Friendship Junior High School
Accelerated Math Program
Mr. Lavine (Room 102A)

A.T.I.M.

Advanced Topics In Mathematics

UNIT 11
Plane Geometry

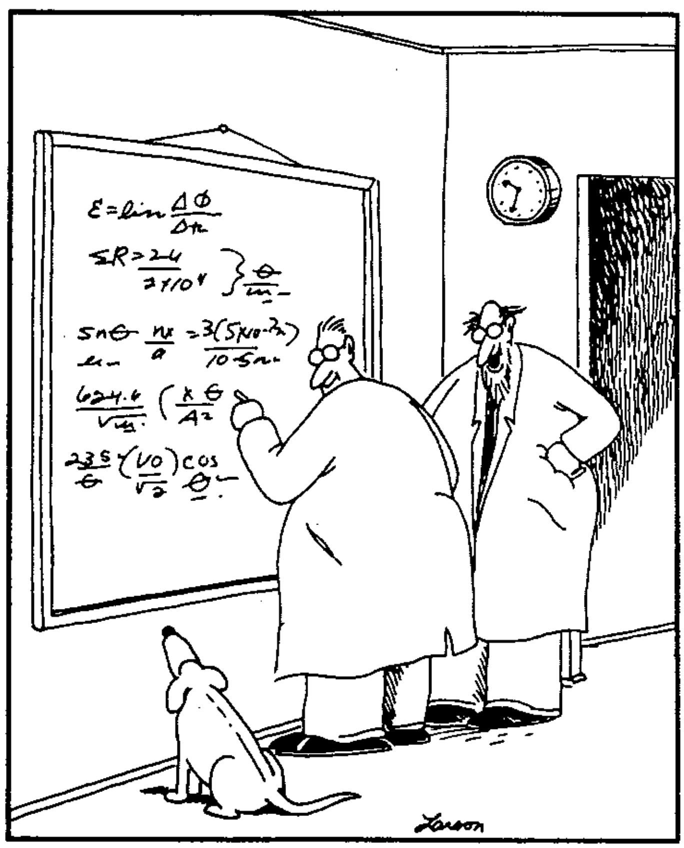
UNIT 12
Solid Geometry

UNIT 13
Introduction to Two-Column Proofs

UNIT 14
Triangle Trigonometry

UNIT 15
Law of Sines & Cosines

UNIT 16
Trigonometric Functions



"Ohhhhh...Look at that, Schuster...Dogs are so cute when they concentrate on trigonometric functions."

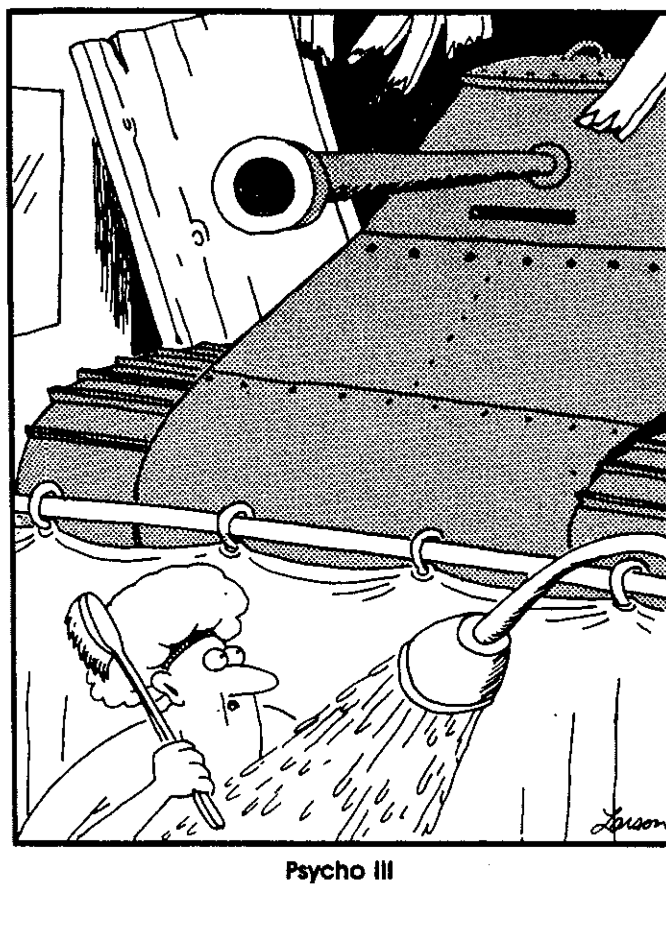
UNIT 11

Plane Geometry

11.1
Polygons, Circles, & Special Triangles

11.2
Applications

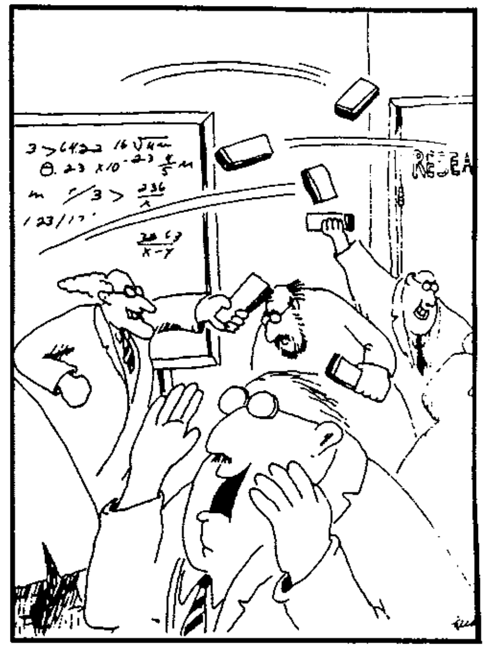
11.3
Sectors & Segments



Psycho III

Plane Geometry

REFERENCE PAGE



"Eraser fight!"

① Parallelogram — $A = (\text{base})(\text{ht})$

② Rhombus, Kite — $A = \frac{1}{2}(\text{prod. of diagonals})$

③ Triangle — $A = \frac{1}{2}(\text{base})(\text{ht})$

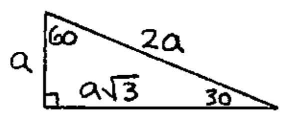
④ Trapezoid — $A = \frac{1}{2}(\text{sum of bases})(\text{ht})$
$A = (\text{median})(\text{ht})$

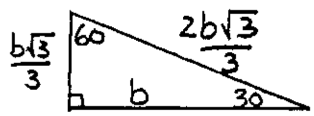
⑤ Circle — $A = \pi r^2 \quad C = 2\pi r$

⑥ Pythagorean Theorem & Triples — $a^2 + b^2 = c^2$

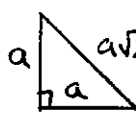
3-4-5 7-24-25
5-12-13 8-15-17

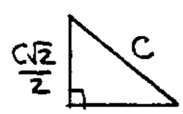
⑦ 30-60-90 Triangle



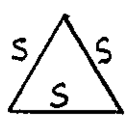


⑧ 45-45-90 Triangle



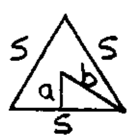


⑨ Equilateral Triangle



$A = \frac{s^2\sqrt{3}}{4}$

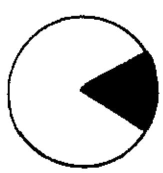
⑩ Regular Polygon



a → apothem
b → radius

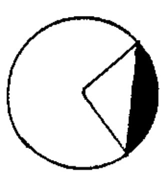
$A = \frac{1}{2}(\text{apoth})(\text{per})$

⑪ Sector of a Circle



$A = \frac{\text{central angle}}{360}(\pi r^2)$

⑫ Segment of a Circle



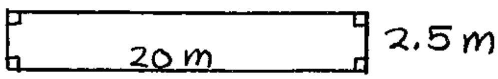
$A = \begin{pmatrix}\text{area of}\\ \text{sector}\end{pmatrix} - \begin{pmatrix}\text{area of}\\ \text{triangle}\end{pmatrix}$

Polygons, Circles, & Special Triangles

DEMONSTRATION 11.1

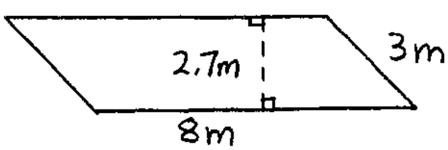
The following is a review of formulas involving polygons, circles, and special right triangles.

① Rectangle



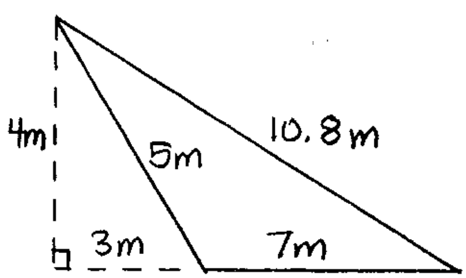
$A = (\text{base})(\text{height})$
$A = (20)(2.5) = 50\,m^2$

② Parallelogram



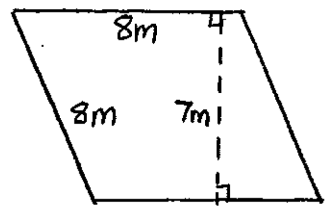
$A = (\text{base})(\text{height})$
$A = (8)(2.7) = 21.6\,m^2$

③ Triangle



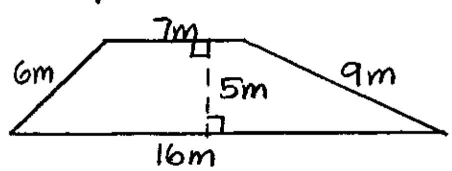
$A = \frac{1}{2}(\text{base})(\text{height})$
$A = \frac{1}{2}(7)(4) = 14\,m^2$

④ Rhombus



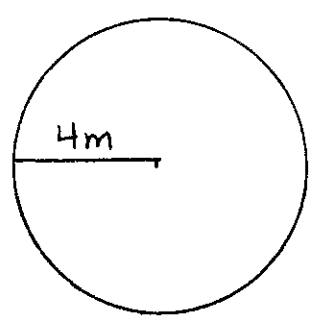
$A = (\text{base})(\text{ht})$
$A = (8)(7) = 56\,m^2$

⑤ Trapezoid



$A = \frac{1}{2}(\text{sum of bases})(\text{height})$
$A = \frac{1}{2}(7+16)(5) = 57.5\,m^2$

⑥ Circle



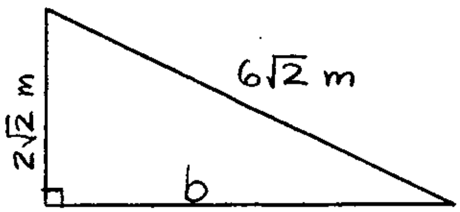
$A = \pi r^2$
$\pi(4)^2 = 16\pi m^2$
$(3.14)(4)^2 = 50.24 m^2$

$C = 2\pi r$
$2\pi(4) = 8\pi m$
$2(3.14)(4) = 25.12\,m$

Polygons, Circles, & Special Triangles

DEMONSTRATION 11.1

⑦ Pythagorean Theorem



$a^2 + b^2 = c^2$

$(2\sqrt{2})^2 + b^2 = (6\sqrt{2})^2$

$8 + b^2 = 72$

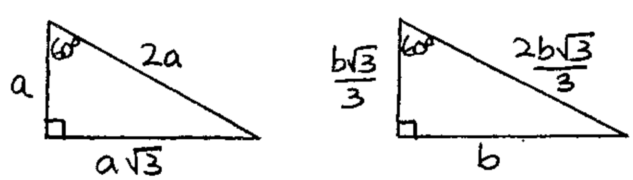
$b^2 = 64$

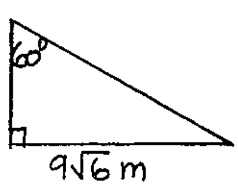
$b = 8$

$A = \frac{1}{2}(2\sqrt{2})(8) = 8\sqrt{2}\ m^2$

$P = (2\sqrt{2}) + (8) + (6\sqrt{2}) = 8 + 8\sqrt{2}\ m$

⑧ 30-60-90 Triangle





Given the long leg, use diagram on the right

$a = 9\sqrt{2}$

$b = 9\sqrt{6}$

$c = 18\sqrt{2}$

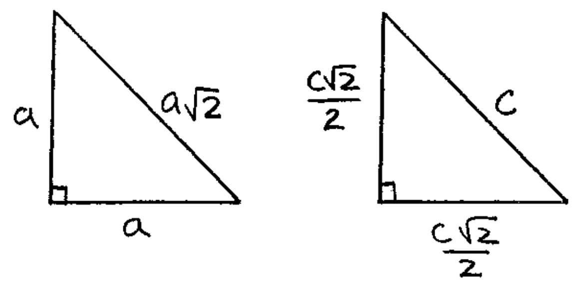
$A = \frac{1}{2}(9\sqrt{2})(9\sqrt{6})$

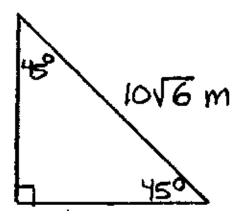
$81\sqrt{3}\ m^2$

$P = (9\sqrt{2}) + (9\sqrt{6}) + (18\sqrt{2})$

$27\sqrt{2} + 9\sqrt{6}\ m$

⑨ 45-45-90 Triangle



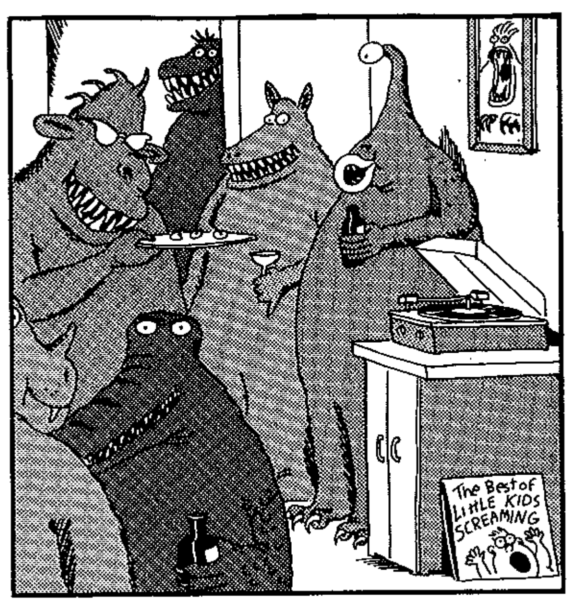


use the diagram at top right given the hypotenuse

hypotenuse $10\sqrt{6}$, leg $10\sqrt{3}$

$A = \frac{1}{2}(10\sqrt{3})(10\sqrt{3}) = 150\ m^2$

$P = 2(10\sqrt{3}) + (10\sqrt{6}) = 20\sqrt{3} + 10\sqrt{6}\ m$



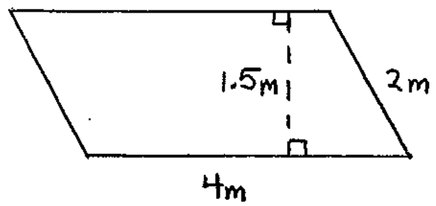
Later, when one of the monsters cranked up the volume, the party really got going.

Polygons, Circles, & Special Triangles

PROBLEM SET 11.1

Determine area and perimeter/circumference:

1



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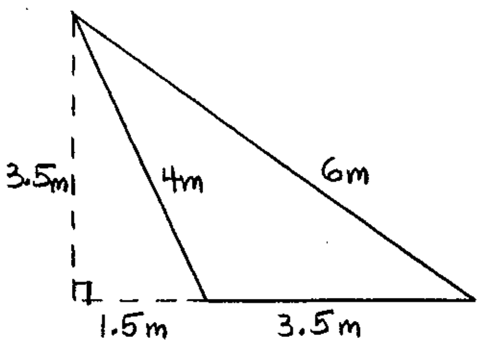

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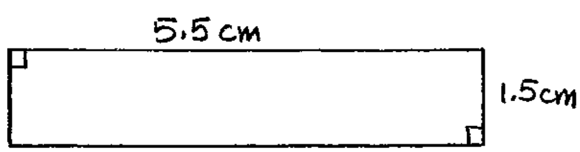

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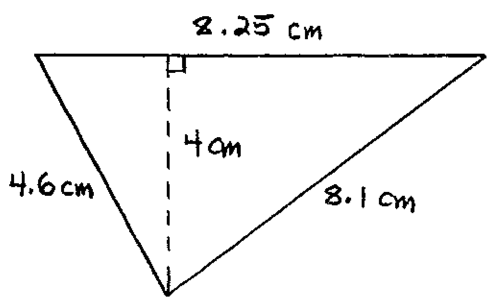

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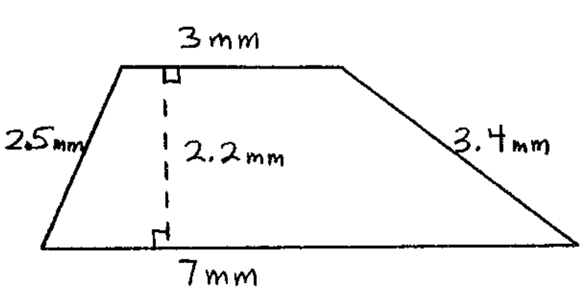

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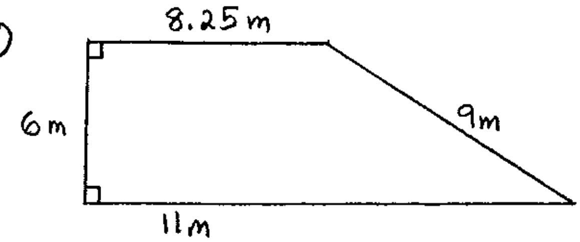

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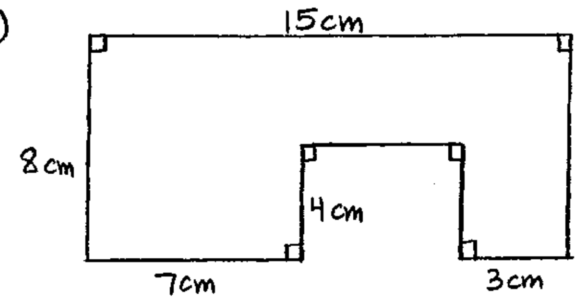

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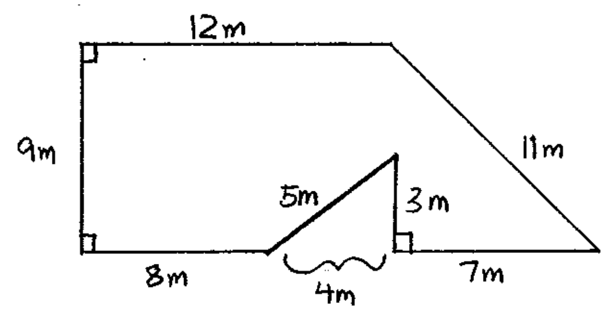

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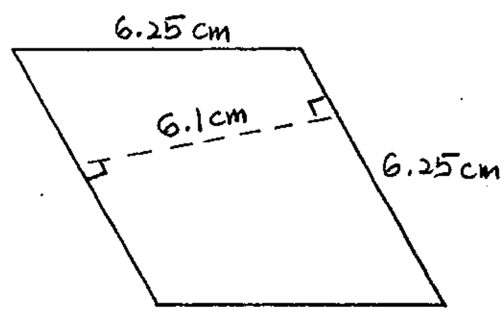

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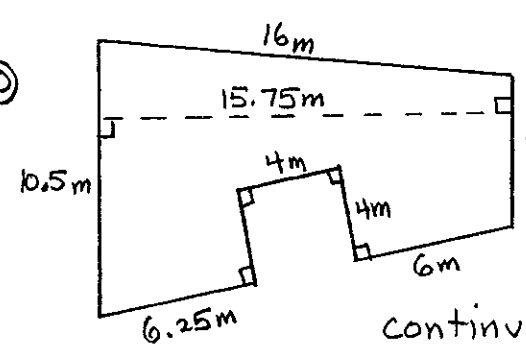
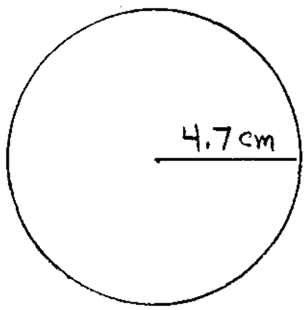


continued

Polygons, Circles, & Special Triangles

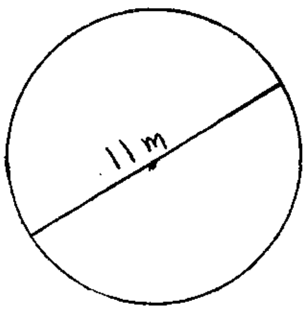
PROBLEM SET 11.1

(11)



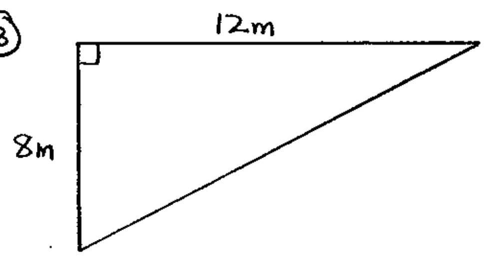
π and
3.14
method

(12)



π and
3.14
method

(13)



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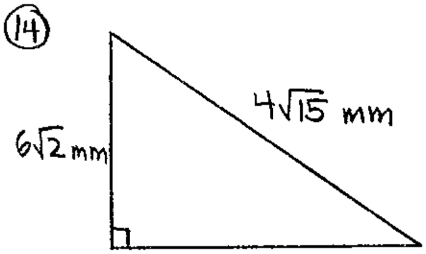

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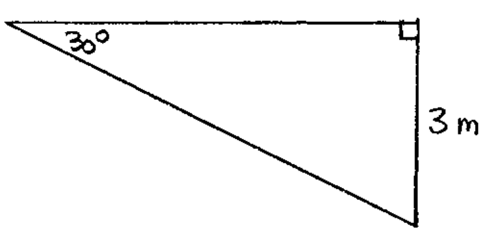

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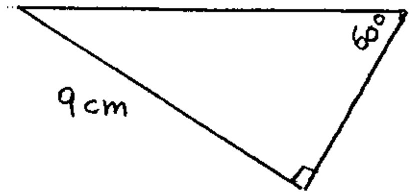

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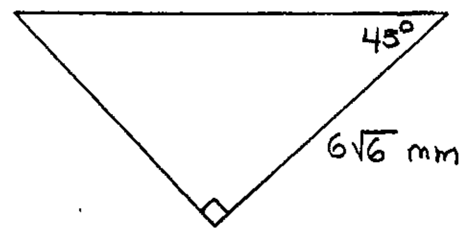

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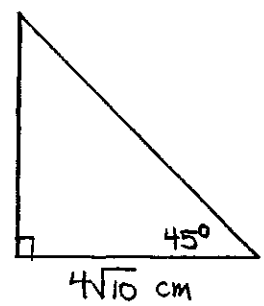

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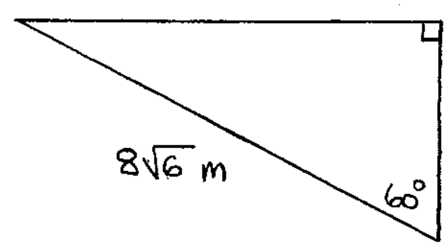

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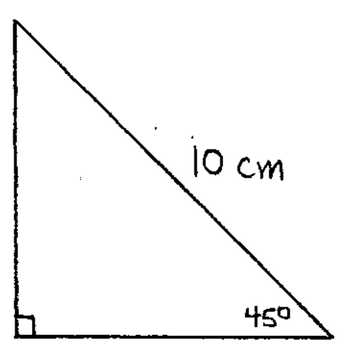
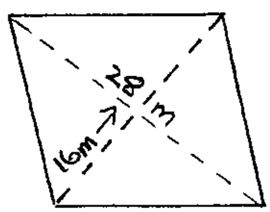


Applications

DEMONSTRATION 11.2

This lesson introduces formulas for determining the area of polygons and regular polygons. The second half of the assignment requires creative application of triangle relationships.

① Using Diagonals



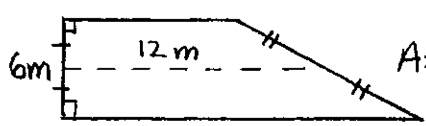
$A=\frac{1}{2}(28)(16)$

$224\ m^2$

$A=\frac{1}{2}$ (prod. of diagonals)

squares, rhombuses, kites

② Using the Median

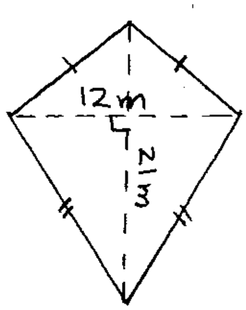


$A=(6)(12)=72m^2$

$A =$ (median)(height)

trapezoids

③ Kites

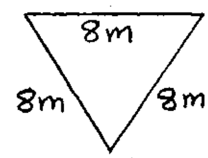


A kite has adjacent congruent sides and perpendicular diagonals

$A=\frac{1}{2}$ (prod. of diag.)

$\frac{1}{2}(12)(21)=126m^2$

④ Equilateral Triangle



$$A=\frac{S^2\sqrt{3}}{4}=\frac{8^2\sqrt{3}}{4}$$

$16\sqrt{3}\ m^2$

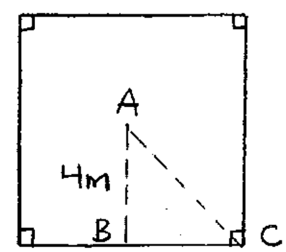
⑤ Regular Polygon

Apothem $\overline{AB}$

radius of the inscribed circle

Radius $\overline{AC}$

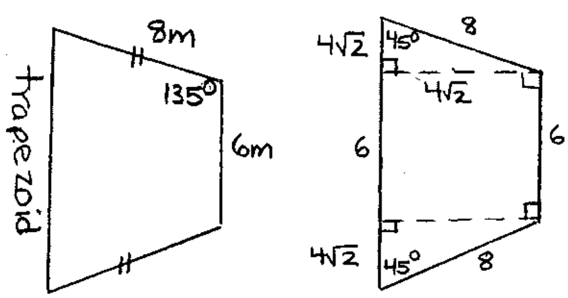
radius of the circumscribed circle



$A=\frac{1}{2}$(apoth)(per)

$A=\frac{1}{2}(4)(32)=64m^2$

⑥ Application (not drawn to scale)



$A=\frac{1}{2}(6+6+8\sqrt{2})(4\sqrt{2})=24\sqrt{2}+32\ m^2$

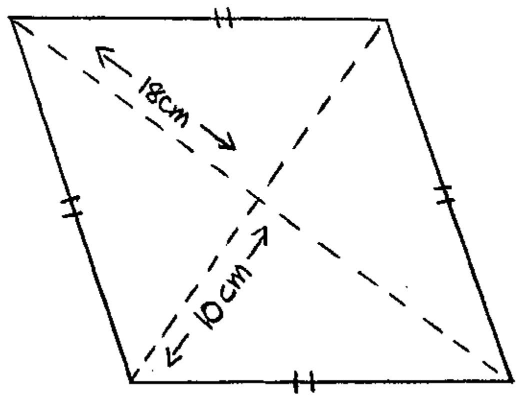
$P=28+8\sqrt{2}\ m$

Applications

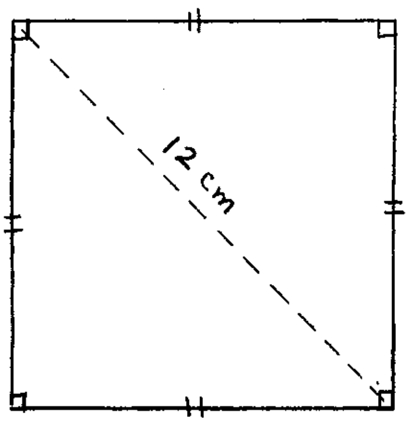
PROBLEM SET 11.2

Determine the area:

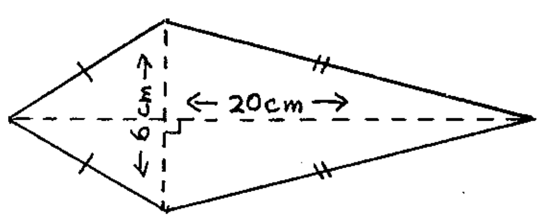
①



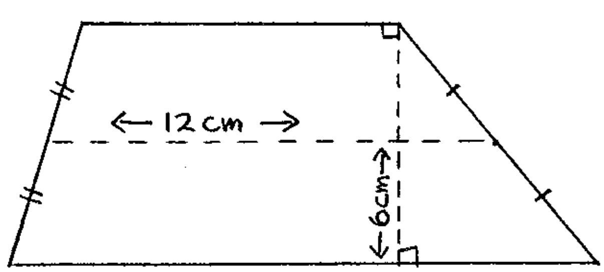
②



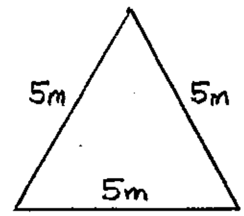
③



④

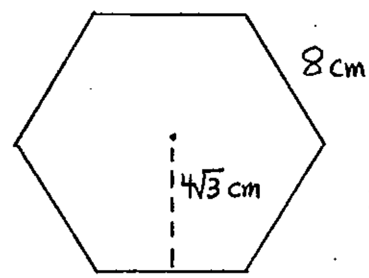


⑤



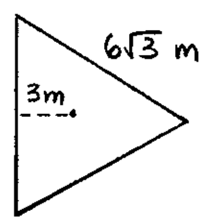
Equilateral Triangle

⑥



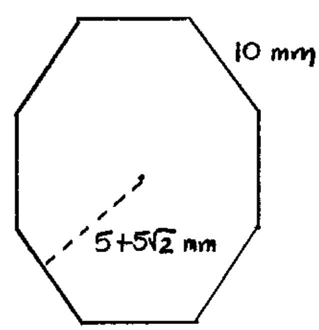
Regular Hexagon

⑦



Equilateral Triangle

⑧



Regular Octagon

Find the area of a square:

⑨ Diagonal 10 m

⑩ Radius 6m

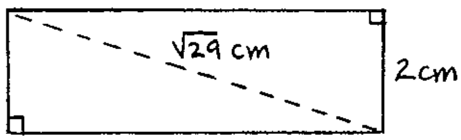
⑪ Perimeter 12m

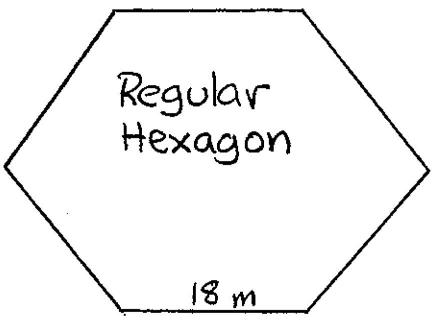
⑫ Apothem 5m

Applications

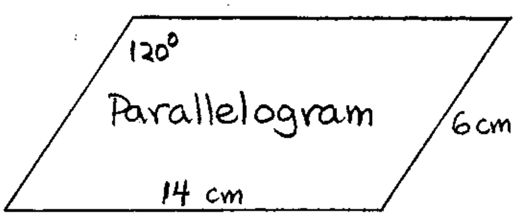
PROBLEM SET 11.2

Find the area and perimeter:

⑬


⑭


Hint: A regular hexagon contains six equilateral triangles. How does this help?

⑮


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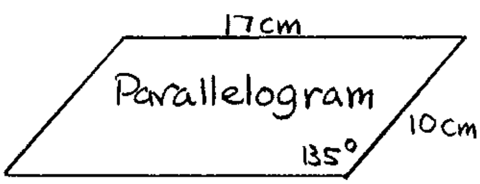

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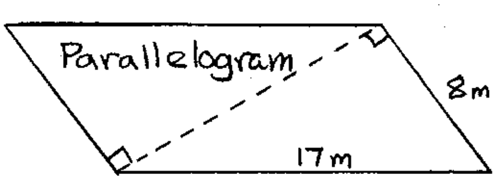

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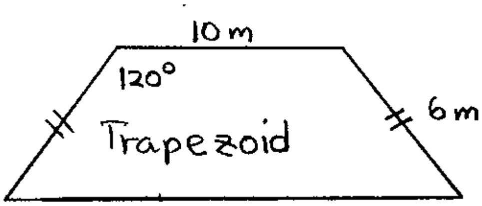

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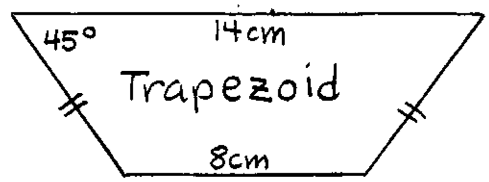

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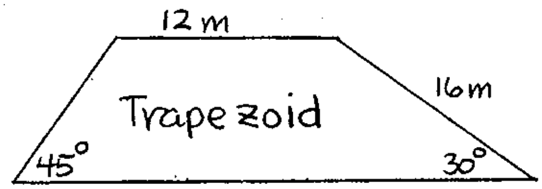
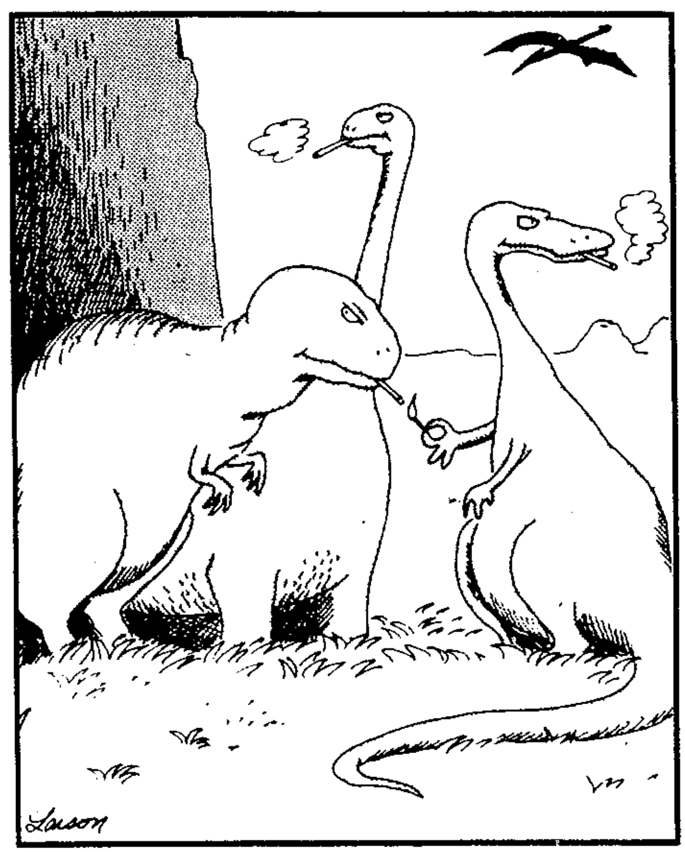




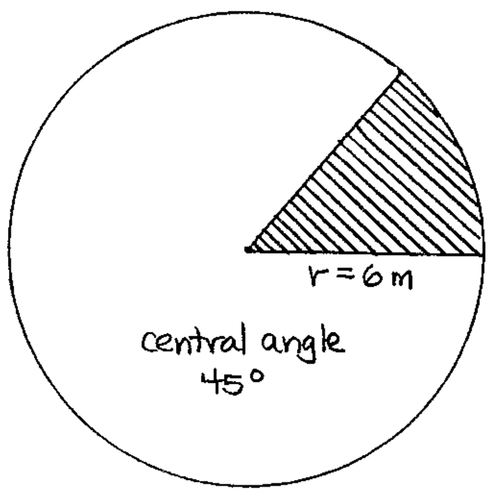
The real reason dinosaurs became extinct

Sectors & Segments

DEMONSTRATION 11.3

A sector is part of a circle bounded by an arc and two radii.
A segment is part of a circle bounded by an arc and a chord.

① Sector of a Circle

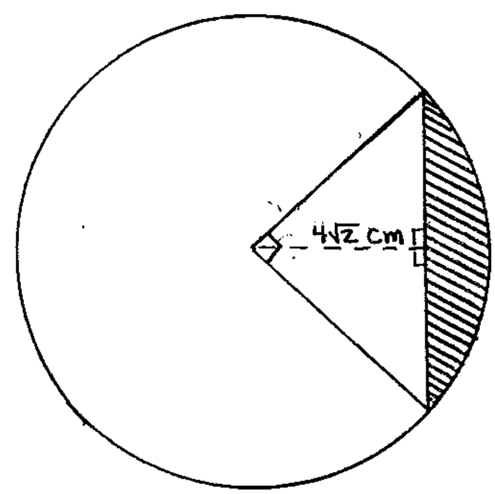


$A = (\text{area of circle})(\text{central}\angle/360)$

$\pi r^2 (\text{part})$

$\pi (6)^2 (45/360) = 4.5\pi\, m^2$

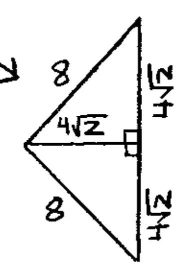
② Segment of a Circle



$A = (\text{area of sector}) - (\text{area of }\triangle)$

$\pi (8)^2 (90/360) - \frac{1}{2}(4\sqrt{2})(8\sqrt{2})$

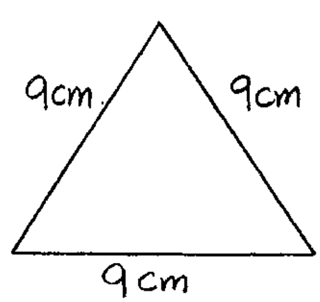
$16\pi - 32\, cm^2$



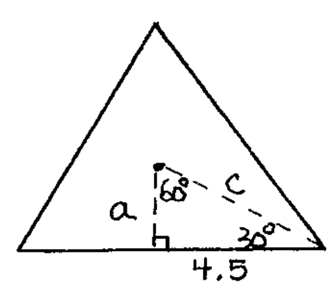
Additional Practice: Applications Involving Polygons and Special Triangles.

Find the radius of the triangle:

③



Use special triangle relationships to solve:



$a = \dfrac{4.5\sqrt{3}}{3}$

radius (c)

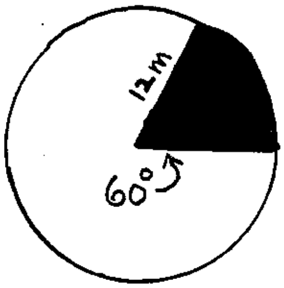
$2a = 3\sqrt{3}$ cm

Sectors & Segments

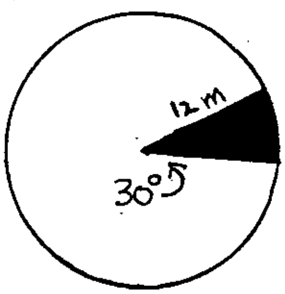
PROBLEM SET 11.3

Solve for area only of all shaded regions. Use π method:

①



②



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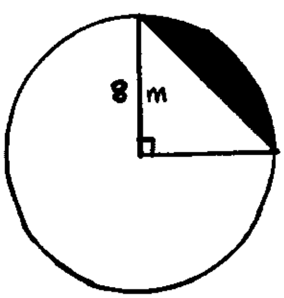

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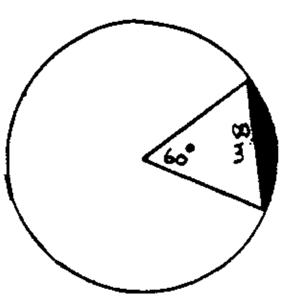
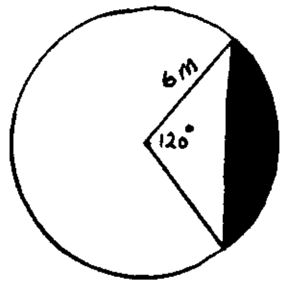


⑤



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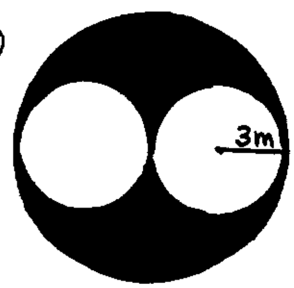

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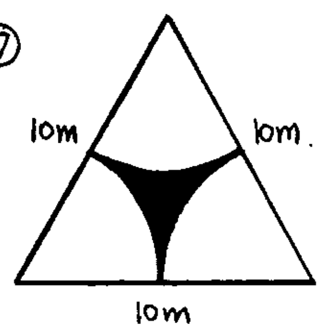

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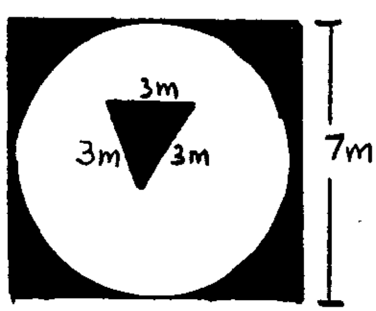

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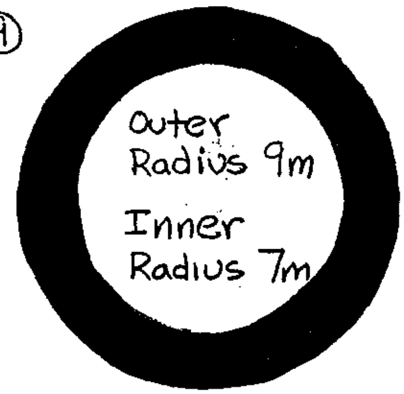

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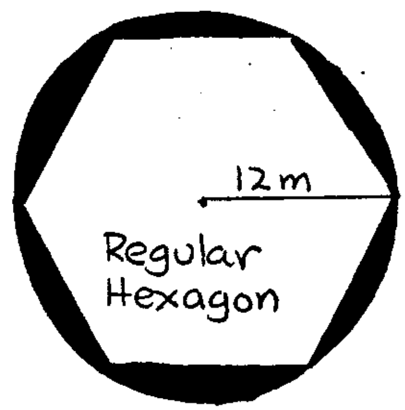

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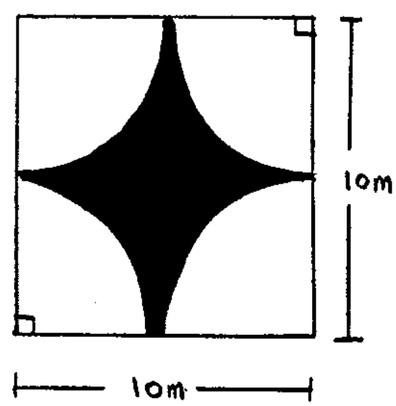

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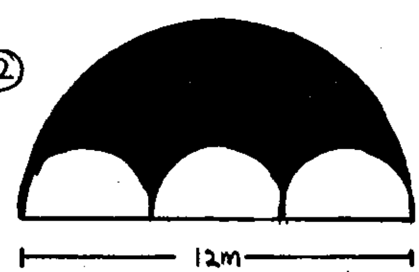
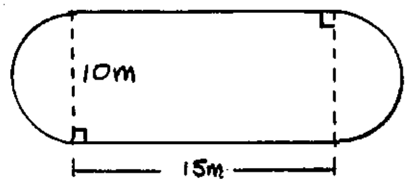


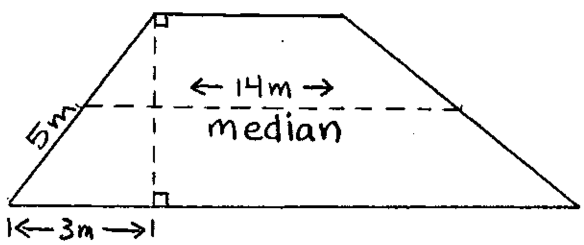
Sectors & Segments

PROBLEM SET 11.3

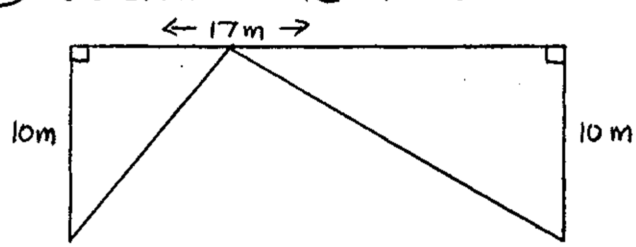
⑬ Determine area and circumference:



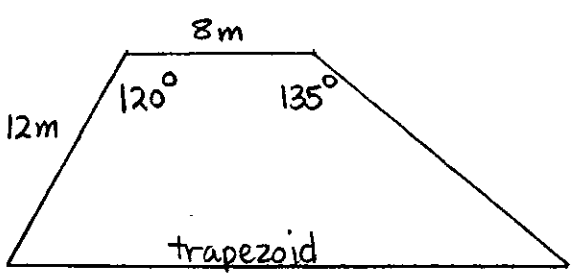
⑭ Determine the area:



⑮ Determine the area:

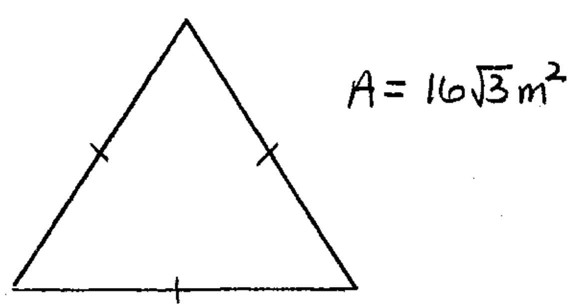


⑯ Determine area and perimeter:



"And now there go the Wilsons! . . . Seems like everyone's evolving except us!"

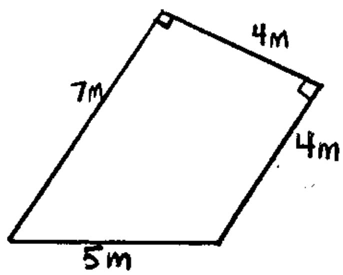
⑰ Find the perimeter of an equilateral triangle that has an area of $16\sqrt{3}\ m^2$



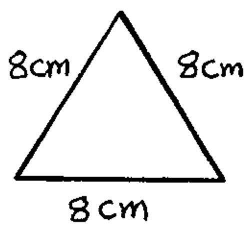
Plane Geometry

UNIT 11 REVIEW & PRACTICE

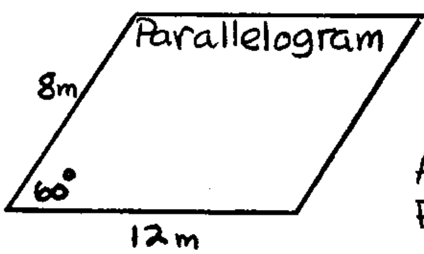
Determine the area (and the perimeter/circumference for selected problems):

① 

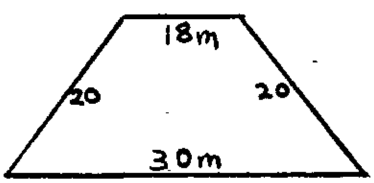
Area
Perimeter

② 

Area
Perimeter

③ 

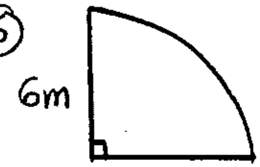
Area
Perimeter

④ 

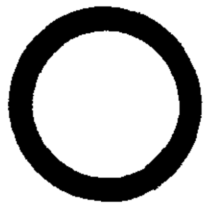
Isosceles Trapezoid

Area
Perimeter

⑤ Find the area of a circle if the circumference is 16π cm

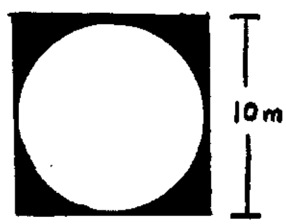
⑥ 

Area
Circumference

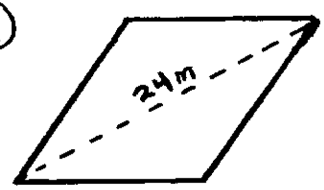
⑦ 

Area (only) of the shaded region

inside r=6m outside r=8m

⑧ 

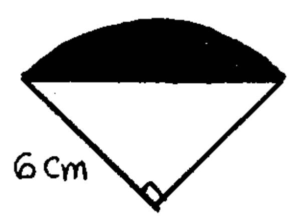
Area (only) of the shaded region

⑨ 

Area (only)

perimeter = 52 m (rhombus)

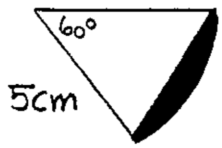
⑩ Find the area of an equilateral triangle whose perimeter is 21 cm.

⑪ 

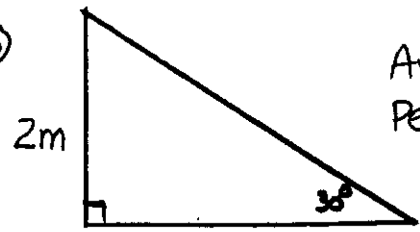
Area (only)

Plane Geometry

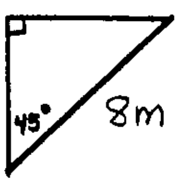
UNIT 11 REVIEW & PRACTICE

12


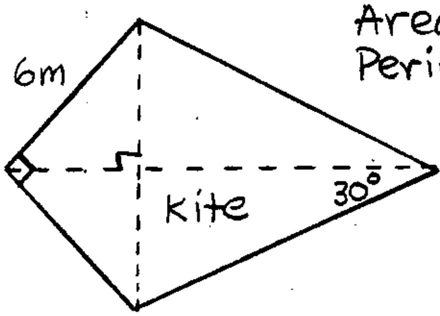
Area (only)

13


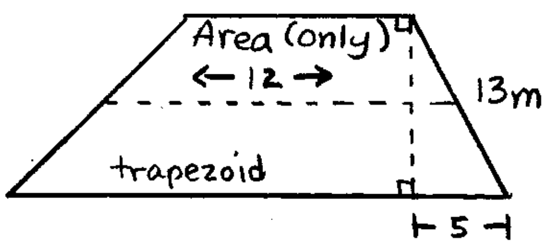
Area
Perimeter

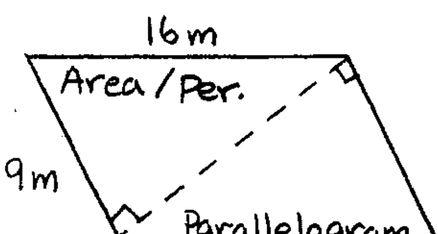
14


Area
Perimeter

15


Area
Perimeter

16


17


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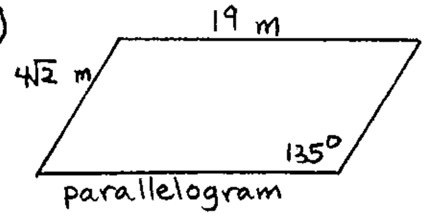
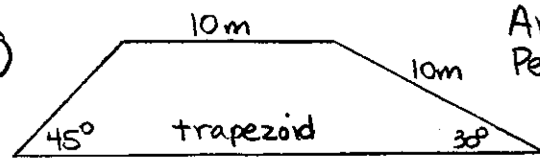
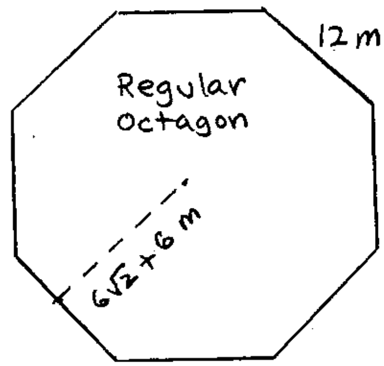


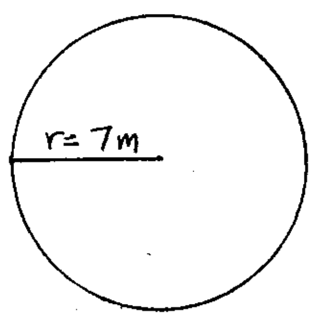
Area
Perimeter

19


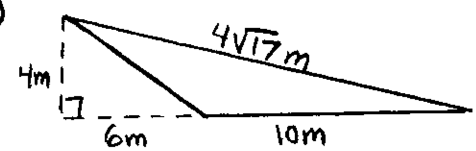
Area
Perimeter

20


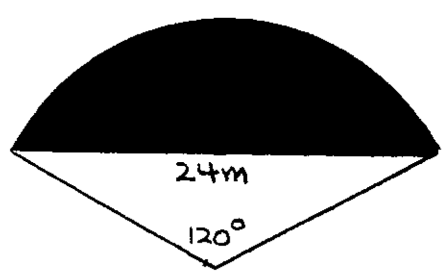
Area
Perimeter

21


Area
Circumference

22


Area
Perimeter

23


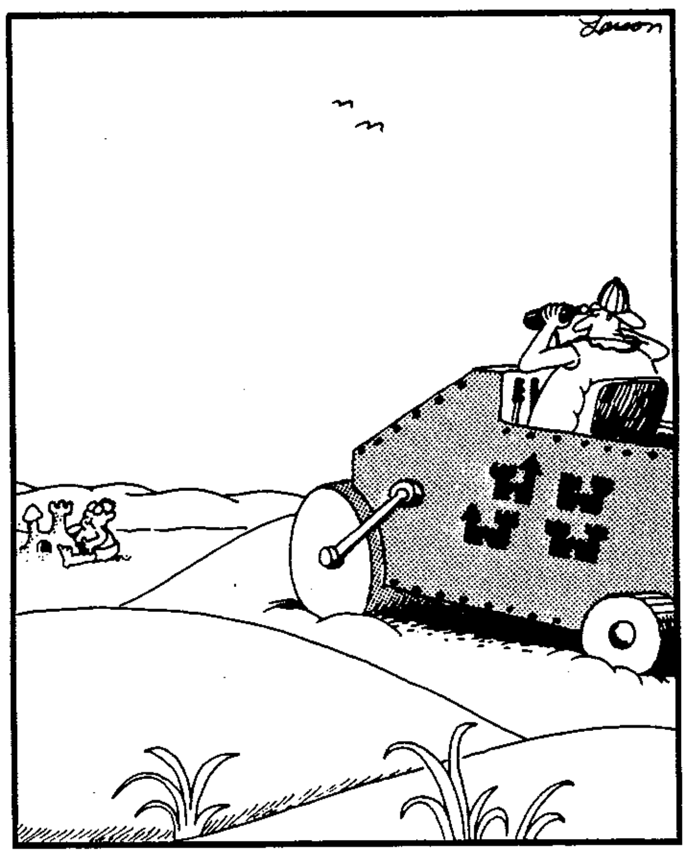
Area (only)
of the
shaded region

UNIT 12

Solid Geometry

12.1
Prisms, Cylinders, & Spheres

12.2
Pyramids & Cones



Solid Geometry

REFERENCE PAGE

① Prism
$V = (\text{base area})(\text{height})$
$SA = 2(\text{base area}) + (\text{perimeter})(\text{height})$

② Cylinder
$V = (\text{base area})(\text{height})$ $V = (\pi r^2)(\text{ht})$
$SA = 2(\text{base area}) + (\text{cir.})(\text{ht})$ $SA = 2(\pi r^2) + (2\pi r)(\text{ht})$

③ Pyramid
$V = \frac{1}{3}(\text{base area})(\text{height})$
$SA = (\text{base area}) + \frac{1}{2}(\text{perimeter})(\text{slant height})$

④ Cone
$V = \frac{1}{3}(\text{base area})(\text{height})$ $V = \frac{1}{3}(\pi r^2)(\text{ht})$
$SA = (\text{base area}) + \frac{1}{2}(\text{cir.})(\text{slant ht})$ $SA = (\pi r^2) + \frac{1}{2}(2\pi r)(\text{slant ht})$

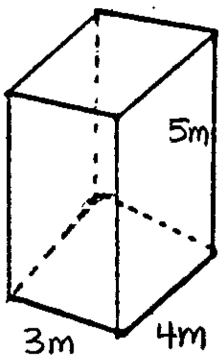
⑤ Sphere
$V = \frac{4}{3}\pi r^3$
$SA = 4\pi r^2$

Prisms, Cylinders, & Spheres

DEMONSTRATION 12.1

For all prisms and cylinders } Vol = (base area)(height)
SA = 2(base area) + (per. or cir.)(height)

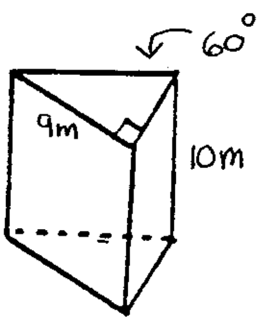
① Rectangular Prism



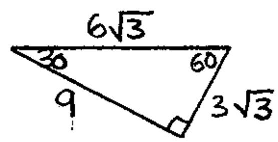
V = (base area)(ht)
$(3\cdot 4)(5)$
$60\ m^3$

SA = 2(base) + (per)(ht)
$(2)(12) + (14)(5)$
$94\ m^2$

② Triangular Prism



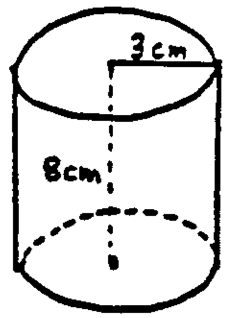
30-60-90



V = (base area)(ht)
$\frac{1}{2}(3\sqrt{3})(9)\cdot(10)$
$135\sqrt{3}\ m^3$

SA = 2(base area) + (per.)(ht)
$2\left(\frac{1}{2}\right)(3\sqrt{3})(9) + (9+9\sqrt{3})(10)$
$27\sqrt{3} + 90 + 90\sqrt{3}$
$117\sqrt{3} + 90\ m^2$

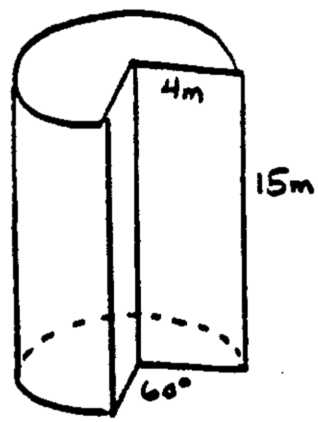
③ Cylinder



V = (base area)(ht)
$(\pi r^2)(ht)$
$(\pi 3^2)(8) = 72\pi\ cm^3$

SA = 2(base) + (cir.)(ht)
$2(\pi r^2) + (2\pi r)(ht)$
$2\pi 3^2 + 2\pi(3)(8)$
$66\pi\ cm^2$

④ Partial Cylinder



central angle 300°

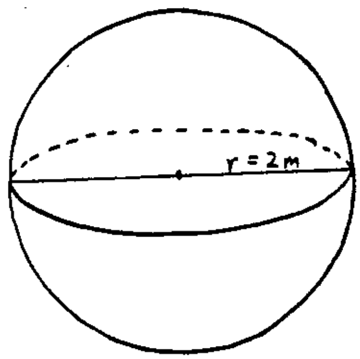
V = (base)(ht)
$\pi r^2(part)(ht)$
$\pi 4^2(300/360)(15)$
$200\pi\ m^3$

SA = 2(base) + (cir.)(ht)
$2(\pi r^2)(part) + \boxed{(2\pi r)(part) + 2r}(ht)$
$2\pi 4^2\left(\frac{300}{360}\right) + \boxed{(2\pi)(4)\left(\frac{300}{360}\right) + 2(4)}(15)$
$26.\bar{6}\pi + [6.\bar{6}\pi + 8](15)$
$126.\bar{6}\pi + 120\ m^2$

Prisms, Cylinders, & Spheres

DEMONSTRATION 12.1

⑤ Sphere



$V = \frac{4}{3}\pi r^3$

$SA = 4\pi r^2$

$V = \frac{4}{3}\pi r^3$

$\frac{4}{3}\pi 2^3$

$10.\overline{6}\pi\, m^3$

$SA = 4\pi r^2$

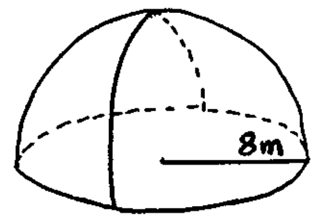
$4\pi 2^2$

$16\pi\, m^2$

A sphere has only one face.

There is no base in a sphere.

⑥ Partial Sphere



$V = \frac{4}{3}\pi r^3 (\text{part})$

$\frac{4}{3}\pi 8^3 (\frac{1}{2})$

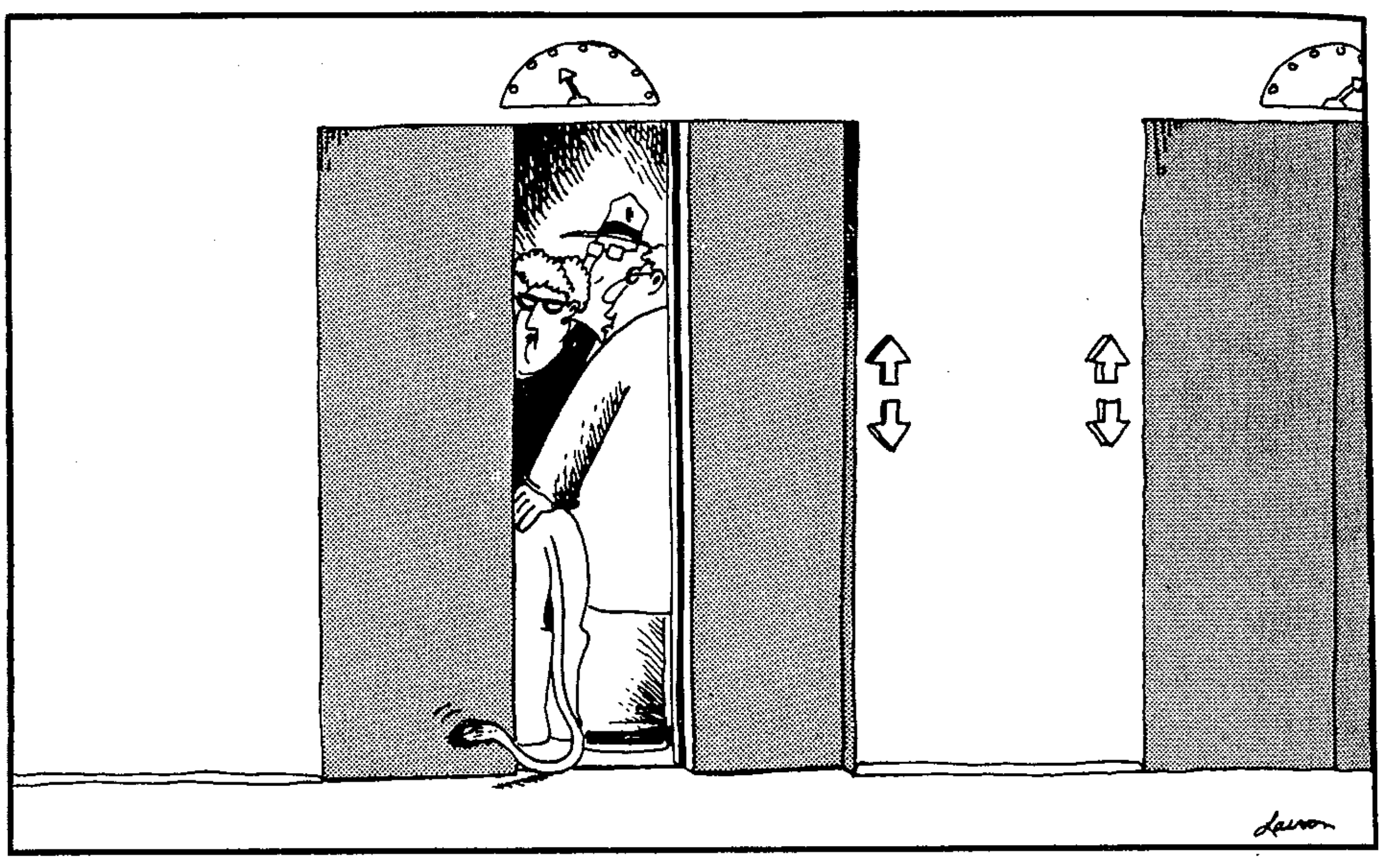
$341.\overline{3}\pi\, m^3$

$SA = 4\pi r^2 (\text{part}) + (\text{base area})$

$4\pi 8^2 (\frac{1}{2}) + (\pi r^2)$

$4\pi 8^2 (\frac{1}{2}) + (\pi 8^2) = 192\pi\, m^2$

Don't forget to include the surface area of the base in this partial sphere.

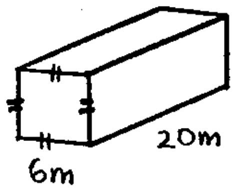


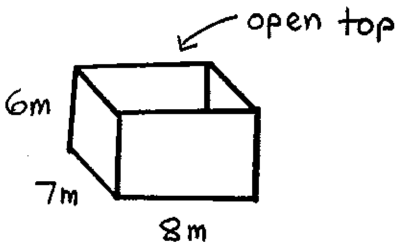
"Don't be alarmed folks . . . He's completely harmless unless something startles him."

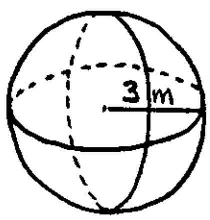
Prisms, Cylinders, & Spheres

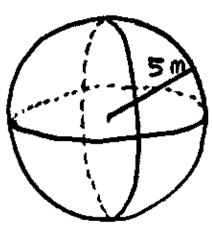
PROBLEM SET 12.1

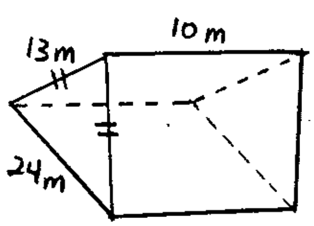
Determine the volume and surface area:

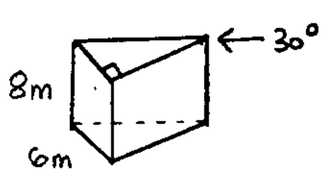
1) 

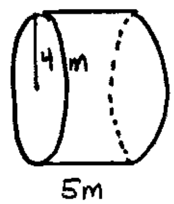
2) 

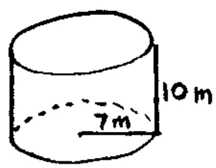
3) 

4) 

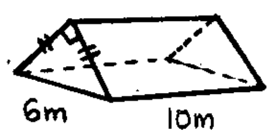
5) 

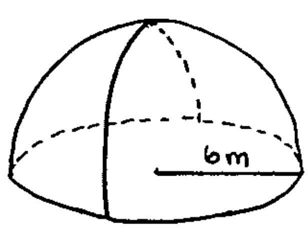
6) 

7) 

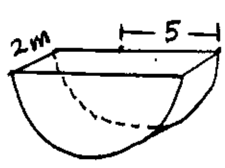
8) 

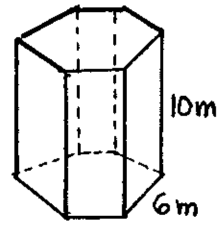
9) Determine the volume of a sphere with a surface area of $144\pi\ m^2$

10) 

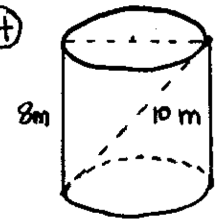
11) 

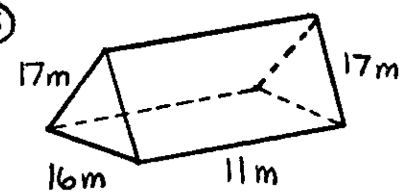
base included

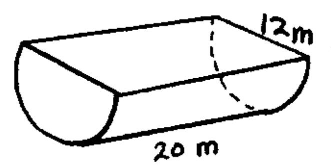
12) 

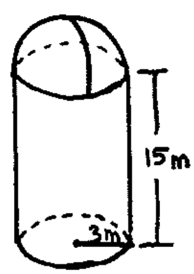
13) 

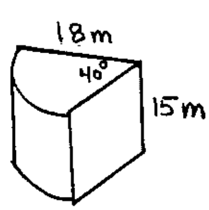
Base is a regular hexagon

14) 

15) 

16) 

17) 

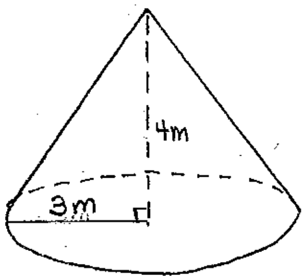
18) 

Pyramids & Cones

DEMONSTRATION 12.2

For all pyramids and cones } Vol = ⅓ (base area) (perpendicular height)
SA = (base area) + ½ (per. or cir.) (slant height)

① Cone



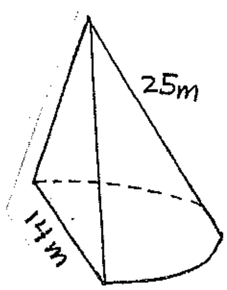
Pyth. Triple
3-4-5

Perp Ht = 4
Slant Ht = 5

V = ⅓ (base area) (ht)
$\frac{1}{3}(\pi)(3)^2(4) = 12\pi \text{ m}^3$

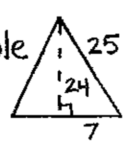
SA = (base area) + ½ (cir)(slant ht)
$(\pi)(3)^2 + \frac{1}{2}(2)(\pi)(3)(5) = 24\pi \text{ m}^2$

② Half Cone



To determine the perp. height:

Pyth. Triple
7-24-25

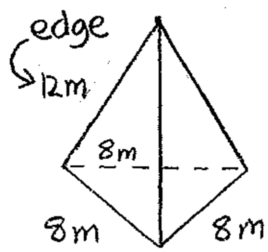


slant ht = 25, r = 7, ht = 24

V = ⅓ (base area) (ht)
$\frac{1}{3}(\frac{1}{2})(\pi)(7)^2(24) = 196\pi \text{ m}^3$

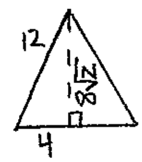
SA = (base) + ½ (arc)(slant ht) + (triangle)
$\frac{1}{2}(\pi)(7)^2 + \frac{1}{2}(\frac{1}{2})(2)(\pi)(7)(25) + (\frac{1}{2})(14)(24)$
$112\pi + 168 \text{ m}^2$

③ Pyramid



Perp Ht : Given
ht = 11 m

Slant Ht:
$4^2 + b^2 = 12^2$
$b = 8\sqrt{2}$

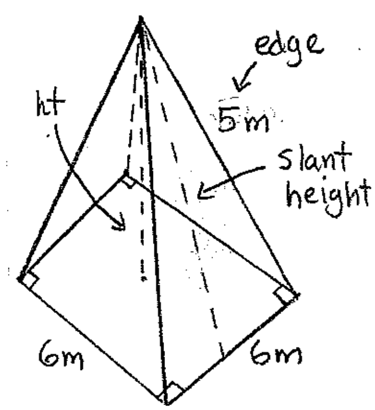


$V = \frac{1}{3}(\text{base})(\text{ht}) = \frac{1}{3}\left(\frac{8^2\sqrt{3}}{4}\right)(11) = \frac{176}{3}\sqrt{3} \text{ m}^3$

SA = (base area) + ½ (per)(slant ht)
$(16\sqrt{3}) + \frac{1}{2}(24)(8\sqrt{2}) = 16\sqrt{3} + 96\sqrt{2} \text{ m}^2$

④ Pyramid

slant ht = 4
Pyth Triple
3-4-5



perp ht = $\sqrt{7}$

$(3)^2 + b^2 = 4^2$
$b^2 = 7$
$b = \sqrt{7}$

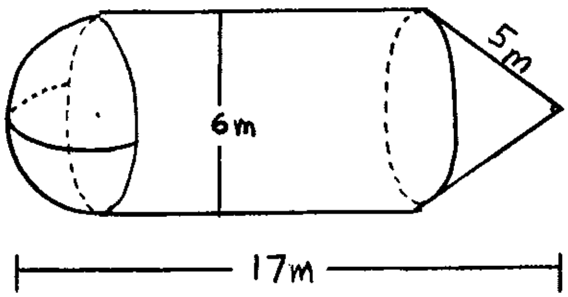
V = ⅓ (base area)(ht)
$\frac{1}{3}(6 \cdot 6)(\sqrt{7}) = 12\sqrt{7} \text{ m}^3$

SA = (base area) + ½ (per.)(slant ht)
$(6 \cdot 6) + \frac{1}{2}(24)(4) = 84 \text{ m}^2$

Pyramids & Cones

DEMONSTRATION 12.2

⑤ Applications and Combinations



Semi-sphere:
$r = 3m$

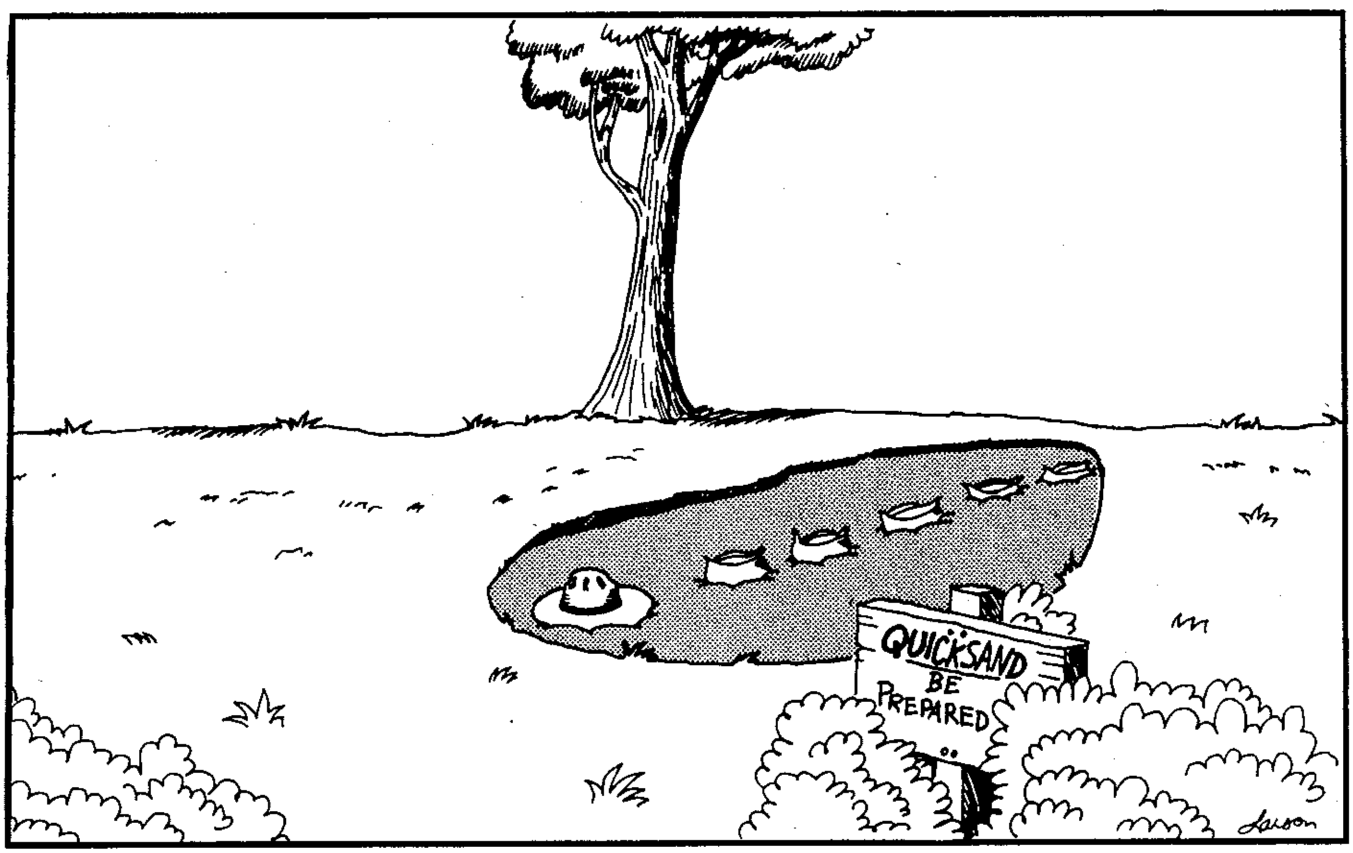
Cylinder:
$r = 3m$
$ht = 10m$

Cone:
$r = 3m$
slant ht $= 5m$
$ht = 4m$

Pyth. Triple 3-4-5

$$V = (\text{semi-sphere}) + (\text{cylinder}) + (\text{cone})$$
$$(1/2)(4/3\,\pi r^3) + (\text{base})(\text{ht}) + (1/3)(\text{base})(\text{ht})$$
$$(1/2)(4/3)(\pi)(3)^3 + (\pi)(3)^2(10) + (1/3)(\pi)(3^2)(4) = 120\pi\, m^3$$

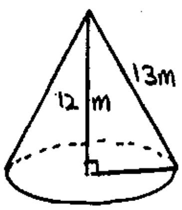
$$SA = (\text{semi-sphere / no base}) + (\text{cylinder / no base}) + (\text{cone / no base})$$
$$(1/2)(4\pi r^2) + (\text{circumference})(\text{ht}) + (\tfrac{1}{2})(\text{circumference})(\text{slant ht})$$
$$(1/2)(4)(\pi)(3)^2 + (2)(\pi)(3)(10) + (1/2)(2)(\pi)(3)(5) = 93\pi\, m^2$$



Pyramids & Cones

PROBLEM SET 12.2

Determine volume and surface area:

①


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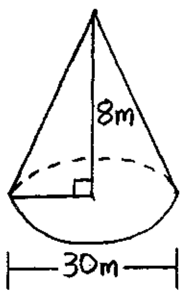

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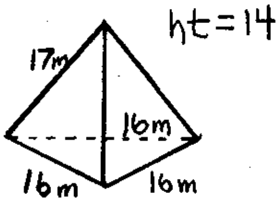

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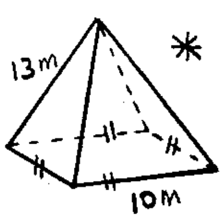

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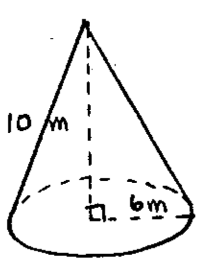

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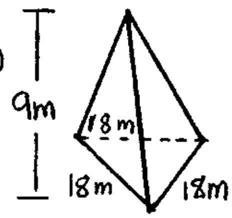

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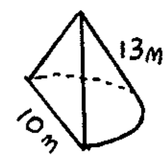

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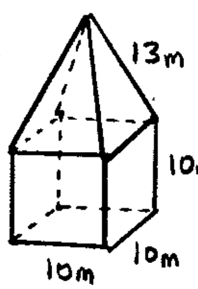
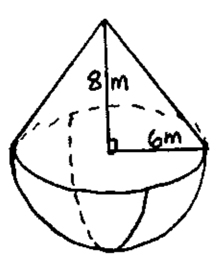


Review Problems

These problems combine lessons 12.1 and 12.2

⑨


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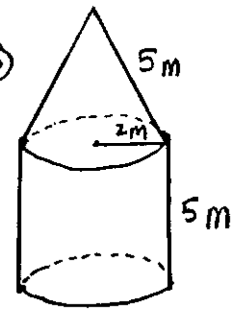

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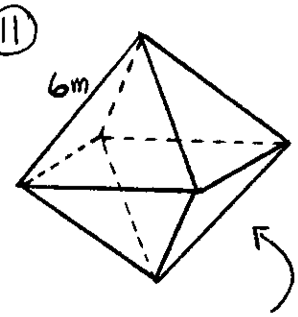
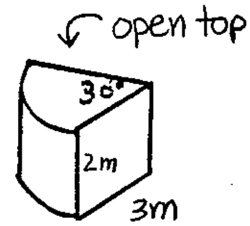


Regular octahedron (all edges are 6 m)

⑫


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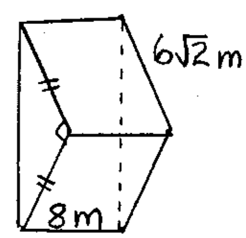
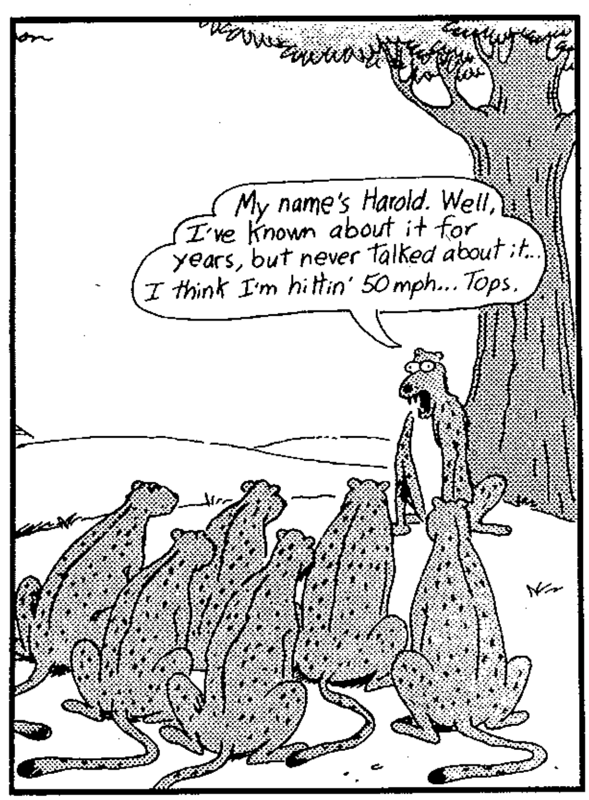


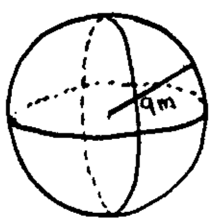


At Slow Cheetahs Anonymous

Solid Geometry

UNIT 12 REVIEW & PRACTICE

Determine volume and surface area:

①


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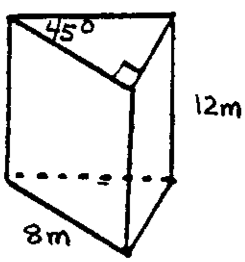

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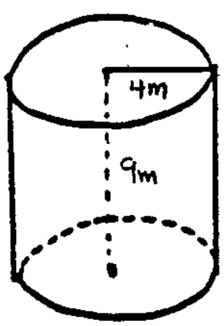

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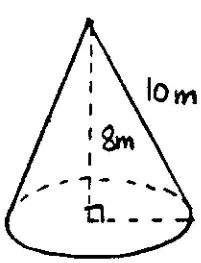

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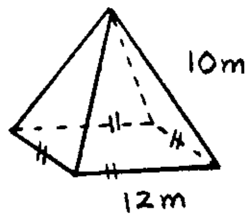

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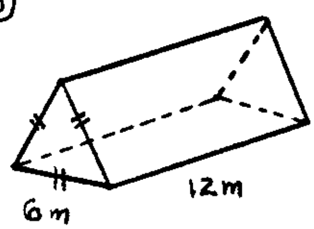

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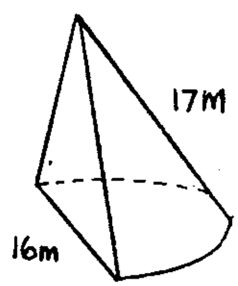

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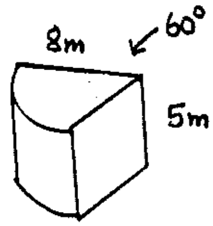
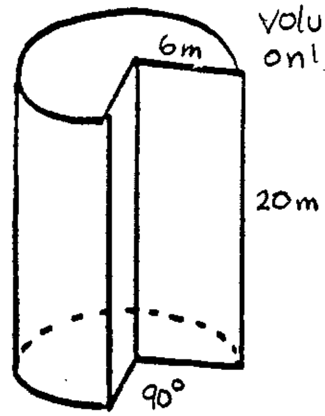


Volume only

⑨


Volume only

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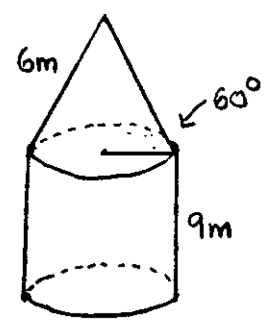

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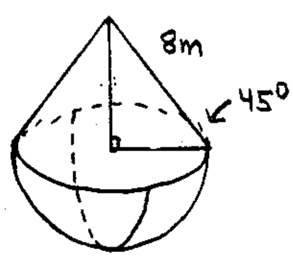


UNIT 13

Introduction to Two-Column Proofs

13.1
Congruent Triangles

13.2
Definitions, Postulates, & Theorems

13.3
Organizing & Constructing Proofs

Introduction to Two-Column Proofs

REFERENCE PAGE

Definitions

1. Segment midpoint	The midpoint separates a segment into two congruent parts
2. Segment Bisector	A segment, line, or plane that intersects at the segment midpoint
3. Right Angle	90 degree angle
4. Linear Pair	Adjacent angles whose noncommon sides form a straight angle
5. Congruent Angles	Angles are congruent iff their measures are exactly equal
6. Congruent Segments	Segments are congruent iff their lengths are exactly equal
7. Complementary Angles	Angles are complementary iff their measures sum to 90°
8. Supplementary Angles	Angles are supplementary iff their measures sum to 180°
9. Vertical Angles	Angles formed on opposite sides of intersecting lines
10. Perpendicular Lines	Lines intersecting at right angles
11. Isosceles Triangle	Triangle with two sides congruent
12. Right Triangle	Triangle with one right angle
13. CPCTC	Corresponding Parts of Congruent Triangles are Congruent

Introduction to Two-Column Proofs

REFERENCE PAGE

Postulates

1. Identity (Reflexive Prop.)
2. Two points form a line
3. Three noncollinear points form a plane
4. A line segment has exactly one midpoint
5. If two angles form a linear pair, then they are supplementary
6. SSS proves congruent triangles
7. SAS proves congruent triangles
8. ASA proves congruent triangles

"OK, folks!...It's a wrap!"

Theorems

1. Vertical angles are equal
2. Complements of equal angles are equal
3. Supplements of equal angles are equal
4. If two equal angles form a linear pair, then they are right angles
5. All right angles are equal
6. Perpendicular lines form congruent adjacent angles
7. The acute angles of a right triangle are complementary
8. AAS proves congruent triangles
9. In a triangle, sides opposite equal angles are equal
10. In a triangle, angles opposite equal sides are equal

Definition:
Explanation of terms

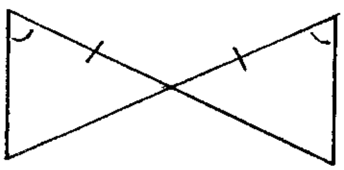
Postulate:
Statement accepted as true

Theorem:
Statement proven to be true

Congruent Triangles

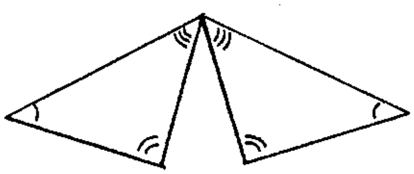
DEMONSTRATION 13.1

If the pair of triangles can be proven to be congruent, state the appropriate postulate: SSS, SAS, or ASA. Indicate "No" if congruence cannot be proven. Use only postulates (not AAS – which is a theorem).

① 

ASA

The vertical angles are equal

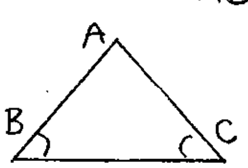
② 

No

AAA is not a postulate

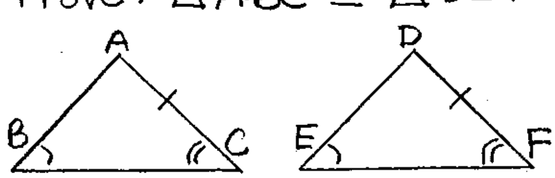
Supply the postulate, definition, or theorem that serves as the reason to support the "Prove" statement.

③ Given: $\angle B \cong \angle C$
Prove: $\overline{AB} \cong \overline{AC}$

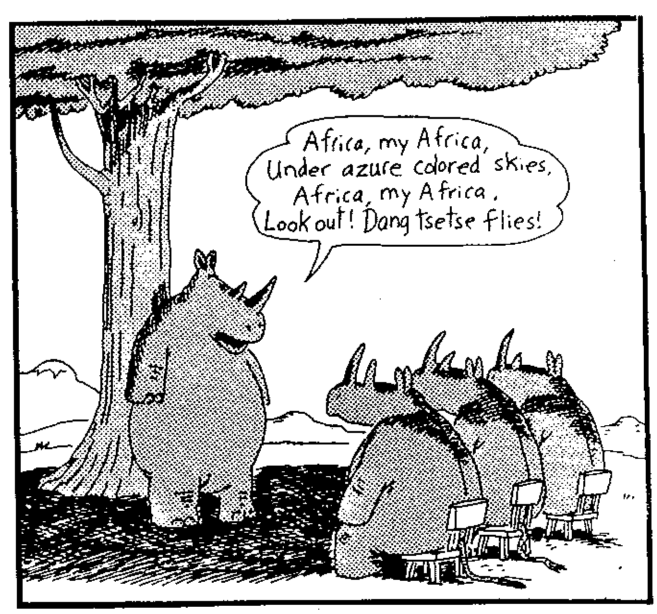


In a triangle, sides opposite equal angles are equal

④ Given: $\angle B \cong \angle E$, $\angle C \cong \angle F$, $\overline{AC} \cong \overline{DF}$
Prove: $\triangle ABC \cong \triangle DEF$



AAS

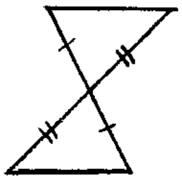


Rhino recitals

Congruent Triangles

PROBLEM SET 13.1

If the pair of triangles can be proven to be congruent, state the appropriate congruence postulate: SSS, SAS, or ASA. Indicate "No" if congruence cannot be proven.

1


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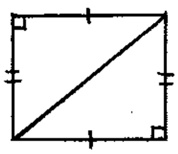

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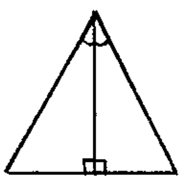

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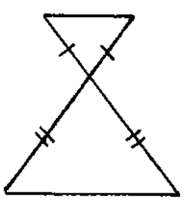

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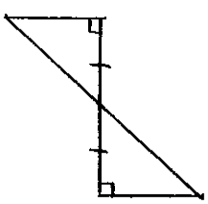

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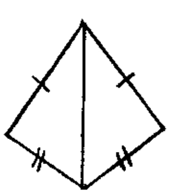

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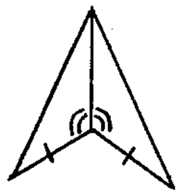

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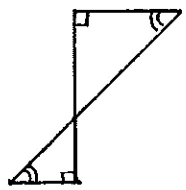

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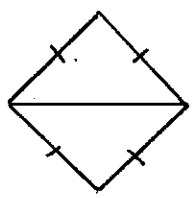

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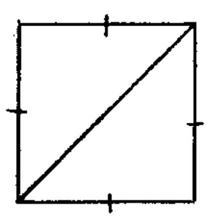

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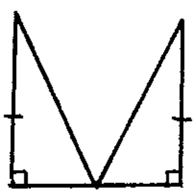

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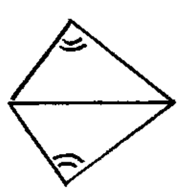

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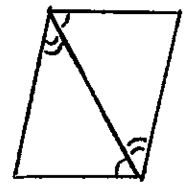

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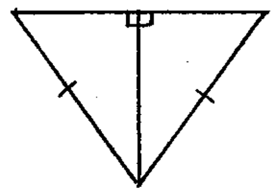

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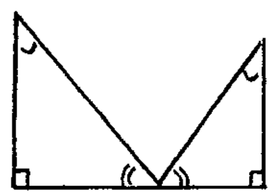
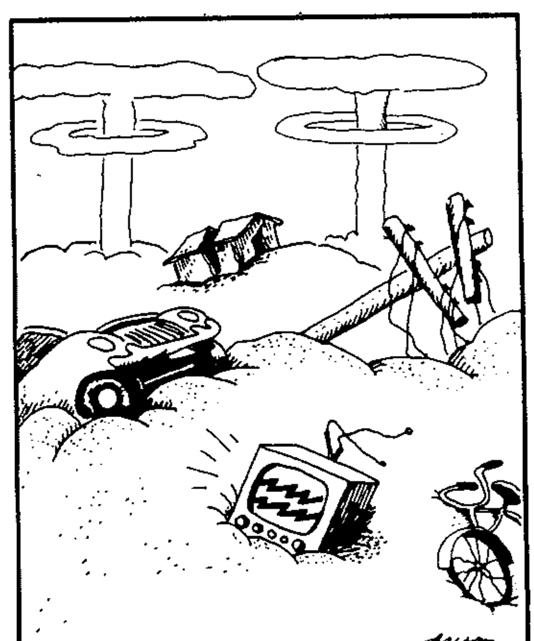


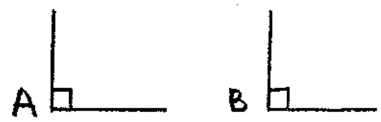

"This is a test. For the next thirty seconds, this station will conduct a test of the emergency broadcast system . . ."

Congruent Triangles

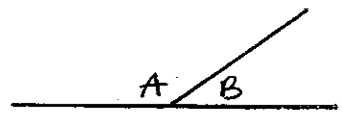
PROBLEM SET 13.1

Supply the postulate, definition, or theorem that serves as the reason to support the "Prove" statement.

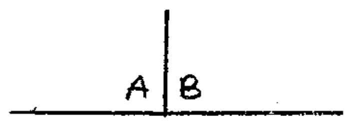
(16) Given: $\angle A$ and $\angle B$ are rt. $\angle$'s
Prove: $\angle A \cong \angle B$



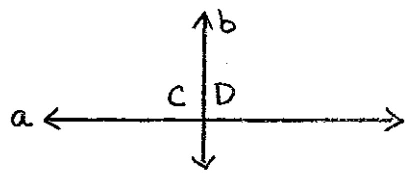
(17) Given: $\angle A$ and $\angle B$ form a linear pair
Prove: $\angle A$ supplementary to $\angle B$



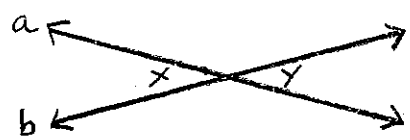
(18) Given: $\angle A$ and $\angle B$ form a linear pair
$\angle A \cong \angle B$
Prove: $\angle A$ and $\angle B$ are rt. $\angle$'s



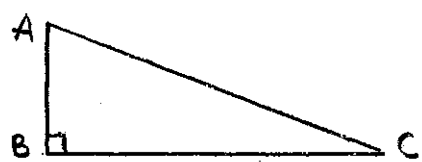
(19) Given: Line a intersects line b
Line a $\perp$ line b
Prove: $\angle C \cong \angle D$



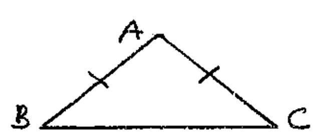
(20) Given: Line a intersects line b
$\angle X$ and $\angle Y$ are vertical $\angle$'s
Prove: $\angle X \cong \angle Y$



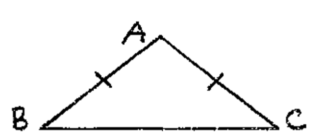
(21) Given: $\triangle ABC$ is a rt. $\triangle$
$\angle B$ is a rt. $\angle$
Prove: $\angle A$ is complementary to $\angle C$



(22) Given: $\overline{AB} \cong \overline{AC}$
Prove: $\angle B \cong \angle C$



(23) Given: $\overline{AB} \cong \overline{AC}$
Prove: $\triangle ABC$ is isosceles

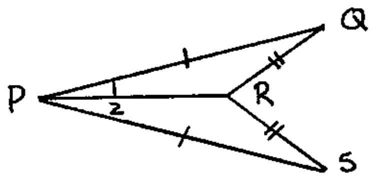


(24) Given: $\overline{AB} \cong \overline{BC}$
Prove: B is the midpoint of $\overline{AC}$

Definitions, Postulates, & Theorems

DEMONSTRATION 13.2

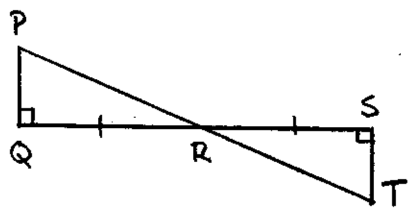
Use the Reference Page to find reasons for each statement in the following proofs:

① 

Given: $\overline{PQ} \cong \overline{PS}$, $\overline{QR} \cong \overline{SR}$

Prove: $\angle 1 \cong \angle 2$

Statements	Reasons
1. $\overline{PQ} \cong \overline{PS}$, $\overline{QR} \cong \overline{SR}$	1.
2. $\overline{PR} \cong \overline{PR}$	2.
3. $\triangle PQR \cong \triangle PSR$	3.
4. $\angle 1 \cong \angle 2$	4.

② 

Given: $\angle Q$ and $\angle S$ are rt. $\angle$'s, $\overline{QR} \cong \overline{SR}$

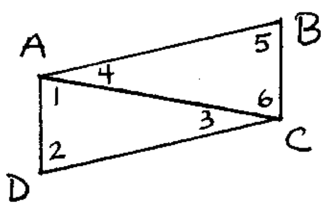
Prove: $\angle P \cong \angle T$

Statements	Reasons
1. $\angle Q$ and $\angle S$ are rt. $\angle$'s, $\overline{QR} \cong \overline{SR}$	1.
2. $\angle Q \cong \angle S$	2.
3. $\angle PRQ$ and $\angle TRS$ are vertical angles	3.
4. $\angle PRQ \cong \angle TRS$	4.
5. $\triangle PRQ \cong \triangle TRS$	5.
6. $\angle P \cong \angle T$	6.

Definitions, Postulates, & Theorems

DEMONSTRATION 13.2

Using the diagram and the "Given" information, organize and complete a two column proof:

③ 

Given: $\angle 1 \cong \angle 6$, $\angle 3 \cong \angle 4$

Prove: $\overline{AD} \cong \overline{CB}$

Statements	Reasons

Answers to Demonstration Problems:

① 1. Given
2. Identity
3. SSS (1, 2)
4. CPCTC

② 1. Given
2. All right angles are equal
3. Definition of vertical angles
4. Vertical angles are equal
5. ASA (1, 2, 4)
6. CPCTC

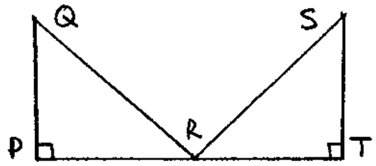
③

Statements	Reasons
1. $\angle 1 \cong \angle 6$, $\angle 3 \cong \angle 4$	1. Given
2. $\overline{AC} \cong \overline{CA}$	2. Identity
3. $\triangle ADC \cong \triangle CBA$	3. ASA (1, 2)
4. $\overline{AD} \cong \overline{CB}$	4. CPCTC

Definitions, Postulates, & Theorems

PROBLEM SET 13.2

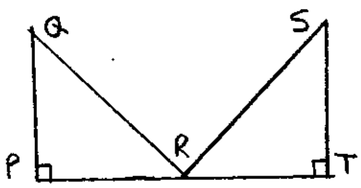
(1)



Given: $\overline{QP} \cong \overline{ST}$, $\angle P$ and $\angle T$ are rt. $\angle$'s, R is midpoint of $\overline{PT}$
Prove: $\overline{QR} \cong \overline{SR}$

Statements	
1. $\overline{QP} \cong \overline{ST}$, $\angle P$ and $\angle T$ are rt. $\angle$'s, R is the midpoint of $\overline{PT}$	1.
2. $\angle P \cong \angle T$	2.
3. $\overline{PR} \cong \overline{TR}$	3.
4. $\triangle QPR \cong \triangle STR$	4.
5. $\overline{QR} \cong \overline{SR}$	5.

(2)



Do not use AAS

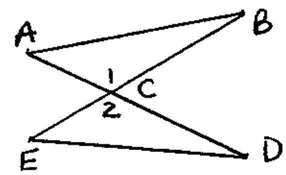
Given: $\angle QRP \cong \angle SRT$, $\overline{QR} \cong \overline{SR}$, $\angle P$ and $\angle T$ are rt. $\angle$'s
Prove: $\overline{PR} \cong \overline{TR}$

Statements	
1. $\angle QRP \cong \angle SRT$, $\overline{QR} \cong \overline{SR}$, $\angle P$ and $\angle T$ are rt. $\angle$'s	1.
2. $\triangle QRP$ and $\triangle SRT$ are rt. triangles	2.

(continued)

Statements (cont.)	
3. $\angle QRP$ and $\angle Q$, $\angle SRT$ and $\angle S$ are complementary	3.
4. $\angle Q \cong \angle S$	4.
5. $\triangle QRP \cong \triangle SRT$	5.
6. $\overline{PR} \cong \overline{TR}$	6.

(3)



Given: $\overline{AC} \cong \overline{EC}$, $\overline{BC} \cong \overline{DC}$
Prove: $\angle B \cong \angle D$

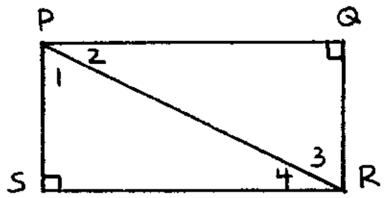
Statements	
1. $\overline{AC} \cong \overline{EC}$, $\overline{BC} \cong \overline{DC}$	1.
2. $\angle 1$ and $\angle 2$ are vertical angles	2.
3. $\angle 1 \cong \angle 2$	3.
4. $\triangle ACB \cong \triangle ECD$	4.
5. $\angle B \cong \angle D$	5.

"Excuse me, but may I assume you're not Dr. Livingstone?"

Definitions, Postulates, & Theorems

PROBLEM SET 13.2

④

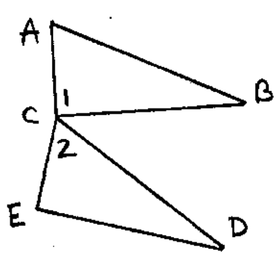


Given: $\angle Q$ and $\angle S$ are rt. $\angle$'s, $\angle 1 \cong \angle 3$

Prove: $\triangle PQR \cong \triangle RSP$

Statements	
1. $\angle Q$ and $\angle S$ are rt. $\angle$'s, $\angle 1 \cong \angle 3$	1.
2. $\angle Q \cong \angle S$	2.
3. $\overline{PR} \cong \overline{RP}$	3.
4. $\triangle PQR \cong \triangle RSP$	4.

⑤

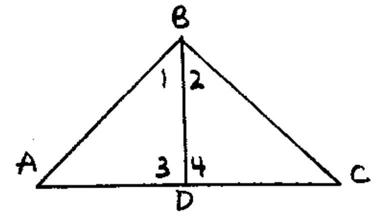


Given: $\overline{BC} \cong \overline{DC}$, $\angle A \cong \angle E$, $\angle 1 \cong \angle 2$

Prove: $\overline{AC} \cong \overline{EC}$

Statements	
1. $\overline{BC} \cong \overline{DC}$, $\angle A \cong \angle E$, $\angle 1 \cong \angle 2$	1.
2. $\triangle ABC \cong \triangle EDC$	2.
3. $\overline{AC} \cong \overline{EC}$	3.

⑥

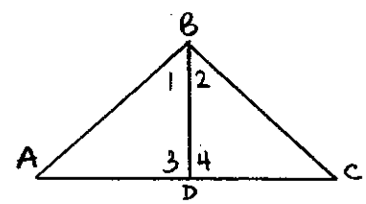


Given: $\angle A \cong \angle C$, $\overline{BD} \perp \overline{AC}$

Prove: $\overline{AB} \cong \overline{CB}$

Statements	
1. $\angle A \cong \angle C$, $\overline{BD} \perp \overline{AC}$	1.
2. $\angle 3 \cong \angle 4$	2.
3. $\overline{BD} \cong \overline{BD}$	3.
4. $\triangle ABD \cong \triangle CBD$	4.
5. $\overline{AB} \cong \overline{CB}$	5.

⑦



Given: $\overline{AB} \cong \overline{CB}$, $\overline{BD}$ bisects $\overline{AC}$

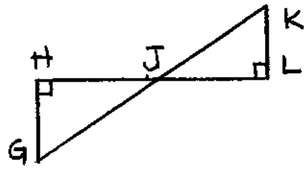
Prove: $\angle A \cong \angle C$

Statements	
1. $\overline{AB} \cong \overline{CB}$, $\overline{BD}$ bisects $\overline{AC}$	1.
2. D is the midpoint of $\overline{AC}$	2.
3. $\overline{AD} \cong \overline{CD}$	3.
4. $\overline{BD} \cong \overline{BD}$	4.
5. $\triangle ABD \cong \triangle CBD$	5.
6. $\angle A \cong \angle C$	6.

Definitions, Postulates, & Theorems

PROBLEM SET 13.2

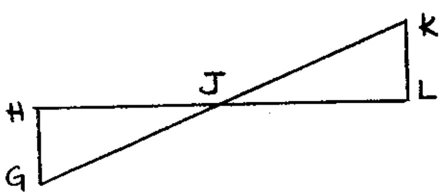
(8)



Given: J is midpoint of $\overline{GK}$,
$\angle H$ and $\angle L$ are rt. $\angle$'s
Prove: $\overline{HG} \cong \overline{LK}$

Statements	
1. J is midpoint of $\overline{GK}$, $\angle H$ and $\angle L$ are rt. $\angle$'s	1.
2. $\overline{GJ} \cong \overline{KJ}$	2.
3. $\angle H \cong \angle L$	3.
4. $\angle HJG$ and $\angle LJK$ are vertical angles	4.
5. $\angle HJG \cong \angle LJK$	5.
6. $\triangle GHJ \cong \triangle KLJ$	6.
7. $\overline{HG} \cong \overline{LK}$	7.

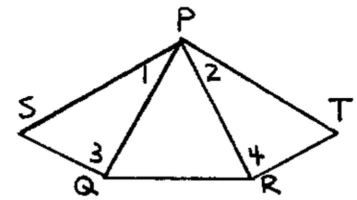
(9)



Given: J is midpoint of $\overline{HL}$,
$\angle G \cong \angle K$
Prove: $\angle H \cong \angle L$

Statements	
1. J is midpoint of $\overline{HL}$, $\angle G \cong \angle K$	1.
2. $\overline{HJ} \cong \overline{LJ}$	2.
3. $\angle HJG$ and $\angle LJK$ are vert. $\angle$'s	3.
4. $\angle HJG \cong \angle LJK$	4.
5. $\triangle GHJ \cong \triangle KLJ$	5.
6. $\angle H \cong \angle L$	6.

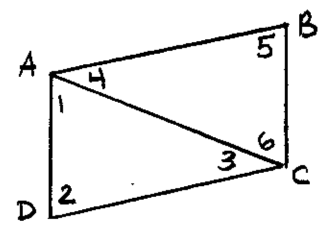
(10)



Given: $\overline{PQ} \perp \overline{SQ}$, $\overline{PR} \perp \overline{TR}$, $\angle 1 \cong \angle 2$,
$\overline{PS} \cong \overline{PT}$
Prove: $\triangle PQR$ is isosceles

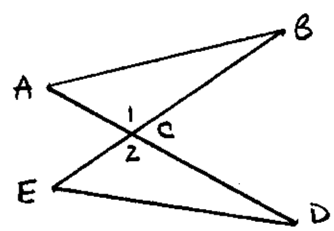
Statements	
1. $\overline{PQ} \perp \overline{SQ}$, $\overline{PR} \perp \overline{TR}$, $\angle 1 \cong \angle 2$, $\overline{PS} \cong \overline{PT}$	1.
2. $\angle 3$ and $\angle 4$ are rt. $\angle$'s	2.
3. $\angle 3 \cong \angle 4$	3.
4. $\triangle PQS \cong \triangle PRT$	4.
5. $\overline{PQ} \cong \overline{PR}$	5.
6. $\triangle PQR$ is isosceles	6.

(11)

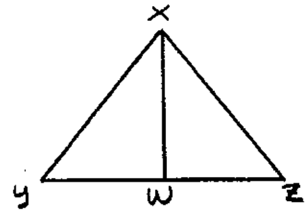


Given: $\angle 3 \cong \angle 4$,
$\overline{DC} \cong \overline{BA}$
Prove: $\angle 1 \cong \angle 6$

(12) Given: $\overline{AC} \cong \overline{EC}$
$\angle A \cong \angle E$
Prove: $\overline{BC} \cong \overline{DC}$



(13)

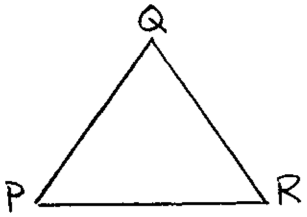


Given: $\overline{YX} \cong \overline{ZX}$, $\angle YXW \cong \angle ZXW$
Prove: W is midpoint of $\overline{YZ}$

Organizing & Constructing Proofs

DEMONSTRATION 13.3

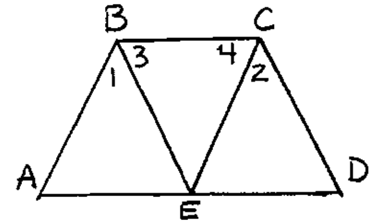
①



Given: $\overline{PQ} \cong \overline{RQ}$
Prove: $\angle P \cong \angle R$

Statements	Reasons

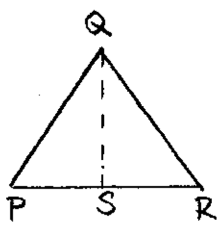
②



Given: $\angle A \cong \angle D$, $\overline{AB} \cong \overline{DC}$, E is midpt. of $\overline{AD}$
Prove: $\angle 3 \cong \angle 4$

Statements	Reasons

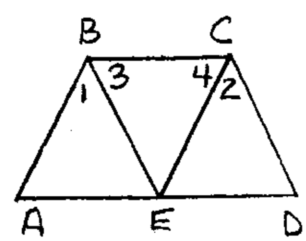
①



Statements	Reasons
1. $\overline{PQ} \cong \overline{RQ}$	1. Given
2. Call S the midpoint of $\overline{PR}$	2. A line segment has exactly one midpoint
3. Draw $\overline{QS}$	3. Two points form a line
4. $\overline{PS} \cong \overline{RS}$	4. Definition of segment midpt.
5. $\overline{QS} \cong \overline{QS}$	5. Identity
6. $\triangle PQS \cong \triangle RQS$	6. SSS (1, 4, 5)
7. $\angle P \cong \angle R$	7. CPCTC

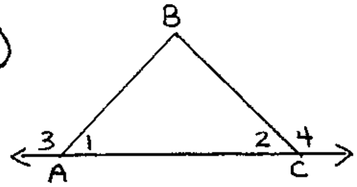
②

Statements	Reasons
1. $\angle A \cong \angle D$, $\overline{AB} \cong \overline{DC}$, E is midpoint of $\overline{AD}$	1. Given
2. $\overline{AE} \cong \overline{DE}$	2. Def. of segment midpoint
3. $\triangle ABE \cong \triangle DCE$	3. SAS (1, 2)
4. $\overline{BE} \cong \overline{CE}$	4. CPCTC
5. $\angle 3 \cong \angle 4$	5. In a triangle, angles opposite equal sides are equal



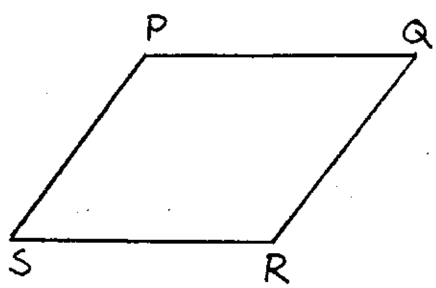
Organizing & Constructing Proofs

PROBLEM SET 13.3

① 

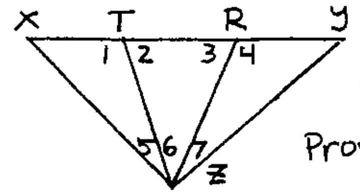
Given: $\overline{AB} \cong \overline{CB}$
Prove: $\angle 3 \cong \angle 4$

Statements	
1. $\overline{AB} \cong \overline{CB}$	1.
2. $\angle 1 \cong \angle 2$	2.
3. $\angle 1$ and $\angle 3$, $\angle 2$ and $\angle 4$ are linear pairs	3.
4. $\angle 1$ and $\angle 3$, $\angle 2$ and $\angle 4$ are supplementary	4.
5. $\angle 3 \cong \angle 4$	5.

② 

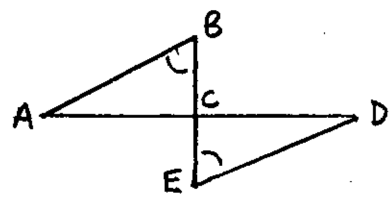
Given: $\overline{PQ} \cong \overline{RS}$, $\overline{PS} \cong \overline{RQ}$
Prove: $\angle P \cong \angle R$

Statements	
1. $\overline{PQ} \cong \overline{RS}$, $\overline{PS} \cong \overline{RQ}$	1.
2. Draw $\overline{SQ}$	2.
3. $\overline{SQ} \cong \overline{QS}$	3.
4. $\triangle QSP \cong \triangle SQR$	4.
5. $\angle P \cong \angle R$	5.

③ 

Given: $\overline{ZT} \cong \overline{ZR}$, $\overline{TX} \cong \overline{RY}$
Prove: $\angle 5 \cong \angle 7$

Statements	
1. $\overline{ZT} \cong \overline{ZR}$, $\overline{TX} \cong \overline{RY}$	1.
2. $\angle 2 \cong \angle 3$	2.
3. $\angle 1$ and $\angle 2$, $\angle 3$ and $\angle 4$ are linear pairs	3.
4. $\angle 1$ and $\angle 2$, $\angle 3$ and $\angle 4$ are supplementary	4.
5. $\angle 1 \cong \angle 4$	5.
6. $\triangle XTZ \cong \triangle YRZ$	6.
7. $\angle 5 \cong \angle 7$	7.

④ 

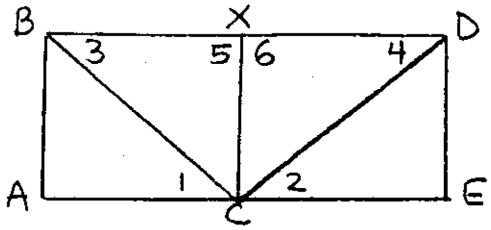
Given: $\overline{BE} \perp \overline{AD}$, C is midpoint of $\overline{AD}$, $\angle B \cong \angle E$
Prove: $\overline{AB} \cong \overline{DE}$

Statements	
1. $\overline{BE} \perp \overline{AD}$, C is midpoint of $\overline{AD}$, $\angle B \cong \angle E$	1.
2. $\angle BCA$ and $\angle ECD$ are rt. $\angle$'s	2.
3. $\angle BCA \cong \angle ECD$	3.
4. $\overline{AC} \cong \overline{DC}$	4.
5. $\triangle ABC \cong \triangle DEC$	5.
6. $\overline{AB} \cong \overline{DE}$	6.

Organizing & Constructing Proofs

PROBLEM SET 13.3

⑤

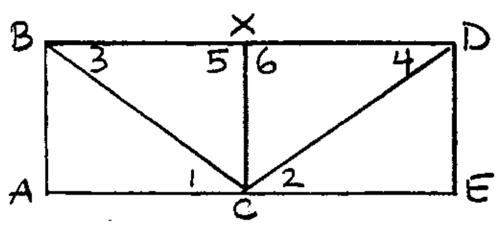


Given: $\overline{BA} \perp \overline{AE}$, $\overline{DE} \perp \overline{AE}$, C is midpoint of $\overline{AE}$, $\angle ABC \cong \angle EDC$

Prove: $\angle 1 \cong \angle 2$

Statements	
1. $\overline{BA} \perp \overline{AE}$, $\overline{DE} \perp \overline{AE}$, C is midpoint of $\overline{AE}$, $\angle ABC \cong \angle EDC$	1.
2. $\overline{AC} \cong \overline{EC}$	2.
3. $\angle A$ and $\angle E$ are rt. $\angle$'s	3.
4. $\angle A \cong \angle E$	4.
5. $\triangle BAC \cong \triangle DEC$	5.
6. $\angle 1 \cong \angle 2$	6.

⑥

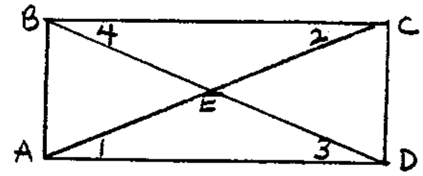


Given: $\overline{CB} \cong \overline{CD}$, $\angle 5 \cong \angle 6$

Prove: $\triangle CBX \cong \triangle CDX$

Statements	
1. $\overline{CB} \cong \overline{CD}$, $\angle 5 \cong \angle 6$	1.
2. $\angle 3 \cong \angle 4$	2.
3. $\triangle CBX \cong \triangle CDX$	3.

⑦

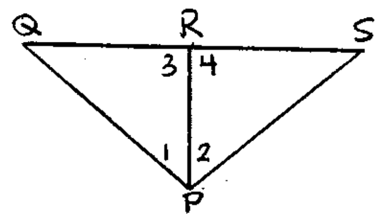


Given: E is midpoint of $\overline{AC}$

$\angle 1 \cong \angle 2$

Prove: $\angle 3 \cong \angle 4$

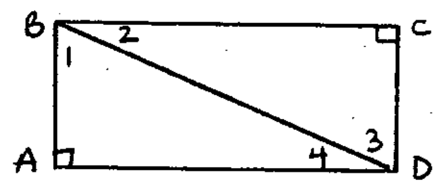
⑧



Given: $\overline{PR} \perp \overline{QS}$, $\angle Q \cong \angle S$

Prove: R is the midpoint of $\overline{QS}$

⑨

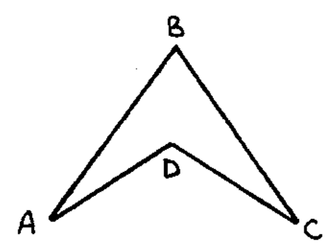


Given: $\angle A$ and $\angle C$ are rt. $\angle$'s

$\angle 2 \cong \angle 4$

Prove: $\overline{AB} \cong \overline{CD}$

⑩



Given: $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$

Prove: $\angle A \cong \angle C$

UNIT 14

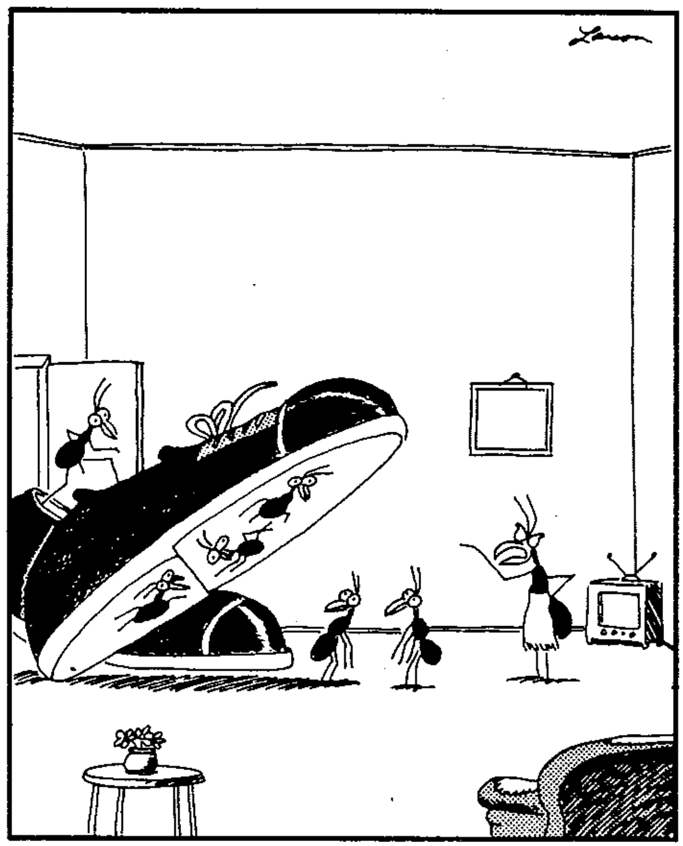
Triangle Trigonometry

14.1
Trigonometric Ratios

14.2
Interpolation

14.3
Solving Right Triangles

14.4
Trigonometric Expressions



"Ernie! Look what you're doing — take those shoes off!"

Trigonometric Ratios

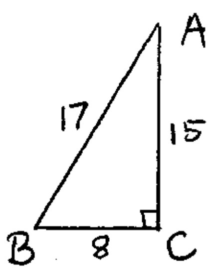
DEMONSTRATION 14.1

For every right triangle, there are trigonometric ratios based on side measures and measures of acute angles.

Trigonometric Ratios		Reciprocals
sin = $\frac{\text{opposite}}{\text{hypotenuse}}$	csc = $\frac{\text{hypotenuse}}{\text{opposite}}$	sine cosecant
cos = $\frac{\text{adjacent}}{\text{hypotenuse}}$	sec = $\frac{\text{hypotenuse}}{\text{adjacent}}$	cosine secant
tan = $\frac{\text{opposite}}{\text{adjacent}}$	cot = $\frac{\text{adjacent}}{\text{opposite}}$	tangent cotangent

Find each value in fraction form:

1. $\sin A = 8/17$
2. $\cos B = 8/17$
3. $\tan A = 8/15$
4. $\csc B = 17/15$
5. $\cot B = 8/15$



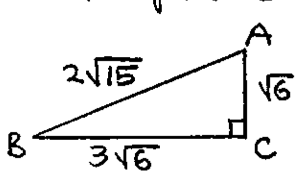
Use a calculator to find angle measures (diagram at left):

9. $\sin A = 8/17 \quad \angle A \approx 28.07^\circ$
10. $\tan B = 15/8 \quad \angle B \approx 61.93^\circ$

Use $\boxed{\text{INV}}$ key on calculator

Round to 4 decimal places:

6. $\sin A \quad \frac{3\sqrt{6}}{2\sqrt{15}}$ .9487



7. $\tan B \quad \frac{\sqrt{6}}{3\sqrt{6}}$ .3333

8. $\sec A \quad \frac{2\sqrt{15}}{\sqrt{6}}$ 3.162

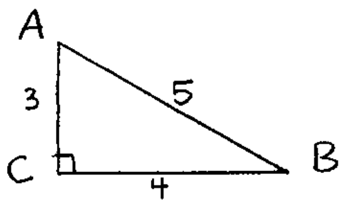
Use a calculator to determine the ratio (round to 2 decimal places):

11. $\tan 40^\circ \approx .84$
12. $\cos 60^\circ = .5$
13. $\csc 20^\circ \approx 2.92 \quad (\sin)(1/x)$
14. $\sec 15^\circ \approx 1.04 \quad (\cos)(1/x)$

Trigonometric Ratios

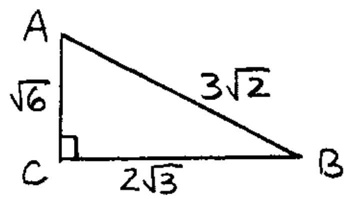
PROBLEM SET 14.1

Find each value in fraction form:



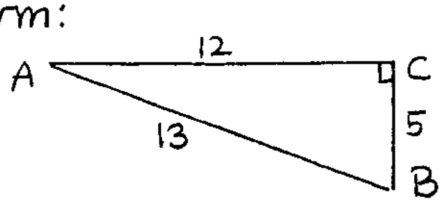
① tan A ⑤ cot B ⑨ csc A
② sin B ⑥ csc B ⑩ cos B
③ cos A ⑦ sec B ⑪ cot A
④ sec A ⑧ tan B ⑫ sin A

Round each value to four significant digits :



⑬ cos A ⑰ csc B ㉑ cos B
⑭ tan B ⑱ sin A ㉒ sin B
⑮ sec B ⑲ tan A ㉓ sec A
⑯ cot A ⑳ csc A ㉔ cot B

Find each value in fraction form:



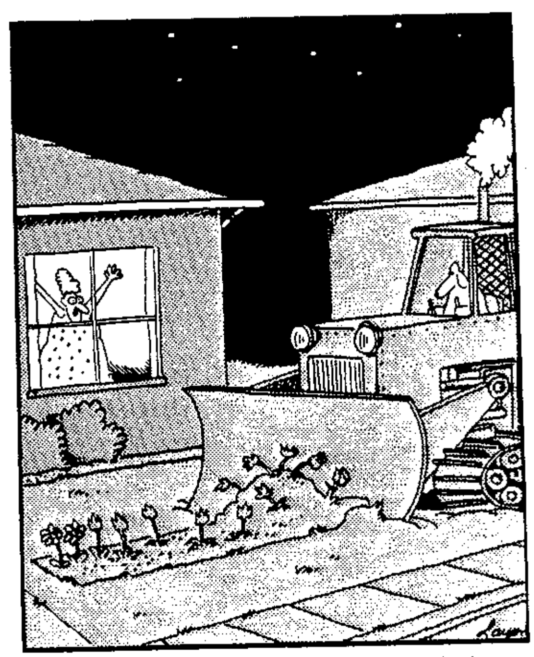
㉕ sin A ㉗ cos B ㉙ tan B
㉖ sec B ㉘ cot A ㉚ csc A

Use a calculator to determine the angle from the diagram (lower left). Round to 2 decimal places :

㉛ sin A ㉝ tan A
㉜ cos B ㉞ sin B

Use a calculator to find the ratio to 2 decimal places:

㉟ sin 30° ㊳ sec 45° ㊶ tan 85°
㊱ tan 20° ㊴ csc 60°
㊲ cos 25° ㊵ cos 10°



Ginger decides to take out Mrs. Talbot's flower bed once and for all.

Interpolation

DEMONSTRATION 14.2

When interpolating angle measures, each degree consists of 60 minutes (60′) and each minute consists of 60 seconds (60″).

Using a calculator, determine the angle to the nearest minute and second:

① cos .3565

$\boxed{\text{INV}}$ cos
69.114596... − 69, =, × 60
6.87576... − 6, =, × 60
52.5457...

69°6′53″

② 34.81°

34.81 − 34, =, × 60
48.6 − 48, =, × 60 → 36

34°48′36″

Interpolation

When using the table:
∠'s 0-45° use top functions
∠'s 45-90° use bottom functions

Using the table, find the ratio or degree measure:

③ tan 41°10′ = .8744

④ csc 48°20′ = 1.339

⑤ cos .7790 = 38°50′

⑥ sin .7862 = 51°50′

Interpolate to find the value. Round to 4th digit (not decimal place):

⑦ sin 40°34′

$$\left.\begin{array}{l} \sin 40°30' = .6494 \\ \sin 40°34' \\ \sin 40°40' = .6517 \end{array}\right\} .0023$$

(.0023)(.4) = .00092
.6494 + .00092 = .65032
.6503

Interpolate to the nearest minute:

⑧ sin A = .7381

$$\left.\begin{array}{l} \left.\begin{array}{l} 47°30' = \sin .7373 \\ \qquad\quad \sin .7381 \end{array}\right\} .0008 \\ 47°40' = \sin .7392 \end{array}\right\} .0019$$

(10′)(8/19) = 4.21...
47°34′

Interpolation

PROBLEM SET 14.2

Using a calculator, determine the angle to the nearest minute and second:

1. sin .4568
2. tan 1.415
3. cos .0269
4. sin .3514
5. $29.\overline{4}^\circ$
6. 42.57°

Using the table, find the indicated ratio:

7. tan $8^\circ 30'$
8. sin $25^\circ 10'$
9. cos $46^\circ 20'$
10. csc $62^\circ 50'$
11. cot $18^\circ 30'$
12. sin $75^\circ 20'$

Using the table, determine the degree measure for each angle:

13. cot A = 1.632
14. cos A = .6862
15. sin A = .9787
16. sec A = 8.614

Using the table, interpolate to find the indicated value (round to 4th <u>digit</u> - not decimal place):

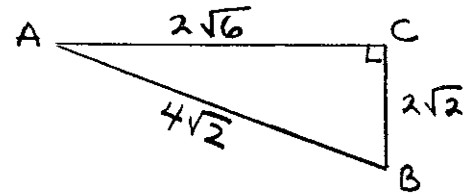
17. sin $43^\circ 23'$
18. csc $47^\circ 15'$
19. cos $31^\circ 42'$
20. cot $47^\circ 18'$

Using the table, interpolate each angle measure to the nearest minute:

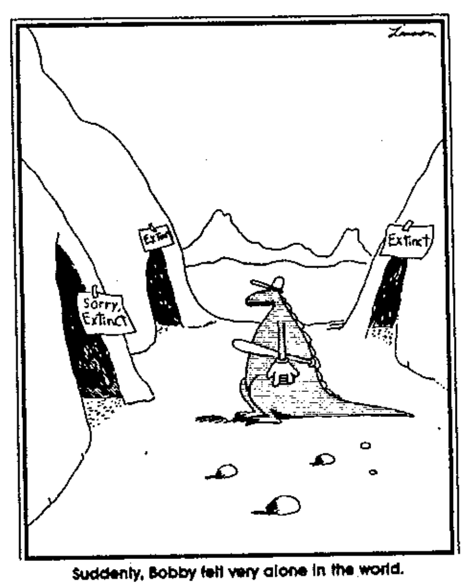
21. sin A = .1111
22. tan A = .2222
23. tan A = 42.71
24. csc A = 1.412

<u>Review</u>

Find each value in fraction form (rationalize when necessary):



25. tan A
26. csc B



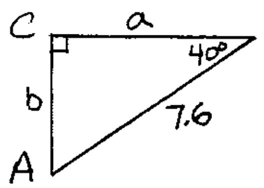
Suddenly, Bobby felt very alone in the world.

Solving Right Triangles

DEMONSTRATION 14.3

To solve a right triangle means to find all the missing measures. You can use trigonometric ratios and the Pythagorean theorem.

① Solve the triangle:

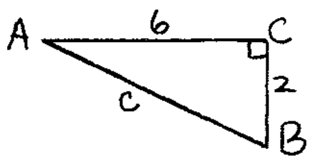


$\angle A = 90 - 40$ $\quad\quad$ $\angle A = 50°$

$\sin 40° = b/7.6$
$(\sin 40°)(7.6) = b$ $\quad\quad$ $b \approx 4.89$

$\sin 50° = a/7.6$
$(\sin 50°)(7.6) = a$ $\quad\quad$ $a \approx 5.82$

② Use the Pythagorean Theorem:

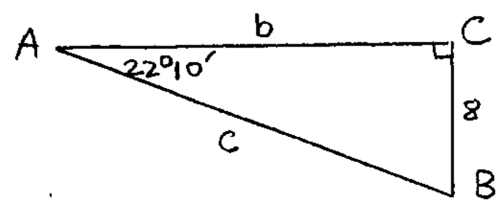


$\tan A = 2/6$
$\angle A \approx 18.°4349\ldots$
$-18, =, \times 60$ $\quad\quad$ $\angle A \approx 18°26'$

$\angle B = 90 - 18°26'$ $\quad\quad$ $\angle B \approx 71°34'$

$2^2 + 6^2 = c^2$
$4 + 36 = c^2$
$c^2 = 40$
$c = \sqrt{40} = 2\sqrt{10}$ $\quad\quad$ $c = 2\sqrt{10}$

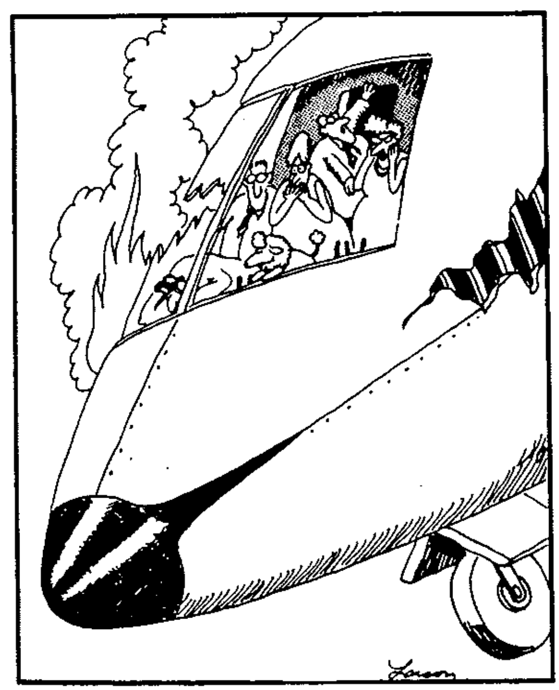
③ Solve (variable in the denominator):



$\angle B = 90 - 22°10'$ $\quad\quad$ $\angle B = 67°50'$

$\tan 22°10' = 8/b$
$b = 8/\tan 22\tfrac{1}{6}°$ $\quad\quad$ $b \approx 19.64$

$\sin 22°10' = 8/c$
$c = 8/\sin 22\tfrac{1}{6}°$ $\quad\quad$ $c \approx 21.20$

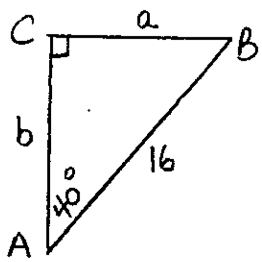


Suddenly, amidst all the confusion, Fifi seized the controls and saved the day.

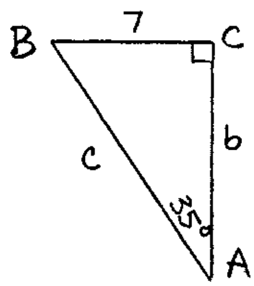
Solving Right Triangles

PROBLEM SET 14.3

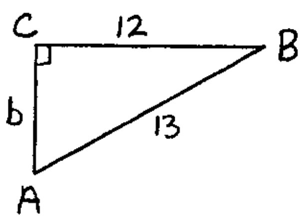
Solve each right triangle.
Round sides to two decimal places
and angles to the nearest minute:



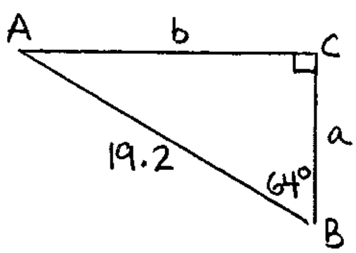
① ∠B
a
b



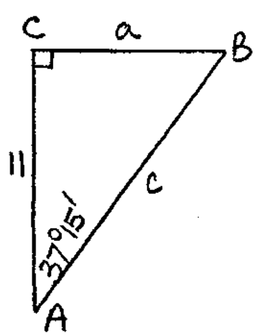
② ∠B
b
c



③ ∠A
∠B
b

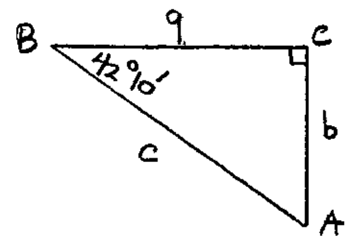


④ ∠A
a
b



⑤ ∠B
a
c

⑥ ∠A
b
c



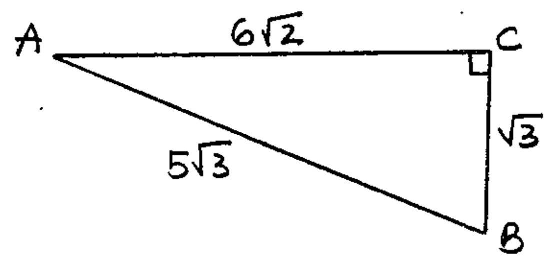
Review

Use the table to interpolate for the indicated ratio or degree measure:

⑦ tan A = 2.15 to the nearest minute (∠A)

⑧ cos 50°42' to four decimal places

Find each value in fraction form: (rationalize if necessary)



⑨ cot B

⑩ cos A

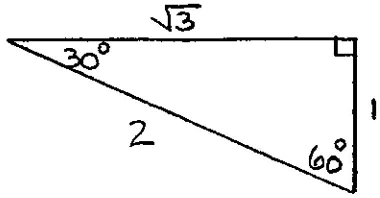
⑪ csc B

Trigonometric Expressions

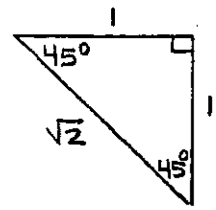
DEMONSTRATION 14.4

Special right triangle relationships can be used to determine the value of expressions that feature trig functions for 30°, 60°, and 45° angles.

30-60-90 Rt. Triangle



45-45-90 Rt. Triangle



Determine the value of each expression in fraction form:

① $\sin 30° - 3\tan 30°$

$(1/2) - 3(1/\sqrt{3})$

$(1/2) - 3/\sqrt{3} \cdot \sqrt{3}/\sqrt{3} = \frac{1}{2} - \sqrt{3}$

② $\cos^2 45° + \sec^2 60°$

$(1/\sqrt{2})^2 + (2/1)^2$

$1/2 + 4 = 9/2$

③ $\tan 30° \cot 30° - 2\sin^2 45°$

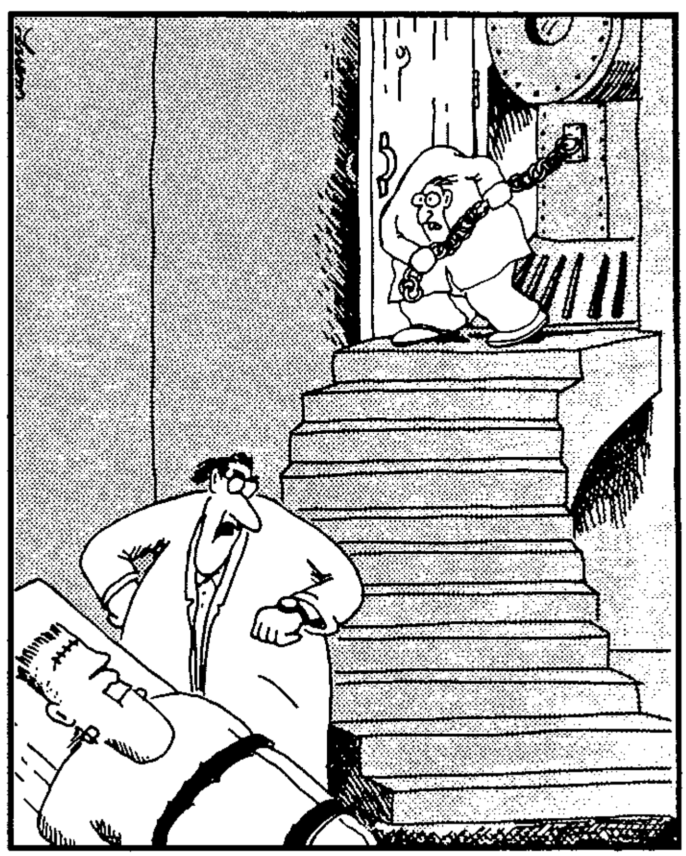
$(1/\sqrt{3})(\sqrt{3}/1) - 2(1/\sqrt{2})^2$

$1 - 2(1/2)$

$1 - 1 = 0$

④ $-2\csc 30°$

$-2(2) = -4$



"Curses! . . . How long does it take Igor to go out and bring back a simple little brain, anyway?"

Trigonometric Expressions

PROBLEM SET 14.4

Find the value of each expression in fraction form:

① $2\cos 30^\circ$

② $\sin^2 45^\circ + \cos^2 45^\circ$

③ $\cos^2 30^\circ - \sin^2 30^\circ$

④ $\csc 45^\circ - \sec 30^\circ$

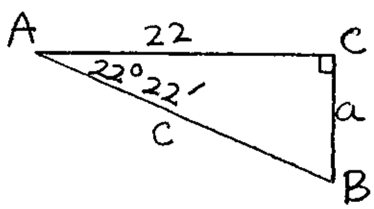
⑤ $-\sin 60^\circ$

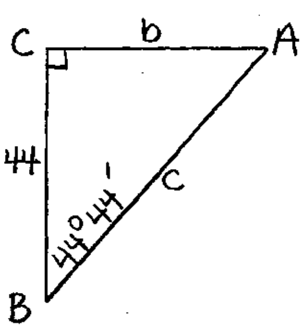
⑥ $2\sin 60^\circ \cos 60^\circ$

⑦ $\sin 30^\circ \cos 60^\circ - \sin 60^\circ \cos 30^\circ$

⑧ $\sec 60^\circ + \cot 30^\circ - 4\cos^2 45^\circ$

Solve each right triangle:

⑨


⑩


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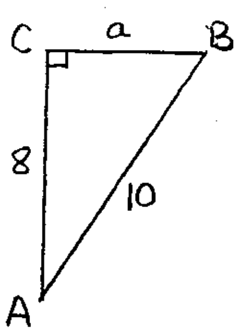


Interpolate to find the ratio or degree measure:

⑫ $\cot A = .5728$

Find $\angle A$ to the nearest minute

⑬ $\tan 22^\circ 34'$

Find the value to 4 decimal places

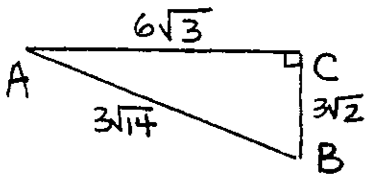
"Well, don't look at me, idiot! . . . I SAID we should've flown!"

Triangle Trigonometry

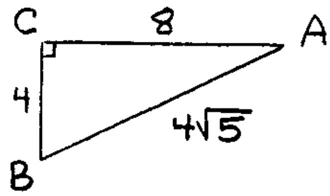
UNIT 14 REVIEW & PRACTICE

Find each value in fraction form (rationalize as needed):

① a) $\tan A$
b) $\csc B$
c) $\sin A$
d) $\cot A$



② a) $\cos B$
b) $\tan A$
c) $\sec B$
d) $\sin A$



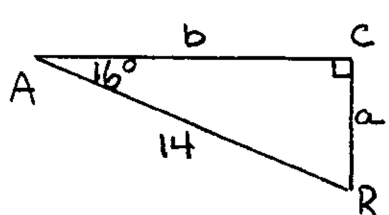
Interpolate to the nearest minute:

③ $\tan A = .4497$

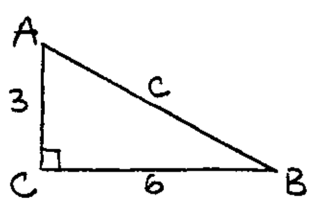
④ $\sin A = .7478$

Solve each triangle. Round side measures to 2 decimal places and angles to the nearest minute:

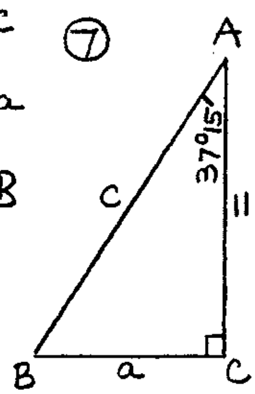
⑤



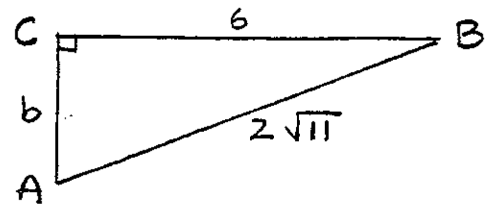
⑥



⑦



⑧



Determine the value of each expression in fraction form:

⑨ $\cos^2 45° - \sin 30°$

⑩ $2\tan^2 60° + (\csc 30°)(\sin^2 45°)$

⑪ $\dfrac{2\cos 60° - \sin^2 45°}{\csc 45°}$

⑫ $\dfrac{(2\tan^2 60°)(\sin 30°)}{\sin 60°}$

Ichabod Crane vs. the Headless Horseman in The People's Court

UNIT 15

Law of Sines & Cosines

15.1
Problem Solving

15.2
Law of Sines

15.3
Law of Cosines

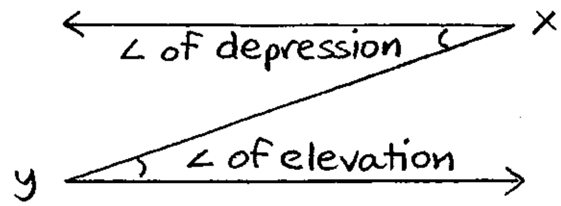
15.4
Review & Practice

"For crying out loud, I was *hibernating*!... Don't you guys ever take a pulse?"

Problem Solving

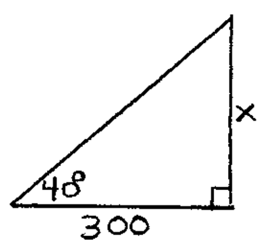
DEMONSTRATION 15.1

The angle of depression and angle of elevation are always measured from the horizontal.



To increase accuracy, use all digits displayed in your calculator when solving:

① Two hikers are 300 m from the base of a radio tower. The angle of elevation to the top of the tower is 40°. How high is the tower?



$\tan 40^\circ = x/300$

$(\tan 40^\circ)(300) = x$

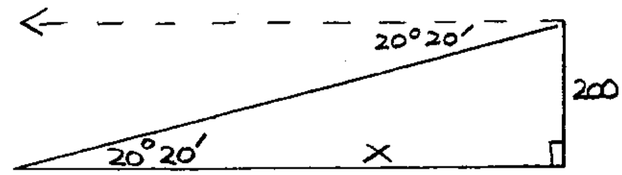
$x \approx 251.73\text{ m}$

② Robert is standing on top of a cliff 200 feet above a lake. The angle of depression to a boat on the lake is 20°20′. How far is the boat from the base of the cliff?

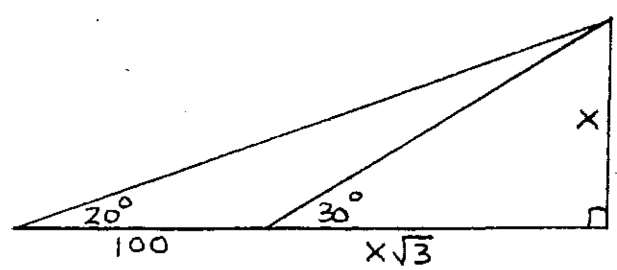
(continued)

$\tan 20^\circ 20' = 200/x$

$x = 200/\tan 20\tfrac{1}{3}^\circ \quad x \approx 539.71\text{ ft.}$



③ The base of a TV antenna and two points on the same side are in a straight line. The two points are 100 feet apart. From the two points, the measures of the angles of elevation to the top of the antenna are 30° and 20°. Find the height of the antenna.



$\tan 20^\circ = x/(100 + x\sqrt{3})$

$(\tan 20^\circ)(100 + x\sqrt{3}) = x$

$36.397023 + .6304149x = x$

$36.397023 = .3695851x$

$x \approx 98.48\text{ ft.}$

Problem Solving

PROBLEM SET 15.1

When solving, round angles to the nearest minute and sides to 2 decimal places:

① At a point on the ground 30 meters from the base of a tree, the angle of elevation to the top of the tree is 65.° How tall is the tree?

② A flagpole casts a shadow 40 feet long when the angle of elevation to the sun is 31°20.′ How tall is the flagpole?

③ The angle of depression from a plane 1000 feet in the air to an aircraft carrier in the water is 63° 18.′ How far is the plane from the carrier?

④ The angle of elevation from the ground to a kite in the air is 70.° It is held by a string 65m long. How high is the kite?

⑤ A 24-foot ladder leans against a building and forms an angle of 18°. How far is the foot of the ladder from the base of the building?

⑥ The top of a lighthouse is 120 meters above sea level. From the top of the lighthouse, the angle of depression to a boat at sea is 43.° Find the distance from the boat to the foot of the lighthouse.

⑦ The measurement of the angle of elevation to the top of a building from a point on the ground is 38°20.′ From a point 50 feet closer to the building, the measurement of the angle of elevation is 45.° What is the height of the building?

Problem Solving

PROBLEM SET 15.1

8. Two observers 200 ft. apart on the same side are in line with a flagpole's base. The angle of elevation from one observer is 30° and it is 60° from the other. How far is the flag pole from each observer?

9. Two buildings are separated by an alley. Joe is looking out a window 60 feet above the ground in one building. The angle of depression from Joe to the base of the other building is 50°. The angle of elevation from Joe to the top of the other building is 40°. How high is the second building?

10. A ship sails due north from port for 90 km, then 40 km east, and then 70 km north. How far is the ship from port?

11. A railroad track rises 20 feet for every 400 feet of track. What is the angle the track makes with the horizontal?

"So! . . . The little sweethearts were going to carve their initials on me, eh?"

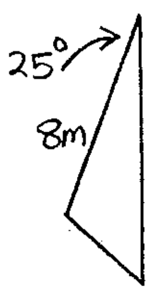
Law of Sines

DEMONSTRATION 15.2

The sine of an angle can be used to compute the area of a triangle that is not a right triangle. In addition, the Law of Sines can solve certain triangles that are not right triangles.

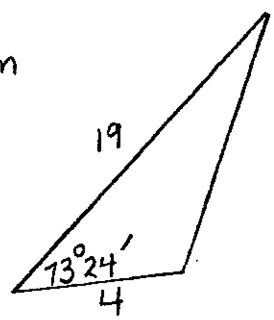
Computing Area

$A = \frac{1}{2}$(prod. of 2 sides)(sin of included angle)

① 

$A = \frac{1}{2}(8)(12)(\sin 25°)$

$A \approx 20.29\ m^2$



② Given:

$b = 4$

$c = 19$

$\angle A = 73°24'$

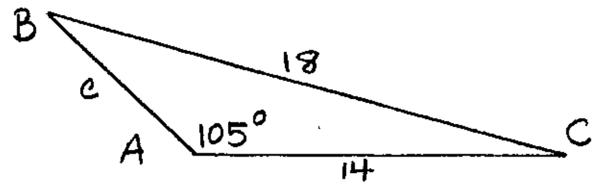
$A = \frac{1}{2}(4)(19)(\sin 73.4)$

$A \approx 36.42$

Note: This formula for area can only be used if the given information includes two sides and the included angle.

Law of Sines

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

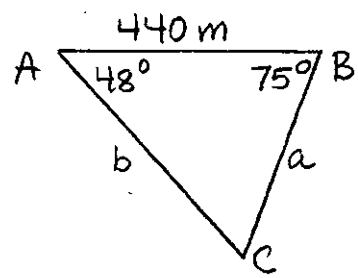
③ 

$$\frac{\sin 105}{18} = \frac{\sin B}{14}$$

$\angle B \approx 48.7°$ $\quad \angle B \approx 48°42'$

$180 - (105 + 48°42')$ $\quad \angle C \approx 26°18'$

$\frac{\sin 105}{18} = \frac{\sin 26.3}{c}$ $\quad c \approx 8.26$

④ 

$\angle C = 57°$

$\frac{\sin 57}{440} = \frac{\sin 75}{b}$ $\quad b \approx 506.76m$

$\frac{\sin 57}{440} = \frac{\sin 48}{a}$ $\quad a \approx 389.88m$

Note: If possible, avoid using rounded values in calculations.

Law of Sines

PROBLEM SET 15.2

Find the area of each triangle:

① $b = 11.5$, $c = 14$, $A = 20°$

② $a = 9.4$, $c = 13.5$, $B = 95°$

③

Solve each triangle:

④ $B = 36°36'$, $C = 119°$, $b = 8$

⑤ $a = 14$, $b = 7.5$, $A = 103°$

Solve each problem:

⑥ An isosceles triangle has a base of 22 cm and a vertex angle of 36° opposite the base. Find the perimeter.

⑦ The longest side of a triangle is 34 yards. Two angles of the triangle are 40° and 65°. Find the length of the other two sides.

⑧ A triangular lot faces two streets that meet at an angle of 85°. The sides of the lot facing the streets are each 160 feet in length. Find the perimeter and area of the lot.

⑨ A ship is sighted at sea between two observation points A and B. These points are 30 miles apart. The angle at A between $\overline{AB}$ and the ship is 34°. The angle at B is 45°34.′ How far is the ship from point B?

⑩ Two planes leave an airport at the same time. Each flies at a speed of 110 miles per hour. One flies 60° east of north. The other flies 40° east of south. How far apart are the planes after 3 hours?

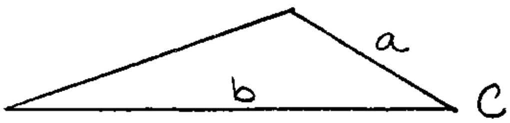
⑪ Points X and Y are on opposite sides of a valley. Point C is 60 km from point X. Angle YXC is 108° and angle YCX is 35°. How wide is the valley?

⑫ A 60-foot building is on top of a hill. A surveyor is on the hill. His angle of elevation to the top of the building is 42° and 18° to the bottom. How far is he from the bottom of the building?

Law of Cosines

DEMONSTRATION 15.3

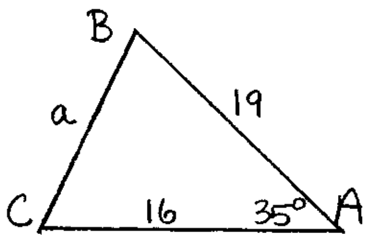
The Law of Cosines can be used to solve triangles that cannot be solved by the Law of Sines.



$$c^2 = a^2 + b^2 - 2ab(\cos C)$$

This formula can be used when sufficient information to complete a ratio in the Law of Sines is not available.

① Given: $A = 35°$, $b = 16$, $c = 19$



Given two sides and included angle

$$a^2 = (16)^2 + (19)^2 - 2(16)(19)(\cos 35)$$

$$a^2 = 118.9\overline{5} \qquad \boxed{a \approx 10.91}$$

$$\frac{\sin 35}{10.906675} = \frac{\sin C}{19}$$

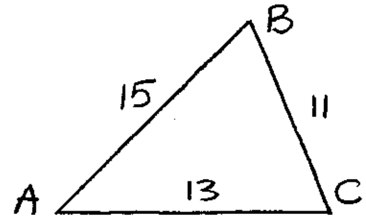
$$\angle C \approx 87.708377° \qquad \boxed{\angle C \approx 87°43'}$$

$$180 - (35 + 87°43')$$

$$\boxed{\angle B \approx 57°17'}$$

② Given: $a = 11$, $b = 13$, $c = 15$

Given three side measures



$$11^2 = (15)^2 + (13)^2 - 2(15)(13)(\cos A)$$

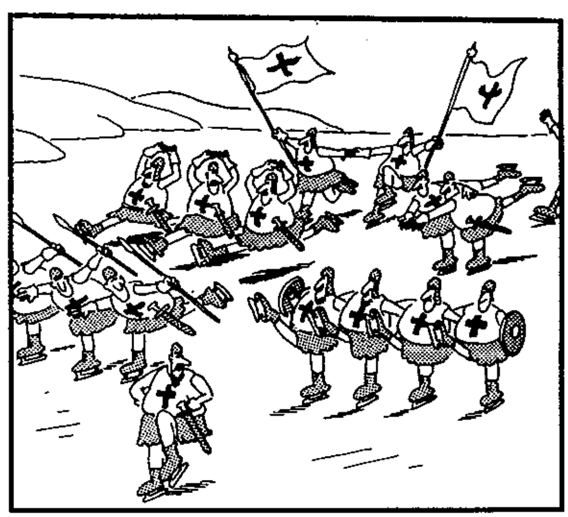
$$\frac{11^2 - (15)^2 - (13)^2}{-2(15)(13)} = \cos A$$

$$\angle A \approx 45.572996 \qquad \boxed{\angle A \approx 45°34'}$$

$$\frac{\sin 45.572996}{11} = \frac{\sin B}{13}$$

$$\angle B \approx 57.563563 \qquad \boxed{\angle B \approx 57°34'}$$

$$\boxed{\angle C \approx 76°52'}$$



The Ice Crusades

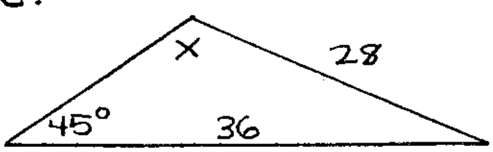
Law of Cosines

DEMONSTRATION 15.3

When dealing with an obtuse angle, this relationship applies:

$$\boxed{\sin A = \sin(\text{supplement of } A)}$$

③ A flower bed is in the shape of a triangle. One angle is 45° and the opposite side is 28 feet. The longest side is 36 feet. Find the angle opposite the longest side.

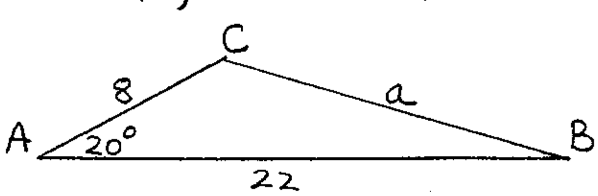


$$\frac{\sin 45}{28} = \frac{\sin x}{36}$$

$x \approx 65.386402$ according to the calculator, but the diagram shows x to be the largest angle:

$$x \approx (180 - 65.386402) \approx \boxed{114°37'}$$

④ Determine $\angle C$ if: $\angle A = 20°$, $b = 8$ cm, $c = 22$ cm



Step (1) Law of Cosines used to solve for a:

$$a^2 = 8^2 + 22^2 - 2(8)(22)(\cos 20)$$

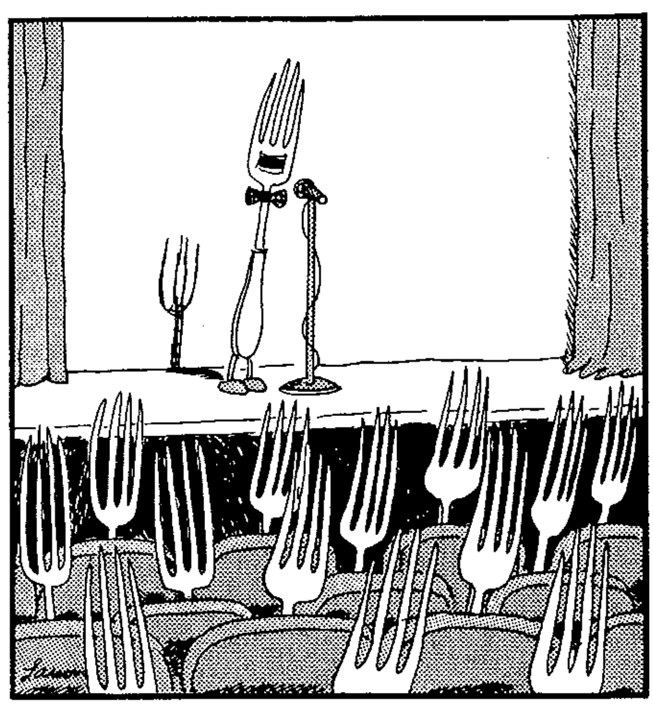
$a^2 = 217.2282 \quad a \approx 14.738663$

Step (2) Law of Sines used to solve for $\angle C$:

$$\frac{\sin 20}{14.738663} = \frac{\sin C}{22}$$

$\angle C \approx 30.698748$ according to the calculator, but $\angle C$ must be the largest angle:

$$\angle C \approx (180 - 30.698748) \approx \boxed{149°18'}$$



"... And so the bartender says, 'Hey! That's not a soup spoon!' ... But seriously, forks ..."

Law of Cosines

PROBLEM SET 15.3

Solve each triangle:

① $a = 20$, $c = 24$, $B = 47°$

② $b = 13$, $a = 21.5$, $C = 39°20'$

③ $A = 40°$, $B = 59°$, $c = 14$

④ $B = 19°$, $a = 51$, $c = 61$

⑤ $a = 345$, $b = 648$, $c = 442$

⑥ $A = 25°26'$, $a = 13.7$, $B = 78°$

Solve each problem:

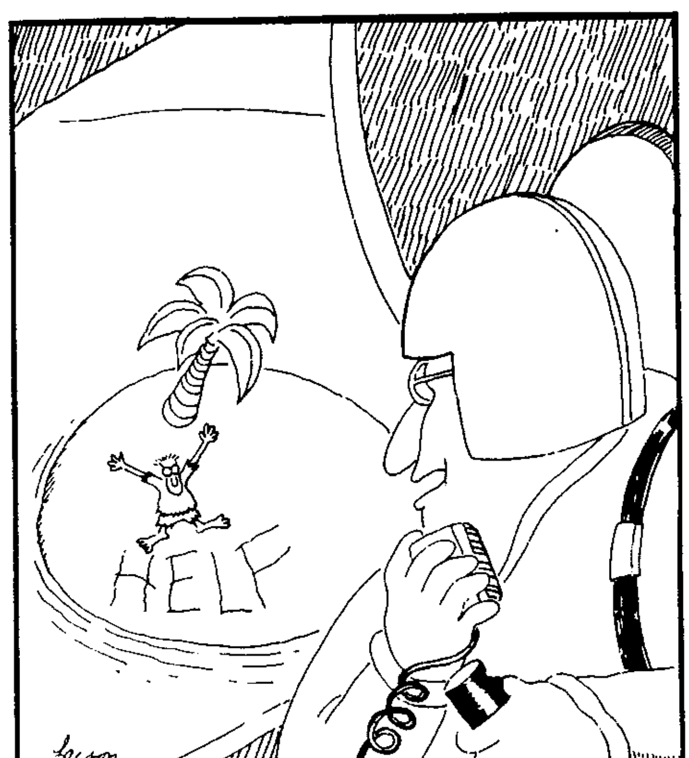
⑦ A triangular plot of land has two sides, 400 and 600 feet, with an angle between them measuring 46°20'. Find the perimeter and area.

⑧ A triangular city lot has sides of 50 m, 70 m, and 80 m. Find the measure of the angle opposite the short side.

⑨ A ship at sea is 70 miles from one radio transmitter and 130 miles from another. The angle between the signals is 130°. How far apart are they?

⑩ The sides of a parallelogram are 55 cm and 71 cm. The larger angle is 106°. Find the longer diagonal and the area.

⑪ A plane flew 1200 km north. It then changed direction by turning 15° clockwise to fly for another 850 km. At the end of its flight, how far was the plane from its starting point?



"Wait! Wait! . . . Cancel that, I guess it says 'helf.'"

Review & Practice

PROBLEM SET 15.4

PART I
Class Assignment

1. From a point on the ground 50 m from the base of a flagpole, the angle of elevation to the top is 48°. How tall is the flagpole?

2. The diagonal of a rectangle is 30 inches long and makes a 15° angle with one of the sides. Find the dimensions of the rectangle.

3. The base of a monument and two points on the same side are in a straight line. The two points are 50 m apart. The measurements of the angles of elevation to the top of the monument are 45° and 25°. Find the height of the monument.

4. An isosceles triangle has a vertex angle of 80° and a base of 20 cm. Determine the perimeter of the triangle.

5. A plane flew 1000 km north. It then changed direction by turning 20° clockwise and flew for another 750 km. How far was the plane from its starting point?

6. Determine the area of a triangle with adjacent sides of 12 in. and 20 in. and an angle of 50° 25′ between them.

7. $\angle A = 20°$, $b = 9$ cm, $c = 18$ cm

   Determine $\angle C$ to the nearest minute.

"Hey! You! . . . No cutting in!"

Law of Sines & Cosines

UNIT 15 REVIEW & PRACTICE

PART II
Skill Check

⑧ A pilot 3000 ft. above the ocean judges the angle of depression to a ship to be $42°$. How far is the plane from the ship?

⑨ A 32-foot ladder leans against a building. The top touches the building 26 ft. above the ground. What is the angle formed by the ladder with the ground?

⑩ From the courtyard wall, the angle of elevation to the top of the school is $40°$. From a point 50 feet closer, the angle of elevation is $60°$. How tall is the school?

⑪ Two sides of a triangle form a $54°30'$ angle. The sides are 12 cm and 12 cm. What is the perimeter of the triangle?

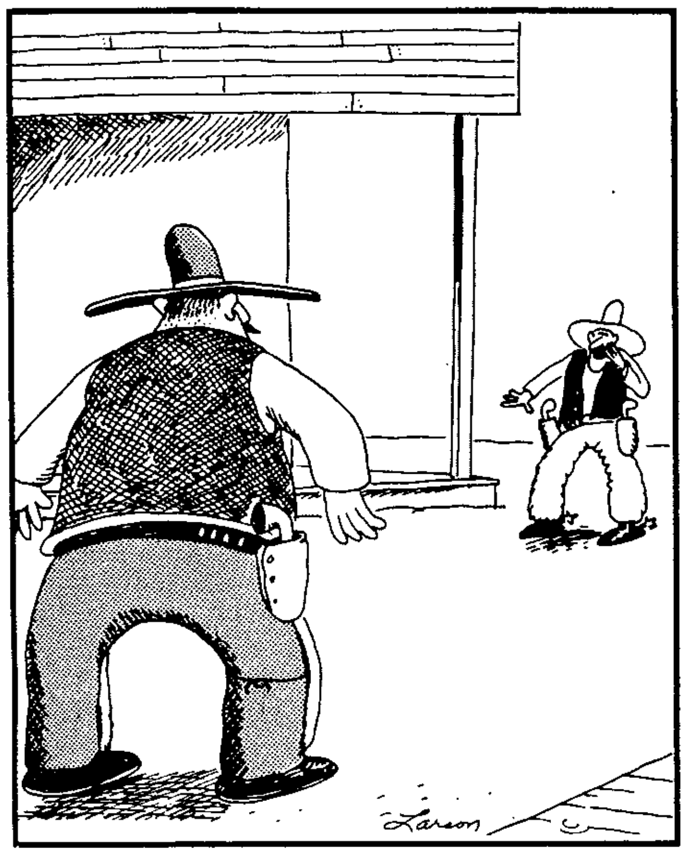
⑫ The sides of a triangle are 4 in., 8 in., and 10 in. Find the smallest angle of the triangle to the nearest minute.

⑬ Determine the area:

$a = 9m$, $c = 14m$,
$\angle B = 24°15'$

⑭ $\angle A = 15°$, $b = 5cm$,
$c = 16cm$

Determine the measure of $\angle C$ to the nearest minute.



"Shoe's untied!"

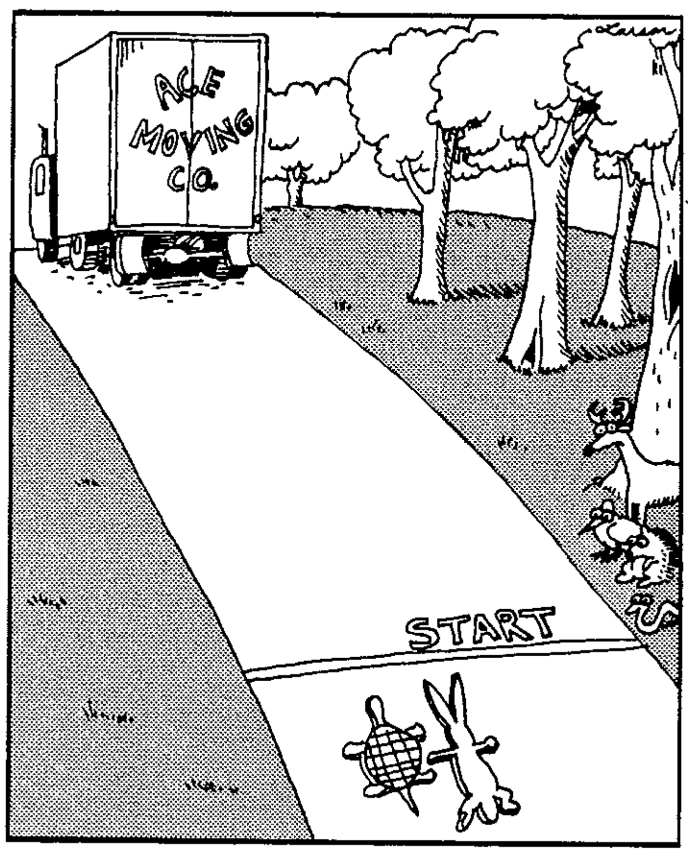
UNIT 16

Trigonometric Functions

16.1
Radians & The Unit Circle

16.2
Sine & Cosine Functions

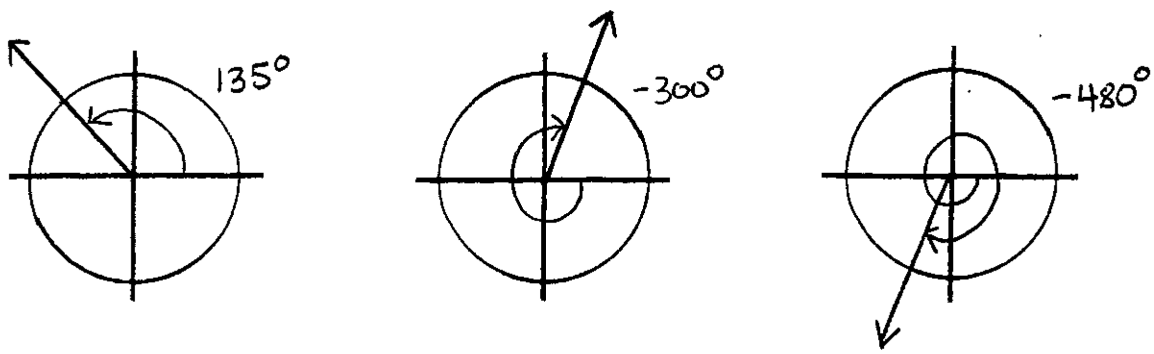
16.3
Graphing Sine & Cosine Curves



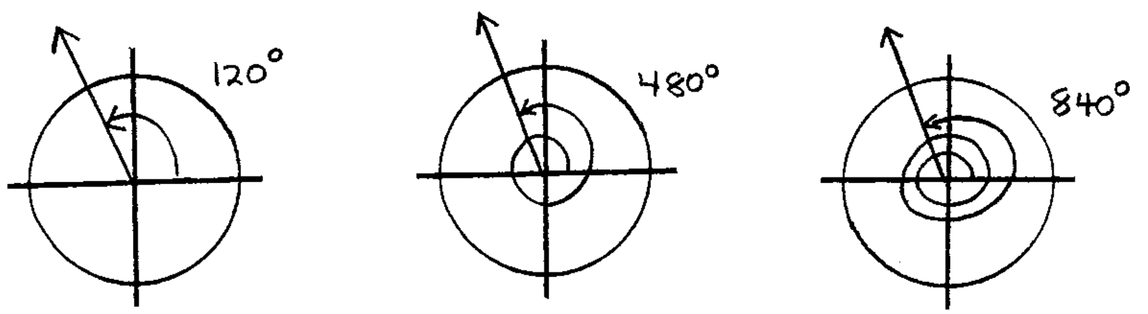
Radians & The Unit Circle

DEMONSTRATION 16.1

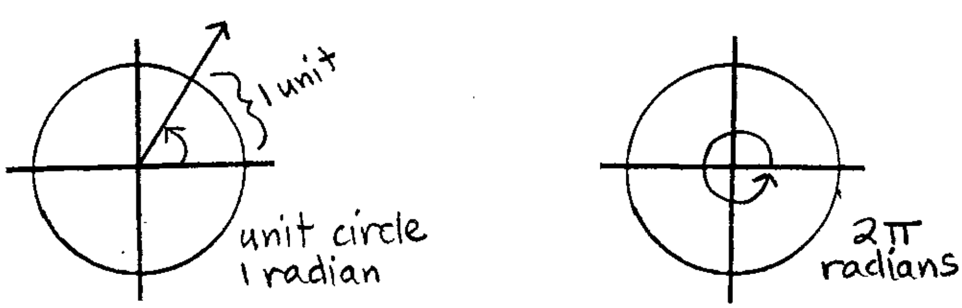
The unit circle has a center at the origin and a radius equal to one unit. The initial side of an angle in standard position is the positive x-axis. A clockwise rotation produces a negative angle and a counterclockwise rotation produces a positive angle.



Angles that differ by one or more rotations are called coterminal angles.



An angle in standard position that intercepts an arc whose length is 1 unit is given the measurement: 1 radian

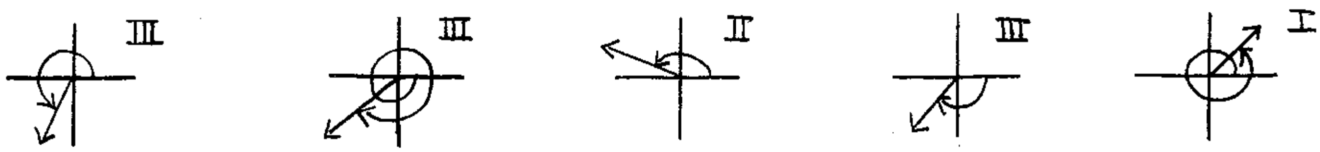


Radians & The Unit Circle

DEMONSTRATION 16.1

Name the quadrant that contains the terminal side of each angle:

① $240°$ ② $-500°$ ③ $\frac{2}{3}\pi$ ④ $-\frac{3}{4}\pi$ ⑤ $\frac{9}{4}\pi$



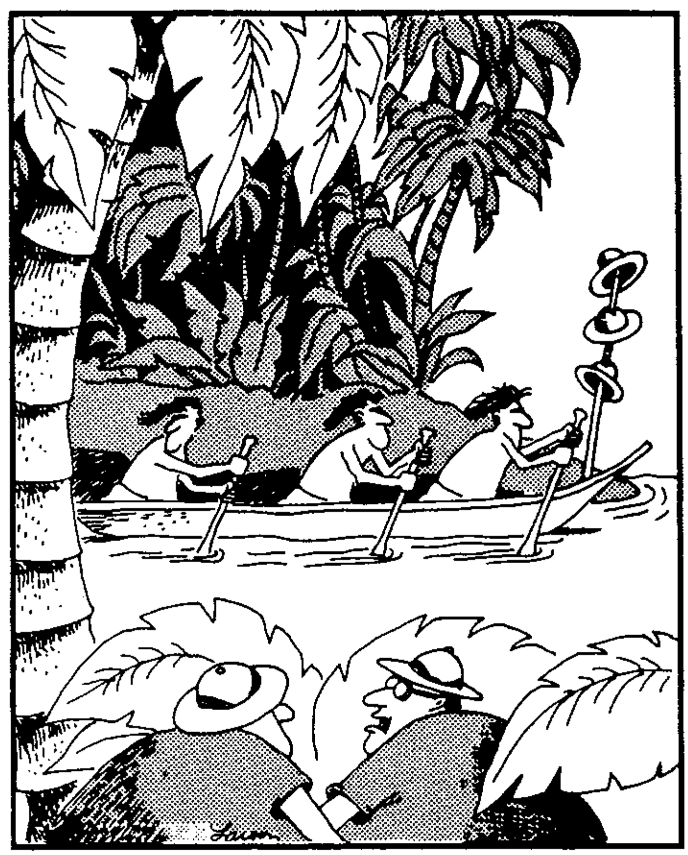
III III II III I

Change to radians:

⑥ $45°$ $(45)(\pi/180) = \pi/4$

⑦ $240°$ $(240)(\pi/180) = 4\pi/3$

⑧ $-150°$ $(-150)(\pi/180) = -5\pi/6$



"Hathunters!"

Change to degrees:

⑨ $5\pi/3$ $(5\pi/3)(180/\pi) = 300°$

⑩ $-4\pi/3$ $(-4\pi/3)(180/\pi) = -240°$

⑪ $3/4$ $(3/4)(180/\pi) = 135/\pi\,°$

Find the least positive coterminal angle:

⑫ $-300° \rightarrow 60°$ 

⑬ $5\pi \rightarrow \pi$ 

⑭ $-\frac{8}{3}\pi \rightarrow \frac{4}{3}\pi$ 

Radians & The Unit Circle

PROBLEM SET 16.1

Name the quadrant containing the terminal side of each angle in standard position:

1. -240°
2. -32°
3. 440°
4. 300°
5. $\frac{5}{3}\pi$
6. $-\frac{12}{5}\pi$
7. $-\frac{4}{7}\pi$
8. $\frac{5}{9}\pi$
9. 945°
10. -210°

Change each of the following degree measures to radians:

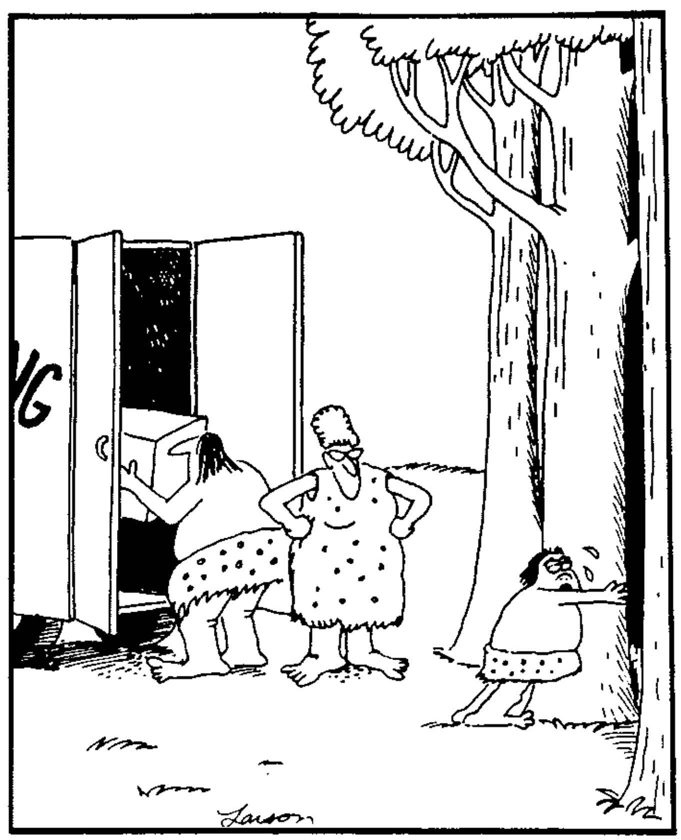
11. 330°
12. -240°
13. 270°
14. -135°
15. 180°
16. -210°
17. 405°
18. 810°
19. -315°
20. -270°

Change each radian measure to degrees:

21. $\frac{-\pi}{4}$
22. $\frac{3}{4}\pi$
23. $\frac{11\pi}{6}$
24. $\frac{7\pi}{4}$
25. 5
26. 2
27. $5\frac{1}{2}\pi$
28. $4\frac{1}{3}\pi$
29. $6\frac{1}{2}\pi$
30. $3\frac{1}{3}\pi$

Find the least positive coterminal angle:

31. -940°
32. $-9/4\,\pi$
33. 11π
34. $-16/3\,\pi$



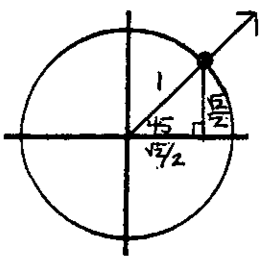
Primitive Man leaves the trees.

Sine & Cosine Functions

DEMONSTRATION 16.2

To determine the value of sine and cosine functions, you can use the coordinates of the point of intersection between the terminal side of the angle and the unit circle. The x-coordinate : cos, the y-coordinate : sin.

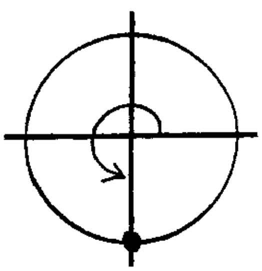
① Find the sin of $45°$



$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right)$

$\sin 45° = \frac{\sqrt{2}}{2}$

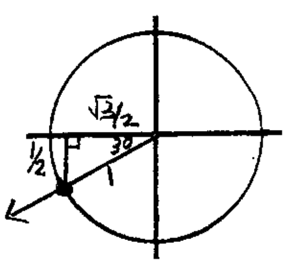
② Find the cos of $270°$



$(0, -1)$

$\cos 270° = 0$

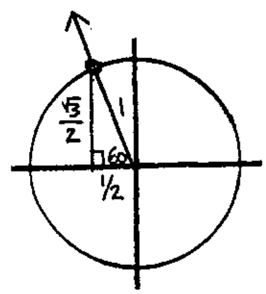
③ Find the cos of $210°$



$\left(\frac{-\sqrt{3}}{2}, \frac{-1}{2}\right)$

$\cos 210° = \frac{-\sqrt{3}}{2}$

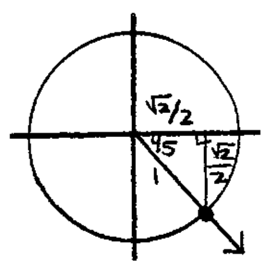
④ Find the sin of $\frac{-10}{3}\pi$



$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

$\sin \frac{-10}{3}\pi = \frac{\sqrt{3}}{2}$

⑤ Evaluate $6(\sin 315°)(\cos 315°)$



$\left(\frac{\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$

$\sin 315° = -\frac{\sqrt{2}}{2}$

$\cos 315° = \frac{\sqrt{2}}{2}$

$6\left(-\frac{\sqrt{2}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = \frac{-12}{4} = -3$

Note: Point of intersection (circle and terminal side): (cos, sin)

Sine & Cosine Functions

PROBLEM SET 16.2

Find each of the values in radical form:

1. $\cos 150^\circ$
2. $\cos -150^\circ$
3. $\cos \frac{11}{3}\pi$
4. $\sin \frac{17}{4}\pi$
5. $\cos \frac{-3}{4}\pi$
6. $\sin \frac{-5}{3}\pi$
7. $\sin \frac{3\pi}{2}$
8. $\cos \frac{7}{4}\pi$
9. $\cos 390^\circ$
10. $\sin -240^\circ$
11. $\cos \frac{-7}{4}\pi$
12. $\sin 660^\circ$
13. $\sin 300^\circ$
14. $\cos 900^\circ$
15. $\cos 330^\circ$
16. $\sin -180^\circ$

Evaluate:

17. $\dfrac{4 \sin 300^\circ + 2 \sin 30^\circ}{3}$
18. $8(\sin 120^\circ)(\cos 120^\circ)$

Review

Name the quandrant that contains the terminal side:

19. $-\frac{8}{3}\pi$
20. 620°

change each degree measure to radians:

21. 510°
22. -630°

change each radian measure to degrees:

23. $\frac{4}{9}\pi$
24. $\frac{8}{3}$

Find the least positive coterminal angle:

25. -1120°
26. $\frac{-17}{5}\pi$

"And the murderer is . . . THE BUTLER! Yes, the butler . . . who, I'm convinced, first gored the Colonel to death before trampling him to smithereens."

Graphing Sine & Cosine Curves

DEMONSTRATION 16.3

The sine and cosine functions can be graphed on a vertical axis when the horizontal axis represents values of an angle in degrees or radians.

SINE CURVE

$a y = b \sin c\,\theta$

$y = \frac{b}{a} \sin c\,\theta$

amplitude $= \left|\frac{b}{a}\right|$

period $= \frac{2\pi}{c}$

COSINE CURVE

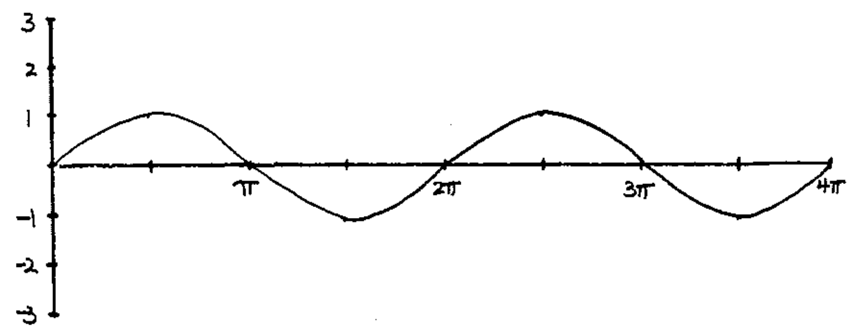
$a y = b \cos c\,\theta$

$y = \frac{b}{a} \cos c\,\theta$

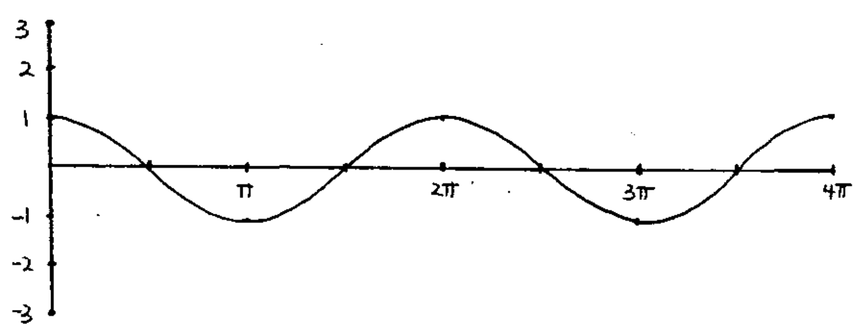
amplitude $= \left|\frac{b}{a}\right|$

period $= \frac{2\pi}{c}$

① $y = \sin\theta$
amplitude $= 1$
period $= 2\pi$



② $y = \cos\theta$
amplitude $= 1$
period $= 2\pi$



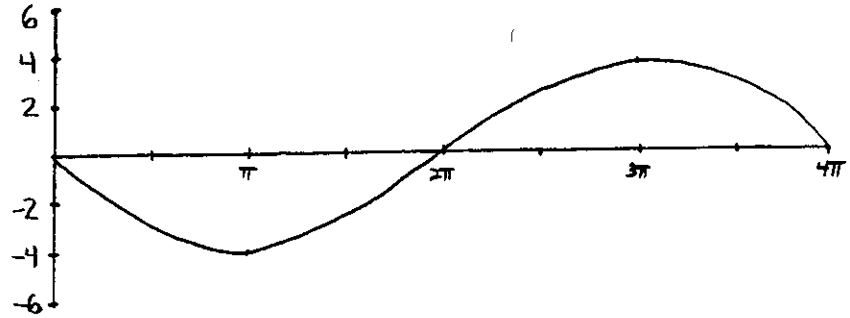
Graphing Sine & Cosine Curves

DEMONSTRATION 16.3

(3) $y = -4 \sin \frac{1}{2}\theta$

amplitude $= |-4| = 4$

period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$



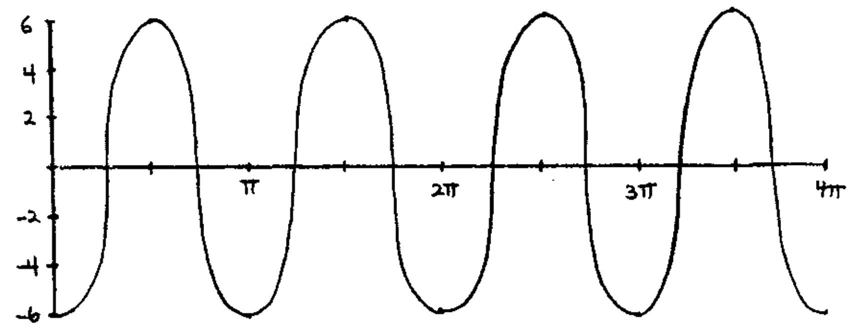
(4) $\frac{1}{3}y = -2\cos 2\theta$

$3\left[\frac{1}{3}y = -2\cos 2\theta\right]$

$y = -6\cos 2\theta$

amplitude $= |-6| = 6$

period $= \frac{2\pi}{2} = \pi$



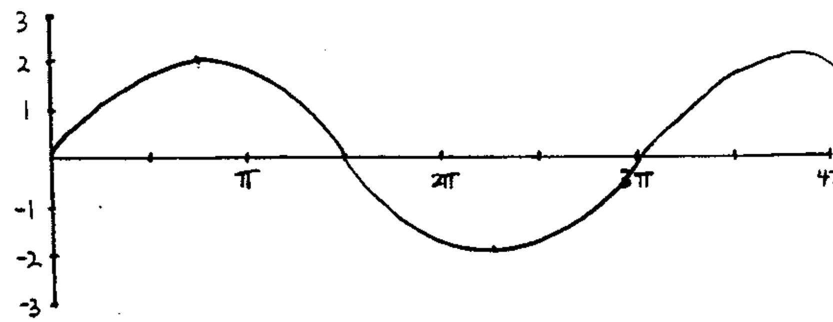
(5) $2y = 4\sin\frac{2}{3}\theta$

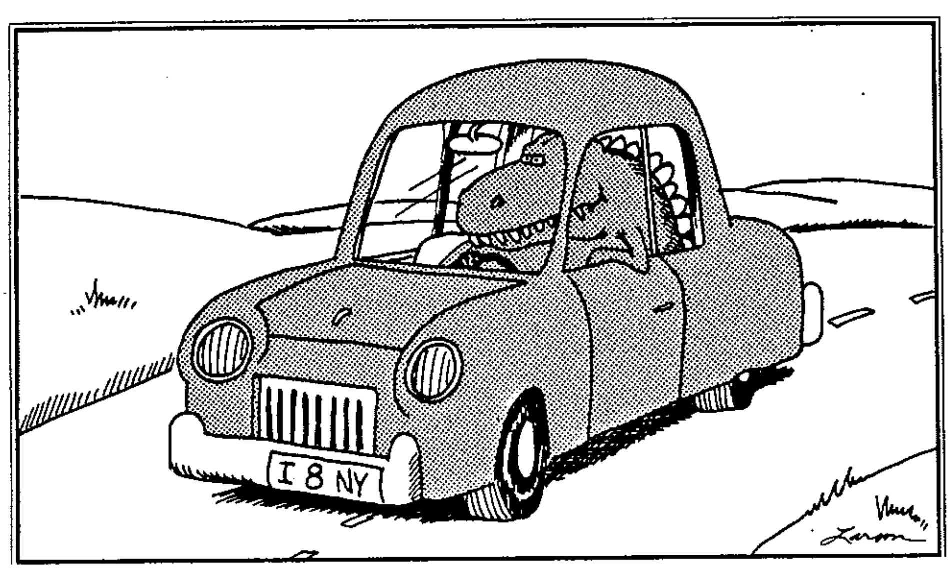
$\frac{1}{2}\left[2y = 4\sin\frac{2}{3}\theta\right]$

$y = 2\sin\frac{2}{3}\theta$

amplitude $= 2$

period $= \frac{2\pi}{\frac{2}{3}} = 3\pi$





Graphing Sine & Cosine Curves

PROBLEM SET 16.3

Graph each of the following:

① $y = \frac{2}{3} \cos \theta$

② $y = 3 \sin \theta$

③ $y = 6 \sin \frac{2}{3} \theta$

④ $y = 3 \cos \frac{1}{2} \theta$

⑤ $y = -2 \sin \theta$

⑥ $y = -3 \sin \frac{2}{3} \theta$

⑦ $\frac{1}{2} y = 3 \sin 2\theta$

⑧ $y = 4 \cos \frac{4}{3} \theta$

⑨ $3y = 2 \sin \frac{1}{2} \theta$

⑩ $\frac{1}{2} y = -2 \cos 2\theta$

Review

Change to radians:

⑪ $80°$ ⑫ $-540°$

Change to degrees:

⑬ $-{}^{5}/_{9}\,\pi$ ⑭ 12

Find the least positive coterminal angle:

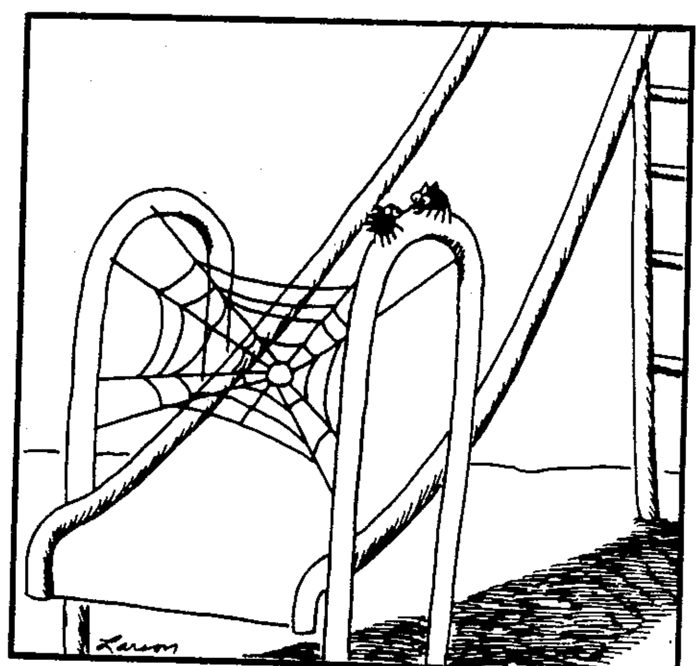
⑮ $-835°$ ⑯ $\frac{9}{2}\pi$

Find the value in radical form:

⑰ $\cos 225°$ ⑱ $\sin -{}^{11}/_{6}\,\pi$

Evaluate:

⑲ $\dfrac{3 \sin 150° - \cos 225°}{\sin 210°}$



"If we pull this off, we'll eat like kings."

Trigonometric Functions

UNIT 16 REVIEW & PRACTICE

Determine the quadrant that contains the terminal side:

① $-690°$ ② $\frac{7}{3}\pi$

Change to radians:

③ $900°$ ④ $-240°$

Change to degrees:

⑤ $-\frac{5}{6}\pi$ ⑥ $\frac{4}{3}\pi$

Find the least positive coterminal angle:

⑦ $-1250°$ ⑧ $-\frac{5}{2}\pi$

Determine each value in radical form:

⑨ $\sin 600°$ ⑪ $\cos \frac{5}{6}\pi$

⑩ $\cos -405°$ ⑫ $\sin -\frac{17}{4}\pi$

Evaluate:

⑬ $$\frac{4(\sin 135°) - (\cos 390°)}{\frac{1}{2}}$$

⑭ $$\frac{2(\cos 150°)(\sin 300°)}{(\cos 180°)}$$

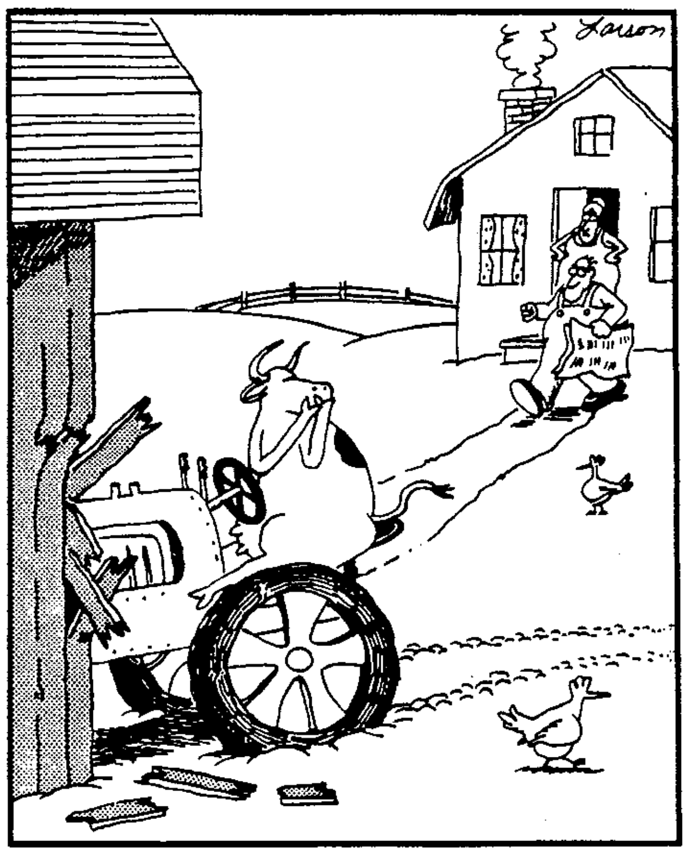
Graph each sine or cosine curve:

⑮ $y = 2\sin\theta$

⑯ $y = -\frac{2}{3}\cos\frac{1}{2}\theta$

⑰ $-2y = 6\sin 2\theta$

⑱ $\frac{1}{2}y = 2\cos\frac{2}{3}\theta$



With a reverberating crash, Lulu's adventure on the tractor had come to an abrupt end.