Friendship Junior High School
Accelerated Math Program
Mr. Lavine (Room 102A)

# A.T.I.M.

## Advanced Topics In Mathematics

UNIT 11
*Plane Geometry*

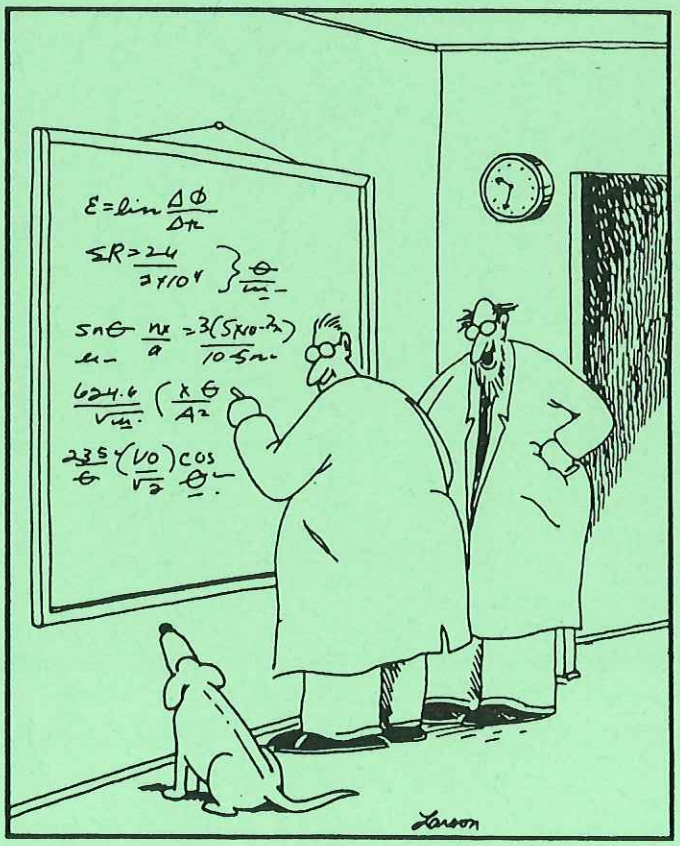
UNIT 12
*Solid Geometry*

UNIT 13
*Introduction to Two-Column Proofs*

UNIT 14
*Triangle Trigonometry*

UNIT 15
*Law of Sines & Cosines*

UNIT 16
*Trigonometric Functions*




"Ohhhhh...Look at that, Schuster...Dogs are so cute when they concentrate on trigonometric functions."

# Polygons, Circles, & Special Triangles

## ANSWER KEY 11.1

① parallelogram

$A = (4)(1.5) = \boxed{6\ m^2}$

$P = (4)+(4)+(2)+(2) = \boxed{12\ m}$

② triangle

$A = \frac{1}{2}(3.5)(3.5) = \boxed{6.125\ m^2}$

$P = (4)+(3.5)+(6) = \boxed{13.5\ m}$

③ rectangle

$A = (5.5)(1.5) = \boxed{8.25\ cm^2}$

$P = (5.5)+(5.5)+(1.5)+(1.5) = \boxed{14\ cm}$

④ triangle

$A = \frac{1}{2}(8.25)(4) = \boxed{16.5\ cm^2}$

$P = (4.6)+(8.1)+(8.25) = \boxed{20.95\ cm}$

⑤ trapezoid

$A = \frac{1}{2}(3+7)(2.2) = \boxed{11\ mm^2}$

$P = (2.5)+(3)+(3.4)+(7) = \boxed{15.9\ mm}$

⑥ trapezoid

$A = \frac{1}{2}(8.25+11)(6) = \boxed{57.75\ m^2}$

$P = (6)+(11)+(9)+(8.25) = \boxed{34.25\ m}$

⑦ rectangle − rectangle

$A = (8)(15) - (4)(5) = \boxed{100\ cm^2}$

$P = (8)+(15)+(8)+(3)+(4)+(5)+(4)+(7) = \boxed{54\ cm}$

⑧ trapezoid − triangle

$A = \frac{1}{2}(19+12)(9) - \frac{1}{2}(3)(4) = \boxed{133.5\ m^2}$

$P = (8)+(9)+(12)+(11)+(7)+(3)+(5) = \boxed{55\ m}$

⑨ rhombus

$A = (6.1)(6.25) = \boxed{38.125\ cm^2}$

$P = 4(6.25) = \boxed{25\ cm}$

⑩ trapezoid − square

$A = \frac{1}{2}(10.5+7)(15.75) - (4)(4) = \boxed{121.8125\ m^2}$

$P = (10.5)+(16)+(7)+(6)+(4)+(4)+(4)+(6.25) = \boxed{57.75\ m}$

⑪ circle

$A = \pi r^2$ $\quad \pi(4.7)^2 = \boxed{22.09\pi\ cm^2}$ $\quad (3.14)(4.7)^2 = \boxed{69.3626\ cm^2}$

$C = 2\pi r$ $\quad 2\pi(4.7) = \boxed{9.4\pi\ cm}$ $\quad 2(3.14)(4.7) = \boxed{29.516\ cm}$

⑫ circle

$A = \pi r^2$ $\quad \pi(5.5)^2 = \boxed{30.25\pi\ m^2}$ $\quad (3.14)(5.5)^2 = \boxed{94.985\ m^2}$

$C = 2\pi r$ $\quad 2\pi(5.5) = \boxed{11\pi\ m}$ $\quad 2(3.14)(5.5) = \boxed{34.54\ m}$

⑬ Pythagorean theorem

$a^2+b^2=c^2$ $\quad c^2 = 208$

$8^2+12^2=c^2$ $\quad c = \sqrt{208} = 4\sqrt{13}\ m^2$

# Polygons, Circles, & Special Triangles

## ANSWER KEY 11.1

(13) continued

$A = \frac{1}{2}(8)(12) = \boxed{48\ m^2}$

$P = (8)+(12)+(4\sqrt{13}) = \boxed{20+4\sqrt{13}\ m}$

(14) Pythagorean Theorem

$a^2+b^2=c^2$ $\quad b^2 = 168$

$(6\sqrt{2})^2 + b^2 = (4\sqrt{15})^2$ $\quad b = 2\sqrt{42}$

$A = \frac{1}{2}(6\sqrt{2})(2\sqrt{42}) = \boxed{12\sqrt{21}\ mm^2}$

$P = \boxed{(6\sqrt{2})+(2\sqrt{42})+(4\sqrt{15})\ mm}$

(15) 30-60-90 Triangle

$a = 3$ short leg

$b = 3\sqrt{3}\ m$ long leg $(a\sqrt{3})$

$c = 6\ m$ hypotenuse $(2a)$

$A = \frac{1}{2}(3)(3\sqrt{3}) = \boxed{9/2\sqrt{3}\ m^2}$

$P = (3)+(3\sqrt{3})+(6) = \boxed{9+3\sqrt{3}\ m}$

(16) 30-60-90 Triangle

$a = 3\sqrt{3}$ short leg $(b\sqrt{3}/3)$

$b = 9$ long leg

$c = 6\sqrt{3}$ hypotenuse $(2a)$

$A = \frac{1}{2}(3\sqrt{3})(9) = \boxed{27/2\sqrt{3}\ cm^2}$

$P = (3\sqrt{3})+(9)+(6\sqrt{3}) = \boxed{9+9\sqrt{3}\ cm}$

(17) 45-45-90 Triangle

$a = 6\sqrt{6}$ leg

$b = 6\sqrt{6}$ leg

$c = 12\sqrt{3}$ hypotenuse $(a\sqrt{2})$

$A = \frac{1}{2}(6\sqrt{6})(6\sqrt{6}) = \boxed{108\ mm^2}$

$P = (6\sqrt{6})+(6\sqrt{6})+(12\sqrt{3}) = \boxed{12\sqrt{6}+12\sqrt{3}\ mm}$

(18) 45-45-90 Triangle

$a = 4\sqrt{10}$ leg

$b = 4\sqrt{10}$ leg

$c = 8\sqrt{5}$ hypotenuse $(a\sqrt{2})$

$A = \frac{1}{2}(4\sqrt{10})(4\sqrt{10}) = \boxed{80\ cm^2}$

$P = (4\sqrt{10})+(4\sqrt{10})+(8\sqrt{5}) = \boxed{8\sqrt{10}+8\sqrt{5}\ cm}$

(19) 30-60-90 Triangle

$a = 4\sqrt{6}$ short leg $(1/2\,c)$

$b = 12\sqrt{2}$ long leg $(a\sqrt{3})$

$c = 8\sqrt{6}$ hypotenuse

$A = \frac{1}{2}(4\sqrt{6})(12\sqrt{2}) = \boxed{48\sqrt{3}\ m^2}$

$P = (4\sqrt{6})+(12\sqrt{2})+(8\sqrt{6}) = \boxed{12\sqrt{6}+12\sqrt{2}\ m}$

(20) 45-45-90 Triangle

$a = 5\sqrt{2}$ leg $(c\sqrt{2}/2)$

$b = 5\sqrt{2}$ leg $(a=b)$

$c = 10$ hypotenuse

$A = \frac{1}{2}(5\sqrt{2})(5\sqrt{2}) = \boxed{25\ cm^2}$

$P = (5\sqrt{2})+(5\sqrt{2})+(10) = \boxed{10\sqrt{2}+10\ cm}$

# *Applications*

## ANSWER KEY 11.2

① $A = \frac{1}{2}(\text{diagonal})(\text{diagonal})$

$\frac{1}{2}(18)(10) = \boxed{90\ cm^2}$

② $A = \frac{1}{2}(\text{diagonal})(\text{diagonal})$

$\frac{1}{2}(12)(12) = \boxed{72\ cm^2}$

③ $A = \frac{1}{2}(\text{diagonal})(\text{diagonal})$

$\frac{1}{2}(6)(20) = \boxed{60cm^2}$

④ $A = (\text{median})(\text{height})$

$(12)(6) = \boxed{72\ cm^2}$

⑤ $A = \frac{a^2\sqrt{3}}{4}$ $\boxed{\frac{25\sqrt{3}}{4}\ m^2}$

⑥ $A = \frac{1}{2}(\text{apothem})(\text{perimeter})$

$\frac{1}{2}(4\sqrt{3})(48) = \boxed{96\sqrt{3}\ cm^2}$

⑦ $A = \frac{1}{2}(\text{apothem})(\text{perimeter})$

$\frac{1}{2}(3)(18\sqrt{3}) = \boxed{27\sqrt{3}\ m^2}$

⑧ $A = \frac{1}{2}(\text{apothem})(\text{perimeter})$

$\frac{1}{2}(5+5\sqrt{2})(80) = \boxed{200+200\sqrt{2}\ mm^2}$

⑨ $A = \frac{1}{2}(\text{diagonal})(\text{diagonal})$

$\frac{1}{2}(10)(10) = \boxed{50\ m^2}$

⑩ radius of square = ½ diagonal

$A = \frac{1}{2}(12)(12) = \boxed{72\ m^2}$

⑪ side of square = ¼ perimeter

$A = 3^2 = \boxed{9\ m^2}$

⑫ $A = \frac{1}{2}(\text{apothem})(\text{perimeter})$

$\frac{1}{2}(5)(40) = \boxed{100\ m^2}$

perimeter is 4-times the side, the side of a square is twice the apothem

⑬ Use the Pythagorean Theorem

$2^2 + b^2 = (\sqrt{29})^2$

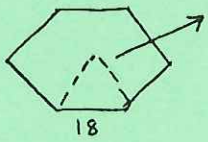
$b^2 = 25$

$b = 5$ (side)

$A = (5)(2) = \boxed{10\ cm^2}$

$P = 2(5) + 2(2) = \boxed{14cm}$

⑭ To determine the area, you can create a triangle:




central angle = 60° (equilateral triangle)

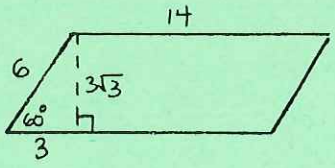
$A = \frac{18^2\sqrt{3}}{4} = 81\sqrt{3}\ m^2$

There are 6 of these triangles in the hexagon:

$A = 6(81\sqrt{3}) = \boxed{486\sqrt{3}\ m^2}$

$P = 6(18) = \boxed{108\ m}$

⑮



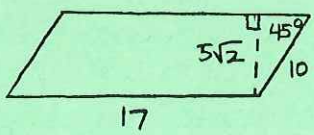

$A = (14)(3\sqrt{3})$

$\boxed{42\sqrt{3}\ cm^2}$

$P = 2(6) + 2(14)$

$\boxed{40cm}$

⑯




$A = (17)(5\sqrt{2})$

$\boxed{85\sqrt{2}\ cm^2}$

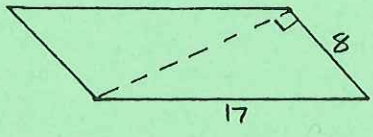
$P = 2(17) + 2(10)$

$\boxed{54\ cm}$

# *Applications*

## ANSWER KEY 11.2

(17) Use the Pythagorean Theorem (Pythagorean triple!)




$8^2+b^2=17^2$

$b=15$

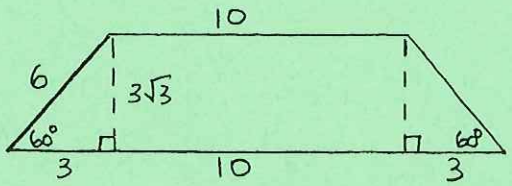
Area = Sum of two triangles

Tri Area = $\frac{1}{2}(8)(15)=60$

$A=\boxed{120\ m^2}$

$P=2(8)+2(17)=\boxed{50\ m}$

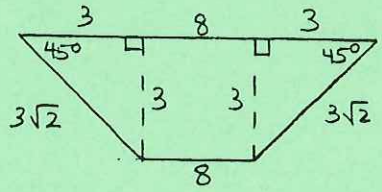
(18)




$A=\frac{1}{2}(10+16)(3\sqrt{3})=\boxed{39\sqrt{3}\ m^2}$

$P=(6)+(10)+(6)+(16)=\boxed{38\ m}$

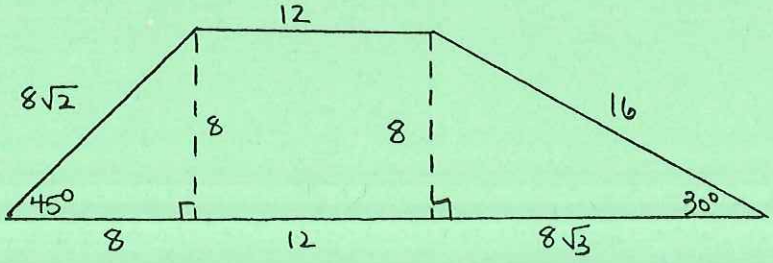
(19)




$A=\frac{1}{2}(8+14)(3)=\boxed{33\ cm^2}$

$P=(3\sqrt{2})+(14)+(3\sqrt{2})+(8)=\boxed{22+6\sqrt{2}\ cm}$

(20) Start on the 30° side

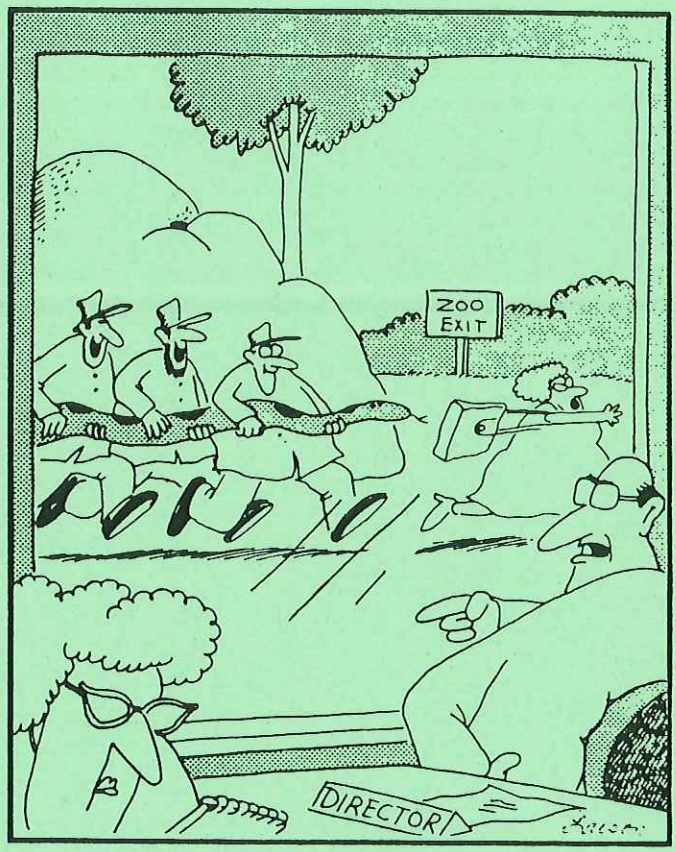



$A=\frac{1}{2}(12+20+8\sqrt{3})(8)$

$(16+4\sqrt{3})(8)=\boxed{128+32\sqrt{3}\ m^2}$

$P=(12)+(16)+(20+8\sqrt{3})+(8\sqrt{2})$

$\boxed{48+8\sqrt{3}+8\sqrt{2}\ m}$




"Take another memo, Miss Wilkens . . . I want to see all reptile personnel in my office first thing tomorrow morning!"

# *Sectors & Segments*

## ANSWER KEY 11.3

① $A = \pi r^2$ (part)

$\pi(12)^2\left(\frac{60}{360}\right) = \boxed{24\pi\ m^2}$

② $A = \pi r^2$ (part)

$\pi(12)^2\left(\frac{30}{360}\right) = \boxed{12\pi\ m^2}$

③ $A$ = sector − triangle

$\pi r^2$(part) $- \frac{1}{2}(8)(8)$

$\pi(8)^2\left(\frac{90}{360}\right) - 32 = \boxed{16\pi - 32\ m^2}$

④ $A$ = sector − triangle

$\pi r^2$(part) $- \frac{\text{side}^2\sqrt{3}}{4}$

$\pi(8)^2\left(\frac{60}{360}\right) - \frac{(8)^2\sqrt{3}}{4}$

$\boxed{10.\overline{6}\,\pi - 16\sqrt{3}\ m^2}$

⑤ $A$ = sector − triangle

$\pi r^2$(part) $- \frac{1}{2}(3)(6\sqrt{3})$ ← use 30-60-90 to determine

$\pi(6)^2\left(\frac{120}{360}\right) - 9\sqrt{3}$

$\boxed{12\pi - 9\sqrt{3}\ m^2}$

⑥ radius of large circle = 6m

$A$ = large circle − 2(small circle)

$\pi r^2 - 2\pi r^2$

$\pi(6)^2 - 2\pi(3)^2 = \boxed{18\pi\ m^2}$

⑦ equilateral tri − $\pi r^2\left(\frac{180}{360}\right)$

$\frac{10^2\sqrt{3}}{4} - \pi 5^2\left(\frac{180}{360}\right)$

$\boxed{25\sqrt{3} - \frac{25}{2}\pi\ m^2}$

⑧ square − circle + equi. triangle

$7^2 - \pi(3.5)^2 + \frac{3^2\sqrt{3}}{4}$

$\boxed{49 - 12.25\pi + \frac{9}{4}\sqrt{3}\ m^2}$

⑨ outer circle − inner circle

$\pi 9^2 - \pi 7^2$

$81\pi - 49\pi = \boxed{32\pi\ m^2}$

⑩ circle − regular hexagon

$\pi 12^2 - 6\left(\frac{12^2\sqrt{3}}{4}\right)$

$\boxed{144\pi - 216\sqrt{3}\ m^2}$

Remember: A regular hexagon consists of 6 equilateral triangles

⑪ square − circle

$10^2 - \pi 5^2 = \boxed{100 - 25\pi\ m^2}$

⑫ semi-circle − 3(small semi-circle)

$\frac{1}{2}\pi r^2 - 3\left(\frac{1}{2}\pi r^2\right)$

$\frac{1}{2}\pi 6^2 - 3\left(\frac{1}{2}\pi 2^2\right) = \boxed{12\pi\ m^2}$

# *Sectors & Segments*

## ANSWER KEY 11.3

(13) circle + rectangle

$\pi 5^2 + (10)(15) = \boxed{150 + 25\pi \text{ m}^2}$

$2\pi r + 15 + 15 = \boxed{30 + 10\pi \text{ m}}$

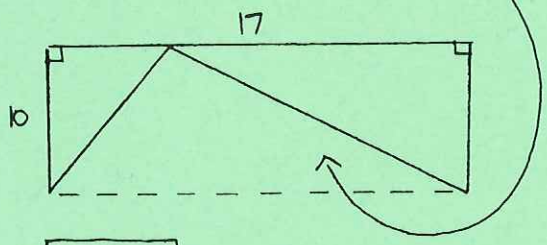
(14) The height of the trapezoid is 4 m (Pythagorean triple)

$A = (\text{median})(\text{height})$

$(14)(4) = \boxed{56 \text{ m}^2}$

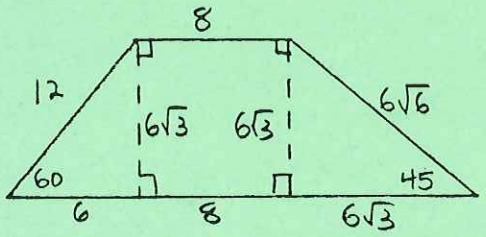
(15) large rectangle − large tri

$(17)(10) - \frac{1}{2}(17)(10)$




$\boxed{85 \text{ m}^2}$

(16)




$A = \frac{1}{2}(8 + 14 + 6\sqrt{3})(6\sqrt{3})$

$(11 + 3\sqrt{3})(6\sqrt{3})$

$\boxed{66\sqrt{3} + 54 \text{ m}^2}$

$P = 12 + 8 + 6\sqrt{6} + 6\sqrt{3} + 8 + 6$

$\boxed{34 + 6\sqrt{6} + 6\sqrt{3} \text{ m}}$

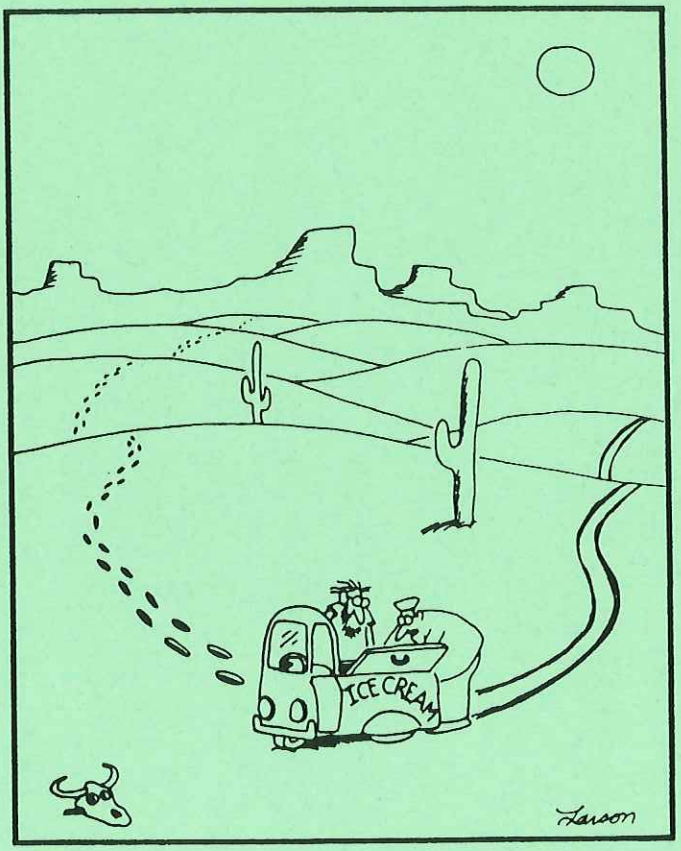
(17) $\frac{\text{side}^2\sqrt{3}}{4} = 16\sqrt{3}$

$\text{side}^2\sqrt{3} = 64\sqrt{3}$

$\text{side}^2 = 64$

$\text{side} = 8$

$\text{perimeter} = \boxed{24 \text{ m}}$




"Let's see . . . No orange . . . no root beer . . . no Fudgsicles . . . Well, for crying out loud! . . . Am I out of everything?"

# *Plane Geometry*

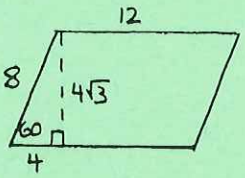
## ANSWER KEY: UNIT 11 REVIEW & PRACTICE

① $A = \frac{1}{2}(7+4)(4) = \boxed{22\ m^2}$

$P = 7+4+4+5 = \boxed{20\ m}$

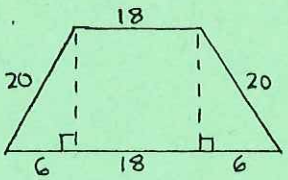
② $A = \frac{s^2\sqrt{3}}{4} = \boxed{16\sqrt{3}\ cm^2}$

$P = \boxed{24\ cm}$

③ 


$A = (12)(4\sqrt{3}) = \boxed{48\sqrt{3}\ m^2}$

$P = 2(8)+2(12) = \boxed{40\ m}$

④ 


Use the Pythagorean Theorem to find the height

$6^2+b^2=20^2$

$b = 2\sqrt{91}$

$A = \frac{1}{2}(18+30)(2\sqrt{91}) = \boxed{48\sqrt{91}\ m^2}$

$P = 20+18+20+30 = \boxed{88\ m}$

⑤ $C = 2\pi r$ $\quad A = \pi r^2$

$16\pi = 2\pi r$ $\quad A = \pi 8^2$

$r = 8$ $\quad A = \boxed{64\pi\ cm^2}$

⑥ $A = \pi r^2(part)$

$\pi 6^2(^{90}/_{360}) = \boxed{9\pi\ m^2}$

$C = 2\pi r(part) + 2r$

$2\pi(6)(^{90}/_{360}) + 2(6)$

$\boxed{3\pi+12\ m}$

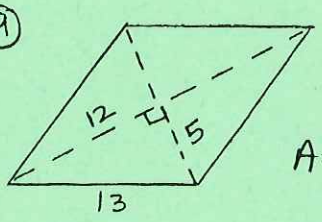
⑦ $A = (large\ circle) - (small\ circle)$

$\pi r^2 - \pi r^2$

$\pi 8^2 - \pi 6^2 = \boxed{28\pi\ m^2}$

⑧ $A = (square) - (circle)$

$10^2 - \pi 5^2 = \boxed{100-25\pi\ m^2}$

⑨ 


5-12-13 is a Pythagorean triple

$A = \frac{1}{2}(24)(10) = \boxed{120\ m^2}$

⑩ $p = 21 \rightarrow side = 7$

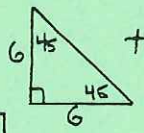
$A = \frac{7^2\sqrt{3}}{4} = \boxed{\frac{49\sqrt{3}}{4}\ cm^2}$

⑪ $A = (sector) - (triangle)$

$\pi r^2(part)$

$\pi 6^2(^{90}/_{360})$

$\boxed{9\pi-18\ cm^2}$

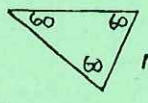



$tri = \frac{1}{2}(6)(6)$

⑫ $A = (sector) - (triangle)$

$\pi r^2(part)$

$\pi 5^2(^{60}/_{360})$




$A = \frac{5^2\sqrt{3}}{4}$

$\boxed{\frac{25}{6}\pi - \frac{25\sqrt{3}}{4}\ cm^2}$

⑬ $a = 2$

$b = 2\sqrt{3}$

$c = 4$

$A = \frac{1}{2}(2)(2\sqrt{3}) = \boxed{2\sqrt{3}\ m^2}$

$P = 2+2\sqrt{3}+4 = \boxed{6+2\sqrt{3}\ m}$

# Plane Geometry

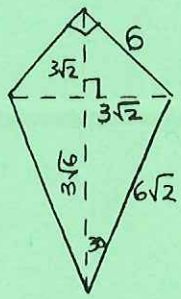
## ANSWER KEY: UNIT 11 REVIEW & PRACTICE

(14) $a = 4\sqrt{2}$, $b = 4\sqrt{2}$, $c = 8$

$A = \frac{1}{2}(4\sqrt{2})^2 = \boxed{16\ m^2}$

$P = 4\sqrt{2} + 4\sqrt{2} + 8 = \boxed{8\sqrt{2} + 8\ m}$

(15)




$A = \frac{1}{2}$ (diagonal)(diagonal)

$\frac{1}{2}(6\sqrt{2})(3\sqrt{6} + 3\sqrt{2})$

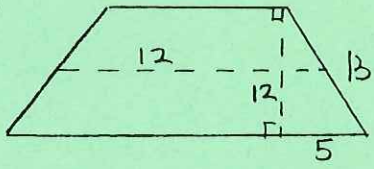
$\frac{1}{2}(18\sqrt{12} + 18\sqrt{4})$

$\boxed{18\sqrt{3} + 18\ m^2}$

$P = 2(6) + 2(6\sqrt{2})$

$\boxed{12 + 12\sqrt{2}\ m}$

(16)




5-12-13 Pyth. triple

$A = (\text{median})(\text{ht}) = (12)(12) = \boxed{144\ m^2}$

(17) Use Pythagorean Theorem to determine the diagonal:

$a^2 + b^2 = 16^2$

$b^2 = 175$

$b = 5\sqrt{7}$

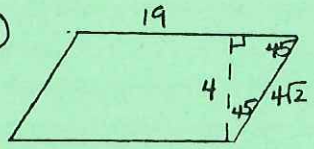
The area of the parallelogram is twice the area of the right triangle:

$\text{Tri} = \frac{1}{2}(9)(5\sqrt{7}) = \frac{45}{2}\sqrt{7}$

$A = 2(\frac{45}{2}\sqrt{7}) = \boxed{45\sqrt{7}\ m^2}$

$P = 2(9) + 2(16) = \boxed{50m}$

(18)

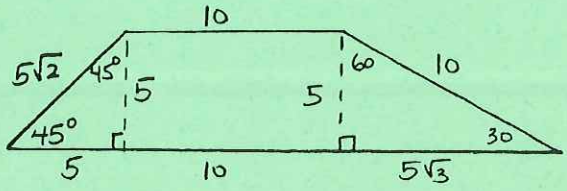



$A = (19)(4) = \boxed{76\ m^2}$

$P = 2(19) + 2(4\sqrt{2})$

$\boxed{38 + 8\sqrt{2}\ m}$

(19)




$A = \frac{1}{2}(10 + 15 + 5\sqrt{3})(5) = \boxed{\frac{125}{2} + \frac{25\sqrt{3}}{2}\ m^2}$

$P = 5\sqrt{2} + 10 + 10 + 5\sqrt{3} + 10 + 5$

$\boxed{35 + 5\sqrt{2} + 5\sqrt{3}\ m}$

(20) $A = \frac{1}{2}$ (perimeter)(apothem)

$\frac{1}{2}(96)(6\sqrt{2} + 6)$

$\boxed{288\sqrt{2} + 288\ m^2}$

$P = \boxed{96\ m}$

(21) $A = \pi r^2$

$\pi(7)^2 = \boxed{49\pi\ m^2}$

$C = 2\pi r$

$2\pi(7) = \boxed{14\pi\ m}$

(22) $4^2 + 6^2 = c^2$ $\quad c = 2\sqrt{13}$

$A = \frac{1}{2}(4)(10) = \boxed{20\ m^2}$

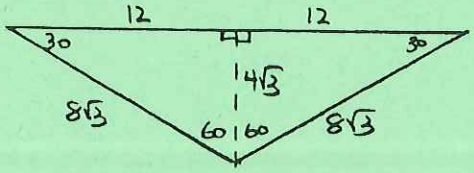
$P = \boxed{2\sqrt{13} + 4\sqrt{17} + 10\ m}$

# *Plane Geometry*

## ANSWER KEY: UNIT 11 REVIEW & PRACTICE

(23) A = (sector) − (triangle)

Determine radius:

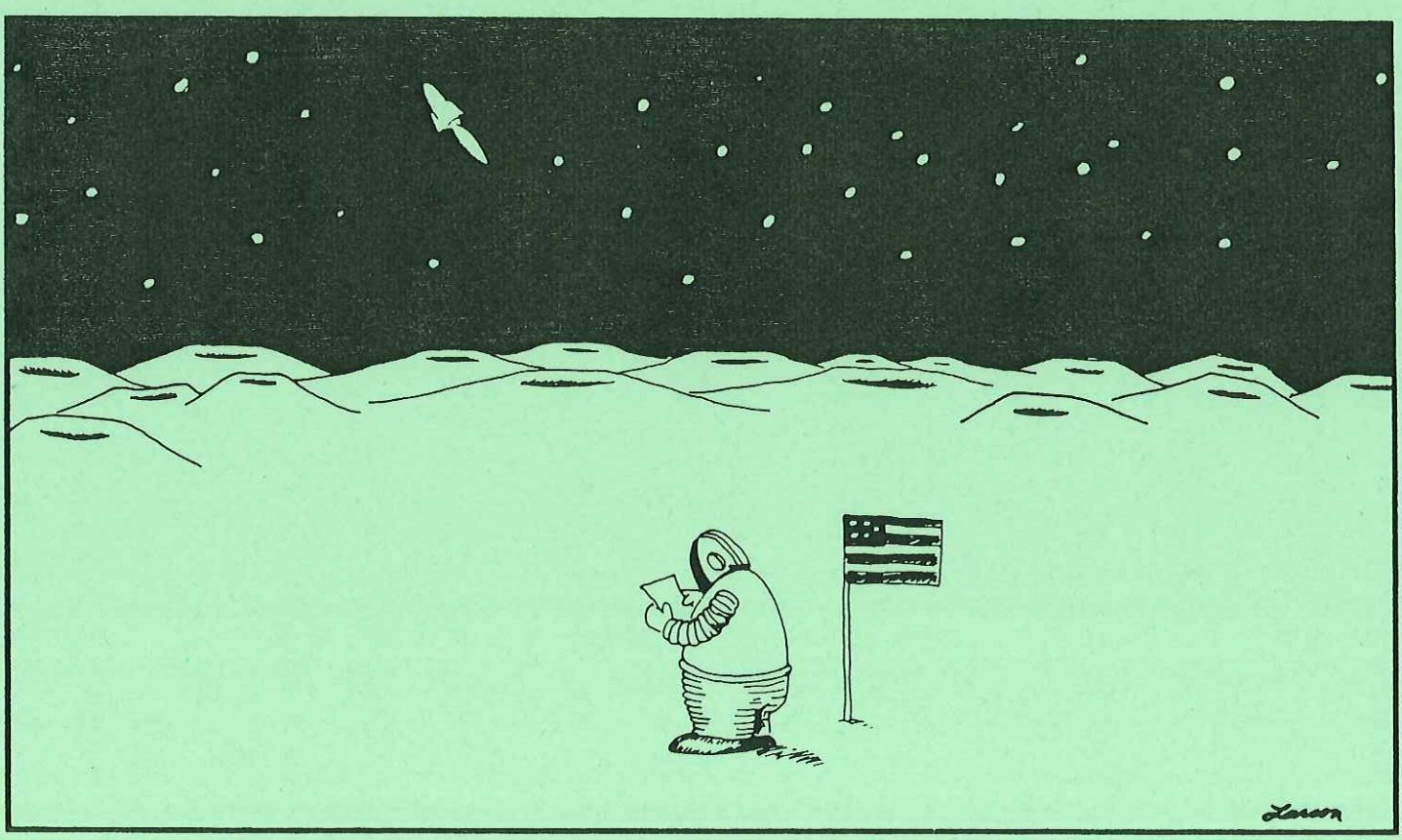



A = (sector) − (triangle)

$\pi r^2$(part) $-\frac{1}{2}$(base)(ht)

$\pi(8\sqrt{3})^2(120/360) - \frac{1}{2}(24)(4\sqrt{3})$

$\boxed{64\pi - 48\sqrt{3}\ m^2}$




"Dear Henry: Where were you? We waited and waited but finally decided that . . ."

# *Prisms, Cylinders, & Spheres*

## ANSWER KEY 12.1

① $V = (\text{base area})(\text{ht})$
$(6)(6) \cdot (20) = \boxed{720\ m^3}$

$SA = 2(\text{base area}) + (\text{per})(\text{ht})$
$(2)(6)(6) + (24)(20) = \boxed{552\ m^2}$

② $V = (\text{base area})(\text{ht})$
$(8)(7) \cdot (6) = \boxed{336\ m^3}$

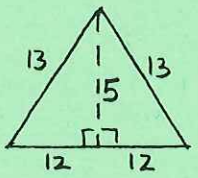
$SA = 1(\text{base area}) + (\text{per})(\text{ht})$
$(1)(8)(7) + (30)(6) = \boxed{236\ m^2}$

③ $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(3)^3 = \boxed{36\pi\ m^3}$

$SA = 4\pi r^2 = 4\pi(3)^2 = \boxed{36\pi m^2}$

④ $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(5)^3 = \boxed{166.\overline{6}\pi\ m^3}$

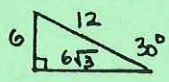
$SA = 4\pi r^2 = 4\pi(5)^2 = \boxed{100\pi\ m^2}$

⑤ 

Pyth. triple 5-12-13
base area $= \frac{1}{2}(5)(24)$

$V = (\text{base area})(\text{ht})$
$\frac{1}{2}(5)(24)(10) = \boxed{600\ m^3}$

$SA = 2(\text{base area}) + (\text{per})(\text{ht})$
$(2)(\frac{1}{2})(5)(24) + (50)(10) = \boxed{620\ m^2}$

⑥ 

$V = (\text{base area})(\text{ht})$
$(\frac{1}{2})(6)(6\sqrt{3})(8) = \boxed{144\sqrt{3}\ m^3}$

$SA = 2(\text{base area}) + (\text{per})(\text{ht})$
$2(\frac{1}{2})(6)(6\sqrt{3}) + (18+6\sqrt{3})(8) = \boxed{84\sqrt{3} + 144 m^2}$

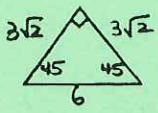
⑦ $V = (\text{base area})(\text{ht})$
$(\pi r^2)(\text{ht}) = \pi 4^2(5) = \boxed{80\pi\ m^3}$

$SA = 2(\text{base area}) + (\text{cir.})(\text{ht})$
$2(\pi r^2) + (2\pi r)(\text{ht})$
$2\pi 4^2 + (2\pi 4)(5) = \boxed{72\pi\ m^2}$

⑧ $V = (\text{base area})(\text{ht})$
$(\pi r^2)(\text{ht}) = \pi(7)^2(10) = \boxed{490\pi\ m^3}$

$SA = 2(\text{base area}) + (\text{cir})(\text{ht})$
$2(\pi r^2) + (2\pi r)(\text{ht})$
$2(\pi 7^2) + (2\pi 7)(10) = \boxed{238\pi\ m^2}$

⑨ $SA = 4\pi r^2 = 144\pi$
$r^2 = 36 \rightarrow r = 6$
$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(6)^3 = \boxed{288\pi\ m^3}$

⑩ 

triangle area $= \frac{1}{2}(3\sqrt{2})^2 = 9$
$V = (\text{base area})(\text{ht})$
$(9)(10) = \boxed{90\ m^3}$

$SA = 2(\text{base area}) + (\text{per})(\text{ht})$
$(2)(9) + (6+6\sqrt{2})(10) = \boxed{78+60\sqrt{2}\ m^2}$

⑪ $V = (\frac{1}{2})(\frac{4}{3}\pi r^3)$
$(\frac{1}{2})(\frac{4}{3})\pi(6)^3 = \boxed{144\pi\ m^3}$

$SA = (\frac{1}{2})(4\pi r^2) + \pi r^2$
$(\frac{1}{2})(4)\pi(6)^2 + \pi(6)^2 = \boxed{108\pi m^2}$

continued

# Prisms, Cylinders, & Spheres

## ANSWER KEY 12.1

(12) $V = (\text{base area})(\text{ht})$

$(\frac{1}{2})(\pi)(5)^2(2) = \boxed{25\pi\ m^3}$

$SA = (2)(\text{base area}) + (\text{cir})(\text{ht})$

$(2)(\frac{1}{2})(\pi r^2) + (\frac{1}{2}2\pi r + 2r)(\text{ht})$

$(2)(\frac{1}{2})(\pi)(5)^2 + [(\frac{1}{2})(2)\pi(5) + 2(5)](2)$

$25\pi + [5\pi + 10](2) = \boxed{35\pi + 20\ m^2}$

(13) Reg. hexagon = 6 equilateral triangles


$\text{area} = \frac{6^2\sqrt{3}}{4} = (9\sqrt{3})(6) = 54\sqrt{3}$ (6 triangles)

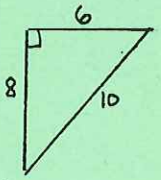
$V = (\text{base area})(\text{ht})$

$(54\sqrt{3})(10) = \boxed{540\sqrt{3}\ m^3}$

$SA = 2(\text{base area}) + (\text{per})(\text{ht})$

$2(54\sqrt{3}) + (36)(10) = \boxed{108\sqrt{3} + 360\ m^2}$

(14) To determine radius:




Pyth. triple 3-4-5 (6-8-10)

diameter = 6 radius = 3

$V = (\text{base area})(\text{ht})$

$(\pi r^2)(\text{ht}) = \pi 3^2(8) = \boxed{72\pi\ m^3}$

$SA = 2(\text{base area}) + (\text{cir})(\text{ht})$

$2(9\pi) + (6\pi)(8) = \boxed{66\pi\ m^2}$

(15) 


Pyth. triple 8-15-17

base area $= \frac{1}{2}(16)(15) = 120$

$V = (\text{base area})(\text{ht})$

$(120)(11) = \boxed{1320\ m^3}$

$SA = 2(\text{base area}) + (\text{per})(\text{ht})$

$2(120) + (50)(11) = \boxed{790\ m^2}$

(16) $V = (\text{base area})(\text{ht})$

$(\frac{1}{2})\pi r^2(\text{ht}) = \frac{1}{2}\pi 6^2(20) = \boxed{360\pi\ m^3}$

$SA = 2(\text{base area}) + (\text{cir})(\text{ht})$

$(2)(\frac{1}{2})\pi r^2 + [(\frac{1}{2})(2\pi r) + 2r](20)$

$(2)(\frac{1}{2})\pi 6^2 + [(\frac{1}{2})(2)(\pi)(6) + 2(6)](20)$

$36\pi + [6\pi + 12](20) = \boxed{156\pi + 240\ m^2}$

(17) $V = (\text{cylinder}) + (\text{half sphere})$

$(\text{base area})(\text{ht}) + (\frac{1}{2})(\frac{4}{3}\pi r^3)$

$(9\pi)(15) + (\frac{1}{2})(\frac{4}{3}\pi 3^3)$

$135\pi + 18\pi = \boxed{153\pi\ m^3}$

$SA = (\text{cylinder} - \text{one base}) + (\text{half sphere} - \text{no base})$

$1(\text{base area}) + (\text{cir})(\text{ht}) + (\frac{1}{2})(4\pi r^2)$

$(1)(9\pi) + (6\pi)(15) + (\frac{1}{2})4\pi 3^2$

$9\pi + 90\pi + 18\pi = \boxed{117\pi\ m^2}$

(18) $V = (\text{base area})(\text{ht})$

$(\pi r^2)(\text{part})(\text{ht})$

$\pi 18^2(\frac{40}{360})(15) = \boxed{540\pi\ m^3}$

$SA = 2(\text{base area}) + (\text{cir})(\text{ht})$

$2(\pi r^2)(\text{part}) + [(2\pi r)(\text{part}) + 2r](\text{ht})$

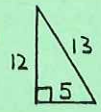
$2(\pi)18^2(\frac{40}{360}) + [2\pi(18)(\frac{40}{360}) + 2(18)](15)$

$72\pi + 60\pi + 540$

$\boxed{132\pi + 540\ m^2}$

# Pyramids & Cones

## ANSWER KEY 12.2

① 

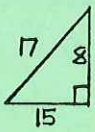

radius = 5

$V = \frac{1}{3}(\text{base})(\text{ht})$

$\frac{1}{3}(\pi)(5)^2(12) = \boxed{100\pi\ m^3}$

$SA = (\text{base}) + \frac{1}{2}(\text{cir})(\text{sl ht})$

$(\pi)(5)^2 + \frac{1}{2}(2)(\pi)(5)(13) = \boxed{90\pi\ m^2}$

② 


radius = 15 slant ht = 17

$V = \frac{1}{3}(\text{base})(\text{ht})$

$\frac{1}{3}(\pi)(15)^2(8) = \boxed{600\pi\ m^3}$

$SA = (\text{base}) + \frac{1}{2}(\text{cir})(\text{sl ht})$

$(\pi)(15)^2 + \frac{1}{2}(2)(\pi)(15)(17) = \boxed{480\pi\ m^2}$

③ 


sl ht = 15

ht = 14 (given)

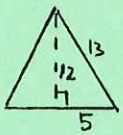
base area $\frac{16^2\sqrt{3}}{4} = 64\sqrt{3}$

$V = \frac{1}{3}(\text{base})(\text{ht})$

$\frac{1}{3}(64\sqrt{3})(14) = \boxed{896/3\sqrt{3}\ m^3}$

$SA = (\text{base}) + \frac{1}{2}(\text{per})(\text{sl ht})$

$(64\sqrt{3}) + \frac{1}{2}(48)(15) = \boxed{64\sqrt{3} + 360\ m^2}$

④ 


ht = $\sqrt{119}$ base area = $10^2 = 100$

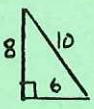
sl ht = 12

$V = \frac{1}{3}(\text{base})(\text{ht})$

$\frac{1}{3}(100)(\sqrt{119}) = \boxed{100/3\sqrt{119}\ m^3}$

$SA = (\text{base}) + \frac{1}{2}(\text{per})(\text{sl ht})$

$(100) + \frac{1}{2}(40)(12) = \boxed{340\ m^2}$

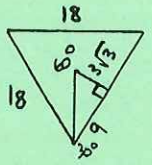
⑤ 


$V = \frac{1}{3}(\text{base})(\text{ht})$

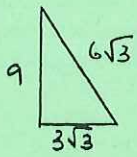
$\frac{1}{3}(\pi)(6)^2(8) = \boxed{96\pi\ m^3}$

$SA = (\text{base}) + (1/2)(\text{cir})(\text{sl ht})$

$(\pi)(6)^2 + (1/2)(2)(\pi)(6)(10) = \boxed{96\pi\ m^2}$

⑥ 


determine apothem ($3\sqrt{3}$) in order to calculate slant height:




sl ht $6\sqrt{3}$

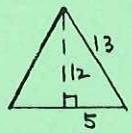
$V = \frac{1}{3}(\text{base})(\text{ht})$

$\frac{1}{3}\left(\frac{18^2\sqrt{3}}{4}\right)(9) = \boxed{243\sqrt{3}\ m^3}$

$SA = (\text{base}) + \frac{1}{2}(\text{per})(\text{sl ht})$

$\left(\frac{18^2\sqrt{3}}{4}\right) + \frac{1}{2}(54)(6\sqrt{3})$

$81\sqrt{3} + 162\sqrt{3} = \boxed{243\sqrt{3}\ m^2}$

⑦ 


ht = 12

sl ht = 13

base area $\frac{1}{2}(\pi)(5)^2 = \frac{25}{2}\pi$

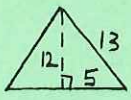
$V = \frac{1}{3}(\text{base})(\text{ht})$

$\frac{1}{3}(25/2\,\pi)(12) = \boxed{50\pi\ m^3}$

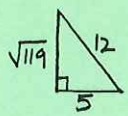
$SA = (\text{base}) + (\text{arc})(\text{sl ht}) + (\text{triangle})$

$(25/2\,\pi) + \frac{1}{2}(2)(\pi)(5)(13) + \frac{1}{2}(10)(12)$

$\boxed{45\pi + 60\ m^2}$

⑧ pyramid slant ht 


sl ht = 12

pyramid height 


ht = $\sqrt{119}$

$V = (\text{cube}) + (\text{pyramid})$

$(10)(10)(10) + \frac{1}{3}(10)(10)(\sqrt{119})$

$\boxed{1000 + 100/3\sqrt{119}\ m^3}$

$SA = (\text{cube - one base}) + (\text{pyramid - no base})$

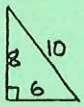
$1(\text{base}) + (\text{per})(\text{ht}) + \frac{1}{2}(\text{per})(\text{sl ht})$

$(10)(10) + (40)(10) + \frac{1}{2}(40)(12) = \boxed{740\ m^2}$

# Pyramids & Cones

## ANSWER KEY 12.2

(9) cone, sl ht




sl ht = 10

V = (cone) + (half sphere)

$\frac{1}{3}$(base)(ht) + $\frac{1}{2}(\frac{4}{3}\pi r^3)$

$\frac{1}{3}(\pi)(6)^2(8) + \frac{1}{2}(\frac{4}{3})(\pi)(6)^3$

$96\pi + 144\pi = \boxed{240\pi\ m^3}$

SA = (cone - no base) + (half sphere - no base)

$\frac{1}{2}$(cir)(sl ht) + $\frac{1}{2}(4\pi r^2)$

$\frac{1}{2}(2)(\pi)(6)(10) + \frac{1}{2}(4)(\pi)(6)^2$

$60\pi + 72\pi = \boxed{132\pi\ m^2}$

(10) cone, height


$2^2 + b^2 = 5^2$

$b = \sqrt{21}$ ht = $\sqrt{21}$

V = (cone) + (cylinder)

$\frac{1}{3}$(base)(ht) + (base)(ht)

$\frac{1}{3}(\pi)(2)^2(\sqrt{21}) + (\pi)(2)^2(5)$

$\boxed{\frac{4}{3}\pi\sqrt{21} + 20\pi\ m^3}$

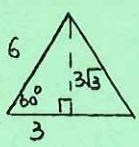
SA = (cone - no base) + (cylinder - 1 base)

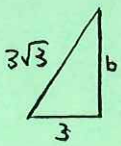
$\frac{1}{2}$(cir)(sl ht) + 1(base) + (cir)(ht)

$\frac{1}{2}(2)(\pi)(2)(5) + (\pi)(2)^2 + (2)(\pi)(2)(5)$

$10\pi + 4\pi + 20\pi = \boxed{34\pi\ m^2}$

(11) To determine pyramid height and slant height:




$3^2 + b^2 = (3\sqrt{3})^2$

$b^2 = 18$

$b = 3\sqrt{2}$

sl ht = $3\sqrt{3}$ ht = $3\sqrt{2}$ continued

V = 2 (pyramid volume)

2 $(\frac{1}{3})$(base)(ht)

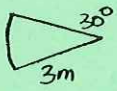
$2(\frac{1}{3})(6)(6)(3\sqrt{2}) = \boxed{72\sqrt{2}\ m^3}$

SA = 2 (pyramid sur area - no base)

2 $(\frac{1}{2})$(per)(sl ht)

$2(\frac{1}{2})(24)(3\sqrt{3}) = \boxed{72\sqrt{3}\ m^2}$

(12) sector




$A = \pi r^2$(part)

$\pi(3)^2(30/360) = .75\pi\ m^2$

$C = 2\pi r$(part) + 2r

$2\pi(3)(30/360) + 2(3) = .5\pi + 6$ m

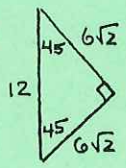
V = (base area)(ht)

$(.75\pi)(2) = \boxed{1.5\pi\ m^3}$

SA = 1(base area) + (cir)(ht)

$1(.75\pi) + (.5\pi + 6)(2) = \boxed{1.75\pi + 12\ m^2}$

(13)




base area = $\frac{1}{2}(6\sqrt{2})^2 = 36\ m^2$

perimeter = $12 + 12\sqrt{2}$ m

V = (base area)(ht)

$(36)(8) = \boxed{288\ m^3}$

SA = 2(base area) + (perimeter)(ht)

$2(36) + (12 + 12\sqrt{2})(8)$

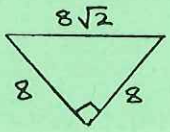
$72 + 96 + 96\sqrt{2} = \boxed{168 + 96\sqrt{2}\ m^2}$

# *Solid Geometry*

## ANSWER KEY: UNIT 12 REVIEW & PRACTICE

① $V = (4/3)(\pi)(9)^3 =$ $\boxed{972\pi\ m^3}$

$SA = (4)(\pi)(9)^2 =$ $\boxed{324\pi\ m^2}$

② 


V = (base area)(ht)

$(\frac{1}{2})(8)(8)(12) =$ $\boxed{384 m^3}$

SA = 2(base area) + (per)(ht)

$(2)(\frac{1}{2})(8)(8) + (16+8\sqrt{2})(12)$

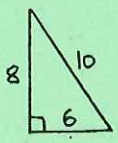
$64 + 192 + 96\sqrt{2} =$ $\boxed{256 + 96\sqrt{2}\ m^2}$

③ V = (base area)(ht)

$(\pi)(4)^2(9) =$ $\boxed{144\pi\ m^3}$

SA = 2(base area) + (cir)(ht)

$(2)(\pi)(4)^2 + (2)(\pi)(4)(9) =$ $\boxed{104\pi\ m^2}$

④ 


Pyth. triple 6-8-10 (3-4-5)

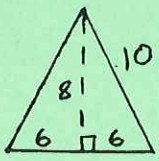
V = 1/3 (base area)(ht)

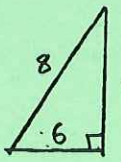
$1/3\ (\pi)(6)^2(8) =$ $\boxed{96\pi\ m^3}$

SA = (base area) + $\frac{1}{2}$(cir)(sl ht)

$(\pi)(6)^2 + (\frac{1}{2})(2)(\pi)(6)(10)$

$36\pi + 60\pi =$ $\boxed{96\pi\ m^2}$

⑤ 


$6^2 + b^2 = 8^2$

$b^2 = 28$

$b = 2\sqrt{7}$

slant height is 8 (10 is an edge)

perp ht = $2\sqrt{7}$

V = $\frac{1}{3}$(base area)(ht)

$(1/3)(12)(12)(2\sqrt{7}) =$ $\boxed{96\sqrt{7}\ m^3}$

SA = (base area) + $\frac{1}{2}$(per)(sl ht)

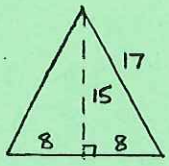
$(12)(12) + (\frac{1}{2})(48)(8) =$ $\boxed{336 m^2}$

⑥ V = (base area)(ht)

$(\frac{6^2\sqrt{3}}{4})(12) =$ $\boxed{108\sqrt{3}\ m^3}$

SA = 2(base area) + (per)(ht)

$2(\frac{6^2\sqrt{3}}{4}) + (18)(12) =$ $\boxed{216 + 18\sqrt{3}\ m^2}$

⑦ 


slant ht = 17

perp ht = 15

radius = 8

15 is both the height of the triangle and the ht of the pyramid.

V = 1/3 (base area)(ht)

$1/3\ (\frac{1}{2})(\pi)(8)^2(15) =$ $\boxed{160\pi\ m^3}$

SA = (base area) + $\frac{1}{2}$(cir)(sl ht) + (triangle)

$\frac{1}{2}(\pi)(8)^2 + \frac{1}{2}[(\frac{1}{2})(2)(\pi)(8)](17) + \frac{1}{2}(16)(15)$

$32\pi + 56\pi + 120 =$ $\boxed{88\pi + 120\ m^2}$

⑧ V = (base area)(ht)

$(\pi)(8)^2(60/360)(5) =$ $\boxed{53.\overline{3}\pi\ m^3}$

continued

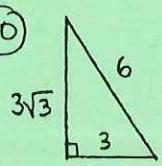
# *Solid Geometry*

## ANSWER KEY: UNIT 12 REVIEW & PRACTICE

(9) $V = (\text{base area})(\text{ht})$

$(\pi)(6)^2(270/360)(20) = \boxed{540\pi\ m^3}$

(10)




cone sl ht $= 6$

cone ht $= 3\sqrt{3}$

radius $= 3$

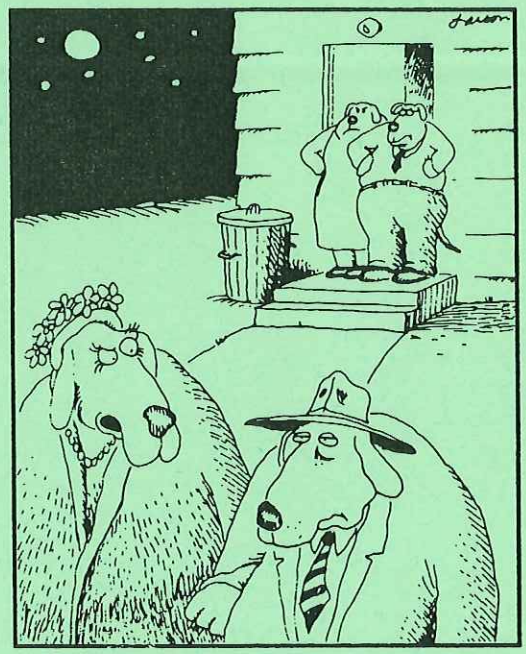
$V = (\text{cone}) + (\text{cylinder})$

$\frac{1}{3}(\text{base})(\text{ht}) + (\text{base})(\text{ht})$

$\frac{1}{3}(\pi)(3)^2(3\sqrt{3}) + (\pi)(3)^2(9)$

$9\pi\sqrt{3} + 81\pi$

$\boxed{9\pi\sqrt{3} + 81\pi\ m^3}$



"One of the nicest evenings I've ever spent at the Wilson's . . . and then you had to go and do that on the rug!"

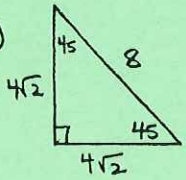
$SA = (\text{cone: no base}) + (\text{cylinder: one base})$

$\frac{1}{2}(\text{cir})(\text{sl ht}) + 1(\text{base}) + (\text{cir})(\text{ht})$

$\frac{1}{2}(2)(\pi)(3)(6) + (\pi)(3)^2 + (2)(\pi)(3)(9)$

$18\pi + 9\pi + 54\pi = \boxed{81\pi\ m^2}$

(11)




cone sl ht $= 8$

cone ht $= 4\sqrt{2}$

radius $= 4\sqrt{2}$

$V = (\text{cone}) + (\text{half sphere})$

$\frac{1}{3}(\text{base})(\text{ht}) + (\frac{1}{2})(4/3\ \pi r^3)$

$\frac{1}{3}(\pi)(4\sqrt{2})^2(4\sqrt{2}) + (\frac{1}{2})(4/3)(\pi)(4\sqrt{2})^3$

$42.\overline{6}\pi\sqrt{2} + 85.\overline{3}\pi\sqrt{2} = \boxed{128\pi\sqrt{2}\ m^3}$

$SA = (\text{cone: no base}) + (\text{1/2 sphere: no base})$

$\frac{1}{2}(\text{cir})(\text{sl ht}) + \frac{1}{2}(4\pi r^2)$

$\frac{1}{2}(2)(\pi)(4\sqrt{2})(8) + \frac{1}{2}(4)(\pi)(4\sqrt{2})^2$

$32\sqrt{2}\pi + 64\pi = \boxed{32\sqrt{2}\pi + 64\pi\ m^2}$

# *Congruent Triangles*

## ANSWER KEY 13.1

| | | | |
|---|---|---|---|
| ① SAS | ⑤ ASA | ⑨ SSS | ⑬ ASA |
| ② SSS or SAS | ⑥ SSS | ⑩ SSS | ⑭ No |
| ③ ASA | ⑦ SAS | ⑪ No | ⑮ No |
| ④ No | ⑧ No | ⑫ No | |

⑯ All right angles are equal

⑰ If two angles form a linear pair, then they are supplementary

⑱ If two equal angles are a linear pair, then they are right angles

⑲ Perpendicular lines form congruent adjacent angles

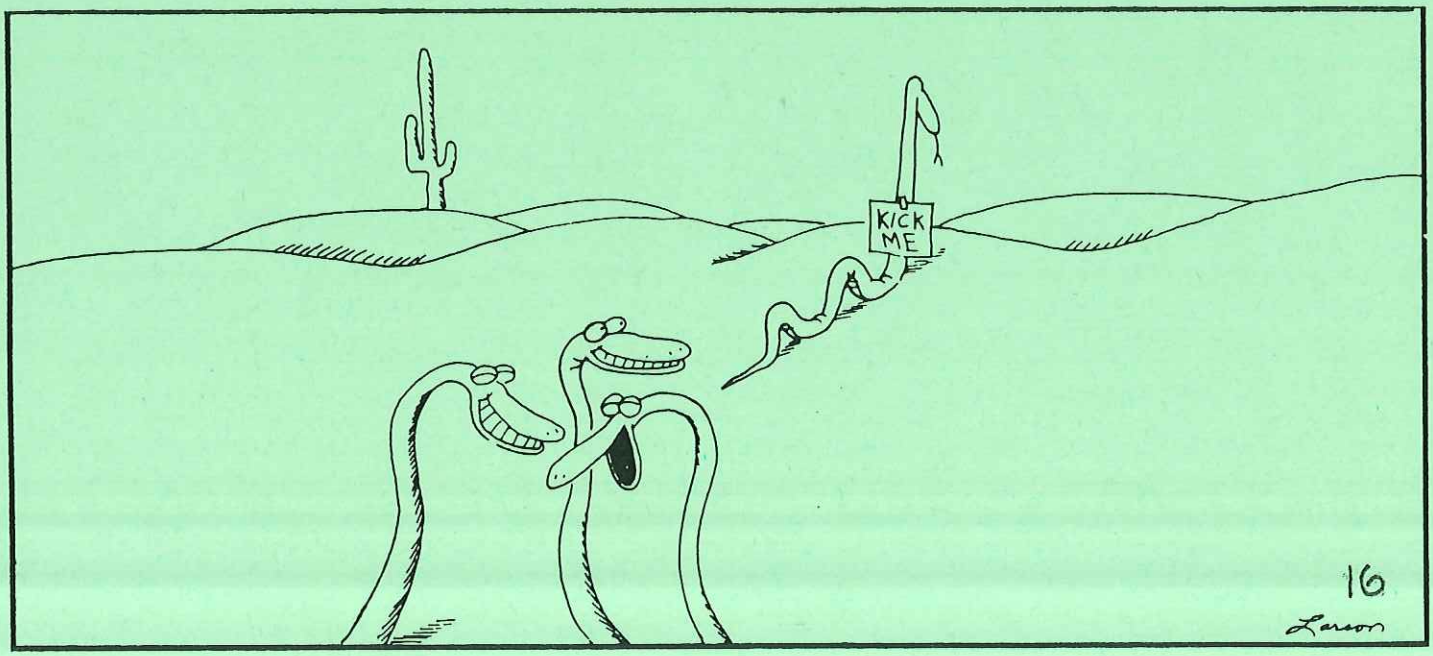
⑳ Vertical angles are equal

㉑ The acute angles of a right triangle are complementary

㉒ In a triangle, angles opposite equal sides are equal

㉓ Definition of isosceles triangle

㉔ Definition of segment midpoint



# *Definitions, Postulates, & Theorems*

## ANSWER KEY 13.2

① 1. Given
2. All rt. ∠'s are equal
3. Definition of segment midpoint
4. SAS (1, 2, 3)
5. CPCTC

② 1. Given
2. Definition of rt. triangle
3. Acute ∠'s of a rt. triangle are complementary
4. Complements of equal ∠'s are equal
5. ASA (1, 4)
6. CPCTC

③ 1. Given
2. Definition of vertical angles
3. Vertical angles are equal
4. SAS (1, 3)
5. CPCTC

④ 1. Given
2. All rt. ∠'s are equal
3. Identity
4. AAS (1, 2, 3)

⑤ 1. Given
2. AAS (1)
3. CPCTC

⑥ 1. Given
2. Perpendicular lines form congruent adjacent angles
3. Identity
4. AAS (1, 2, 3)
5. CPCTC

⑦ 1. Given
2. Definition of segment bisector
3. Definition of segment midpoint
4. Identity
5. SSS (1, 3, 4)
6. CPCTC

⑧ 1. Given
2. Definition of segment midpoint
3. All right angles are equal
4. Definition of vertical angles
5. Vertical angles are equal
6. AAS (2, 3, 5)
7. CPCTC

⑨ 1. Given
2. Definition of segment midpoint
3. Definition of vertical angles
4. Vertical angles are equal
5. AAS (1, 2, 4)
6. CPCTC

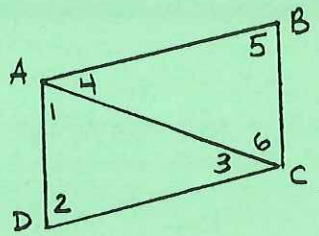
⑩ 1. Given
2. Definition of perpendicular lines
3. All right angles are equal
4. AAS (1, 3)
5. CPCTC
6. Definition of isosceles triangle

Continued

# *Definitions, Postulates, & Theorems*

## ANSWER KEY 13.2

(11)

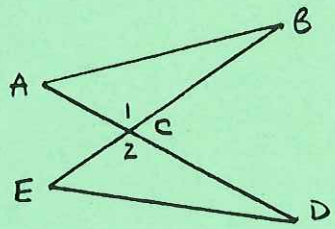



Given: $\angle 3 \cong \angle 4$, $\overline{DC} \cong \overline{BA}$

Prove: $\angle 1 \cong \angle 6$

| STATEMENTS | REASONS |
|---|---|
| 1. $\angle 3 \cong \angle 4$, $\overline{DC} \cong \overline{BA}$ | 1. Given |
| 2. $\overline{AC} \cong \overline{CA}$ | 2. Identity |
| 3. $\triangle ACD \cong \triangle CAB$ | 3. SAS (1, 2) |
| 4. $\angle 1 \cong \angle 6$ | 4. CPCTC |

(12)

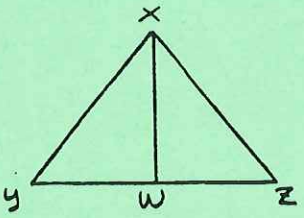



Given: $\overline{AC} \cong \overline{EC}$, $\angle A \cong \angle E$

Prove: $\overline{BC} \cong \overline{DC}$

| STATEMENTS | REASONS |
|---|---|
| 1. $\overline{AC} \cong \overline{EC}$, $\angle A \cong \angle E$ | 1. Given |
| 2. $\angle 1$ and $\angle 2$ are vertical angles | 2. Definition of vertical angles |
| 3. $\angle 1 \cong \angle 2$ | 3. Vertical angles are equal |
| 4. $\triangle ACB \cong \triangle ECD$ | 4. ASA (1, 3) |
| 5. $\overline{BC} \cong \overline{DC}$ | 5. CPCTC |

(13)




Given: $\overline{YX} \cong \overline{ZX}$, $\angle YXW \cong \angle ZXW$

Prove: W is the midpoint of $\overline{YZ}$

| STATEMENTS | REASONS |
|---|---|
| 1. $\overline{YX} \cong \overline{ZX}$, $\angle YXW \cong \angle ZXW$ | 1. Given |
| 2. $\overline{XW} \cong \overline{XW}$ | 2. Identity |
| 3. $\triangle YXW \cong \triangle ZXW$ | 3. SAS (1, 2) |
| 4. $\overline{YW} \cong \overline{ZW}$ | 4. CPCTC |
| 5. W is the midpoint of $\overline{YZ}$ | 5. Definition of segment midpoint |

# *Organizing & Constructing Proofs*

## ANSWER KEY 13.3

① 1. Given
2. In a triangle, angles opposite equal sides are equal
3. Definition of linear pair
4. If two angles form a linear pair, then they are supplementary
5. Supplements of equal angles are equal

② 1. Given
2. Two points form a line
3. Identity
4. SSS (1,3)
5. CPCTC

③ 1. Given
2. In a triangle, angles opposite equal sides are equal
3. Definition of linear pair
4. If two angles form a linear pair, then they are supplementary
5. Supplements of equal angles are equal
6. SAS (1,5)
7. CPCTC

④ 1. Given
2. Definition of perpendicular lines
3. All right angles are equal
4. Definition of segment midpoint
5. AAS (1,3,4)
6. CPCTC

⑤ 1. Given
2. Definition of segment midpoint
3. Definition of perpendicular lines
4. All right angles are equal
5. AAS (1,2,4)
6. CPCTC

⑥ 1. Given
2. In a triangle, angles opposite equal sides are equal
3. AAS (1,2)

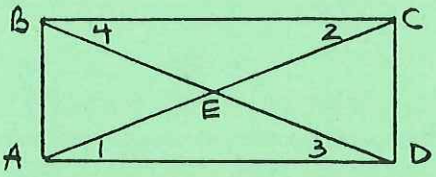
continued


"So now tell the court, if you will, Mrs. Potato Head, exactly what transpired on the night your husband chased you with the Vegomatic."

# *Organizing & Constructing Proofs*

## ANSWER KEY 13.3

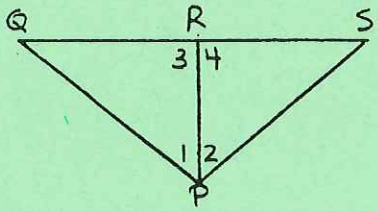
(7)




Given: E is midpoint of $\overline{AC}$, $\angle 1 \cong \angle 2$
Prove: $\angle 3 \cong \angle 4$

| STATEMENTS | REASONS |
|---|---|
| 1. E is midpoint of $\overline{AC}$, $\angle 1 \cong \angle 2$ | 1. Given |
| 2. $\overline{AE} \cong \overline{CE}$ | 2. Definition of segment midpoint |
| 3. $\angle BEC$ and $\angle DEA$ are vertical $\angle$'s | 3. Definition of vertical angles |
| 4. $\angle BEC \cong \angle DEA$ | 4. Vertical angles are equal |
| 5. $\triangle BEC \cong \triangle DEA$ | 5. ASA (1, 2, 4) |
| 6. $\angle 3 \cong \angle 4$ | 6. CPCTC |

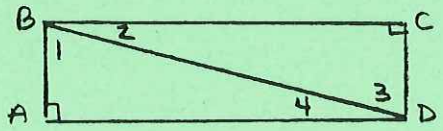
(8)




Given: $\overline{PR} \perp \overline{QS}$, $\angle Q \cong \angle S$
Prove: R is the midpoint of $\overline{QS}$

| STATEMENTS | REASONS |
|---|---|
| 1. $\overline{PR} \perp \overline{QS}$, $\angle Q \cong \angle S$ | 1. Given |
| 2. $\angle 3 \cong \angle 4$ | 2. Perpendicular lines form congruent adjacent $\angle$s |
| 3. $\overline{PR} \cong \overline{PR}$ | 3. Identity |
| 4. $\triangle QRP \cong \triangle SRP$ | 4. AAS (1, 2, 3) |
| 5. $\overline{SR} \cong \overline{QR}$ | 5. CPCTC |
| 6. R is the midpoint of $\overline{QS}$ | 6. Definition of segment midpoint |

(9)



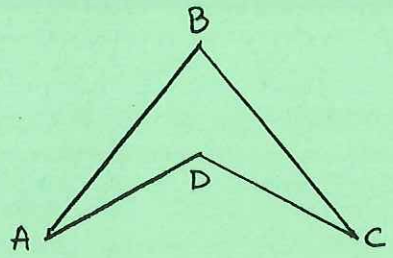

Given: $\angle A$ and $\angle C$ are rt. $\angle$'s, $\angle 2 \cong \angle 4$
Prove: $\overline{AB} \cong \overline{CD}$

| STATEMENTS | REASONS |
|---|---|
| 1. $\angle A$ and $\angle C$ are rt. $\angle$'s, $\angle 2 \cong \angle 4$ | 1. Given |
| 2. $\angle A \cong \angle C$ | 2. All right angles are equal |
| 3. $\overline{BD} \cong \overline{DB}$ | 3. Identity |
| 4. $\triangle ABD \cong \triangle CDB$ | 4. AAS (1, 2, 3) |
| 5. $\overline{AB} \cong \overline{CD}$ | 5. CPCTC |

# *Organizing & Constructing Proofs*

## ANSWER KEY 13.3

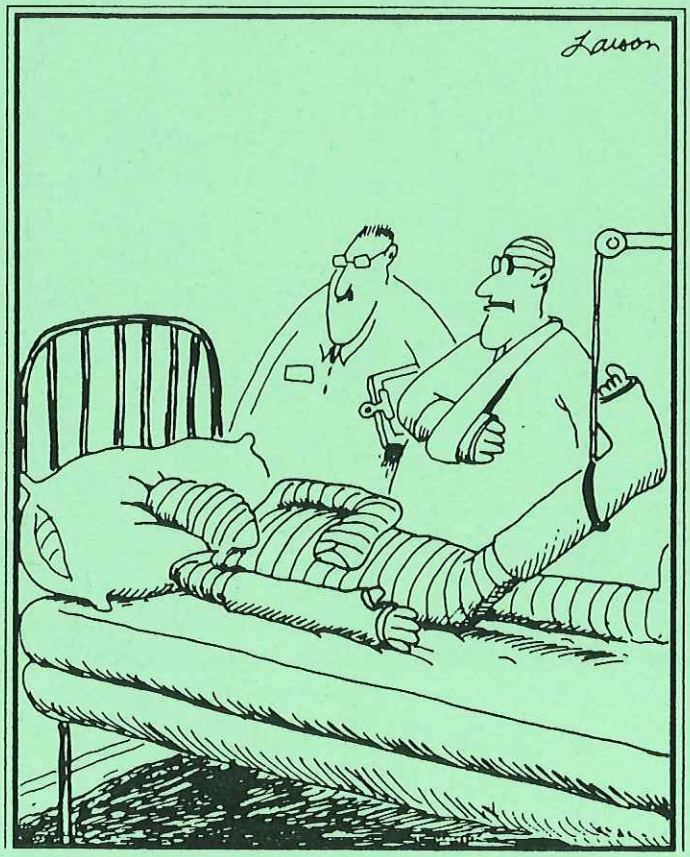
(10)



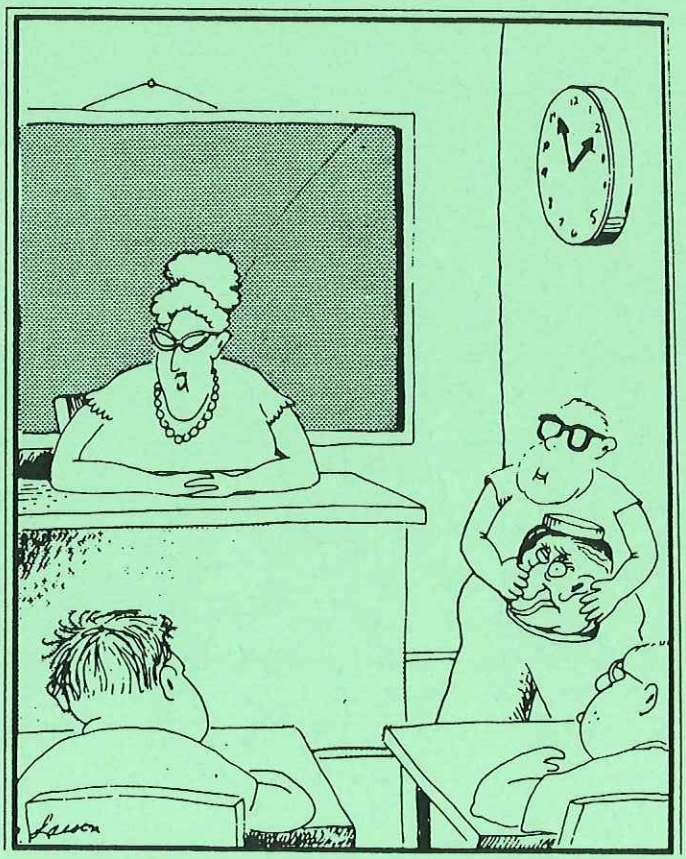

Given: $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$

Prove: $\angle A \cong \angle C$

| STATEMENTS | REASONS |
|---|---|
| 1. $\overline{AB} \cong \overline{CB}$, $\overline{AD} \cong \overline{CD}$ | 1. Given |
| 2. Draw $\overline{BD}$ | 2. Two points form a line |
| 3. $\overline{BD} \cong \overline{BD}$ | 3. Identity |
| 4. $\triangle ABD \cong \triangle CBD$ | 4. SSS (1, 3) |
| 5. $\angle A \cong \angle C$ | 5. CPCTC |




"So there he was — this big gorilla just laying there. And Jim says, 'Do you suppose it's dead or just asleep?'"




"And next, for show and tell, Bobby Henderson says he has something he found on the beach last summer . . ."

# *Trigonometric Ratios*

## ANSWER KEY 14.1

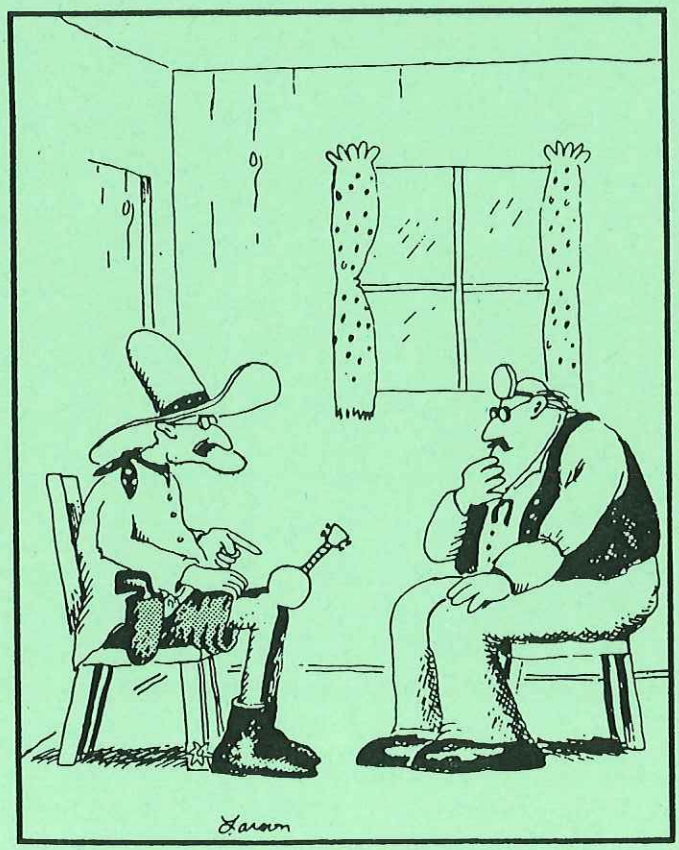
(1) $\tan A = 4/3$
(2) $\sin B = 3/5$
(3) $\cos A = 3/5$
(4) $\sec A = 5/3$
(5) $\cot B = 4/3$
(6) $\csc B = 5/3$
(7) $\sec B = 5/4$
(8) $\tan B = 3/4$
(9) $\csc A = 5/4$
(10) $\cos B = 4/5$
(11) $\cot A = 3/4$
(12) $\sin A = 4/5$

(13) $\cos A = \sqrt{6}/3\sqrt{2} \approx .5774$
(14) $\tan B = \sqrt{6}/2\sqrt{3} \approx .7071$
(15) $\sec B = 3\sqrt{2}/2\sqrt{3} \approx 1.225$
(16) $\cot A = \sqrt{6}/2\sqrt{3} \approx .7071$
(17) $\csc B = 3\sqrt{2}/\sqrt{6} \approx 1.732$
(18) $\sin A = 2\sqrt{3}/3\sqrt{2} \approx .8165$
(19) $\tan A = 2\sqrt{3}/\sqrt{6} \approx 1.414$
(20) $\csc A = 3\sqrt{2}/2\sqrt{3} \approx 1.225$
(21) $\cos B = 2\sqrt{3}/3\sqrt{2} \approx .8165$
(22) $\sin B = \sqrt{6}/3\sqrt{2} \approx .5774$
(23) $\sec A = 3\sqrt{2}/\sqrt{6} \approx 1.732$
(24) $\cot B = 2\sqrt{3}/\sqrt{6} \approx 1.414$

(25) $\sin A = 5/13$
(26) $\sec B = 13/5$
(27) $\cos B = 5/13$
(28) $\cot A = 12/5$
(29) $\tan B = 12/5$
(30) $\csc A = 13/5$

(31) $\sin A = 5/13$ $A \approx 22.62°$
(32) $\cos B = 5/13$ $B \approx 67.38°$
(33) $\tan A = 5/12$ $A \approx 22.62°$
(34) $\sin B = 12/13$ $B \approx 67.38°$

(35) $\sin 30° = .5$
(36) $\tan 20° \approx .36$
(37) $\cos 25° \approx .91$
(38) $\sec 45° \approx 1.41$ (cos)(1/x)
(39) $\csc 60° \approx 1.15$ (sin)(1/x)
(40) $\cos 10° \approx .98$
(41) $\tan 85° \approx 11.43$




"I've had it, Doc! . . . I've come all the way from Alabama with this danged thing on my knee!"

# *Interpolation*

## ANSWER KEY 14.2

① sin .4568

27.180809... −27, =, ×60
10.84856... −10, =, ×60
50.913596...
**27°10′51″**

② tan 1.415

54.750625... −54, =, ×60
45.037476... −45, =, ×60
2.2485672...
**54°45′2″**

③ cos .0269

88.468558... −88, =, ×60
27.513456... −27, =, ×60
30.807335...
**88°27′31″**

④ sin .3514

20.572969... −20, =, ×60
34.378158... −34, =, ×60
22.689473...
**20°34′23″**

⑤ $29.\overline{4}°$

$29.\overline{4}°$ −29, =, ×60
$26.\overline{6}$ −26, =, ×60
39.9984...
**29°26′40″**

⑥ 42.57°

42.57 −42, =, ×60
34.2 −34, =, ×60
12
**42°34′12″**

⑦ tan 8°30′ = **.1495**

⑧ sin 25°10′ = **.4253**

⑨ cos 46°20′ = **.6905**

⑩ csc 62°50′ = **1.124**

⑪ cot 18°30′ = **2.989**

⑫ sin 75°20′ = **.9674**

⑬ cot (1.632) = **31°30′**

⑭ cos (.6862) = **46°40′**

⑮ sin (.9787) = **78°10′**

⑯ sec (8.614) = **83°20′**

⑰ sin 43°20′ = .6862
sin 43°23′
sin 43°30′ = .6884
} .0022

(.0022)(.3) = .00066
.6862 + .00066 = .68686 ≈ **.6869**

⑱ csc 47°10′ = 1.364
csc 47°15′
csc 47°20′ = 1.360
} .004

(.004)(.5) = .002
1.364 − .002 = **1.362**

⑲ cos 31°40′ = .8511
cos 31°42′
cos 31°50′ = .8496
} .0015

(.0015)(.2) = .0003
.8511 − .0003 = **.8508**

# *Interpolation*

## ANSWER KEY 14.2

(20) $\cot 47°10' = .9271$
$\cot 47°18' =$
$\cot 47°20' = .9217$ } .0054

$(.0054)(.8) = .00432$

$.9271 - .00432 = .92278 \approx$ **.9228**

(21) $6°20' = \sin(.1103)$
$\sin(.1111)$ } .0029 } .0008
$6°30' = \sin(.1132)$

$(10')(8/29) = 2.758... \approx$ **$6°23'$**

(22) $12°30' \quad \tan(.2217)$
$\tan(.2222)$ } .003 } .0005
$12°40' \quad \tan(.2247)$

$(10')(5/30) = 1.\overline{6} \approx$ **$12°32'$**

(23) $88°30 \quad \tan(38.19)$
$\tan(42.71)$ } 4.77 } 4.52
$88°40 \quad \tan(42.96)$

$(10')(4.52/4.77) = 9.47... \approx$ **$88°39'$**

(24) $45°00' \quad \csc(1.414)$
$\csc(1.412)$ } .004 } .002
$45°10' \quad \csc(1.410)$

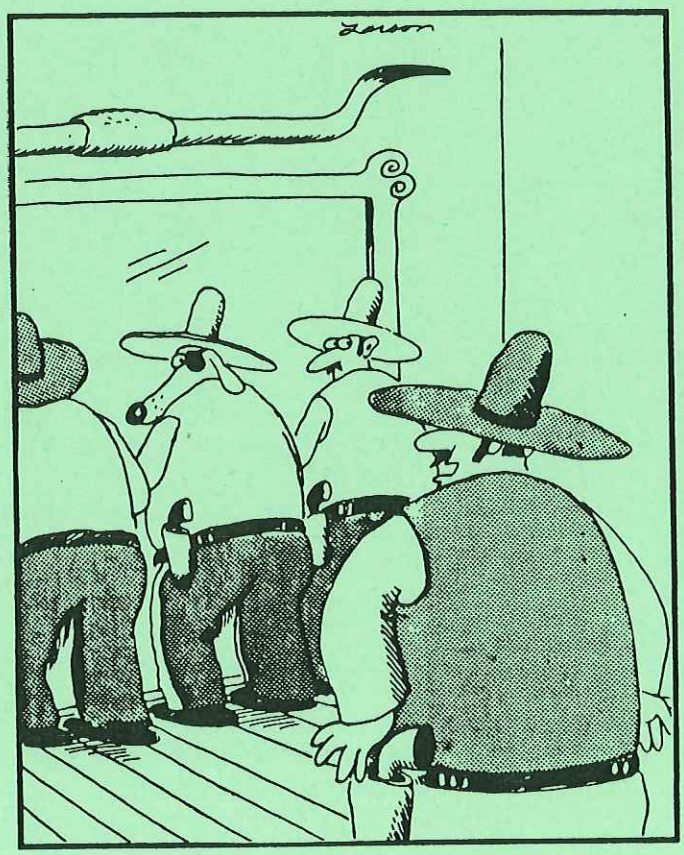
$(10')(1/2) = 5'$ **$45°5'$**

(25) $\tan A = \frac{\cancel{2}\sqrt{2}}{\cancel{2}\sqrt{6}} = \frac{\sqrt{2}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{12}}{6}$

$\frac{2\sqrt{3}}{6} =$ **$\frac{\sqrt{3}}{3}$**

(26) $\csc B = \frac{4\sqrt{2}}{2\sqrt{6}} = \frac{2\sqrt{2}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{2\sqrt{12}}{6}$

$\frac{4\sqrt{3}}{6} =$ **$\frac{2\sqrt{3}}{3}$**



"Okay . . . which of you is the one they call 'Old-One-Eyed-Dog-Face'?"

# Solving Right Triangles

## ANSWER KEY 14.3

① $\sin 40° = a/16$
$(\sin 40°)(16) = a$ → $a \approx 10.28$

$\angle B = 90 - 40$ → $\angle B = 50°$

$\cos 40° = b/16$
$(\cos 40°)(16) = b$ → $b \approx 12.26$

② $\angle B = 90 - 35$ → $\angle B = 55°$

$\tan 55° = b/7$
$(\tan 55°)(7) = b$ → $b \approx 10.00$

$\sin 35° = 7/c$
$c = 7/\sin 35°$ → $c \approx 12.20$

③ $\sin A = 12/13$
$\angle A = 67.380135\ldots$
$-67, =, \times 60$ → $\angle A \approx 67°23'$

$\angle B = 90 - 67°23'$ → $\angle B \approx 22°37'$

$b^2 + 12^2 = 13^2$
$b^2 = 25$ → $b = 5$

④ $\angle A = 90 - 64$ → $\angle A = 26°$

$\sin B = b/19.2$
$(\sin 64°)(19.2) = b$ → $b \approx 17.26$

$\sin A = a/19.2$
$(\sin 26°)(19.2) = a$ → $a \approx 8.42$

⑤ $\angle B = 90 - 37°15'$ → $\angle B = 52°45'$

$\tan 37°15' = 9/11$
$(\tan 37° \frac{15}{60})(11) = a$ → $a \approx 8.36$

$\cos 37°15' = 11/c$
$c = 11/\cos 37° \frac{15}{60}$ → $c \approx 13.82$

⑥ $\angle A = 90 - 42°10'$ → $\angle A = 47°50'$

$\tan 42°10' = b/9$
$(\tan 42° \frac{1}{6})(9) = b$ → $b \approx 8.15$

$\cos 42°10' = 9/c$
$c = 9/\cos 42° \frac{1}{6}$ → $c \approx 12.14$

⑦ $65°$ $\tan 2.146$ } .016 } .005
$\tan 2.15$
$65°10'$ $\tan 2.161$

$(10')(5/16) = 3.125$ → $65°3'$

⑧ $\cos 50°40' = .6338$ } .0022
$\cos 50°42'$
$\cos 50°50' = .6316$

$(.0022)(2/10) = .00044$
$.6338 - .00044 = .63336 \approx .6334$

⑨ $\cot B = \frac{\sqrt{3}}{6\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{6}}{12}$

⑩ $\cos A = \frac{6\sqrt{2}}{5\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{6\sqrt{6}}{15} = \frac{2\sqrt{6}}{5}$

⑪ $\csc B = \frac{5\sqrt{3}}{6\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{5\sqrt{6}}{12}$

# *Trigonometric Expressions*

## ANSWER KEY 14.4

① $2\cos 30°$

$2\left(\sqrt{3}/2\right) = \boxed{\sqrt{3}}$

② $\sin^2 45° + \cos^2 45°$

$\left(1/\sqrt{2}\right)^2 + \left(1/\sqrt{2}\right)^2$

$1/2 + 1/2 = \boxed{1}$

③ $\cos^2 30° - \sin^2 30°$

$\left(\sqrt{3}/2\right)^2 - \left(1/2\right)^2$

$3/4 - 1/4 = \boxed{1/2}$

④ $\csc 45° - \sec 30°$

$\left(\sqrt{2}/1\right) - \left(2/\sqrt{3}\right)$

$\sqrt{2} - \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \boxed{\sqrt{2} - \frac{2\sqrt{3}}{3}}$

⑤ $-\sin 60° = \boxed{-\sqrt{3}/2}$

⑥ $2\sin 60°\cos 60°$

$2\left(\sqrt{3}/2\right)\left(1/2\right) = \boxed{\sqrt{3}/2}$

⑦ $\sin 30°\cos 60° - \sin 60°\cos 30°$

$\left(1/2\right)\left(1/2\right) - \left(\sqrt{3}/2\right)\left(\sqrt{3}/2\right)$

$1/4 - 3/4 = \boxed{-1/2}$

⑧ $\sec 60° + \cot 30° - 4\cos^2 45°$

$(2) + (\sqrt{3}) - 4\left(1/\sqrt{2}\right)^2$

$(2) + (\sqrt{3}) - 4\left(1/2\right)$

$(2) + (\sqrt{3}) - (2) = \boxed{\sqrt{3}}$

⑨ $\angle B = 90 - 22°22'$ $\boxed{\angle B = 67°38'}$

$\tan 22°22' = a/22$

$\left(\tan 22\tfrac{22}{60}\right)(22) = a$ $\boxed{a \approx 9.05}$

$\cos 22°22' = 22/c$

$c = 22/\cos 22\tfrac{22}{60}°$ $\boxed{c \approx 23.79}$

⑩ $\angle A = 90 - 44°44'$ $\boxed{\angle A = 45°16'}$

$\tan 44°44' = b/44$

$\left(\tan 44\tfrac{44}{60}°\right)(44) = b$ $\boxed{b \approx 43.59}$

$\cos 44°44' = 44/c$

$c = 44/\cos 44\tfrac{44}{60}°$ $\boxed{c \approx 61.94}$

⑪ 6-8-10 Pyth triple $\boxed{a = 6}$

$\sin B = 8/10$

$\angle B \approx 53.130...°$ $\boxed{\angle B \approx 53°8'}$

$-53, =, \times 60$

$\cos A = 8/10$

$\angle A \approx 36.8698...°$ $\boxed{\angle A \approx 36°52'}$

$-36, =, \times 60$

⑫ $60°10' \approx \cot .5735$

$\cot .5728$ } $.0039$ } $.0007$

$60°20'$ $\cot .5696$

$(10')(7/39) \approx 1.7948...$

$10 + 2 = 12'$ $\boxed{60°12'}$

⑬ $22°30' \approx \tan .4142$ } $.0034$

$22°34'$

$22°40' \approx \tan .4176$

$(.0034)(4/10) = .00136$

$.4142 + .00136 = .41556$ $\boxed{.4156}$

# Triangle Trigonometry

## ANSWER KEY: UNIT 14 REVIEW & PRACTICE

① a) $\tan A = \frac{3\sqrt{2}}{6\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{6}}{18} = \boxed{\frac{\sqrt{6}}{6}}$

b) $\csc B = \frac{3\sqrt{14}}{6\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{3\sqrt{42}}{18} = \boxed{\frac{\sqrt{42}}{6}}$

c) $\sin A = \frac{3\sqrt{2}}{3\sqrt{14}} \cdot \frac{\sqrt{14}}{\sqrt{14}} = \frac{3\sqrt{28}}{42} = \frac{6\sqrt{7}}{42} = \boxed{\frac{\sqrt{7}}{7}}$

d) $\cot A = \frac{6\sqrt{3}}{3\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{6\sqrt{6}}{6} = \boxed{\sqrt{6}}$

② a) $\cos B = \frac{4}{4\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{4\sqrt{5}}{20} = \boxed{\frac{\sqrt{5}}{5}}$

b) $\tan A = \frac{4}{8} = \boxed{\frac{1}{2}}$

c) $\sec B = \frac{4\sqrt{5}}{4} = \boxed{\sqrt{5}}$

d) $\sin A = \frac{4}{4\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{4\sqrt{5}}{20} = \boxed{\frac{\sqrt{5}}{5}}$

③ $24°10'$ tan .4487
tan .4497 } .0035 } .0010
$24°20'$ tan .4522

$(10')(\frac{10}{35}) \approx 2.857\ldots$ $\boxed{24°13'}$

④ $48°20'$ sin .7470
sin .7478 } .0020 } .0008
$48°30'$ sin .7490

$(10')(\frac{8}{20}) = 4$ $\boxed{48°24'}$

⑤ $\angle B = 90 - 16$ $\boxed{\angle B = 74°}$

$\sin 16° = a/14$
$(\sin 16°)(14) = a$ $\boxed{a \approx 3.86}$

$\cos 16° = b/14$
$(\cos 16°)(14) = b$ $\boxed{b \approx 13.46}$

⑥ $\tan A = 6/3$
$\angle A \approx 63.4349\ldots$
$-63, =, \times 60$ $\boxed{\angle A \approx 63°26'}$

$\angle B = 90 - 63°26'$ $\boxed{\angle B \approx 26°34'}$

$3^2 + 6^2 = c^2$
$c^2 = 45$
$c = 3\sqrt{5}$ $\boxed{c \approx 6.71}$

⑦ $\angle B = 90 - 37°15'$ $\boxed{\angle B = 52°45'}$

$\tan 37°15' = a/11$
$(\tan 37\frac{1}{4}°)(11) = a$ $\boxed{a \approx 8.36}$

$\cos 37°15' = 11/c$
$c = 11/\cos 37\frac{1}{4}°$ $\boxed{c \approx 13.82}$

⑧ $\sin A = 6/2\sqrt{11}$
$\angle A \approx 64.7605\ldots$
$-64, =, \times 60$ $\boxed{\angle A \approx 64°46'}$

$\angle B = 90 - 64°46'$ $\boxed{\angle B \approx 25°14'}$

$b^2 + 6^2 = (2\sqrt{11})^2$
$b^2 + 36 = 44$
$b^2 = 8 \quad b = 2\sqrt{2}$ $\boxed{b \approx 2.83}$

# *Triangle Trigonometry*

## ANSWER KEY: UNIT 14 REVIEW & PRACTICE

⑨ $\cos^2 45° - \sin 30°$

$(1/\sqrt{2})^2 - (1/2)$

$1/2 - 1/2 = \boxed{0}$

⑩ $2\tan^2 60° + (\csc 30°)(\sin^2 45°)$

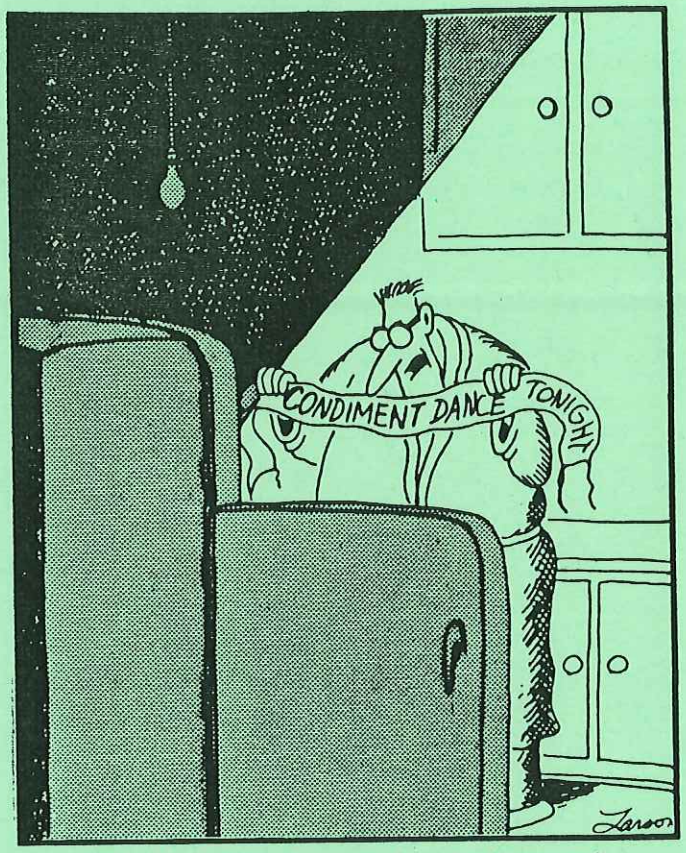
$2\,(\sqrt{3}/1)^2 + (2/1)(1/\sqrt{2})^2$

$2\,(3) + (2)(1/2) = \boxed{7}$

⑪ $\dfrac{2\cos 60° - \sin^2 45°}{\csc 45°}$

$\dfrac{2(1/2) - (1/\sqrt{2})^2}{(\sqrt{2}/1)} = \dfrac{1 - 1/2}{\sqrt{2}}$

$\dfrac{\frac{1}{2}}{\sqrt{2}} = \dfrac{1}{2}\cdot\dfrac{1}{\sqrt{2}} = \dfrac{1}{2\sqrt{2}}\cdot\dfrac{\sqrt{2}}{\sqrt{2}} = \boxed{\dfrac{\sqrt{2}}{4}}$




"I'll just take THIS, thank you! . . . And knock off that music!"

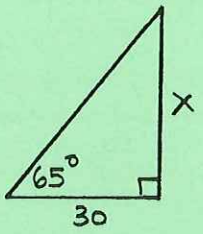
⑫ $\dfrac{(2\tan^2 60°)(\sin 30°)}{\sin 60°}$

$\dfrac{2\,(\sqrt{3}/1)^2(1/2)}{(\frac{\sqrt{3}}{2})} = \dfrac{3}{(\frac{\sqrt{3}}{2})}$

$3\cdot\dfrac{2}{\sqrt{3}} = \dfrac{6}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{\sqrt{3}} = \dfrac{6\sqrt{3}}{3} = \boxed{2\sqrt{3}}$

# *Problem Solving*

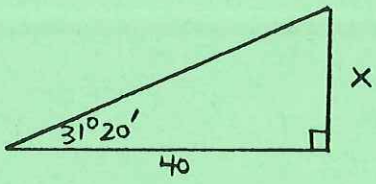
## ANSWER KEY 15.1

① 


$\tan 65^\circ = x/30$

$(\tan 65^\circ)(30) = x$

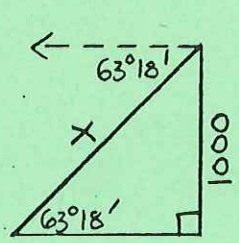
$x \approx$ **64.34 m.**

② 


$\tan 31\frac{1}{3}^\circ = x/40$

$(\tan 31\frac{1}{3}^\circ)(40) = x$

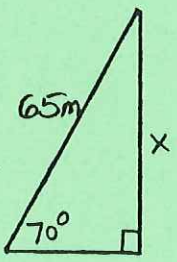
$x \approx$ **24.35 ft.**

③ 


$\sin 63.3^\circ = 1000/x$

$x = 1000/\sin 63.3^\circ$

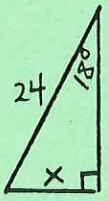
$x \approx$ **1119.36 ft**

④ 


$\sin 70^\circ = x/65$

$(\sin 70^\circ)(65) = x$

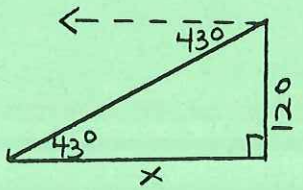
$x \approx$ **61.08 m.**

⑤ 


$\sin 18^\circ = x/24$

$(\sin 18^\circ)(24) = x$

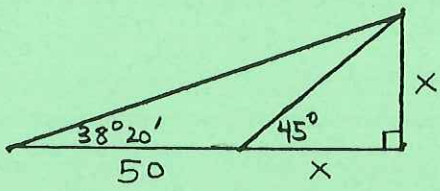
$x \approx$ **7.42 ft.**

⑥ 


$\tan 43^\circ = 120/x$

$x = 120/\tan 43^\circ$

$x \approx$ **128.68 m**

⑦ 

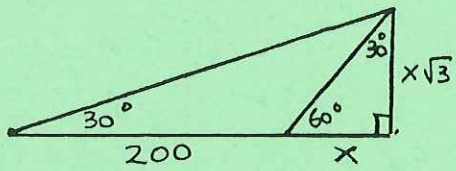

$\tan 38°20' = x/50+x$

$(\tan 38\frac{1}{3}^\circ)(50+x) = x$

$39.534874... + .7906975x = x$

$39.534874 = .2093025x$

$x \approx$ **188.89 ft**

⑧ 

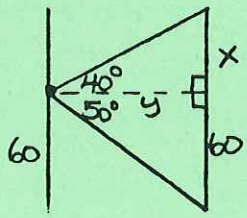

$\tan 30^\circ = x\sqrt{3}/200+x$

$(\tan 30^\circ)(200+x) = x\sqrt{3}$

$115.47005 + .5773503x = 1.7320508x$

$115.47005 = 1.1547005x$

$x \approx$ **100 ft** $\quad 200+x \approx$ **300 ft**

⑨ 


$\tan 50^\circ = 60/y$

$y = 60/\tan 50^\circ$

$y \approx 50.345978$

$\tan 40^\circ = x/50.345978$

$(\tan 40^\circ)(50.345978) = x$

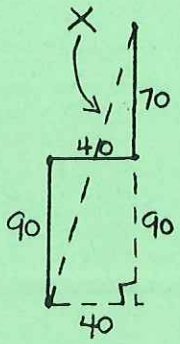
$x \approx 42.245292$

$+ 60 \approx$ **102.25 ft**

# *Problem Solving*

## ANSWER KEY 15.1

(10)




x is distance to port

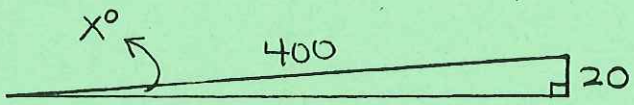
Use Pythagorean Theorem:

$40^2 + 160^2 = x^2$

$x^2 = 27200$

$x \approx$ **164.92 km**

(11)



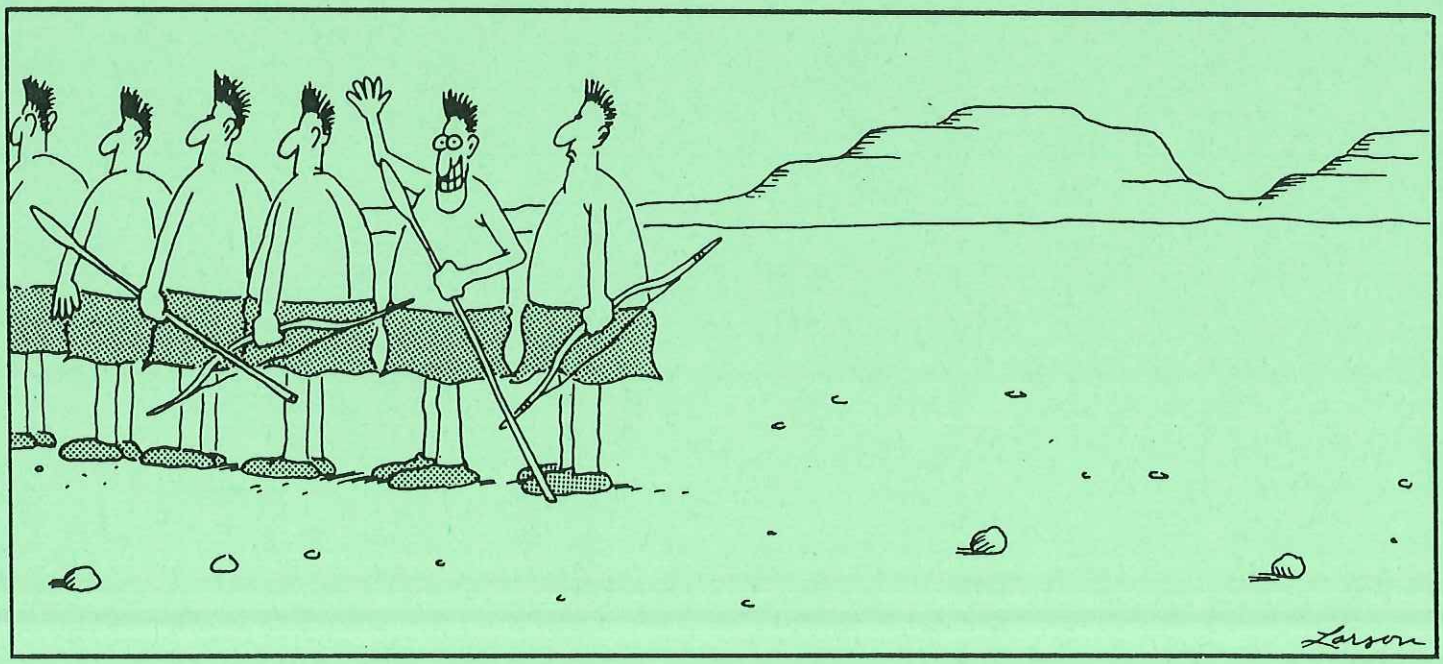

$\sin x = 20/400$

$x = 2.865983...^\circ$

$-2 = ,\times 60$

**$2^\circ 52'$**


"Oh, wonderful! Look at this, Etta — another mouth to feed."




Second to last of the Mohicans

# *Law of Sines*

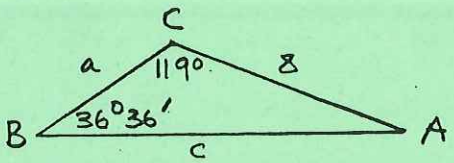
## ANSWER KEY 15.2

① $A = \frac{1}{2}(11.5)(14)(\sin 20^\circ) = \boxed{27.53}$

② $A = \frac{1}{2}(9.4)(13.5)(\sin 95^\circ) = \boxed{63.21}$

③ $A = \frac{1}{2}(6)(15)(\sin 50^\circ) = \boxed{34.47\ m^2}$

④



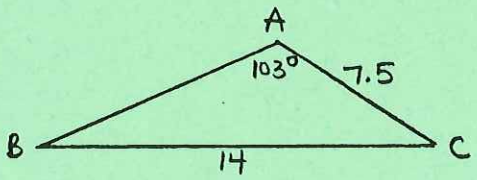

$\frac{\sin 36.6^\circ}{8} = \frac{\sin 119^\circ}{c}$ $c \approx \boxed{11.74}$

$\angle A = 180 - (119 + 36.6) = 24.4$

$\angle A = 24.4^\circ = \boxed{24^\circ 24'}$

$\frac{\sin 36.6}{8} = \frac{\sin 24.4}{a}$ $a \approx \boxed{5.54}$

⑤




$\frac{\sin 103^\circ}{14} = \frac{\sin B}{7.5}$ $\angle B = 31.465426\ldots$

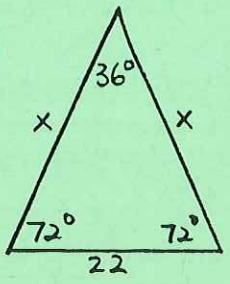
$\boxed{\angle B \approx 31^\circ 28'}$

$\angle C = 180 - (103 + 31.465426\ldots)$

$\angle C = 45.534574\ldots$ $\boxed{\angle C \approx 45^\circ 32'}$

$\frac{\sin 103^\circ}{14} = \frac{\sin 45.534574^\circ}{c}$ $\boxed{c \approx 10.25}$

⑥




$180 - 36 = 144 \div 2 = 72$

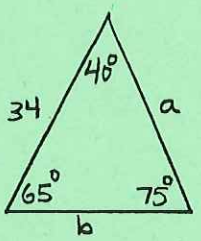
base angles $= 72^\circ$

$\frac{\sin 36^\circ}{22} = \frac{\sin 72^\circ}{x}$

$x = 35.596748\ldots$

$P = 22 + 2(35.596748) \approx \boxed{93.19\ cm}$

⑦

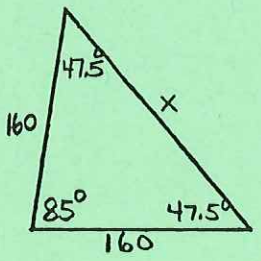



$\frac{\sin 75^\circ}{34} = \frac{\sin 40^\circ}{b}$

$b \approx \boxed{22.63\ yards}$

$\frac{\sin 75^\circ}{34} = \frac{\sin 65^\circ}{a}$ $a \approx \boxed{31.9\ yards}$

⑧




$\frac{\sin 85^\circ}{x} = \frac{\sin 47.5^\circ}{160}$

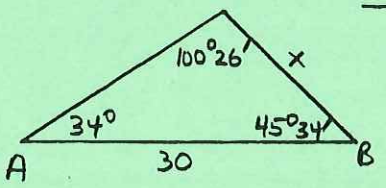
$x \approx 216.19$

$P \approx 216.19 + 2(160)$

$\boxed{P \approx 536.19\ feet}$

$A = \frac{1}{2}(160)^2(\sin 85^\circ) \approx \boxed{12{,}751.29\ ft^2}$

⑨



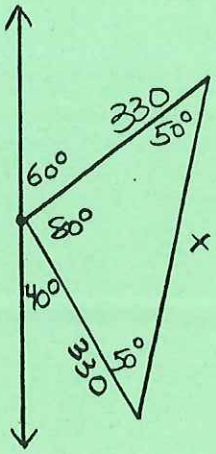

$\frac{\sin 100.4\overline{3}^\circ}{30} = \frac{\sin 34^\circ}{x}$

$\boxed{x \approx 17.06\ miles}$

# *Law of Sines*

## ANSWER KEY 15.2

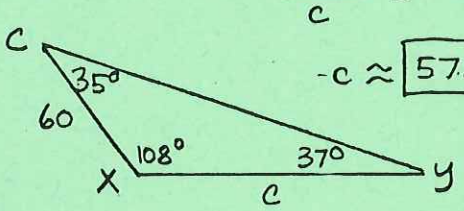
(10)




$$\frac{\sin 50^\circ}{330} = \frac{\sin 80^\circ}{x}$$

$$x \approx \boxed{424.24 \text{ miles}}$$

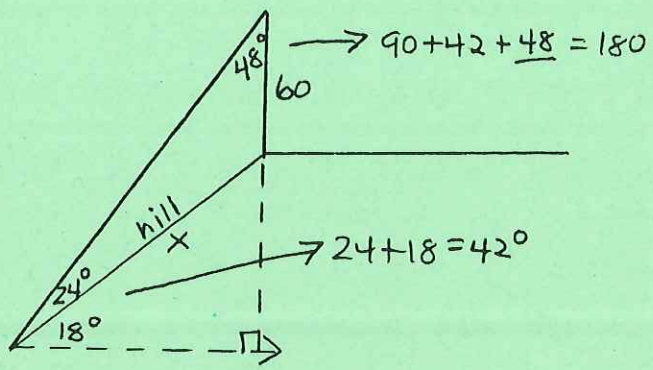
(11)




$$\frac{\sin 35^\circ}{c} = \frac{\sin 37^\circ}{60}$$

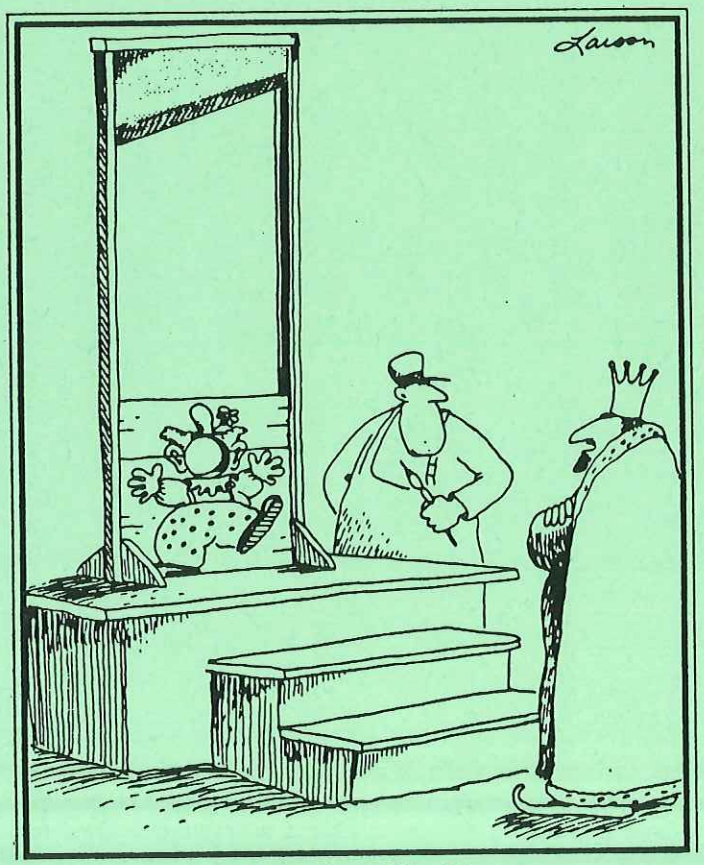
$$c \approx \boxed{57.18 \text{ km}}$$

(12)

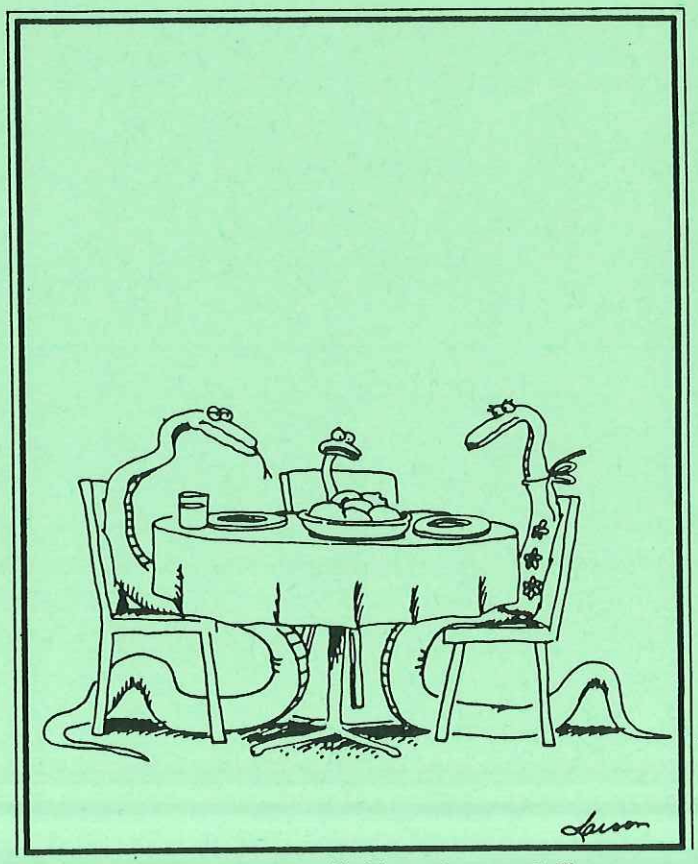



$$\frac{\sin 24^\circ}{60} = \frac{\sin 48^\circ}{x}$$

$$x \approx \boxed{109.63 \text{ feet}}$$



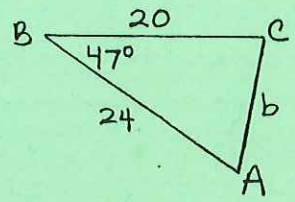

"I like it . . . I like it."




"Oh, brother! . . . Not hamsters again!"

# *Law of Cosines*

## ANSWER KEY 15.3

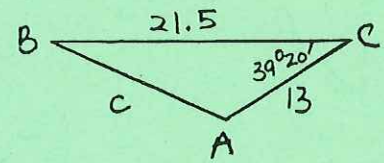
① 


$b^2 = (20)^2 + (24)^2 - 2(20)(24)(\cos 47^\circ)$

$b^2 = 321.28157...$ $\boxed{b \approx 17.92}$

$\frac{\sin 47^\circ}{17.924329} = \frac{\sin A}{20}$ $\angle A = 54.6909...$ $\boxed{\angle A \approx 54^\circ 41'}$

$180 - (47 + 54^\circ 41')$ $\boxed{\angle C \approx 78^\circ 19'}$

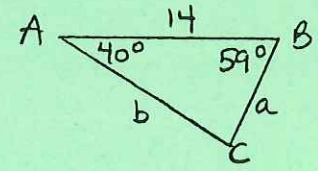
② 


$c^2 = (21.5)^2 + (13)^2 - 2(21.5)(13)(\cos 39\frac{1}{3}^\circ)$

$c^2 = 198.87938...$ $\boxed{C \approx 14.10}$

$\frac{\sin 39\frac{1}{3}^\circ}{14.10246} = \frac{\sin B}{13}$ $\angle B \approx 35.752228$ $\boxed{\angle B \approx 35^\circ 45'}$

$180 - (39^\circ 20' + 35^\circ 45')$ $\boxed{\angle A \approx 104^\circ 55'}$

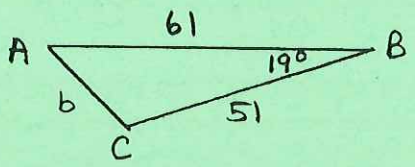
③ 


$\boxed{\angle C = 81^\circ}$

$\frac{\sin 81^\circ}{14} = \frac{\sin 59^\circ}{b}$

$\frac{\sin 81^\circ}{14} = \frac{\sin 40^\circ}{a}$ $\boxed{b \approx 12.15}$

$\boxed{a \approx 9.11}$

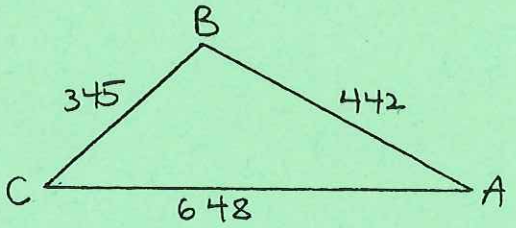
④ 


$b^2 = (61)^2 + (51)^2 - 2(61)(51)(\cos 19^\circ)$

$b^2 = 438.98342...$ $\boxed{b \approx 20.95}$

$\frac{\sin 19^\circ}{20.951931} = \frac{\sin A}{61}$ $\angle A \approx 52.417832^\circ$ $\boxed{\angle A \approx 52^\circ 25'}$

$180 - (19 + 52^\circ 25')$ $\boxed{\angle C \approx 108^\circ 35'}$

⑤ 


$345^2 = (442)^2 + (648)^2 - 2(442)(648)(\cos A)$

$\frac{345^2 - (442)^2 - (648)^2}{-2(442)(648)} = (\cos A)$

$\angle A \approx 29.968791...$ $\boxed{\angle A \approx 29^\circ 58'}$

$\frac{\sin 29.968791^\circ}{345} = \frac{\sin C}{442}$

$\angle C \approx 39.789975...$ $\boxed{\angle C \approx 39^\circ 47'}$

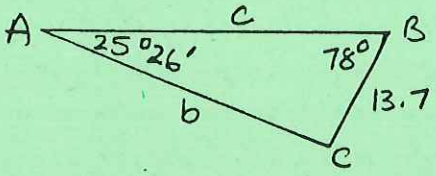
$180 - (29^\circ 58' + 39^\circ 47')$

$\boxed{\angle B \approx 110^\circ 15'}$

# *Law of Cosines*

## ANSWER KEY 15.3

⑥

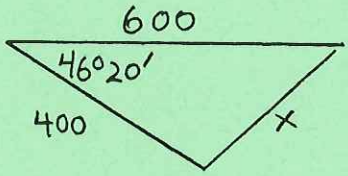



$\frac{\sin 25.4\overline{3}}{13.7} = \frac{\sin 78^\circ}{b}$ $\boxed{b \approx 31.20}$

$180 - (25°26' + 78°)$ $\boxed{\angle C = 76°34'}$

$\frac{\sin 25.4\overline{3}}{13.7} = \frac{\sin 76.5\overline{6}}{c}$ $\boxed{c \approx 31.03}$

⑦




$x^2 = (400)^2 + (600)^2 - 2(400)(600)(\cos 46\frac{1}{3}^\circ)$

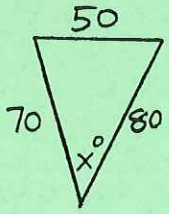
$x^2 \approx 188578.33\ldots$ $x \approx 434.26$

$P = 400 + 600 + 434.26 =$ $\boxed{1434.26 \text{ ft}}$

$A = \frac{1}{2}(400)(600)(\sin 46\frac{1}{3}^\circ)$

$A = \boxed{86,804.28 \text{ ft}^2}$

⑧



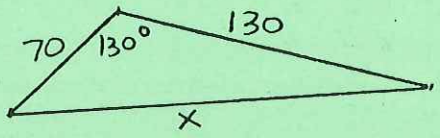

$50^2 = (70)^2 + (80)^2 - 2(70)(80)(\cos x^\circ)$

$\frac{50^2 - (70)^2 - (80)^2}{-2(70)(80)} = \cos x^\circ$

$x \approx 38.213211\ldots$

$\boxed{x \approx 38°13'}$

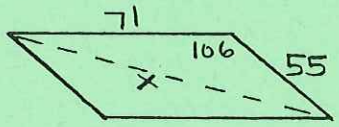
⑨




$x^2 = (70)^2 + (130)^2 - 2(70)(130)(\cos 130^\circ)$

$x^2 = 33,498.734$ $x \approx \boxed{183.03 \text{ miles}}$

⑩



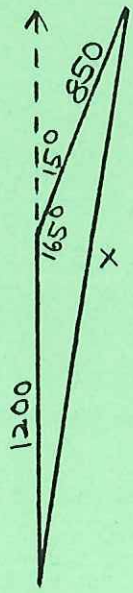

$x^2 = (71)^2 + (55)^2 - 2(71)(55)(\cos 106^\circ)$

$x^2 = 10218.728$ $\boxed{x \approx 101.09 \text{ cm}}$

$A = \left[\frac{1}{2}(71)(55)(\sin 106^\circ)\right](2)$

$\boxed{A \approx 3753.73 \text{ cm}^2}$

⑪



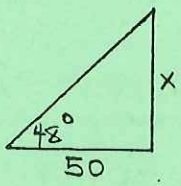

$x^2 = (850)^2 + (1200)^2 - 2(850)(1200)(\cos 165^\circ)$

$x^2 \approx 4132988.7$

$x \approx \boxed{2032.98 \text{ km}}$

# Review & Practice

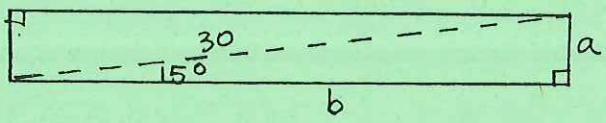
## ANSWER KEY 15.4

① 


$\tan 48^\circ = x/50$

$(\tan 48^\circ)(50) = x$

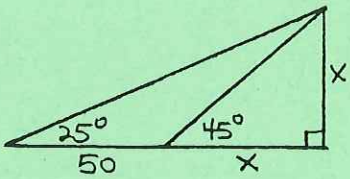
$x \approx \boxed{55.53 \text{ m}}$

② 


$\sin 15^\circ = a/30$ $\quad \cos 15^\circ = b/30$

$(\sin 15^\circ)(30) = a$ $\quad (\cos 15^\circ)(30) = b$

$a \approx \boxed{7.76 \text{ in.}}$ $\quad b \approx \boxed{28.98 \text{ in.}}$

③ 

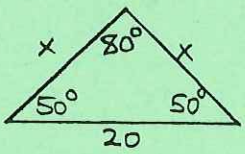

$\tan 25^\circ = \dfrac{x}{50+x}$

$(\tan 25^\circ)(50+x) = x$

$23.315383 + .4663077x = x$

$23.315383 = .5336923x$

$x \approx \boxed{43.69 \text{ m}}$

④ 

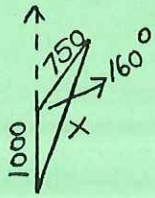

$\dfrac{\sin 80^\circ}{20} = \dfrac{\sin 50^\circ}{x}$

$x \approx 15.56$

$P = 20 + 2(15.56) \approx \boxed{51.11 \text{ cm}}$ or $\boxed{51.12 \text{ cm}}$

using all digits of x

using 15.56 value of x

⑤ 

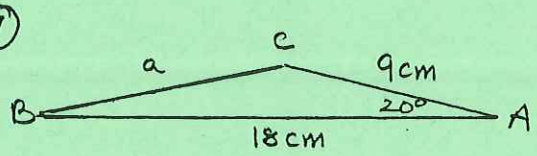

$x^2 = (1000)^2 + (750)^2 - 2(1000)(750)(\cos 160^\circ)$

$x^2 \approx 2973038.9$ $\quad x \approx \boxed{1723.96 \text{ Km}}$

⑥ $A = \frac{1}{2}(12)(20)(\sin 50^\circ 25')$

$A = \frac{1}{2}(12)(20)(\sin 50.416^\circ)$

$A \approx \boxed{92.48 \text{ in}^2}$

⑦ 


$a^2 = (9^2) + (18)^2 - 2(9)(18)(\cos 20^\circ)$

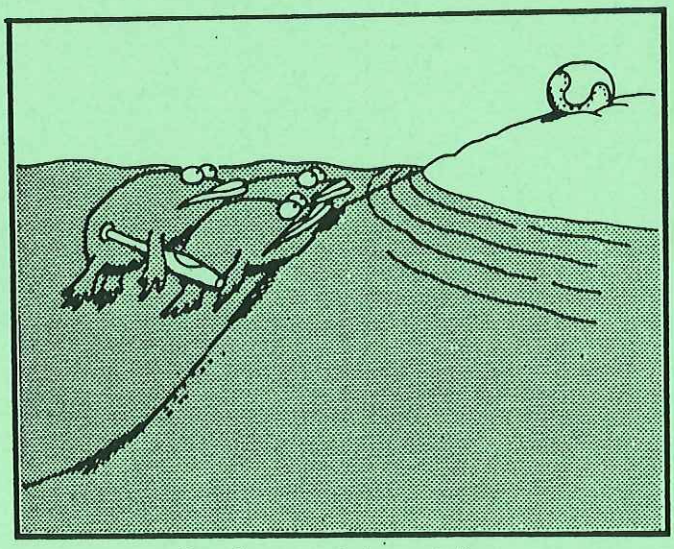
$a^2 = 100.53959$ $\quad a \approx 10.026943$

$\dfrac{\sin 20^\circ}{10.026943} = \dfrac{\sin C}{18}$

According to the calculator, $\angle C$ is $\approx 37.877988^\circ$

Because $\angle C$ must be obtuse, use the supplement $180 - 37.877988$

$\angle C \approx 142.12201$ $\quad \boxed{\angle C \approx 142^\circ 7'}$

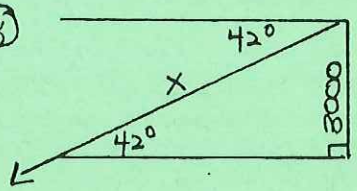


Great moments in evolution

# *Law of Sines & Cosines*

## ANSWER KEY: UNIT 15 REVIEW & PRACTICE

(8)

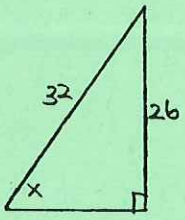



$\sin 42° = 3000/x$

$x = 3000/\sin 42°$

$x \approx$ **4488.43 ft.**

(9)

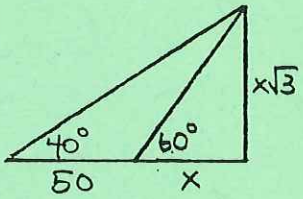



$\sin x = 26/32$

$\angle x \approx 54.340912$

**$\angle x \approx 54°20'$**

(10)




$\tan 40° = x\sqrt{3}/50+x$

$(\tan 40°)(50+x) = x\sqrt{3}$

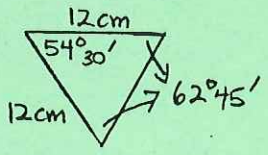
$41.954982 + .8390996x = 1.7320508x$

$41.954982 = .8929512x$

$x \approx 46.98463$

$x\sqrt{3} \approx$ **81.38 ft.**

(11)

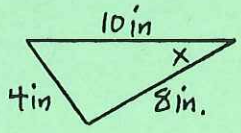



$\frac{\sin 62.75}{12} = \frac{\sin 54.5}{x}$

$x \approx 10.988974$

$P = 12+12+10.98974 \approx$ **34.99 cm**

(12)




**$\angle x \approx 22°20'$**

$4^2 = 10^2+8^2-2(10)(8)(\cos x)$

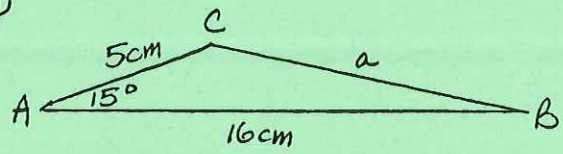
$\frac{4^2-10^2-8^2}{-2(10)(8)} = \cos x$ $\quad x \approx 22.331645$

(13) $A = \frac{1}{2}(9)(14)(\sin 24°15')$

$A = \frac{1}{2}(9)(14)(\sin 24\frac{1}{4}°)$

$A \approx$ **25.88 $m^2$**

(14)




$a^2 = 5^2+16^2-2(5)(16)(\cos 15°)$

$a^2 = 126.45187$ $\quad a \approx 11.245082$

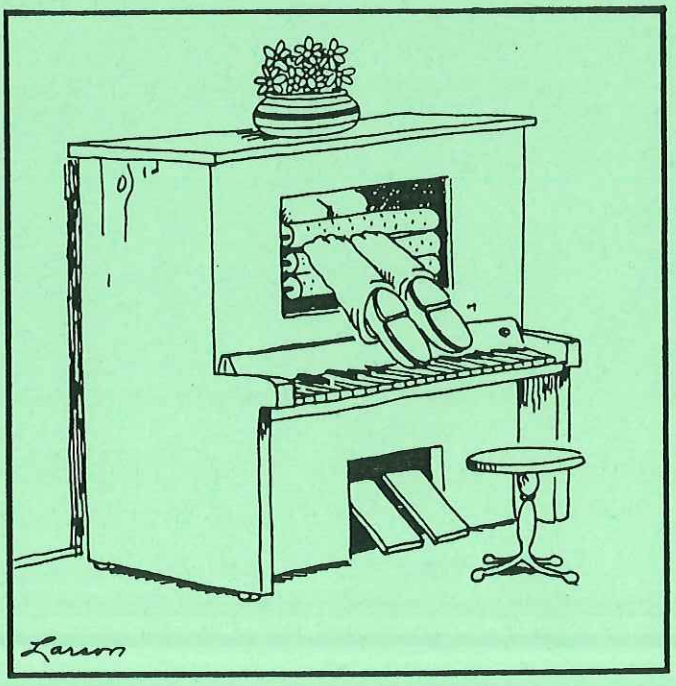
$\frac{\sin 15}{11.245082} = \frac{\sin C}{16}$

Calculator indicates $\angle C \approx 21.608297$

but $\angle C$ must be the largest angle.

use supplement $(180 - 21.608297)$

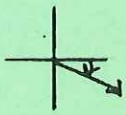
$\angle C \approx 158.3917$ **$\angle C \approx 158°24'$**



# Radians & The Unit Circle

## ANSWER KEY 16.1

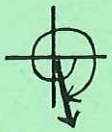
① II  $-240^\circ$

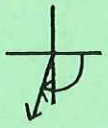
② IV  $-32^\circ$

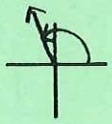
③ I  $440^\circ$

④ IV  $300^\circ$

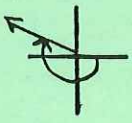
⑤ IV  $\frac{5}{3}\pi$

⑥ IV  $-\frac{12}{5}\pi$

⑦ III  $-\frac{4}{7}\pi$

⑧ II  $\frac{5}{9}\pi$

⑨ III  $945^\circ$

⑩ II  $-210^\circ$

⑪ $(330)(\pi/180) = \frac{11}{6}\pi$

⑫ $(-240)(\pi/180) = -\frac{4}{3}\pi$

⑬ $(270)(\pi/180) = \frac{3}{2}\pi$

⑭ $(-135)(\pi/180) = -\frac{3}{4}\pi$

⑮ $(180)(\pi/180) = \pi$

⑯ $(-210)(\pi/180) = -\frac{7}{6}\pi$

⑰ $(405)(\pi/180) = \frac{9}{4}\pi$

⑱ $(810)(\pi/180) = \frac{9}{2}\pi$

⑲ $(-315)(\pi/180) = -\frac{7}{4}\pi$

⑳ $(-270)(\pi/180) = -\frac{3}{2}\pi$

㉑ $(-\pi/4)(180/\pi) = -45^\circ$

㉒ $(3/4\,\pi)(180/\pi) = 135^\circ$

㉓ $(11/6\,\pi)(180/\pi) = 330^\circ$

㉔ $(7/4\,\pi)(180/\pi) = 315^\circ$

㉕ $(5)(180/\pi) = 900/\pi^\circ$

㉖ $(2)(180/\pi) = 360/\pi^\circ$

㉗ $(5\frac{1}{2})(180/\pi) = 990^\circ$

㉘ $(4\frac{1}{3})(180/\pi) = 780^\circ$

㉙ $(6\frac{1}{2}\pi)(180/\pi) = 1170^\circ$

㉚ $(3\frac{1}{3}\pi)(180/\pi) = 600^\circ$

㉛ $-940^\circ$ $\boxed{140^\circ}$

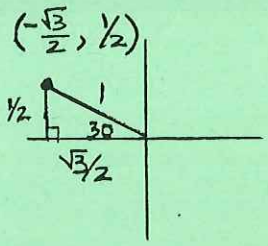
㉜ $-9/4\,\pi$ $\boxed{7/4\,\pi}$

㉝ $11\pi$ $\boxed{\pi}$

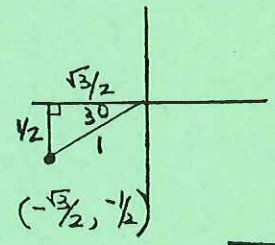
㉞ $-16/3\,\pi$ $\boxed{2/3\,\pi}$

# *Sine & Cosine Functions*

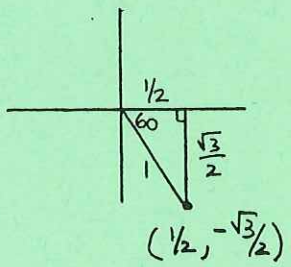
## ANSWER KEY 16.2

① 

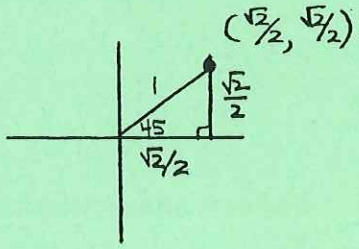

$\cos 150° = \boxed{-\frac{\sqrt{3}}{2}}$

② 

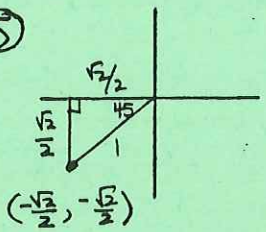

$\cos -150° = \boxed{-\frac{\sqrt{3}}{2}}$

③ 

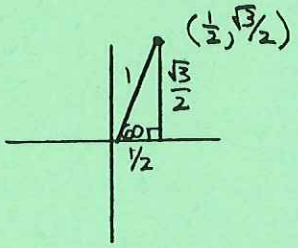

$\cos 1\frac{1}{3}\pi = \boxed{\frac{1}{2}}$

④ 

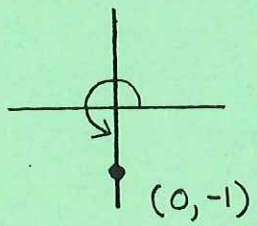

$\sin 1\frac{7}{4}\pi = \boxed{\frac{\sqrt{2}}{2}}$

⑤ 

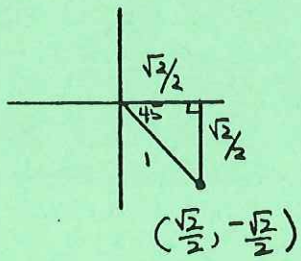

$\cos -\frac{3}{4}\pi = \boxed{\frac{-\sqrt{2}}{2}}$

⑥ 

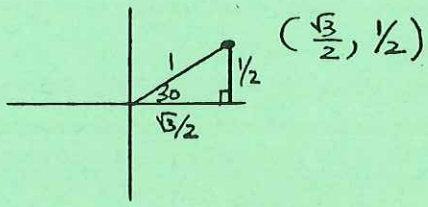

$\sin -5\frac{1}{3}\pi = \boxed{\frac{\sqrt{3}}{2}}$

⑦ 

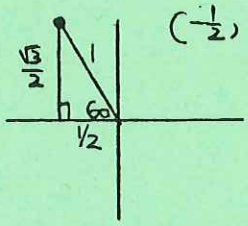

$\sin \frac{3\pi}{2} = \boxed{-1}$

⑧ 

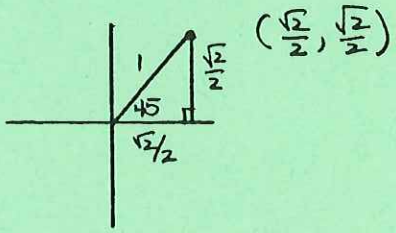

$\cos \frac{7}{4}\pi = \boxed{\frac{\sqrt{2}}{2}}$

⑨ 

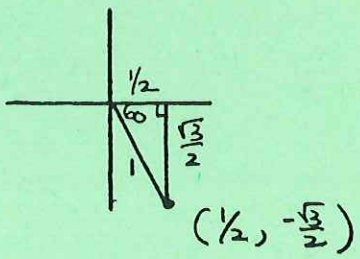

$\cos 390° = \boxed{\frac{\sqrt{3}}{2}}$

⑩ 


$\sin -240° = \boxed{\frac{\sqrt{3}}{2}}$

⑪ 


$\cos -1\frac{7}{4}\pi = \boxed{\frac{\sqrt{2}}{2}}$

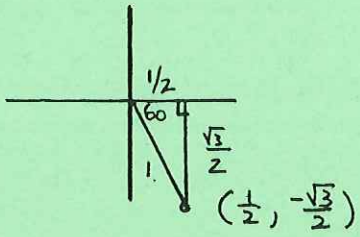
⑫ 


$\sin 660° = \boxed{-\frac{\sqrt{3}}{2}}$

# *Sine & Cosine Functions*

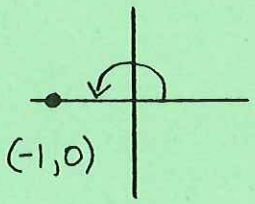
## ANSWER KEY 16.2

(13)



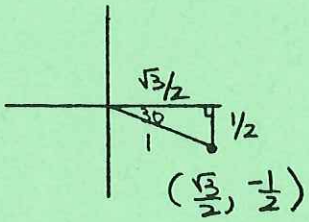

$\sin 300° = \boxed{\frac{-\sqrt{3}}{2}}$

(14)



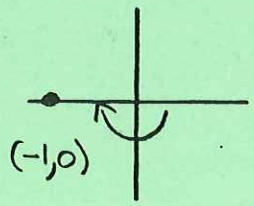

$\cos 900° = \boxed{-1}$

(15)



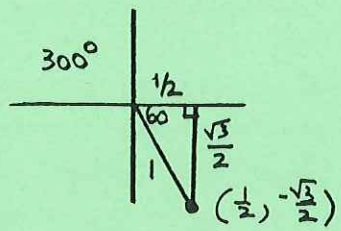

$\cos 330° = \boxed{\sqrt{3}/2}$

(16)

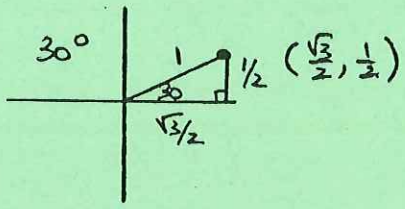



$\sin -180° = \boxed{0}$

(17)



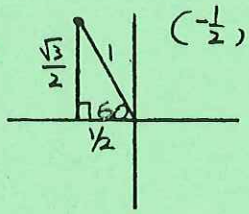

$\sin 300° = \frac{-\sqrt{3}}{2}$




$\sin 30° = 1/2$

$$\frac{4\sin 300° + 2\sin 30°}{3} = \frac{4(\frac{-\sqrt{3}}{2}) + 2(1/2)}{3} = \boxed{\frac{-2\sqrt{3}+1}{3}}$$

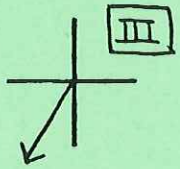
(18)



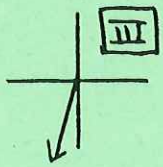

$8(\sin 120°)(\cos 120°)$

$8(\sqrt{3}/2)(-1/2) = \boxed{-2\sqrt{3}}$

(19) $-\frac{8}{3}\pi$




(20) $620°$




(21) $510°$

$(510)(\pi/180)$

$\boxed{17/6\,\pi}$

(22) $-630° = (-630)(\pi/180) = \boxed{-7/2\,\pi}$

(23) $\frac{4}{9}\pi = (\frac{4}{9}\pi)(\frac{180}{\pi}) = \boxed{80°}$

(24) $8/3 = (8/3)(180/\pi) = \boxed{\frac{480°}{\pi}}$

(25) $-1120° \rightarrow \boxed{320°}$

(26) $\frac{-17}{5}\pi \rightarrow \boxed{\frac{3}{5}\pi}$

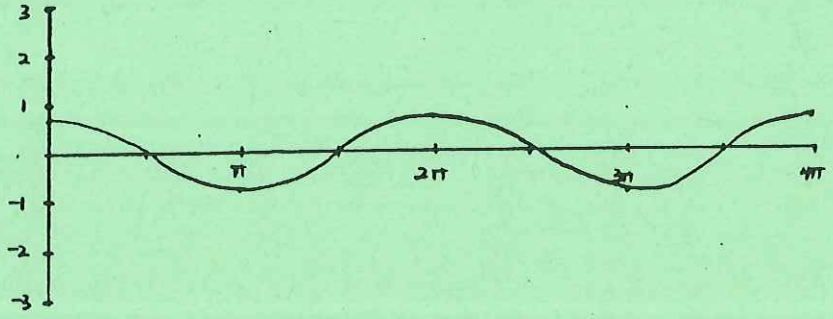
# *Graphing Sine & Cosine Curves*

## ANSWER KEY 16.3

① $y = \frac{2}{3}\cos\theta$

amplitude $= \frac{2}{3}$

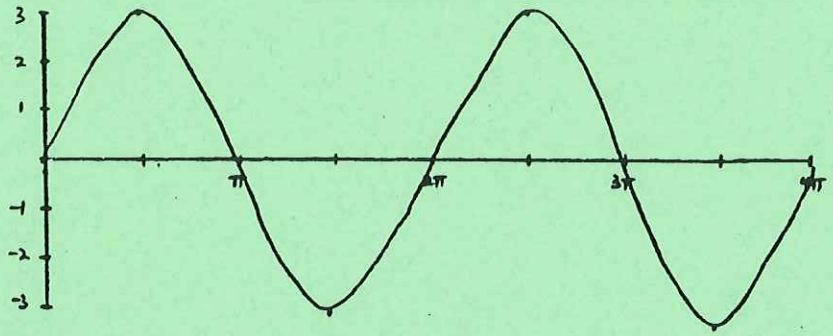
period $= 2\pi$




② $y = 3\sin\theta$

amplitude $= 3$

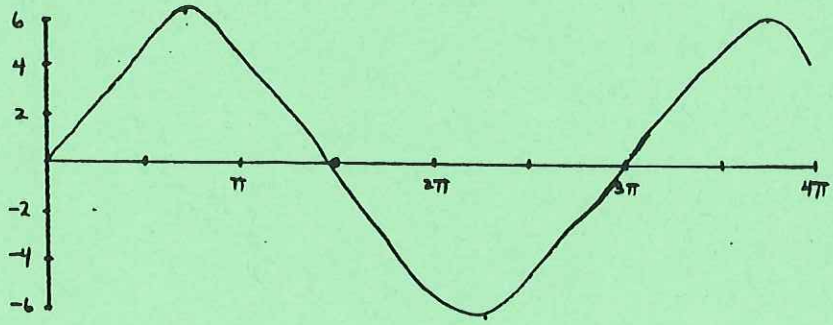
period $= 2\pi$




③ $y = 6\sin\frac{2}{3}\theta$

amplitude $= 6$

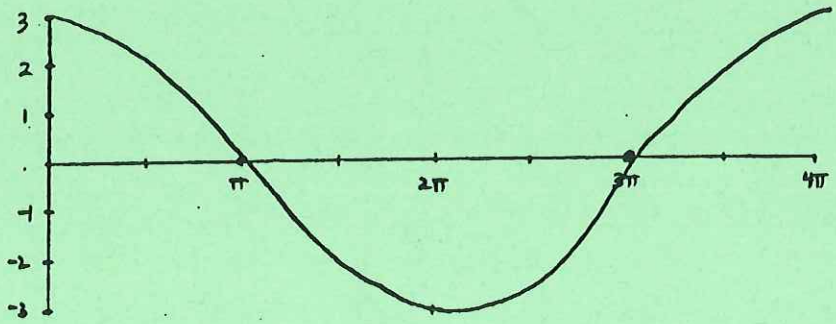
period $= \frac{2\pi}{\frac{2}{3}} = 3\pi$




④ $y = 3\cos\frac{1}{2}\theta$

amplitude $= 3$

period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$



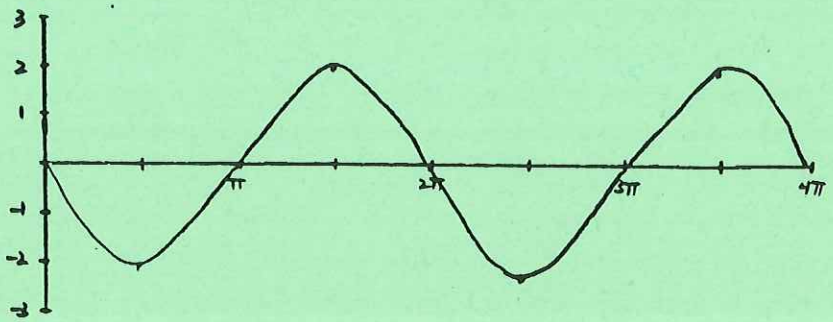
# *Graphing Sine & Cosine Curves*

## ANSWER KEY 16.3

⑤ $y = -2 \sin \theta$

amplitude $= |-2| = 2$

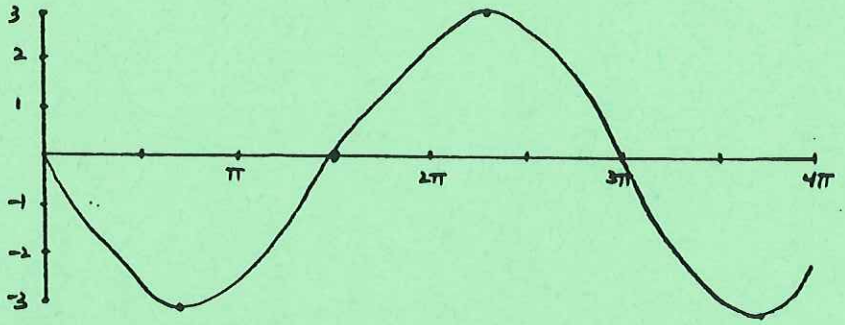
period $= 2\pi$




⑥ $y = -3 \sin \frac{2}{3}\theta$

amplitude $= |-3| = 3$

period $= \frac{2\pi}{\frac{2}{3}} = 3\pi$



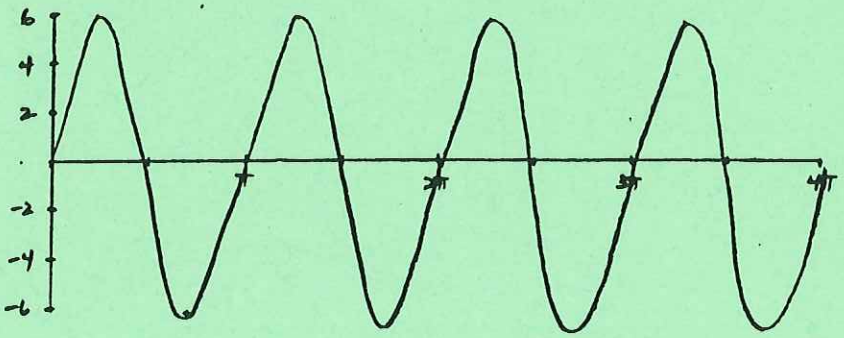

⑦ $\frac{1}{2}y = 3 \sin 2\theta$

$2\left[\frac{1}{2}y = 3 \sin 2\theta\right]$

$y = 6 \sin 2\theta$

amplitude $= 6$

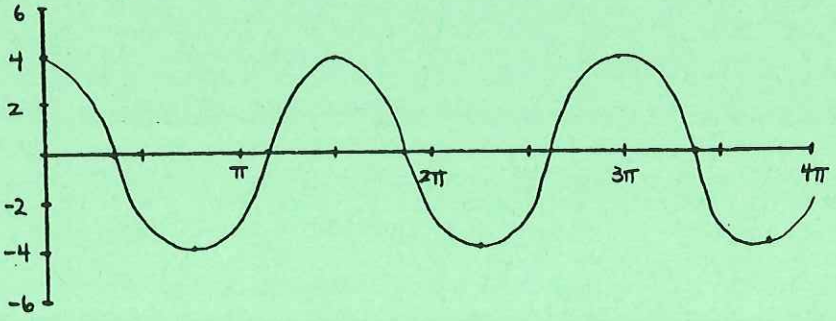
period $= \frac{2\pi}{2} = \pi$




⑧ $y = 4 \cos \frac{4}{3}\theta$

amplitude $= 4$

period $= \frac{2\pi}{\frac{4}{3}} = \frac{3\pi}{2}$



# Graphing Sine & Cosine Curves

## ANSWER KEY 16.3

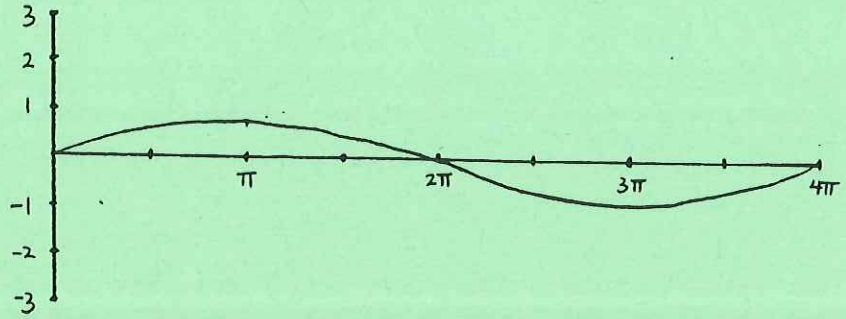
⑨ $3y = 2\sin\frac{1}{2}\theta$

$\frac{1}{3}\left[3y = 2\sin\frac{1}{2}\theta\right]$

$y = \frac{2}{3}\sin\frac{1}{2}\theta$

amplitude $= \frac{2}{3}$

period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$



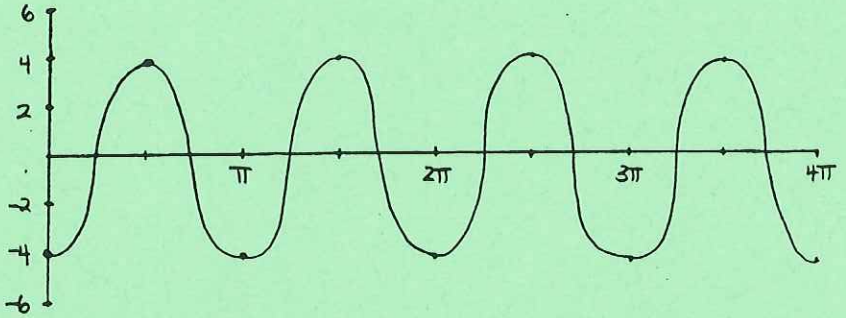

⑩ $\frac{1}{2}y = -2\cos 2\theta$

$2\left[\frac{1}{2}y = -2\cos 2\theta\right]$

$y = -4\cos 2\theta$

amplitude $= |-4| = 4$

period $= \frac{2\pi}{2} = \pi$




⑪ $(80)(\pi/180) = \boxed{\frac{4}{9}\pi}$

⑫ $(-540)(\pi/180) = \boxed{-3\pi}$

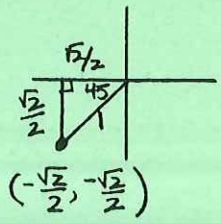
⑬ $(-\frac{5}{9}\pi)(180/\pi) = \boxed{-100°}$

⑭ $(12)(180/\pi) = \boxed{2160/\pi°}$

⑮ $-835 \rightarrow \boxed{245°}$

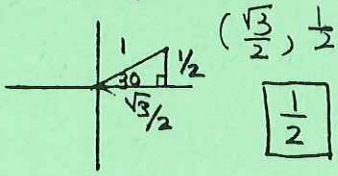
⑯ $9/2\,\pi \rightarrow \boxed{1/2\,\pi}$

⑰ $\cos 225°$




$\boxed{-\frac{\sqrt{2}}{2}}$

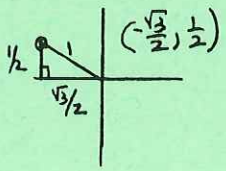
⑱ $\sin^{-1}1/6\,\pi$




$\boxed{\frac{1}{2}}$

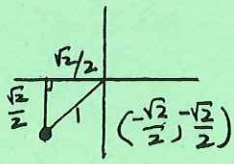
⑲ $\dfrac{3\sin 150° - \cos 225°}{\sin 210°}$

$\sin 150°$



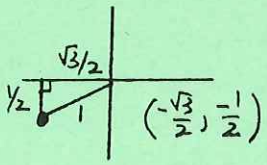

$\sin 150° = \frac{1}{2}$

$\cos 225°$




$\cos 225° = -\frac{\sqrt{2}}{2}$

$\sin 210°$



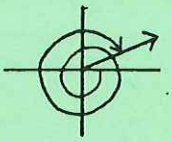

$\sin 210° = -\frac{1}{2}$

$$\frac{3(\frac{1}{2}) - (-\frac{\sqrt{2}}{2})}{-\frac{1}{2}} = \frac{\frac{3+\sqrt{2}}{2}}{-\frac{1}{2}} = \frac{6+2\sqrt{2}}{-2} = \boxed{-3-\sqrt{2}}$$

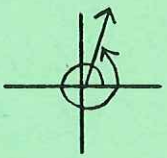
# *Trigonometric Functions*

## ANSWER KEY: UNIT 16 REVIEW & PRACTICE

① $-690°$ **I**



② $7/3\pi$ **I**



③ $(900)(\pi/180) = \boxed{5\pi}$

④ $(-240)(\pi/180) = \boxed{-4/3\pi}$

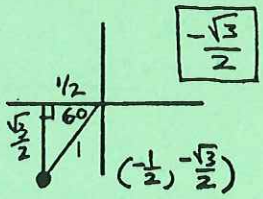
⑤ $(-5/6\pi)(180/\pi) = \boxed{-150°}$

⑥ $(4/3\pi)(180/\pi) = \boxed{240°}$

⑦ $-1250° = \boxed{190°}$

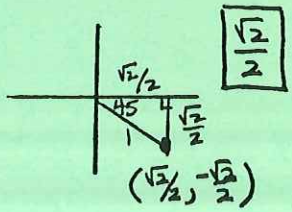
⑧ $-5/2\pi = \boxed{3/2\pi}$

⑨ $\sin 600°$



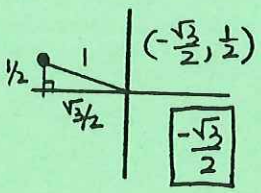

$\boxed{\frac{-\sqrt{3}}{2}}$

⑩ $\cos -405°$



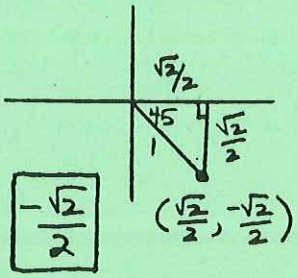

$\boxed{\frac{\sqrt{2}}{2}}$

⑪ $\cos 5/6\pi$



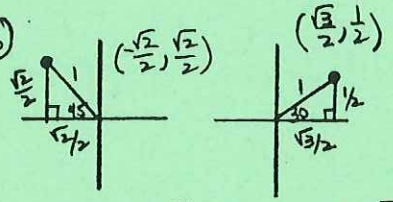

$\boxed{\frac{-\sqrt{3}}{2}}$

⑫ $\sin -17/4\pi$




$\boxed{\frac{-\sqrt{2}}{2}}$

⑬

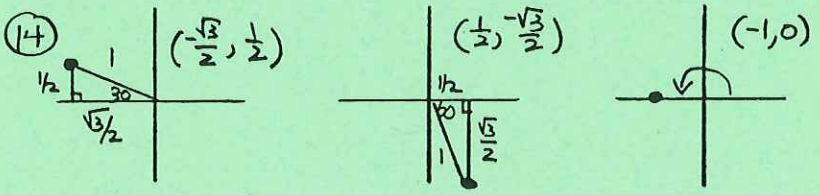



$\sin 135° = \frac{\sqrt{2}}{2}$ $\cos 390° = \frac{\sqrt{3}}{2}$

$$\frac{4\left(\frac{\sqrt{2}}{2}\right)-\left(\frac{\sqrt{3}}{2}\right)}{\frac{1}{2}}$$

$\boxed{4\sqrt{2}-\sqrt{3}}$

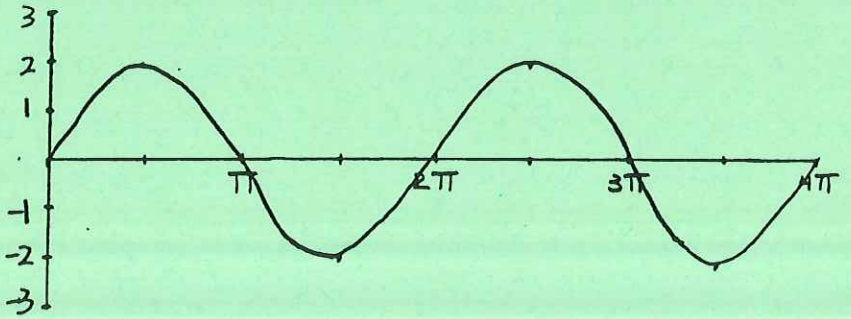
⑭




$\cos 150° = \frac{-\sqrt{3}}{2}$ $\sin 300° = \frac{-\sqrt{3}}{2}$ $\cos 180° = -1$

$$\frac{2\left(\frac{-\sqrt{3}}{2}\right)\left(\frac{-\sqrt{3}}{2}\right)}{(-1)} = \boxed{\frac{-3}{2}}$$

⑮ $y = 2\sin\theta$ amplitude $= 2$ period $= 2\pi$



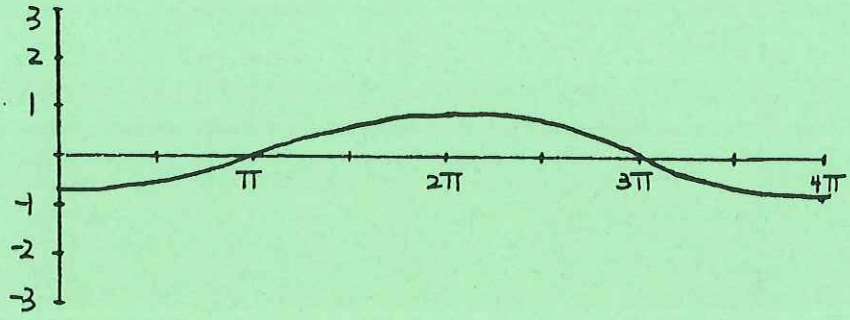
# *Trigonometric Functions*

## ANSWER KEY: UNIT 16 REVIEW & PRACTICE

(16) $y = -\frac{2}{3}\cos\frac{1}{2}\theta$

amplitude $= \frac{2}{3}$

period $= \frac{2\pi}{\frac{1}{2}} = 4\pi$

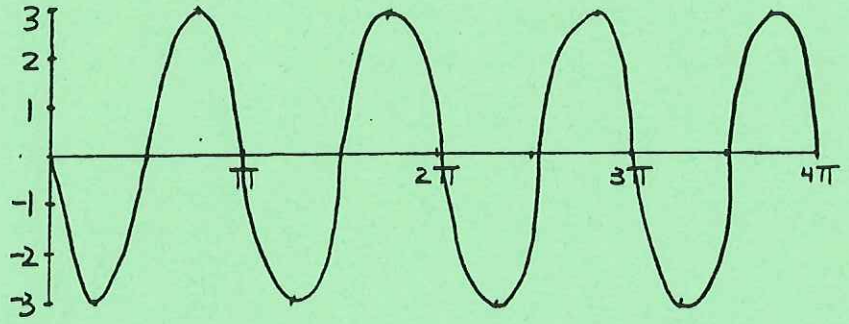



(17) $-2y = 6\sin 2\theta$

$y = -3\sin 2\theta$

amplitude $= 3$

period $= \frac{2\pi}{2} = \pi$

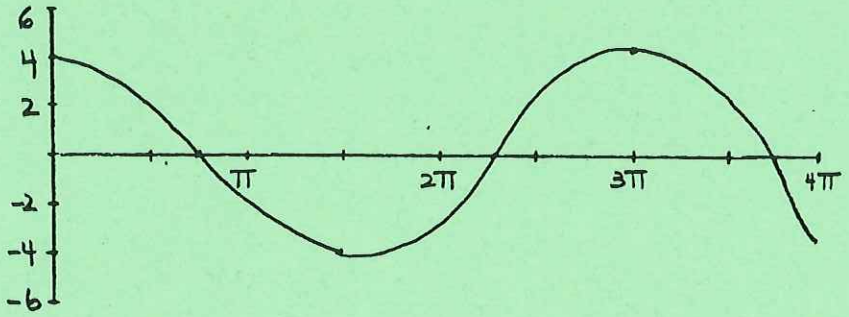


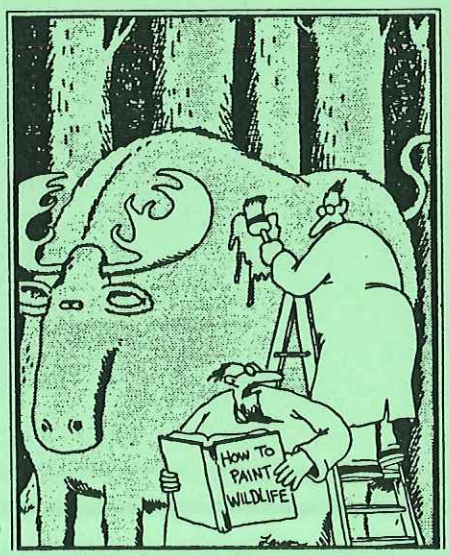

(18) $\frac{1}{2}y = 2\cos\frac{2}{3}\theta$

$y = 4\cos\frac{2}{3}\theta$

amplitude $= 4$

period $= \frac{2\pi}{\frac{2}{3}} = 3\pi$




"Hold on there, Dale. It says we should sand between coats."


"Hey! ... You kids!"