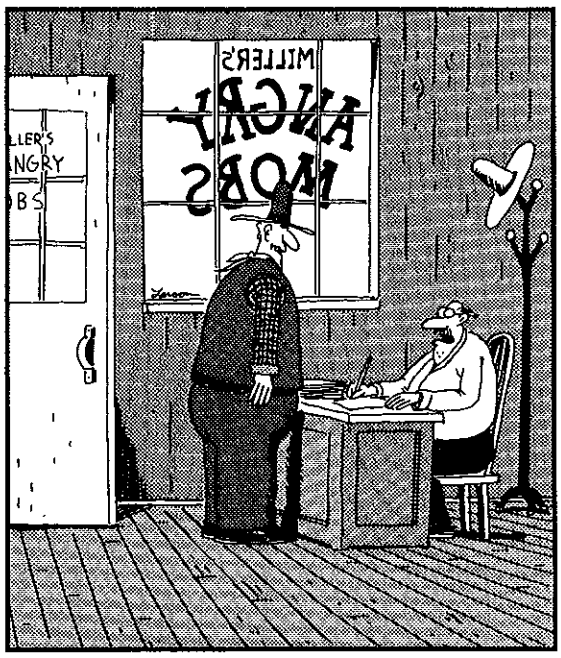
Solve the following equations in the quadratic form:

⑨ $x-6\sqrt{x}+8=0$

⑩ $x-5\sqrt{x}-6=0$

⑪ $x^{2/5}-4=0$

⑫ $x^{1/3}-4=0$



"OK, Bill. Tuesday night, 8 o'clock, over at the sheriff's office where they're holdin' your brother's killer. ... You want that with extra hollerin'?"

The Discriminant, Roots, & Quadratic Form

ANSWER KEY (Review Unit 7B)

① $2x^2+3x+5=0$
$b^2-4ac=-31$
2 imaginary roots

② $3x^2-4x+1=0$
$b^2-4ac=4$
2 rational roots

③ $4x^2-12x+9=0$
$b^2-4ac=0$
1 rational root

④ $3x^2-2x-3=0$
$b^2-4ac=40$
2 irrational roots

⑤ $-1/3, 4$ sum: $11/3$ prod: $-4/3$
$x^2-11/3x-4/3=0$
$3x^2-11x-4=0$

⑥ $2/5, 1/4$ sum: $13/20$ prod: $1/10$
$x^2-13/20x+1/10=0$
$20x^2-13x+2=0$

⑦ $3\pm4i$ sum: 6 prod: 25
$x^2-6x+25=0$

⑧ $\sqrt{3}, -\frac{2}{3}\sqrt{3}$ sum: $\frac{1}{3}\sqrt{3}$ prod: -2
$x^2-\frac{1}{3}\sqrt{3}x-2=0$
$3x^2-x\sqrt{3}-6=0$

⑨ $x-6\sqrt{x}+8=0$
$(\sqrt{x})^2-6\sqrt{x}+8=0$
$(\sqrt{x}-4)(\sqrt{x}-2)=0$
$\sqrt{x}=4$, $\sqrt{x}=2$
$x=16$, $x=4$ ← both answers check

⑩ $x-5\sqrt{x}-6=0$
$(\sqrt{x})^2-5\sqrt{x}-6=0$
$(\sqrt{x}-6)(\sqrt{x}+1)=0$
$\sqrt{x}=6$, $\sqrt{x}=-1$
$x=36$, ~~$x=1$~~ ← does not check

⑪ $x^{2/5}-4=0$
$(x^{1/5})^2-4=0$
$(x^{1/5}+2)(x^{1/5}-2)=0$
$x^{1/5}=-2$, $x^{1/5}=2$
$x=-32$, $x=32$ ← both check

⑫ $x^{1/3}-4=0$
$(x^{1/6})^2-4=0$
$(x^{1/6}+2)(x^{1/6}-2)=0$
$x^{1/6}=-2$, $x^{1/6}=2$
$x=64$

⑬ $x^4-5x^2+4=0$
$(x^2)^2-5x^2+4=0$
$(x^2-4)(x^2-1)=0$
$(x+2)(x-2)(x+1)(x-1)=0$
$x=\pm2, \pm1$

The Discriminant, Roots, & Quadratic Form

ANSWER KEY (Review Unit 7B)

(14) $x^4-10x^2+9=0$

$(x^2)^2-10x^2+9=0$

$(x^2-9)(x^2-1)=0$

$(x+3)(x-3)(x+1)(x-1)=0$

$x=\pm 3, \pm 1$

(15) $x^4-25=0$

$(x^2)^2-25=0$

$(x^2+5)(x^2-5)=0$

$x^2=-5,\ x^2=5$

$x=\pm i\sqrt{5},\ x=\pm\sqrt{5}$

$x=\pm i\sqrt{5}, \pm\sqrt{5}$

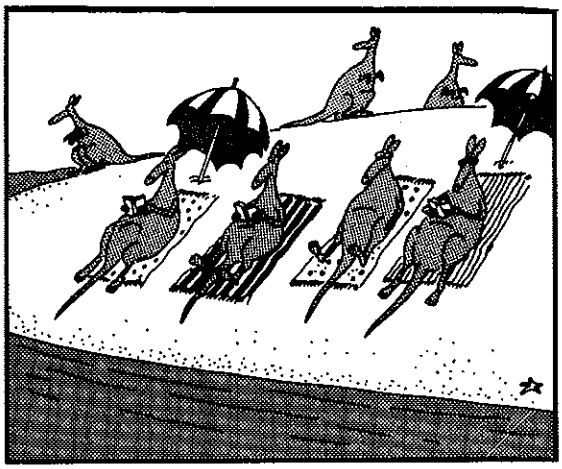
(16) $x^4-9=0$

$(x^2)^2-9=0$

$(x^2+3)(x^2-3)=0$

$x^2=-3,\ x^2=3$

$x=\pm i\sqrt{3}, \pm\sqrt{3}$



Incredibly, Morty had forgotten to bring a pocketbook.

(17) $x^6-28x^3+27=0$

$(x^3)^2-28x^3+27=0$

$(x^3-27)(x^3-1)=0$

$(x-3)(x^2+3x+9)(x-1)(x^2+x+1)=0$

$\frac{-3\pm\sqrt{3^2-4(1)(9)}}{2(1)}$ $\quad$ $\frac{-1\pm\sqrt{1^2-4(1)(1)}}{2(1)}$

$\frac{-3\pm\sqrt{-27}}{2}$ $\quad$ $\frac{-1\pm\sqrt{-3}}{2}$

$\frac{-3\pm 3i\sqrt{3}}{2}$ $\quad$ $\frac{-1\pm i\sqrt{3}}{2}$

$x=3$ $\quad$ $x=1$

$x=3,\ \frac{-3\pm 3i\sqrt{3}}{2},\ 1,\ \frac{-1\pm i\sqrt{3}}{2}$

(18) $x^6-64=0$

$(x^3)^2-64=0$

$(x^3+8)(x^3-8)=0$

$(x+2)(x^2-2x+4)(x-2)(x^2+2x+4)=0$

$\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(4)}}{2(1)}$ $\quad$ $\frac{-2\pm\sqrt{2^2-4(1)(4)}}{2(1)}$

$\frac{2\pm\sqrt{-12}}{2}$ $\quad$ $\frac{-2\pm\sqrt{-12}}{2}$

$1\pm i\sqrt{3}$ $\quad$ $-1\pm i\sqrt{3}$

$x=-2$ $\quad$ $x=2$

$x=\pm 2, \pm 1\pm i\sqrt{3}$

Graphing Quadratic Functions

DEMONSTRATION (Review Unit 7C)

Name the axis of symmetry and the turning point. Draw the graph. Indicate the roots (real or imaginary).

① $y = x^2 - 2x + 7$

axis: $x = \frac{-b}{2a}$ $\boxed{x=1}$ axis

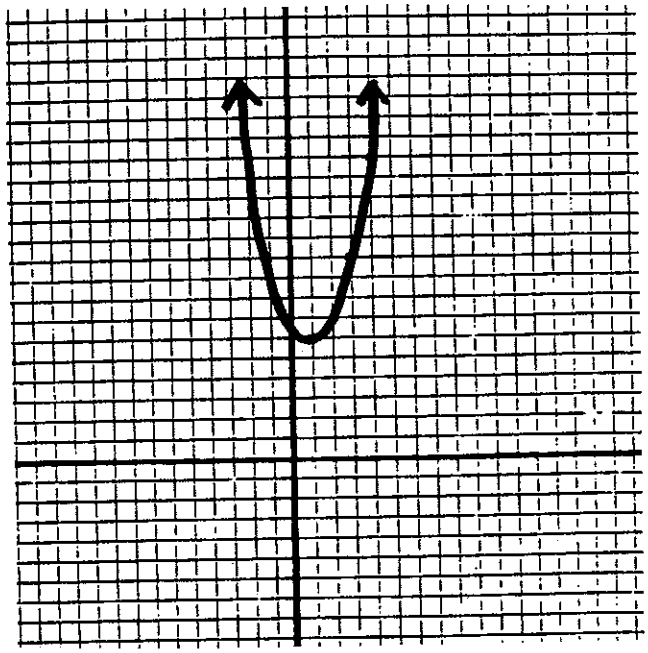
$(1)^2 - 2(1) + 7 = 6$ $\boxed{(1,6)}$ min. pt.

x	y	
0	7	$(0)^2 - 2(0) + 7 = 7$
-1	10	$(-1)^2 - 2(-1) + 7 = 10$
-2	15	$(-2)^2 - 2(-2) + 7 = 15$

$x^2 - 2x + 7 = 0$

$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(7)}}{2(1)}$$

$\boxed{x = 1 \pm i\sqrt{6}}$ roots



② $y > -x^2 + 6x - 5$

axis: $x = \frac{-b}{2a}$ $\boxed{x=3}$ axis

$-(3)^2 + 6(3) - 5 = 4$ $\boxed{(3,4)}$ max. pt.

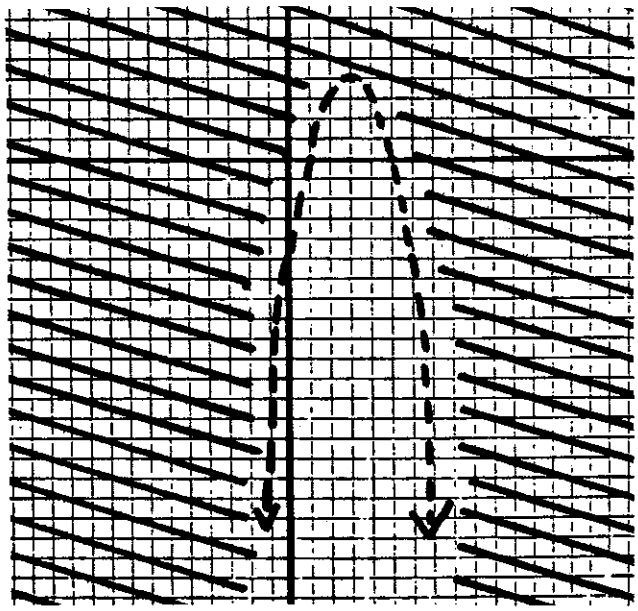
x	y	
2	3	$-(2)^2 + 6(2) - 5 = 3$
1	0	$-(1)^2 + 6(1) - 5 = 0$
0	-5	$-(0)^2 + 6(0) - 5 = -5$

$-x^2 + 6x - 5 = 0$

$x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0$

$x = 5, 1$ $\boxed{x<1 \text{ or } x>5}$ roots



Additional Practice Problems in: 8.4

Graphing Quadratic Functions

PROBLEM SET (Review Unit 7C)

Name the axis of symmetry and the turning point. Draw the graph. Indicate the roots (real or imaginary).

① $y = x^2 - 6x + 8$

② $y = -x^2 - 6x - 9$

③ $y = x^2 + 2x + 6$

④ $y = -x^2 + 4x - 8$

⑤ $y \leq -x^2 - 10x - 24$

⑥ $y < x^2 - 8x + 12$

Inside tours of Acme Fake Vomit Inc.

Sumo temporaries

Graphing Quadratic Functions

ANSWER KEY (Review Unit 7C)

① $y = x^2 - 6x + 8$

axis: $x = \frac{-b}{2a}$ $\boxed{x=3}$ axis

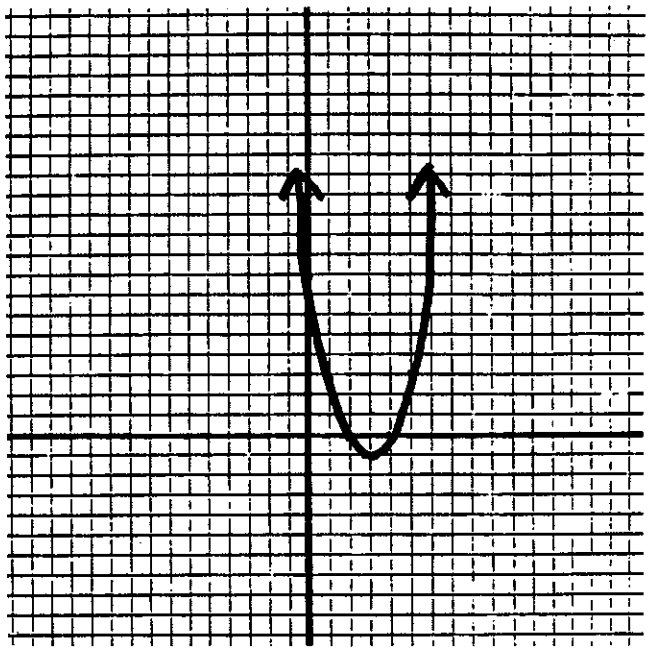
$(3)^2 - 6(3) + 8 = -1$ $\boxed{(3,-1)}$ min. pt.

x	y	
2	0	$(2)^2 - 6(2) + 8 = 0$
1	3	$(1)^2 - 6(1) + 8 = 3$
0	8	$(0)^2 - 6(0) + 8 = 8$

$x^2 - 6x + 8 = 0$

$$\frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(8)}}{2(1)} = \frac{6 \pm \sqrt{4}}{2}$$

$\boxed{x = 2, 4}$ roots



② $y = -x^2 - 6x - 9$

axis: $x = \frac{-b}{2a}$ $\boxed{x=-3}$ axis

$-(-3)^2 - 6(-3) - 9 = 0$ $\boxed{(-3,0)}$ max. pt.

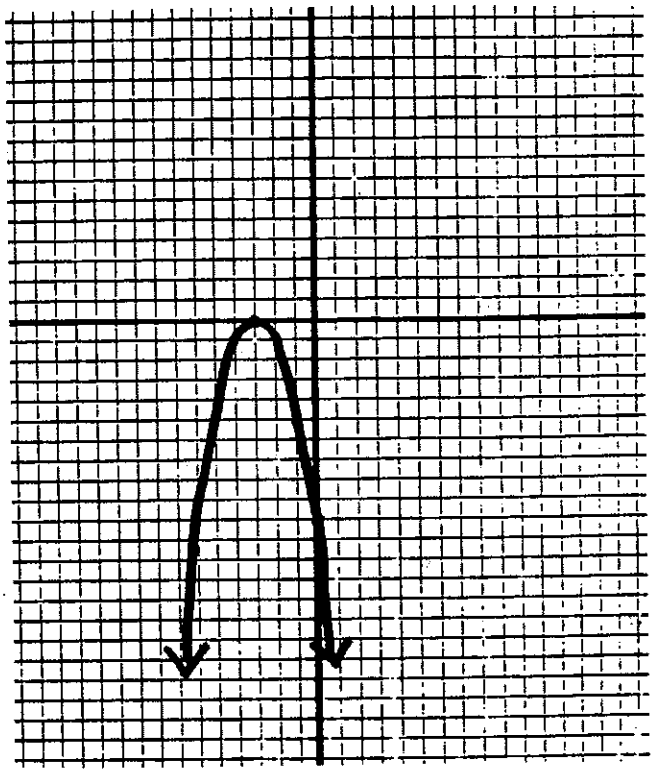
x	y	
-2	-1	$-(-2)^2 - 6(-2) - 9 = -1$
-1	-4	$-(-1)^2 - 6(-1) - 9 = -4$
0	-9	$-(0)^2 - 6(0) - 9 = -9$

$-x^2 - 6x - 9 = 0$

$x^2 + 6x + 9 = 0$

$(x+3)^2 = 0$

$\boxed{x = -3}$ roots



Graphing Quadratic Functions

ANSWER KEY (Review Unit 7C)

③ $y = x^2 + 2x + 6$

$x = \frac{-b}{2a}$ $\boxed{x = -1}$ axis

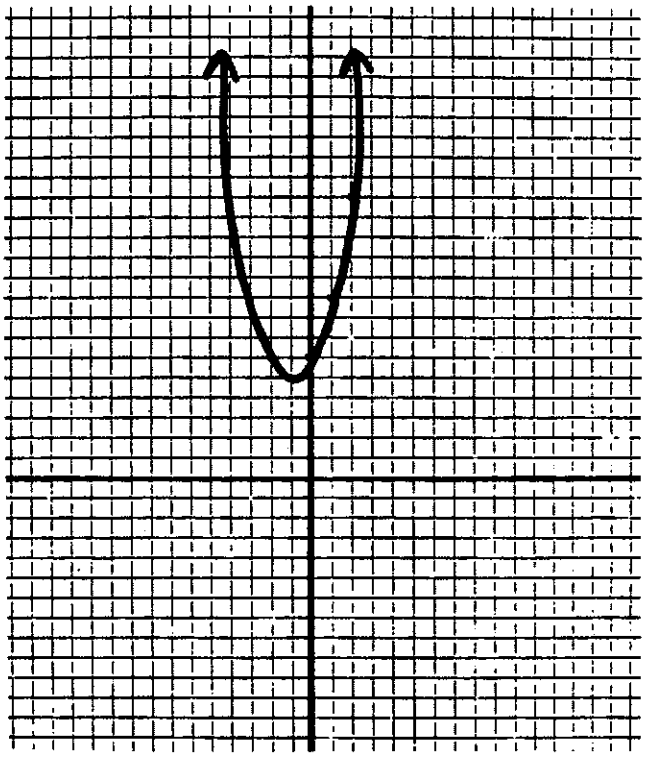
$(-1)^2 + 2(-1) + 6 = 5$ $\boxed{(-1, 5)}$ min. pt.

x	y	
0	6	$(0)^2 + 2(0) + 6 = 6$
1	9	$(1)^2 + 2(1) + 6 = 9$
2	14	$(2)^2 + 2(2) + 6 = 14$

$x^2 + 2x + 6 = 0$

$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(6)}}{2(1)} = \frac{-2 \pm 2i\sqrt{5}}{2}$

$\boxed{x = -1 \pm i\sqrt{5}}$ roots



④ $y = -x^2 + 4x - 8$

$x = \frac{-b}{2a}$ $\boxed{x = 2}$ axis

$-(2)^2 + 4(2) - 8 = -4$ $\boxed{(2, -4)}$ max pt.

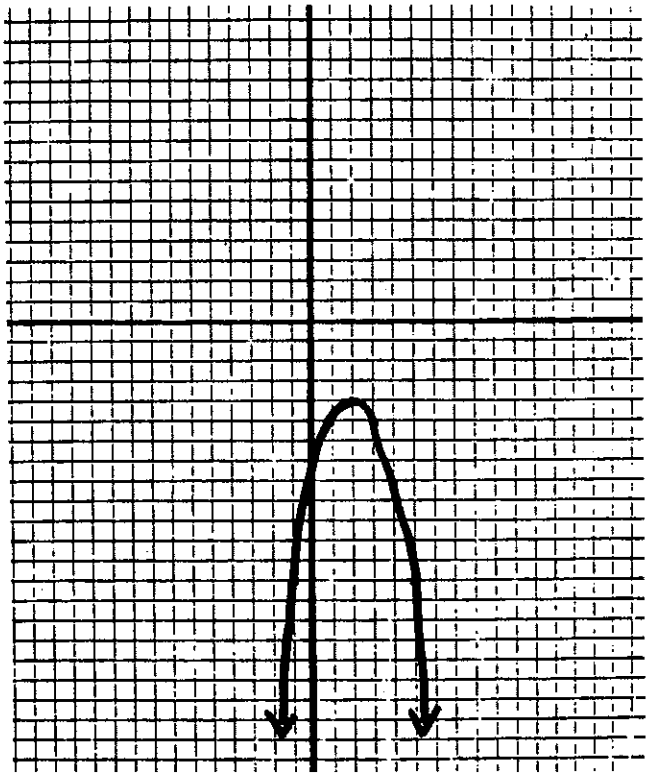
x	y	
1	−5	$-(1)^2 + 4(1) - 8 = -5$
0	−8	$-(0)^2 + 4(0) - 8 = -8$
−1	−13	$-(-1)^2 + 4(-1) - 8 = -13$

$-x^2 + 4x - 8 = 0$

$x^2 - 4x + 8 = 0$

$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(8)}}{2(1)} = \frac{4 \pm 4i}{2}$

$\boxed{x = 2 \pm 2i}$ roots



Graphing Quadratic Functions

ANSWER KEY (Review Unit 7C)

⑤ $y \leq -x^2 - 10x - 24$

$x = \frac{-b}{2a}$ $\boxed{x = -5}$ axis

$-(-5)^2 - 10(-5) - 24 = 1$ $\boxed{(-5, 1)}$ max. pt.

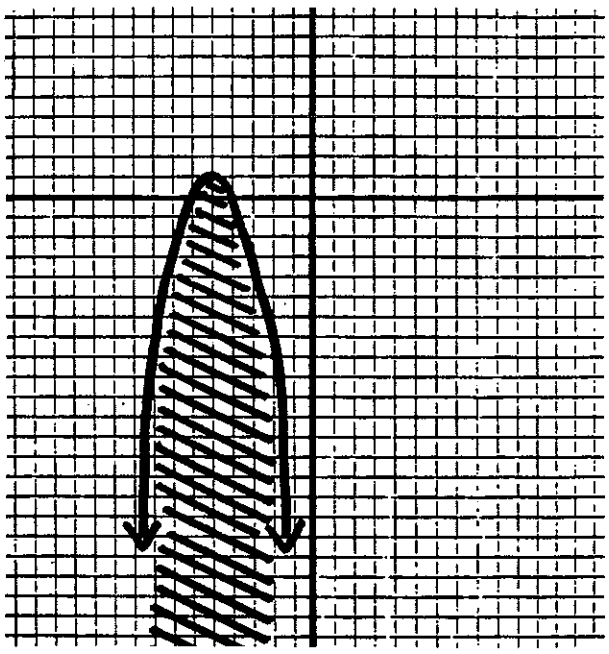
x	y	
-4	0	$-(-4)^2 - 10(-4) - 24 = 0$
-3	-3	$-(-3)^2 - 10(-3) - 24 = -3$
-2	-8	$-(-2)^2 - 10(-2) - 24 = -8$

$-x^2 - 10x - 24 = 0$

$x^2 + 10x + 24 = 0$

$(x+6)(x+4) = 0$ $x = -6, -4$

$\boxed{-6 \leq x \leq -4}$ roots



⑥ $y < x^2 - 8x + 12$

$x = \frac{-b}{2a}$ $\boxed{x = 4}$ axis

$(4)^2 - 8(4) + 12 = -4$ $\boxed{(4, -4)}$ min. pt.

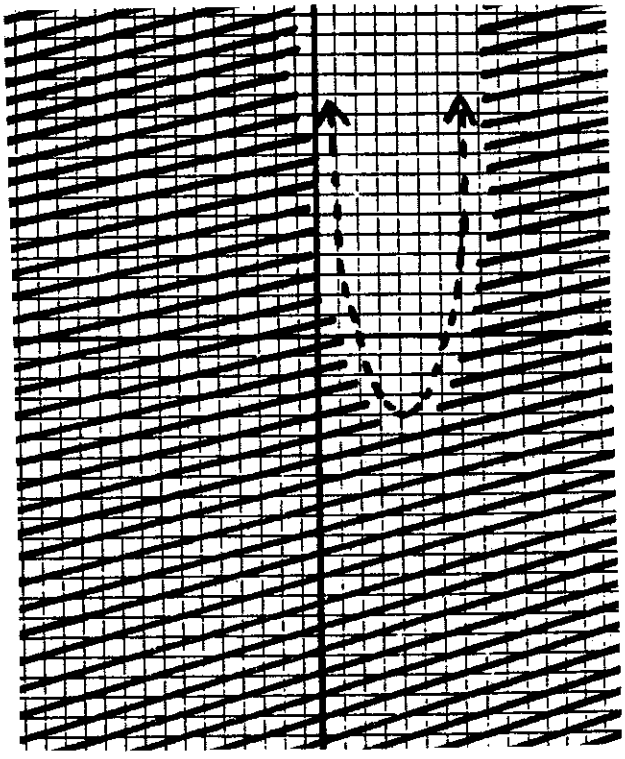
x	y	
3	-3	$(3)^2 - 8(3) + 12 = -3$
2	0	$(2)^2 - 8(2) + 12 = 0$
1	5	$(1)^2 - 8(1) + 12 = 5$

$x^2 - 8x + 12 = 0$

$(x-6)(x-2) = 0$

$x = 6, 2$

$\boxed{x < 2 \text{ or } x > 6}$ roots



Rational Expressions

DEMONSTRATION (Review Unit 8A)

For the purpose of this review unit, it is assumed that all values are real and that variables do not have to be qualified. Denominators $\neq 0$.

① DIVIDE AND SIMPLIFY

$$\frac{x^2}{x^2-25y^2} \div \frac{x}{x+5y}$$

$$\frac{x^2}{x^2-25y^2} \cdot \frac{x+5y}{x}$$

$$\frac{x^2(x+5y)}{(x+5y)(x-5y)x} = \frac{x}{x-5y}$$

② SUBTRACT AND SIMPLIFY

$$\frac{x+4}{2x-8} - \frac{x+12}{4x-16}$$

$$\frac{x+4}{2(x-4)} - \frac{x+12}{4(x-4)}$$

$$\frac{2(x+4) - (x+12)}{4(x-4)}$$

$$\frac{2x+8-x-12}{4(x-4)}$$

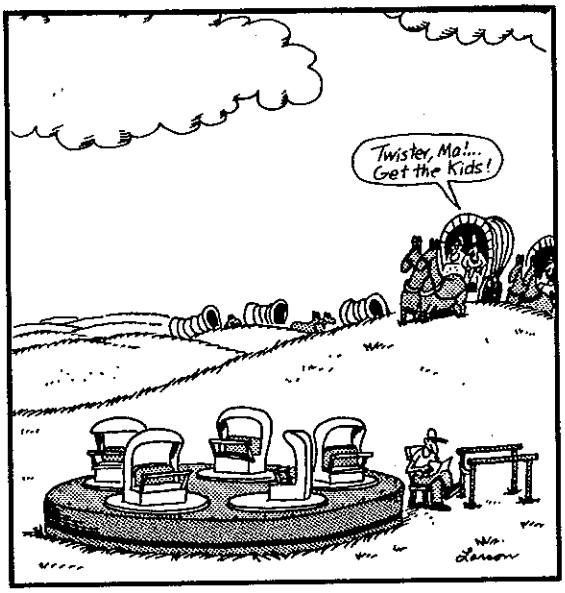
$$\frac{(x-4)}{4(x-4)} = \frac{1}{4}$$

③ COMPLEX FRACTION

$$\frac{y+2+\frac{10}{y-5}}{\frac{4}{y-5}+2} = \frac{\frac{(y+2)(y-5)+10}{(y-5)}}{\frac{4+2(y-5)}{(y-5)}}$$

$$\frac{\frac{(y^2-3y)}{(y-5)}}{\frac{(2y-6)}{(y-5)}} = \frac{(y^2-3y)}{(y-5)} \cdot \frac{(y-5)}{(2y-6)}$$

$$\frac{y^2-3y}{2y-6} = \frac{y(y-3)}{2(y-3)} = \frac{y}{2}$$



Additional Practice: 9.1 and 9.2

Rational Expressions

PROBLEM SET (Review Unit 8A)

Calculate and simplify:

① $\dfrac{x^2+4x+3}{x^2-1} \cdot \dfrac{x-1}{x+4}$

② $\dfrac{n^2+7n+12}{n^2+8n+15} \cdot \dfrac{n+5}{n+4}$

③ $\dfrac{2n^2-7n+3}{n^2-9} \div \dfrac{n-3}{n+3}$

④ $\dfrac{x^2-16}{6x^2-x-2} \div \dfrac{x+4}{2x+1}$

⑤ $\dfrac{n+3}{n-3} - \dfrac{3n-2}{4(3-n)}$

⑥ $\dfrac{4x}{2x-y} + \dfrac{8x+2y}{3(y-2x)}$

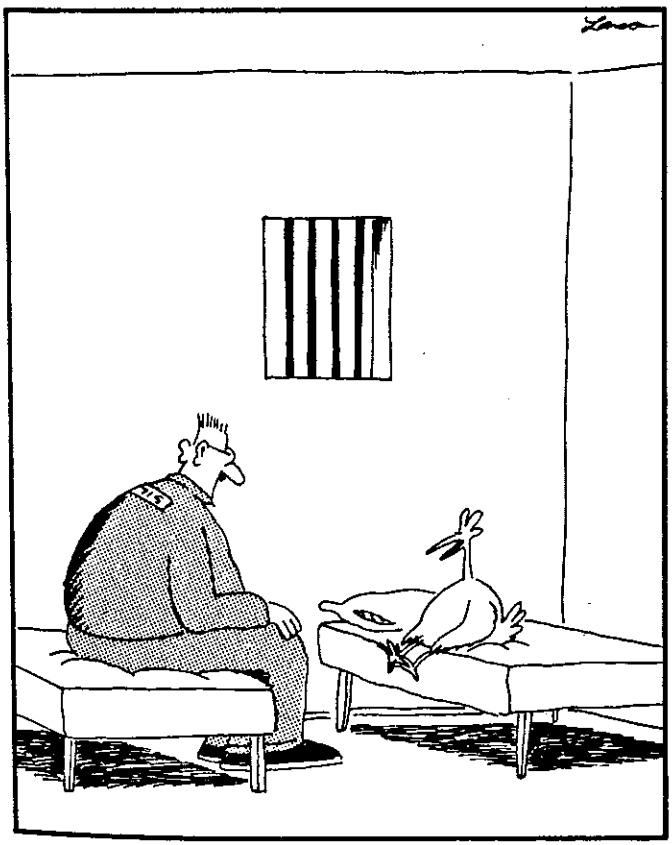
Simplify each complex fraction:

⑦ $\dfrac{\dfrac{1}{n+2} + \dfrac{1}{n-3}}{\left(\dfrac{2n^2+n-1}{n^2+n-2}\right)}$

⑧ $\dfrac{\dfrac{2}{x-4} - \dfrac{1}{2x-1}}{\left(\dfrac{3x^2-10x-8}{x^2-8x+16}\right)}$

⑨ $\dfrac{2+\dfrac{4}{n-1}}{\left(\dfrac{n^2-2n-3}{2n^2-n-1}\right)}$

⑩ $\dfrac{n+\dfrac{2}{n-1}}{\left(\dfrac{2n^2+2n-12}{n^2+2n-3}\right)}$



"So, then, when Old MacDonald turned his back, I took that ax, and with a whack whack here and a whack whack there, I finished him off."

Rational Expressions

ANSWER KEY (Review Unit 8A)

① $\frac{x^2+4x+3}{x^2-1} \cdot \frac{x-1}{x+4}$

$\frac{(x+1)(x+3)(x-1)}{(x+1)(x-1)(x+4)} = \frac{x+3}{x+4}$

② $\frac{n^2+7n+12}{n^2+8n+15} \cdot \frac{n+5}{n+4}$

$\frac{(n+3)(n+4)(n+5)}{(n+5)(n+3)(n+4)} = 1$

③ $\frac{2n^2-7n+3}{n^2-9} \div \frac{n-3}{n+3}$

$\frac{(2n-1)(n-3)}{(n-3)(n+3)} \cdot \frac{(n+3)}{(n-3)} = \frac{2n-1}{n-3}$

④ $\frac{x^2-16}{6x^2-x-2} \div \frac{x+4}{2x+1}$

$\frac{(x-4)(x+4)}{(2x+1)(3x-2)} \cdot \frac{(2x+1)}{(x+4)} = \frac{x-4}{3x-2}$

⑤ $\frac{n+3}{n-3} - \frac{3n-2}{4(3-n)} = \frac{4(n+3)+3n-2}{4(n-3)}$

$\frac{4n+12+3n-2}{4n-12} = \frac{7n+10}{4n-12}$

⑥ $\frac{4x}{2x-y} + \frac{8x+2y}{3(y-2x)} = \frac{3(4x)-(8x+2y)}{3(2x-y)}$

$\frac{12x-8x-2y}{3(2x-y)} = \frac{2(2x-y)}{3(2x-y)} = \frac{2}{3}$

⑦ $\frac{\frac{1}{n+2}+\frac{1}{n-3}}{\left(\frac{2n^2+n-1}{n^2+n-2}\right)} = \frac{\frac{(n-3)+(n+2)}{(n+2)(n-3)}}{\frac{(2n-1)(n+1)}{(n+2)(n-1)}}$

$\frac{(2n-1)}{(n+2)(n-3)} \cdot \frac{(n+2)(n-1)}{(2n-1)(n+1)} = \frac{n-1}{n^2-2n-3}$

⑧ $\frac{\frac{2}{x-4}-\frac{1}{2x-1}}{\left(\frac{3x^2-10x-8}{x^2-8x+16}\right)} = \frac{\frac{2(2x-1)-1(x-4)}{(x-4)(2x-1)}}{\frac{(3x+2)(x-4)}{(x-4)(x-4)}}$

$\frac{3x+2}{(x-4)(2x-1)} \cdot \frac{(x-4)(x-4)}{(3x+2)(x-4)} = \frac{1}{2x-1}$

⑨ $\frac{2+\frac{4}{n-1}}{\left(\frac{n^2-2n-3}{2n^2-n-1}\right)} = \frac{\frac{2(n-1)+4}{n-1}}{\frac{(n+1)(n-3)}{(n-1)(2n+1)}} = \frac{\frac{2n+2}{n-1}}{\frac{(n+1)(n-3)}{(n-1)(2n+1)}}$

$\frac{2(n+1)}{(n-1)} \cdot \frac{(n-1)(2n+1)}{(n+1)(n-3)} = \frac{2(2n+1)}{n-3}$

⑩ $\frac{n+\frac{2}{n-1}}{\left(\frac{2n^2+2n-12}{n^2+2n-3}\right)} = \frac{\frac{n(n-1)+2}{(n-1)}}{\frac{2(n-2)(n+3)}{(n-1)(n+3)}} = \frac{\frac{n^2-n+2}{(n-1)}}{\frac{2(n-2)}{(n-1)}}$

$\frac{(n-2)(n+1)}{(n-1)} \cdot \frac{(n-1)}{2(n-2)} = \frac{n+1}{2}$

Rational Equations

DEMONSTRATION (Review Unit 8B)

① RATIONAL EQUATION

$$\frac{c}{c-4} - \frac{6}{4-c} = c$$

$$(c-4)\left(\frac{c}{c-4} + \frac{6}{c-4} = c\right)$$

$$c+6 = c(c-4)$$
$$c+6 = c^2-4c$$
$$c^2-5c-6=0$$
$$(c-6)(c+1)=0 \qquad \boxed{c=6,-1}$$

② EXCLUDED VALUE

$$\frac{7}{x-3} = \frac{x+4}{x-3}$$

$$(x-3)\left(\frac{7}{x-3} = \frac{x+4}{x-3}\right)$$

$$7 = x+4$$
$$x = 3 \leftarrow \text{3 is an excluded value}$$

$\boxed{\text{no solutions}}$

③ EXCLUDED VALUE

$$w + \frac{w}{w-1} = \frac{4w-3}{w-1}$$

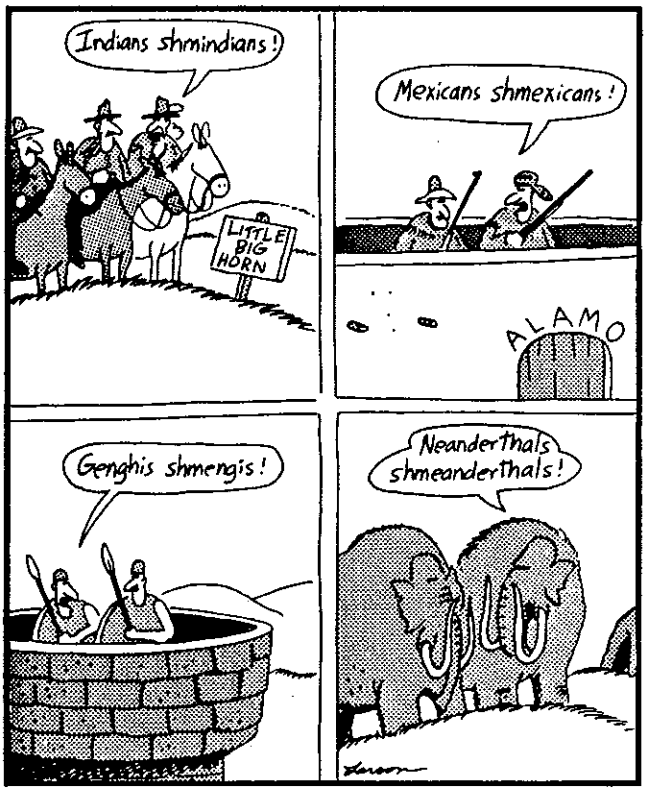
continued

$$(w-1)\left(w + \frac{w}{w-1} = \frac{4w-3}{w-1}\right)$$

$$w(w-1)+w = 4w-3$$
$$w^2-w+w = 4w-3$$
$$w^2-4w+3=0$$
$$(w-3)(w-1)=0$$
$$w = 3, \not{1} \leftarrow \text{1 is an excluded value}$$

$\boxed{w=3}$

Additional Practice Problems in: 9.3



History shmistory

Rational Equations

PROBLEM SET (Review Unit 8B)

Solve each rational equation:

① $\frac{2}{n-4} - \frac{2n}{n+4} = \frac{2}{n-4}$

② $\frac{x}{x+3} - \frac{2x}{x-1} = \frac{-3}{x+3}$

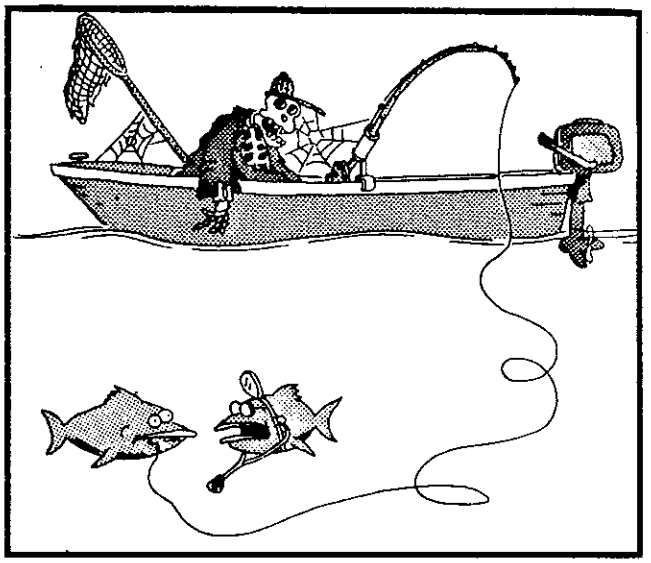
③ $\frac{5x+9}{x^2-4} - \frac{2}{x+2} = \frac{x-1}{x-2}$

④ $\frac{6}{n^2-1} + \frac{2}{n+1} = \frac{3n}{n-1}$

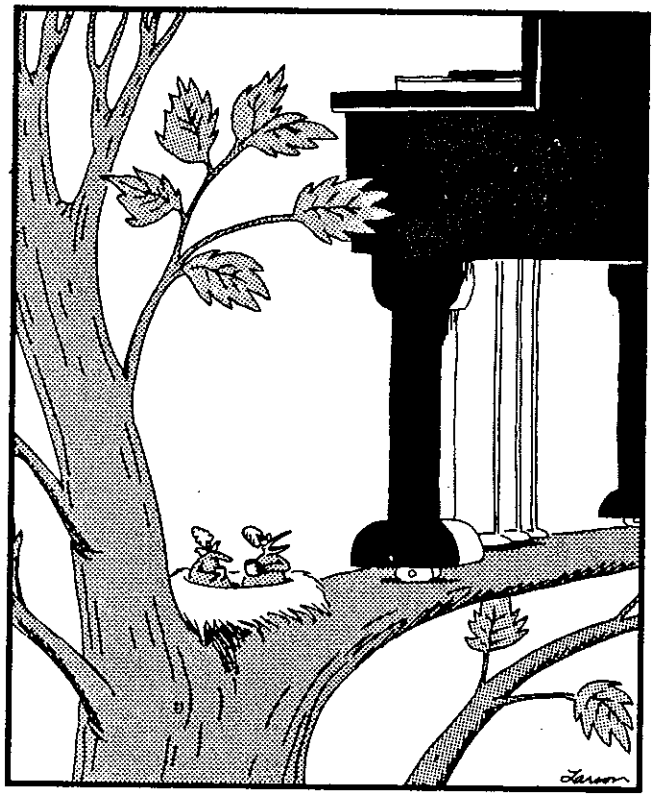
⑤ $\frac{x+1}{x+3} - \frac{7}{x^2-9} = 2$

⑥ $\frac{n+1}{n-4} - \frac{2}{n-2} = \frac{2n}{n-4}$

Be careful about excluded values.



"Well, first the bad news—you're definitely hooked."



"Oh, my word, Helen! You play, *too*? ... And here I always thought you were just a songbird."

Rational Equations

ANSWER KEY (Review Unit 8B)

① $\left(\frac{2}{n-4} - \frac{2n}{n+4} = \frac{2}{n-4}\right)(n-4)(n+4)$

$2(n+4) - 2n(n-4) = 2(n+4)$

$2n+8 - 2n^2+8n = 2n+8$

$2n^2 - 8n = 0$

$2n(n-4) = 0$

$n = 0, \cancel{4}$ ← 4 is an excluded value

$\boxed{n=0}$

② $\left(\frac{x}{x+3} - \frac{2x}{x-1} = \frac{-3}{x+3}\right)(x+3)(x-1)$

$x(x-1) - 2x(x+3) = -3(x-1)$

$x^2 - x - 2x^2 - 6x = -3x+3$

$x^2 + 4x + 3 = 0$

$(x+3)(x+1) = 0$

$x = \cancel{-3}, -1$ ← −3 is an excluded value

$\boxed{x=-1}$

③ $\left(\frac{5x+9}{x^2-4} - \frac{2}{x+2} = \frac{x-1}{x-2}\right)(x^2-4)$

$5x+9 - 2(x-2) = (x-1)(x+2)$

$5x+9 - 2x+4 = x^2+x-2$

$x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$\boxed{x = 5, -3}$

④ $\left(\frac{6}{n^2-1} + \frac{2}{n+1} = \frac{3n}{n-1}\right)(n^2-1)$

$6 + 2(n-1) = 3n(n+1)$

$6 + 2n - 2 = 3n^2 + 3n$

$3n^2 + n - 4 = 0$

$3n^2 + 4n - 3n - 4 = 0$

$n(3n+4) - 1(3n+4) = 0$

$(3n+4)(n-1) = 0$

$n = -4/3, \cancel{1}$ ← 1 is an excluded value

$\boxed{n = -4/3}$

⑤ $\left(\frac{x+1}{x+3} - \frac{7}{x^2-9} = 2\right)(x^2-9)$

$(x+1)(x-3) - 7 = 2(x^2-9)$

$x^2 - 2x - 3 - 7 = 2x^2 - 18$

$x^2 + 2x - 8 = 0$

$(x+4)(x-2) = 0$

$\boxed{x = -4, 2}$

⑥ $\left(\frac{n+1}{n-4} - \frac{2}{n-2} = \frac{2n}{n-4}\right)(n-4)(n-2)$

$(n-2)(n+1) - 2(n-4) = 2n(n-2)$

$(n^2 - n - 2) - 2n + 8 = 2n^2 - 4n$

$n^2 - n - 6 = 0$

$(n-3)(n+2) = 0$

$\boxed{n = 3, -2}$