Friendship Junior High School
Accelerated Math Program
Mr. Lavine (Room 102A)

# A.T.I.M.

# Advanced Topics In Mathematics

UNIT 4
*Inequalities & Coordinate Graphing*

UNIT 5
*Linear Programming*

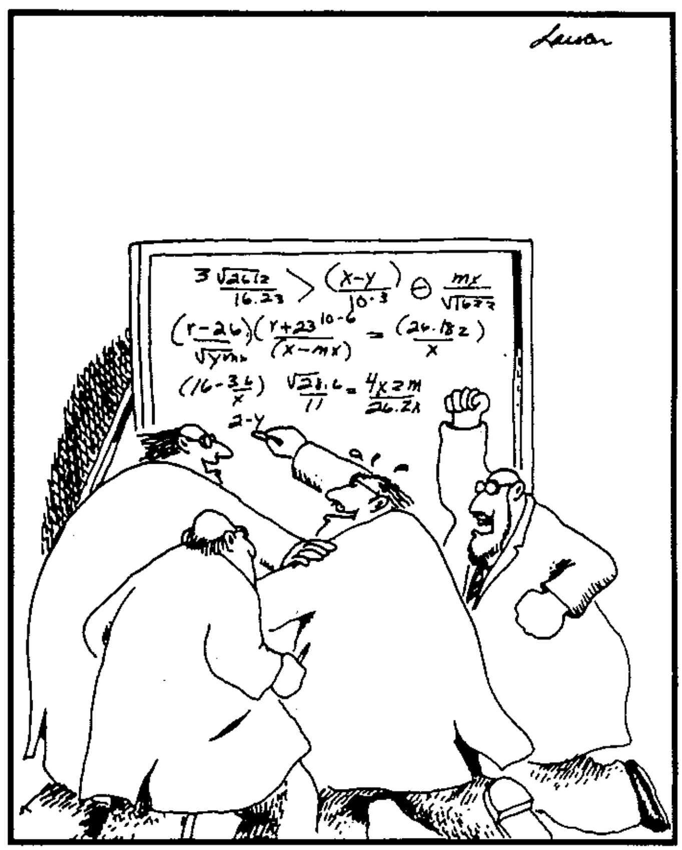
UNIT 6
*Roots & Radicals*

UNIT 7
*Rational Exponents & Complex Numbers*

UNIT 8
*Quadratics*

UNIT 9
*Rational Expressions*

UNIT 10
*Problem Solving*



**"Go for it, Sidney! You've got it! You've got it! Good hands! Don't choke!"**

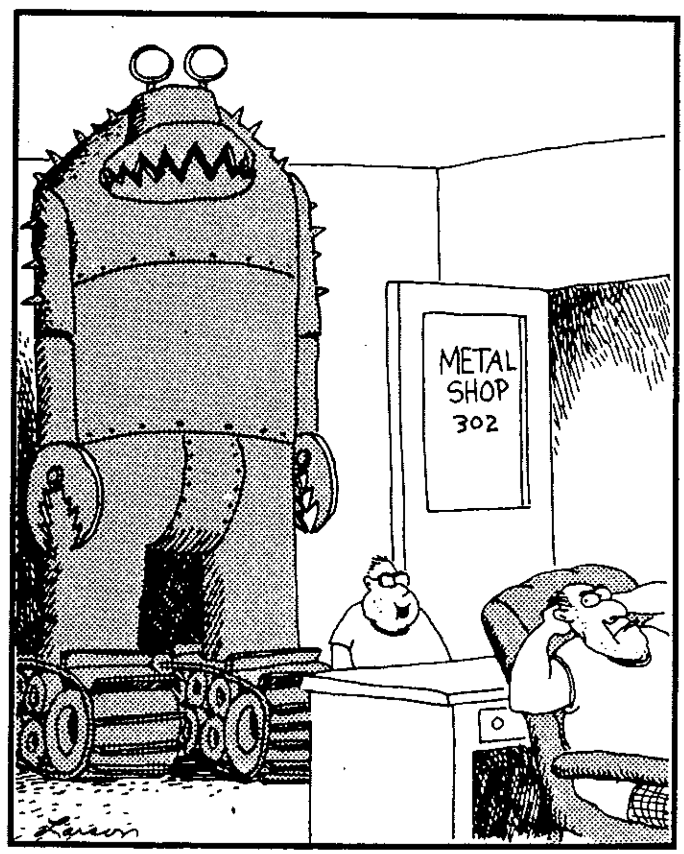
# UNIT 4

# *Inequalities & Coordinate Graphing*

4.1
*Inequalities & Absolute Value*

4.2
*Graphing Linear Equations*

4.3
*Graphing Systems of Inequalities*



"My project's ready for grading, Mr. Big Nose . . . Hey! I'm talkin' to YOU, squidbrain!"

# *Inequalities & Absolute Value*

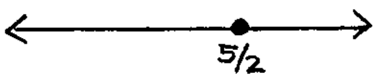
## DEMONSTRATION 4.1

Solve each equation and inequality. Graph the solution on a number line.

① $4(2n-3)=8$

$8n-12=8$

$8n=20$ $\boxed{n=5/2}$

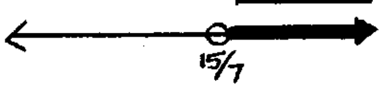



② $3-\frac{x}{3}<2(x-1)$

$3-\frac{x}{3}<2x-2$

$9-x<6x-6$

$-7x<-15$ $\boxed{x>15/7}$

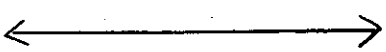



③ $3n-2>2(n-1)+n$

$3n-2>2n-2+n$

$3n-2>3n-2$

$-2>-2$ $\boxed{\text{no solutions}}$



④ $2(2x-2)>4(x-2)$

$4x-4>4x-8$

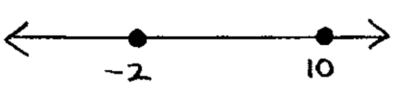
$-4>-8$ $\boxed{\text{all solutions}}$

⑤ Absolute Value Eq.

$|x-4|=6$

$x-4=6$ or $x-4=-6$

$\boxed{x=10 \text{ or } x=-2}$ union




⑥ Compound Inequality

$3<x+5\leq 11$

$3<x+5$ and $x+5\leq 11$ intersection

$-2<x$ and $x\leq 6$

$\boxed{-2<x\leq 6}$ Always write an intersection in compound form


⑦ Absolute Value - Intersection (<)

$|3n-2|+1\leq 5$

$|3n-2|\leq 4$

$3n-2\leq 4$ and $3n-2\geq -4$

$3n\leq 6$ and $3n\geq -2$

$n\leq 2$ and $n\geq -2/3$ $\boxed{-\frac{2}{3}\leq n\leq 2}$


⑧ Absolute Value - Union (>)

$|4x-2|>10$

$4x-2>10$ or $4x-2<-10$

$4x>12$ or $4x<-8$

$x>3$ or $x<-2$

$\boxed{x>3 \text{ or } x<-2}$

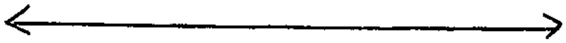
# *Inequalities & Absolute Value*

## DEMONSTRATION 4.1

⑨ Absolute Value – False Ineq.

$|4n+3|+3<1$

$|4n+3|<-2$ $\boxed{\text{no solutions}}$



Absolute Value must be $\geq 0$

⑩ Absolute Value – Identity

$|3a+1|+4\geq 2$

$|3a+1|\geq -2$ $\boxed{\text{all solutions}}$

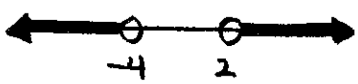
Absolute Value must be $\geq 0$

⑪ Intersection of Absolute Value Systems

$|n+1|>3$ <u>and</u> $|n|\neq 6$

$(n+1>3$ <u>or</u> $n+1<-3)$ <u>and</u> $(n\neq 6$ <u>and</u> $n\neq -6)$

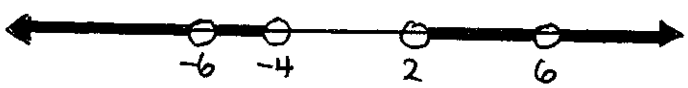
$\boxed{(n>2 \text{ or } n<-4) \quad \text{and} \quad (n\neq 6 \text{ and } n\neq -6)}$





<u>and</u>







⑫ Intersection of Absolute Value Systems

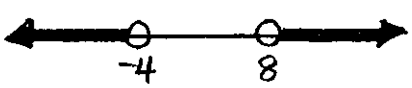
$6<|x-2|\leq 10$

$|x-2|>6$ <u>and</u> $|x-2|\leq 10$

$(x-2>6$ <u>or</u> $x-2<-6)$ <u>and</u> $(x-2\leq 10$ <u>and</u> $x-2\geq -10)$

$(x>8$ <u>or</u> $x<-4)$ <u>and</u> $(x\leq 12$ <u>and</u> $x\geq -8)$

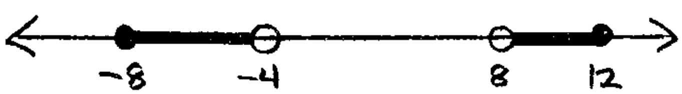
$\boxed{(x>8 \text{ or } x<-4) \quad \text{and} \quad (-8\leq x\leq 12)}$





<u>and</u>





# *Inequalities & Absolute Value*

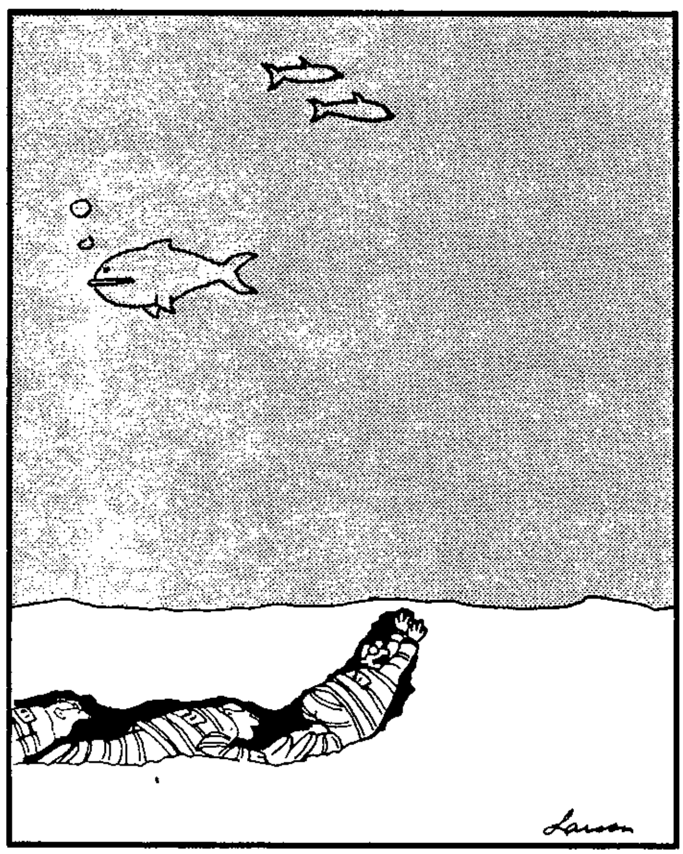
## PROBLEM SET 4.1

Solve and graph on a number line:

1. $3(2n-5) = 7$
2. $3x - \frac{2x}{5} = 3(x-1)$
3. $3n+1 < n-7$
4. $1+2(x+4) \geq 1+3(x+2)$
5. $2(m-5)-3(2m-5) < 5m+1$
6. $2n - \frac{n}{4} \leq 2(3n+1)$
7. $3n-9 > 3(n-1)-6$
8. $4(n-1) \leq 2n+2(n+3)$
9. $\frac{3x-3}{5} < \frac{2(2x-1)}{6}$
10. $-4 < 3x-1 < 8$

Solve and graph these absolute value inequalities on a number line:

11. $|n-4| \leq 8$
12. $|x+3| > 17$
13. $|4x-3|+3 \geq 15$
14. $6 + |3x| > 0$
15. $|2n-5| < 7$
16. $|6a+25| + 14 < 6$
17. $|n-3| < 5$ and $|n| \neq 1$
18. $|2n+1| > 3$ and $|n| \leq 5$
19. $4 < |2n-2| \leq 12$
20. $2 \leq |4n+2| < 18$



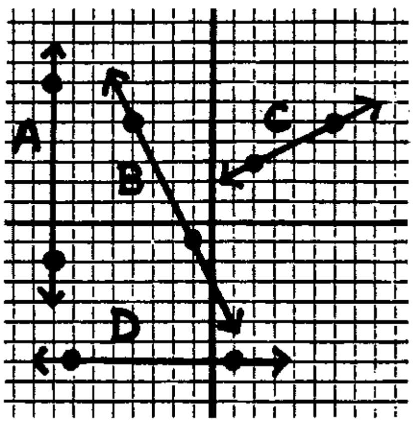
"We're almost free, everyone! . . . I just felt the first drop of rain."

# *Graphing Linear Equations*

## DEMONSTRATION 4.2

This lesson is a review of Algebra I work on linear equations.

### ① Slope




A = undef.

B = −2

C = ½

D = 0

Slope = rise over run

A line containing (-6, 2) and (−4, 10) has a slope of:

$$\frac{y_1 - y_2}{x_1 - x_2} = \frac{(2)-(10)}{(-6)-(-4)} = \frac{-8}{-2} = 4$$

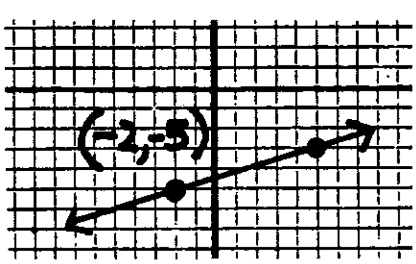
To graph a line given the slope and a point:

(−2, −5)  $m = 2/7$

First plot the point

Count up 2 (rise)

Count right 7 (run)




### ② Slope-Intercept Form

$\boxed{y = mx + b}$  $y = \frac{1}{2}x + 3$

| | | | |
|---|---|---|---|
| slope = | $m$ | $m = 1/2$ | |
| y-int = | $b$ | $b = 3$ | (0,3) |
| x-int = | $-b/m$ | $-b/m = -6$ | (-6,0) |

### ③ Standard Form

$\boxed{Ax + By = C}$  $x - 2y = -6$

| | | | |
|---|---|---|---|
| slope = | $-A/B$ | $-A/B = 1/2$ | |
| y-int = | $C/B$ | $C/B = 3$ | (0,3) |
| x-int = | $C/A$ | $C/A = -6$ | (-6,0) |

conditions: no fractions, "A" must be positive, GCF must be factored out

### ④ Point-Slope Form

$\boxed{y - y_1 = m(x - x_1)}$  $y - 1/3 = 2(x - 4)$

slope = $m$  $m = 2$

$(x_1, y_1)$ is on the line  $(4, 1/3)$ is on the line

To determine intercepts, you must change to another form

# Graphing Linear Equations

## DEMONSTRATION 4.2

⑤ Graphing Review

Horizontal line: $y =$
(no $x$ term)

Vertical line: $x =$
(no $y$ term)

Line through origin
(constant = 0)

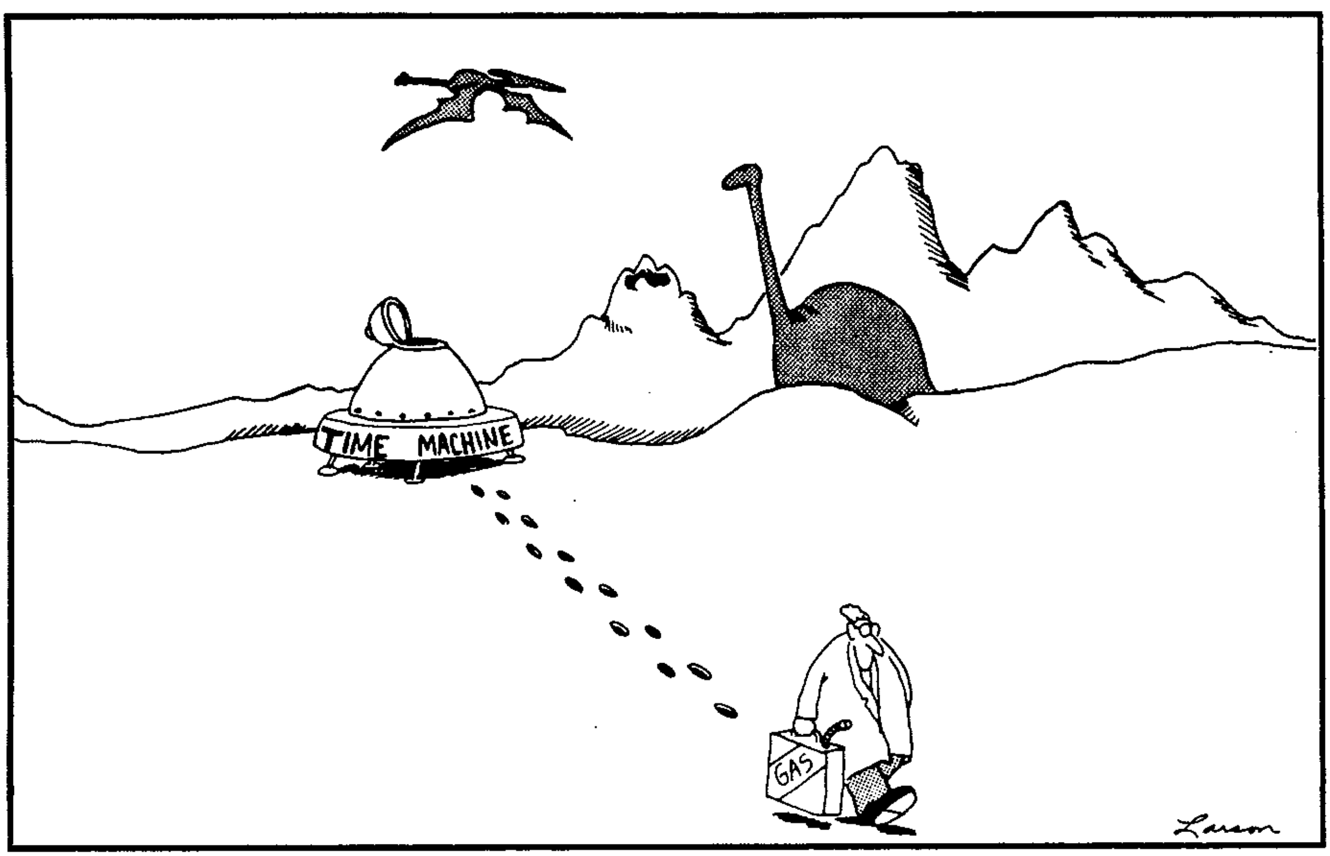
$(x, y)$ (horiz, vert)

Parallel lines (same slope)

Perpendicular lines
(slopes are negative reciprocals)

Positive slope
(slants upward to the right)

Negative slope
(slants downward to the right)



# Graphing Linear Equations

## PROBLEM SET 4.2

Write an equation for each:

1. Through $(-6,4)$ and $(2,-3)$ in slope-intercept form
2. Through $(5,-2)$ and $(1,-6)$ in standard form
3. Through $(4,9)$ and $(-5,-2)$ in point-slope form
4. Parallel to $y=\frac{2}{3}x+6$ through $(4,-2)$ in standard form
5. Parallel to $3x-4y=12$ through $(-6,3)$ in slope-intercept form
6. Parallel to $x-5y=6$ through $(6,4)$ in point-slope form
7. Perpendicular to $y=\frac{4}{5}x-12$ through $(0,6)$ in standard form
8. Perpendicular to $2x-y=8$ through $(9,-4)$ in slope-intercept form
9. Perpendicular to $3x-2y=8$ through $(-3,1)$ in point-slope form
10. Perpendicular to $x=4$ through $(6,-7)$ in any form
11. Parallel to $y=-4x-3$ through the origin in any form

Change to slope-intercept form:

12. $3x-y=4$
13. $x+3y=8$
14. $2x+5y=10$
15. $5x-3y=9$

Indicate the slope and both intercepts. Draw a graph for each:

16. $2x-3y=18$
17. $y=-\frac{2}{3}x+4$
18. $y+3=\frac{3}{4}(x-6)$
19. $3x+y=7$
20. $y=-\frac{1}{4}x-2$

Review Problems

Graph on a number line:

21. $2<|2x-4|<12$
22. $|n-3|>5$ and $|n|\le 12$

# *Graphing Systems of Inequalities*

## DEMONSTRATION 4.3

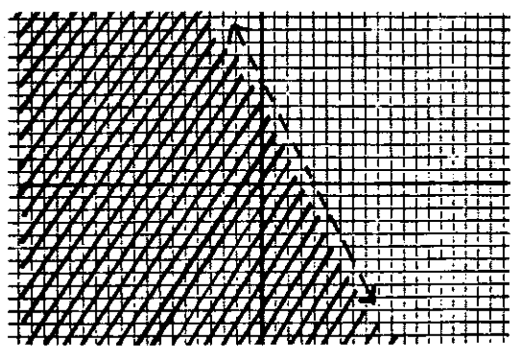
A system of inequalities involves shading in all solutions that satisfy the system.

### Graphing An Inequality

① $2x + y < 8$

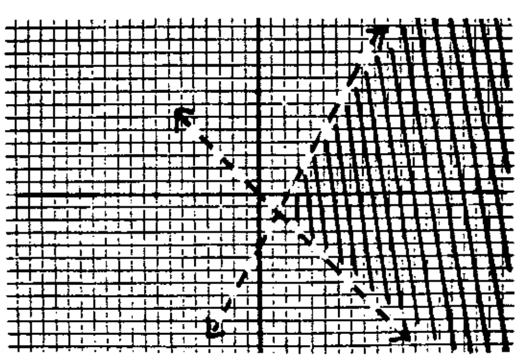
Note: Always change to slope-intercept form.

$y < -2x + 8$

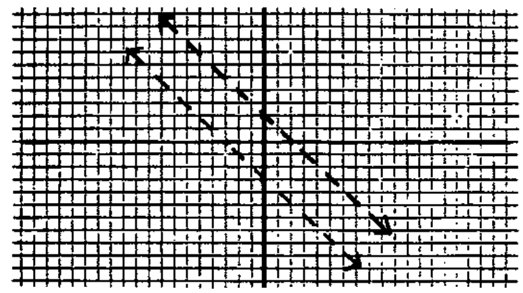


### Graphing A System

② $2x - y > 4 \longrightarrow y < 2x - 4$
$y > -x$



③ $y > -x + 2$
$y < -x - 3$

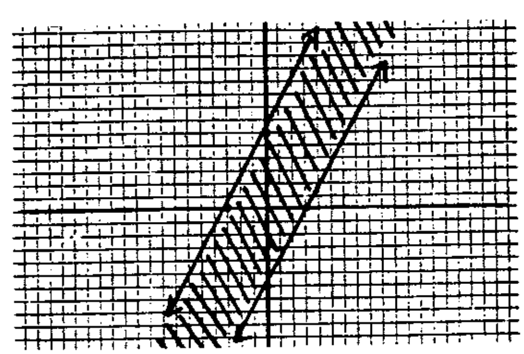


No solution for this system

### Absolute Value Systems

④ $|2x - y| \leq 6$

$2x - y \leq 6$ and $2x - y \geq -6$
$-y \leq -2x + 6$ and $-y \geq -2x - 6$
$y \geq 2x - 6$ and $y \leq 2x + 6$



Absolute value: $<, \leq$ indicates "intersection"

# *Graphing Systems of Inequalities*

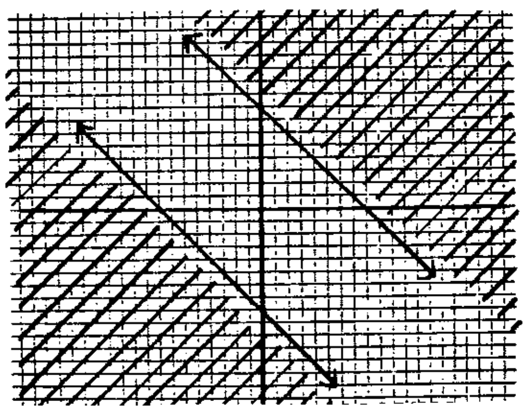
## DEMONSTRATION 4.3

⑤ Absolute Value : Union

$|x+y| \geq 8$

$x+y \geq 8$ or $x+y \leq -8$

$y \geq -x+8$ or $y \leq -x-8$



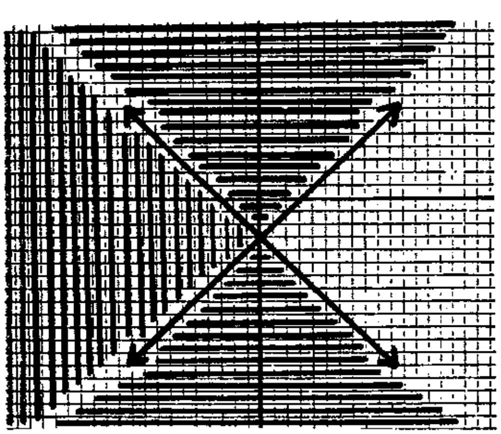
Absolute value $>, \geq$ indicates union (all points)

⑥ Absolute Value : Union

$|y+3| \geq x$

$y+3 \geq x$ or $y+3 \leq -x$

$y \geq x-3$ or $y \leq -x-3$



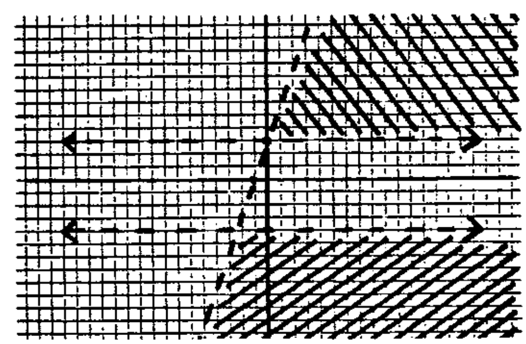
⑦ Intersection and Union

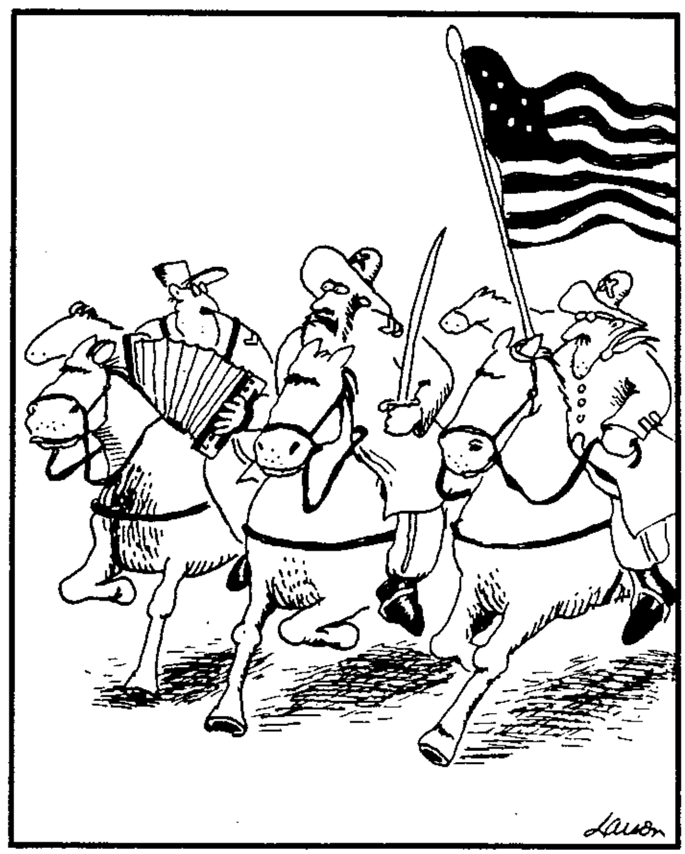
$|2y+1| > 7$ and $y < 3x+3$

$(2y+1 > 7$ or $2y+1 < -7)$ and $(y < 3x+3)$

$(2y > 6$ or $2y < -8)$ and $(y < 3x+3)$

$(y > 3$ or $y < -4)$ and $(y < 3x+3)$





"You heard me, Simmons! . . . You get that cursed bugle fixed!"

# *Graphing Systems of Inequalities*

## PROBLEM SET 4.3

Graph each inequality:

① $y \le 2x + 6$

② $2x - 3y < 12$

Graph each system:

③ $y \ge x - 3$
$y \ge -x - 3$

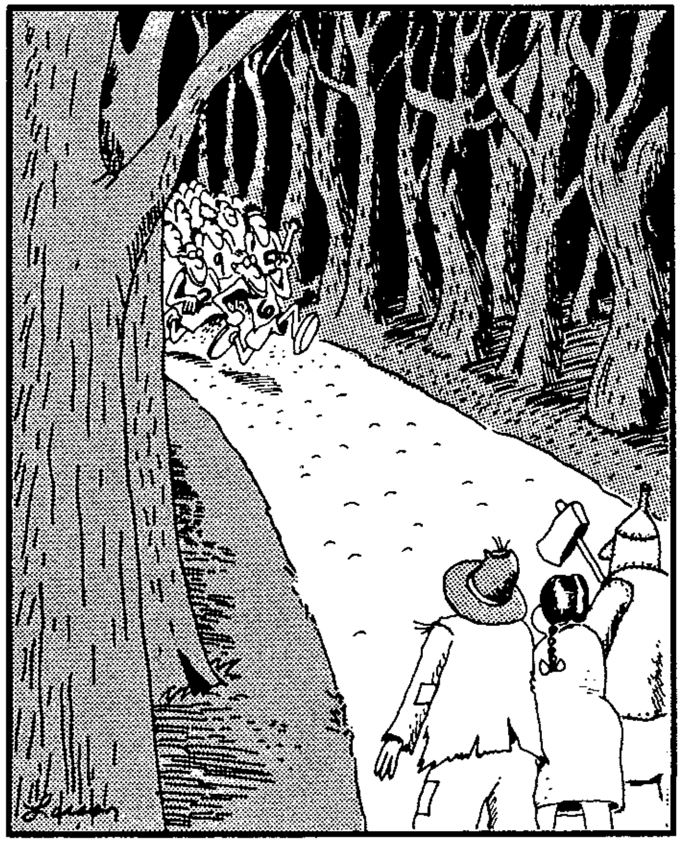
④ $y < -x - 3$
$x + 2 > y$

⑤ $x + y \le 3$
$2x - 3y \ge 6$

⑥ $x + 2y > 8$
$3x - 4y < 12$

⑦ $x + y > -3$
$x + y < 10$

⑧ $y > x + 1$
$y < x - 5$



⑨ $|y - x| \le 4$

⑩ $|x - 1| < y$

⑪ $|2x + y| > 4$

⑫ $|x| \ge y$

⑬ $|x| \le 5$
$x + y < 6$

⑭ $|y| < 4$
$2x - y < 6$

⑮ $|y| > 3$
$3x - y < 6$

⑯ $|y - x| > 3$
$|y| < 10$

# *Inequalities & Coordinate Graphing*

## UNIT 4 REVIEW & PRACTICE

Solve and graph on a number line:

1. $2(x+4) + \frac{x}{3} < 4x - 1$
2. $\frac{x+3}{4} \geq \frac{2(x-3)}{3}$
3. $3(2x-3) \geq 2(3x+2)$
4. $4(x-1) < 4x+5$
5. $-1 < n+4 < 9$
6. $|3x+6| > 12$
7. $3 \leq |n-4| \leq 10$
8. $|2x+1| > 5$ and $|x| < 10$

Linear equations:

9. Write an equation in slope-intercept form for a line parallel to $5x-3y=4$ through $(6,-2)$. Identify the intercepts.
10. Write an equation in standard form for a line perpendicular to $y=-\frac{3}{4}x+7$ through $(2,5)$. Identify the intercepts.
11. Write an equation in point-slope form for a line through $(-2,6)$ and $(10,-1)$.
12. Change $2x+3y=-4$ to slope-intercept form

Graph the inequality:

13. $3x - y < 8$

Graph the system:

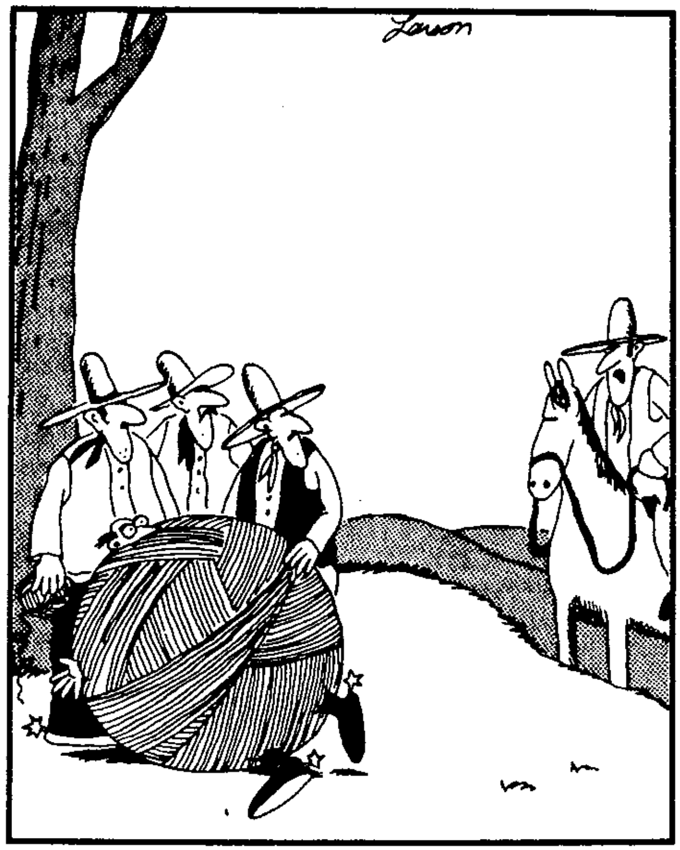
14. $2x - y > 6$  
    $y > -2x$
15. $|3x - y| \leq 4$
16. $|x - y| > 5$
17. $|x| \leq 4$  
    $x - 2y \leq 6$
18. $|y-2| > 8$  
    $2x + y < 6$
19. $|x+1| > 14$  
    $|y-2| > 10$

# UNIT 5

# *Linear Programming*

5.1
*Graphing Systems*

5.2
*Problem Solving*




"Hang him, you idiots! Hang him! . . . 'String-him-up' is a figure of speech!"

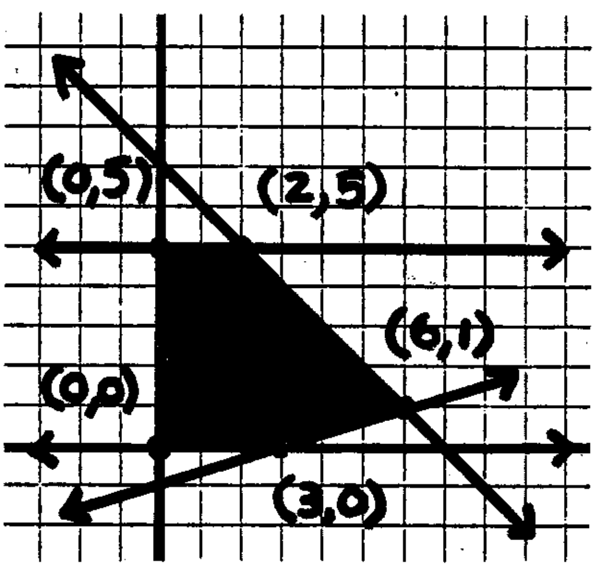
# *Graphing Systems*

## DEMONSTRATION 5.1

Linear programming is a procedure for finding the maximum or minimum value of a function in two variables subject to given conditions on the variables.

① Function:
$f(x,y) = 5x - 3y$

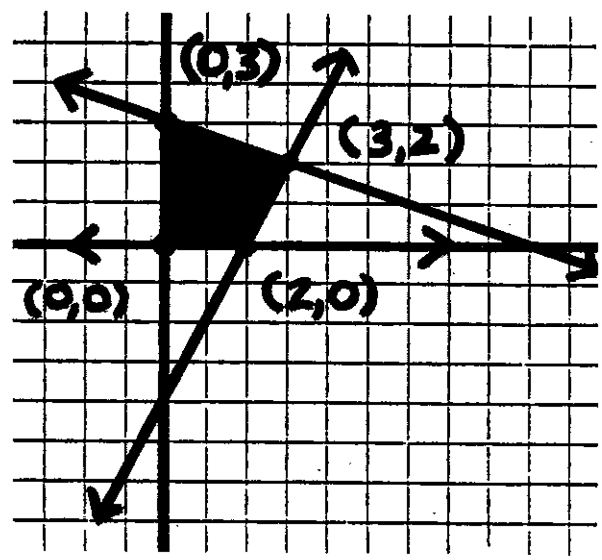
Conditions:
$x \geq 0$
$x + y \leq 7 \longrightarrow y \leq -x + 7$
$0 \leq y \leq 5$
$3y \geq x - 3 \longrightarrow y \geq \frac{1}{3}x - 1$




| | | | |
|---|---|---|---|
| $\boxed{(0,5)}$ | $5(0) - 3(5) =$ | $\boxed{-15}$ | min |
| (0,0) | $5(0) - 3(0) =$ | 0 | |
| (3,0) | $5(3) - 3(0) =$ | 15 | |
| $\boxed{(6,1)}$ | $5(6) - 3(1) =$ | $\boxed{27}$ | max |
| (2,5) | $5(2) - 3(5) =$ | −5 | |

② Function:
$F(x,y) = x + 2y$

Conditions:
$x \geq 0$
$y \geq 0$
$2x - y \leq 4 \longrightarrow y \geq 2x - 4$
$x + 3y \leq 9 \longrightarrow y \leq -\frac{1}{3}x + 3$




| | | | |
|---|---|---|---|
| $\boxed{(0,0)}$ | $(0) + 2(0) =$ | $\boxed{0}$ | min |
| (2,0) | $(2) + 2(0) =$ | 2 | |
| $\boxed{(3,2)}$ | $(3) + 2(2) =$ | $\boxed{7}$ | max |
| (0,3) | $(0) + 2(3) =$ | 6 | |

The max. and min. points are always among the vertices of the shaded region. To find non-integer vertices, use a system.

# *Graphing Systems*

## PROBLEM SET 5.1

Graph each system of inequalities. Identify the vertices of the polygon. Find the maximum and minimum values:

① Function:
$f(x,y) = 3x - 2y$

Conditions:
$y \geq 2$
$1 \leq x \leq 5$
$y \leq x + 3$

② Function:
$f(x,y) = 4x + 3y$

Conditions:
$x + y \geq 2$
$4y \leq x + 8$
$y \geq 2x - 5$

③ Function:
$f(x,y) = 2x - 3y$

Conditions:
$y \leq 7$
$y \leq x + 4$
$y \geq -x + 6$
$x \leq 5$

④ Function:
$f(x,y) = x - 2y$

Conditions:
$y \leq x + 5$
$y \geq x$
$x \geq -3$
$y + 2x \leq 5$

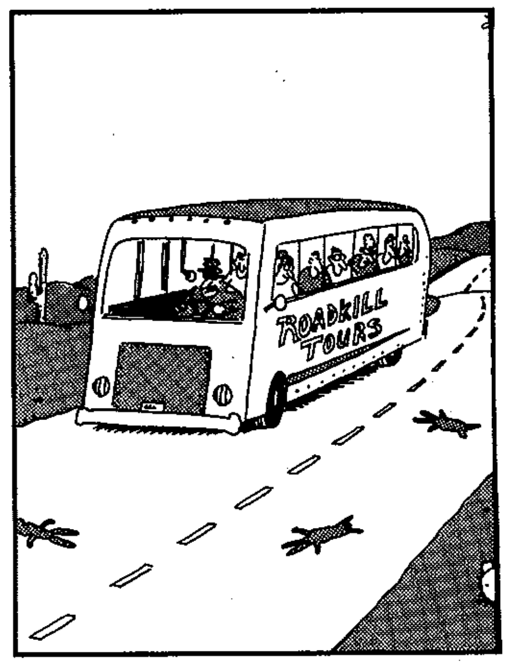
⑤ Function:
$f(x,y) = 3x + y$

Conditions:
$y \leq x + 6$
$y + 2x \geq 6$
$2 \leq x \leq 6$

⑥ Function:
$f(x,y) = 3y + x$

Conditions:
$x + y \geq 2$
$4y \leq x + 8$
$2y \geq 3x - 6$

Use "large" graph sheets for this assignment to make it easier to identify vertices.




"As you can see, most of these things are jackrabbits, but keep your eyes peeled for armadillo as well. ...We're about five miles now from the dead steer."

Be sure to use elimination or substitution to find non-integer vertices in problems #2 and #4. In all problems, identify both the ordered pair and the function value for the maximum and minimum.

# *Problem Solving*

## DEMONSTRATION 5.2

Use linear programming to solve:

① The Blair Company makes two kinds of pianos: spinets and consoles. The equipment in the factory allows for making at most 450 spinets and 200 consoles in one month. During June, the company can spend $360,000 to make its allotment of pianos (see chart). How many of each type should be made to maximize profits? What will the profit be?

| | Unit Cost | Unit Profit |
|---|---|---|
| Spinets | $600 | $125 |
| Consoles | $900 | $200 |

$x$ = Spinets  $y$ = Consoles

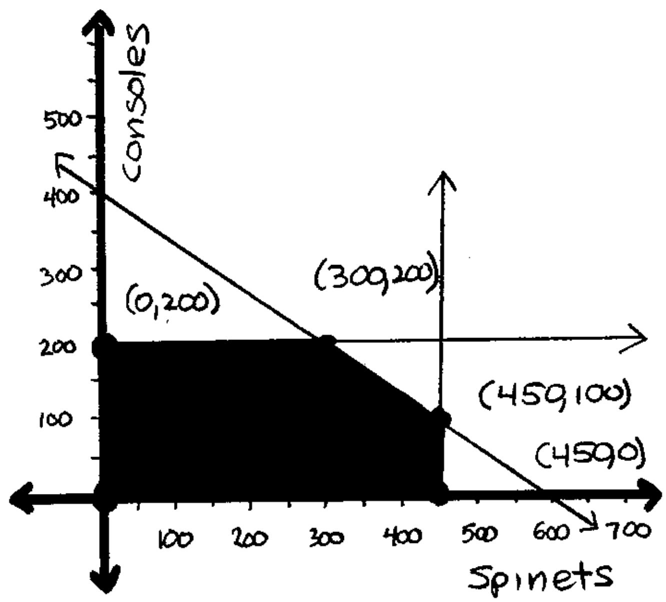
$x \leq 450$
$y \leq 200$
$600x + 900y \leq 360{,}000$

In slope-intercept form:

$x \geq 0$
$y \geq 0$
$y \leq -\frac{2}{3}x + 400$

Graph the inequalities:

Establish numbers on the axis that are practical:




Function:
$f(x,y) = 125x + 200y$

| | | |
|---|---|---|
| (0,200) | $125(0) + 200(200) = 40{,}000$ |
| * (300,200) | $125(300) + 200(200) = 77{,}500$ |
| (450,100) | $125(450) + 200(100) = 76{,}250$ |
| (450,0) | $125(450) + 200(0) = 56{,}250$ |
| (0,0) | $125(0) + 200(0) = 0$ |

To maximize profits:

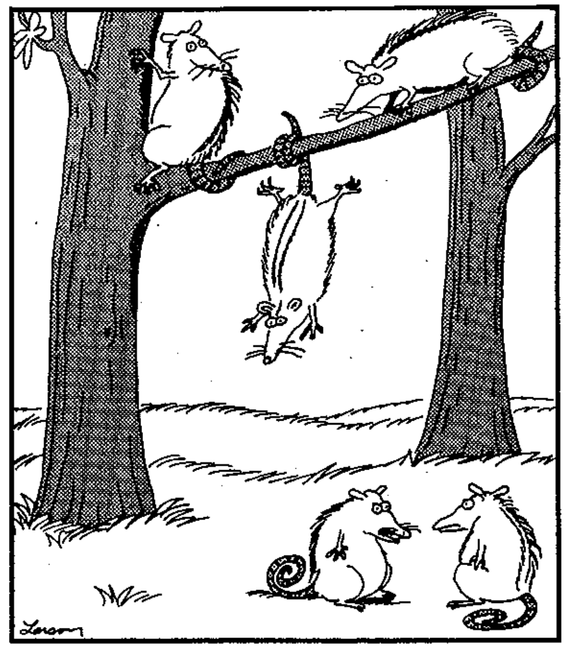
300 spinets
200 consoles

$77,500 profit

# *Problem Solving*

## DEMONSTRATION 5.2

② A painter has exactly 32 units of yellow dye and 54 units of green dye. He plans to mix as many gallons as possible of Color A and Color B. Each gallon of Color A requires 4 units of yellow dye and 1 unit of green dye. Each gallon of Color B requires 1 unit of yellow dye and 6 units of green dye.



"My dad can act deader than your dad."

$x$ = gallons of Color A
$y$ = gallons of Color B

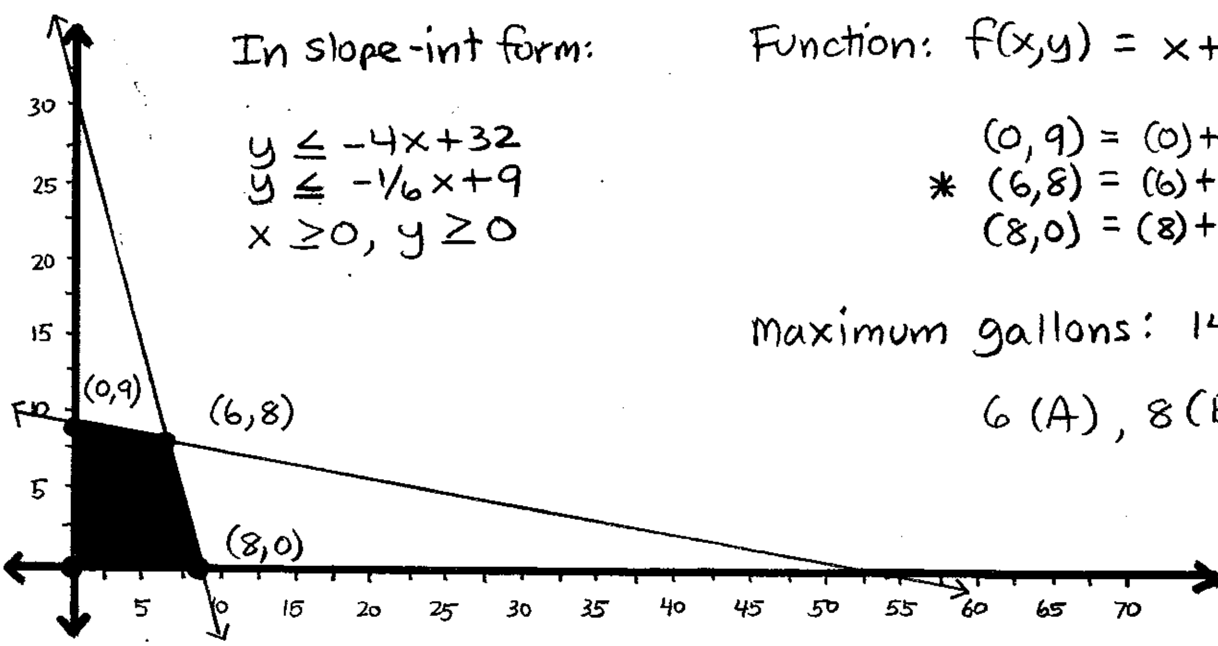
Conditions:
$x \geq 0$
$y \geq 0$
yellow: $4x + y \leq 32$
green: $x + 6y \leq 54$

Note: To find vertex (6,8) use a system

$$4x + y = 32$$
$$x + 6y = 54$$

In slope-int form:

$y \leq -4x + 32$
$y \leq -\frac{1}{6}x + 9$
$x \geq 0,\ y \geq 0$




Function: $f(x,y) = x + y$

$(0,9) = (0) + (9) = 9$
∗ $(6,8) = (6) + (8) = 14$
$(8,0) = (8) + (0) = 8$

Maximum gallons: 14

6 (A), 8 (B)

# *Problem Solving*

## PROBLEM SET 5.2

Use linear programming to solve:

① The area of a parking lot is 600 $m^2$. A car requires 6 $m^2$ and a bus requires 30 $m^2$. The attendant can handle no more than 60 vehicles. If a car is charged \$2.50 and a bus \$7.50, how many of each should be accepted to maximize profit?

② A dress making shop makes dresses and pantsuits. The equipment in the shop allows for making at most 30 dresses and 20 pantsuits in one week. It takes 10 worker hours to make a dress and 20 hours to make a pantsuit. There are 500 worker hours available per week.

How many of each should be made to maximize profit:

a) if the profit is \$40 for both dresses and pantsuits?

b) if the profit for a dress is \$15 and a pantsuit \$45?

③ Fashion Furniture makes two kinds of chairs: rockers and swivels. Two operations are used in construction. A rocker requires 2 hours of Operation A and 3 hours of Operation B. Each swivel requires 4 hours of A and 1 hour of B. Operation A may be used at most 20 hours per day. Operation B is limited to 15 hours per day. The company makes \$12 profit on each rocker and \$10 on each swivel. How many of each should be made per day to maximize profit?

④ Recreation Unlimited produces footballs and basketballs. Each football requires 4 hours on machine A and 2 hours on machine B. Each basketball requires 6 hours on A and 6 hours on B and 1 hour on C. Each machine has limits on its use:

Machine A is available 120 hrs.
Machine B is available 72 hrs.
Machine C is available 10 hrs.

If the company makes \$3 profit on a football and \$2 on a basketball, how

# *Problem Solving*

## PROBLEM SET 5.2

many of each should be produced to maximize profit?

⑤ A builder has 60 lots on which he can build houses with one house on each lot. He builds two types of houses: colonial and ranch. Sales experience has taught him that he should plan to build at least 3 times as many ranch-style houses as colonial. If he makes a profit of \$5000 on each colonial and \$4500 on each ranch, how many of each kind should be built to maximize profit?

⑥ The Ajax Furniture Company makes two kinds of wood table legs, one plain and one fancy. Each plain leg takes 2 hours on a lathe and 1 hour of sanding. Each fancy leg takes 1 hour on a lathe and 4 hours of sanding. The <u>4</u> lathes in the factory can be operated up to 11½ hours each per day. The <u>6</u> sanding machines can be operated up to 12 hours each per day. Each plain leg nets a \$3 profit and each fancy leg nets a \$5 profit. If the company can sell all the table legs it produces, how many of each kind should be produced each day to maximize profits?


That night, their revenge was meted out on both Farmer O'Malley and his wife. The next day, police investigators found a scene that they could describe only as "grisly, yet strangely hilarious."

# *Linear Programming*

## UNIT 5 REVIEW & PRACTICE

Graph each system of inequalities. Identify the vertices. Find the max. and min. values:

① Function: $f(x,y)=5x-3y$

Conditions:
$y \geq 0$
$0 \leq x \leq 5$
$-x+y \leq 2$
$x+y \leq 6$

② Function: $f(x,y)=2x+4y$

Conditions:
$x \geq 0$
$y \geq 0$
$x+y \leq 3$
$3x+y \leq 6$

Solve each problem by establishing a graph based on a system of inequalities subject to conditions. Use a function to evaluate:

③ A carpenter makes bookcases in 2 sizes: large and small. It takes 4 hrs. to make a large one and 2 hrs. for a small one. The carpenter can spend up to 24 hours per week working on them. He makes a profit of \$50 on a large bookcase and \$20 on a small one. To satisfy customers, he must make at least 2 of each size per week. How many of each size per week should be made to maximize profit?

④ A tire manufacturer has 800 units of rubber to use in producing radial tires and tractor tires. Each radial tire requires 5 units of rubber while each tractor tire requires 20 units. Labor costs are \$12 for a radial and \$12 for a tractor tire. The company budgets a maximum of \$1200 for labor. Customer needs require the company to make at least 40 radial tires. If each radial results in a \$10 profit and each tractor tire results in a \$25 profit, how many of each should the company make to maximize profit?

Note: On the unit test, points are awarded for the graph as well as the answer.

# UNIT 6

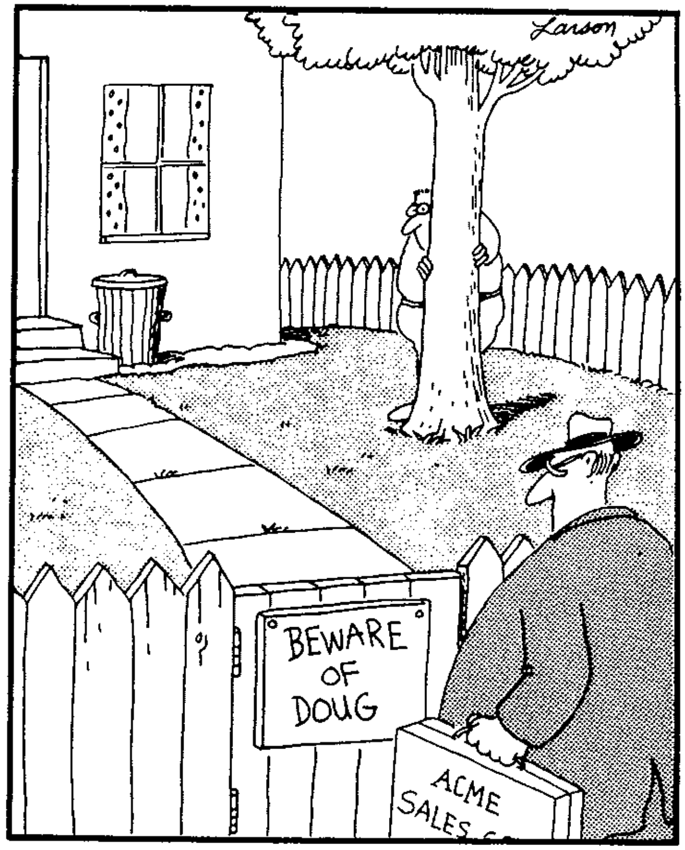
# *Roots & Radicals*

6.1
*Simplifying Radicals*

6.2
*Radical Operations*

6.3
*Dividing & Rationalizing*

6.4
*Radical Equations*



# *Simplifying Radicals*

## DEMONSTRATION 6.1

Over the past two years, you have learned how to simplify radicals with an index of 2. This lesson extends the procedure to include index values greater than 2.

Terms

radical sign

$\sqrt[3]{216}$

index    radicand

$\sqrt[3]{216} = 6$    principal root

Simplify each radical:

① $\sqrt{100}$

$\sqrt{100} = 10$

② $\sqrt{32x^2}$

$\sqrt{32x^2} = 4|x|\sqrt{2}$

③ $\sqrt[3]{8x^4y^6}$

$\sqrt[3]{8x^4y^6} = 2xy^2\sqrt[3]{x}$

④ $\sqrt[3]{-24a^3b^7}$

$\sqrt[3]{-24a^3b^7} = -2ab^2\sqrt[3]{3b}$

⑤ $\sqrt[4]{a^5b^9c^8}$

$\sqrt[4]{a^5b^9c^8} = |a|b^2c^2\sqrt[4]{ab}$

⑥ $\sqrt[4]{x^7y^5z^8}$

$\sqrt[4]{x^7y^5z^8} = xyz^2\sqrt[4]{x^3y}$

⑦ $\sqrt{(a+b)^2}$

$\sqrt{(a+b)^2} = |a+b|$

⑧ $\sqrt{n^2-14n+49}$

$\sqrt{n^2-14n+49} = |n-7|$

⑨ $\sqrt[3]{(2x-1)^3}$

$\sqrt[3]{(2x-1)^3} = 2x-1$

⑩ $\sqrt[3]{-.001a^6b^4}$

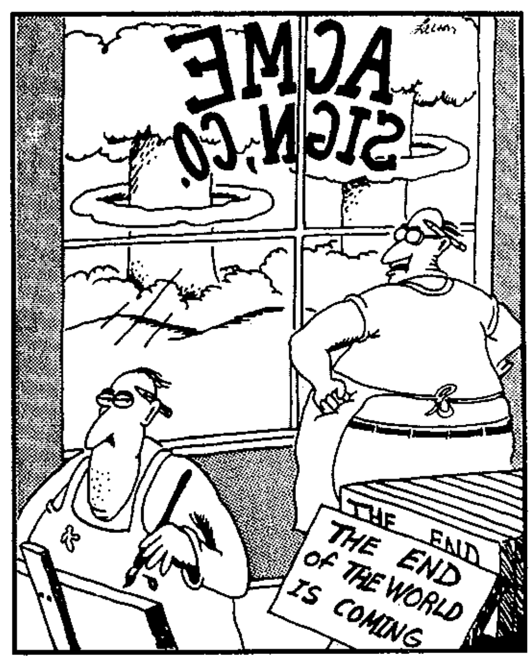
$\sqrt[3]{-.001a^6b^4} = -.1a^2b\sqrt[3]{b}$

# *Simplifying Radicals*

## PROBLEM SET 6.1

Simplify:

1. $\sqrt{169}$
2. $\sqrt{225}$
3. $\sqrt{121n^2}$
4. $-\sqrt{144x^6}$
5. $-\sqrt{64a^2b^4}$
6. $\sqrt{121b^2c^6}$
7. $\sqrt{80a^4b^3}$
8. $\sqrt{60x^3y^2z^4}$
9. $-\sqrt{48a^8b^7c^5}$
10. $\sqrt{12a^9b^7c^4}$
11. $\sqrt[3]{-8b^3m^3}$
12. $\sqrt[3]{-27n^3s^6}$
13. $\sqrt[3]{64a^7b^4}$
14. $\sqrt[3]{80x^4y^9}$
15. $\sqrt[4]{48a^4b^5}$
16. $\sqrt[4]{162a^6b^9c^5}$
17. $\sqrt{(x+y)^2}$
18. $\sqrt{(3a+b)^2}$
19. $\sqrt[3]{(x+y)^3}$
20. $\sqrt{(x+3)^4}$
21. $\sqrt[5]{(2n-3)^5}$
22. $\sqrt{x^2+10x+25}$
23. $\sqrt{n^2+6n+9}$
24. $\sqrt[3]{.008n^4}$
25. $\sqrt[5]{-32}$
26. $\sqrt[4]{a^6b^5c^3d^2}$
27. $\sqrt[3]{x^4y^5z^7}$
28. $\sqrt[6]{a^7b^9c^{11}}$
29. $\sqrt[7]{-x^{10}y^{15}}$
30. $-\sqrt{9x^8y^6}$
31. $\sqrt[3]{-1}$
32. $\sqrt[4]{a^4b^4}$
33. $\sqrt[4]{x^4y^8z^5}$
34. $\sqrt[4]{n^9m^{10}s^5}$
35. $\sqrt[6]{x^9y^{13}z^{12}}$
36. $\sqrt[3]{a^4b^7c^{12}}$
37. $\sqrt[4]{x^5y^8z^{13}}$
38. $\sqrt[5]{a^8b^6c}$
39. $\sqrt[4]{a^{10}b^{13}c^9}$
40. $\sqrt{a^3b^6c^2d^5}$




"Wouldn't you know it! . . . There goes our market for those things!"

# *Radical Operations*

## DEMONSTRATION 6.2

Multiplication: Radicands can only be multiplied if they have the same index. Addition / Subtraction: Radicals can only be combined if they are "like terms" – having the same index and radicand.

### Absolute Value

① $\sqrt{x} \cdot \sqrt{x} = \sqrt{x^2} = x$

No absolute value is needed because each radicand must be positive in the original multiplication.

② $\sqrt[4]{2m} \cdot \sqrt[4]{5m^3}$

$\sqrt[4]{2m} \cdot \sqrt[4]{5m^3} = \sqrt[4]{10m^4} = m\sqrt[4]{10}$

③ $\sqrt[4]{16x^2y} \cdot \sqrt[4]{96x^4y^5}$

$\sqrt[4]{2^4x^2y} \cdot \sqrt[4]{2^5 \cdot 3 \cdot x^4y^5}$

$\sqrt[4]{2^9 \cdot 3x^6y^6} = 4|x|y\sqrt[4]{6x^2y^2}$

x could be negative, y cannot

### Distributive Property

④ $\sqrt{x}\,(\sqrt{x} - \sqrt{y})$

$\sqrt{x}\,(\sqrt{x} - \sqrt{y}) = x - \sqrt{xy}$

⑤ $5\sqrt{27} + 2\sqrt{3} - 7\sqrt{48}$

$5\sqrt{3^3} + 2\sqrt{3} - 7\sqrt{2^4 \cdot 3}$

$15\sqrt{3} + 2\sqrt{3} - 28\sqrt{3} = -11\sqrt{3}$

⑥ $\sqrt[3]{40a} + \sqrt[3]{135a}$

$\sqrt[3]{2^3 \cdot 5a} + \sqrt[3]{3^3 \cdot 5a}$

$2\sqrt[3]{5a} + 3\sqrt[3]{5a} = 5\sqrt[3]{5a}$

⑦ $\sqrt[4]{2} - \sqrt[4]{2x^4}$

$\sqrt[4]{2} - |x|\sqrt[4]{2} = (1-|x|)\sqrt[4]{2}$

must have parenthesis

### Multiply With FOIL

⑧ $(6+\sqrt{2})(\sqrt{10}+\sqrt{5})$

$6\sqrt{10} + 6\sqrt{5} + \sqrt{20} + \sqrt{10}$

$6\sqrt{10} + 6\sqrt{5} + 2\sqrt{5} + \sqrt{10}$

$7\sqrt{10} + 8\sqrt{5}$

# *Radical Operations*

## DEMONSTRATION 6.2

Multiplying Binomial Conjugates

⑨ $(\sqrt{6}+\sqrt{3})(\sqrt{6}-\sqrt{3})$

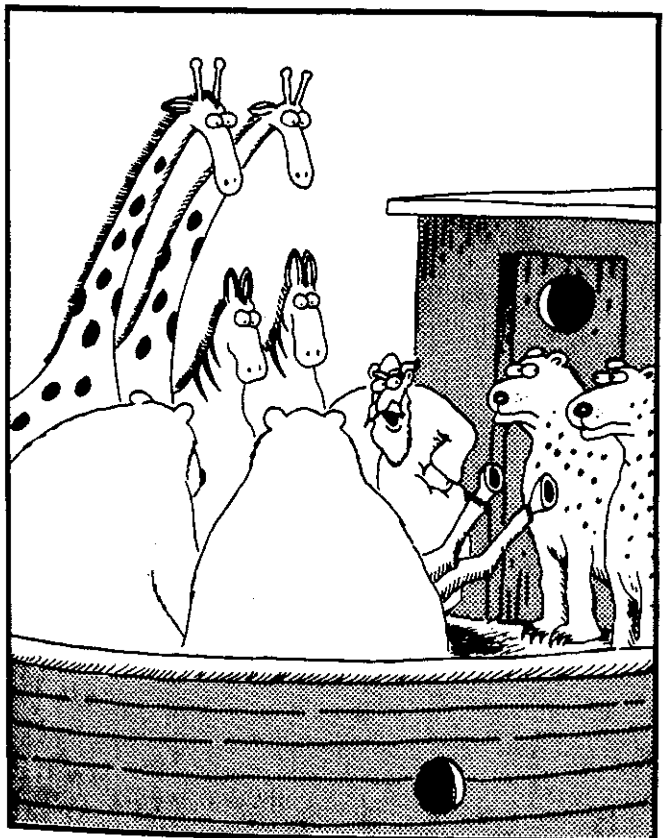
$(\sqrt{6})^2-(\sqrt{3})^2 = 6-3 = 3$

Conjugates are binomials with the sum and difference of identical terms.

The product is the difference of their squares.

$(\sqrt{2}-2\sqrt{5})(\sqrt{2}+2\sqrt{5})$

$(\sqrt{2})^2-(2\sqrt{5})^2 = 2-20 = -18$



"Well, so much for the unicorns . . . But from now on, all carnivores will be confined to 'C' deck."

The Product of a Binomial and Trinomial

⑩ $(x-\sqrt[3]{y})(x^2+x\sqrt[3]{y}+\sqrt[3]{y^2})$

Use the column method

$$\begin{array}{rrrr} x^3 & +\,x^2\sqrt[3]{y} & +\,x\sqrt[3]{y^2} & \\ & -\,x^2\sqrt[3]{y} & -\,x\sqrt[3]{y^2} & -\,\sqrt[3]{y^3} \\ \hline x^3 & & & -\,\sqrt[3]{y^3} \end{array}$$

$x^3-\sqrt[3]{y^3}$

$x^3-y$

# *Radical Operations*

## PROBLEM SET 6.2

Multiply and simplify:

1. $\sqrt[4]{5m^3b^5} \cdot \sqrt[4]{125m^2b^3}$
2. $\sqrt[4]{3b^6r^7} \cdot \sqrt[4]{81b^2r^2}$
3. $\sqrt[3]{54r^4s^3} \cdot \sqrt[3]{16rs}$
4. $\sqrt{125m^2n} \cdot \sqrt{32m^4n^6}$
5. $\sqrt[4]{32a^5b^3} \cdot \sqrt[4]{162a^3b^2}$
6. $\sqrt{r}\,(\sqrt{r} + r\sqrt{5})$
7. $\sqrt{b}\,(b + a\sqrt{b})$
8. $\sqrt{m}\,(\sqrt{p} + \sqrt{mq})$

Compute and simplify:

9. $8\sqrt[3]{2a} + 3\sqrt[3]{2a} - 8\sqrt[3]{2a}$
10. $\sqrt[3]{54} - \sqrt[3]{128x^3}$
11. $\sqrt[4]{x^2} + \sqrt[4]{x^6}$
12. $-\sqrt{2x^2y^4} + \sqrt{8x^2y^4}$
13. $(7 + \sqrt{11p})(7 - \sqrt{11p})$
14. $(\sqrt{3} + \sqrt{5})(\sqrt{3} - \sqrt{5})$
15. $(3 - \sqrt[3]{4})(\sqrt[3]{2} + \sqrt[3]{16})$
16. $(4 - \sqrt[3]{9})(\sqrt[3]{3} + \sqrt[3]{81})$
17. $(y + \sqrt[3]{4})(y^2 - y\sqrt[3]{4} + \sqrt[3]{16})$
18. $(x - \sqrt[3]{3})(x^2 + x\sqrt[3]{3} + \sqrt[3]{9})$
19. $(m + \sqrt[3]{a})(m^2 - m\sqrt[3]{a} + \sqrt[3]{a^2})$
20. $(2 + \sqrt[3]{K})(4 - 2\sqrt[3]{K} + \sqrt[3]{K^2})$

Late at night, and without permission, Reuben would often enter the nursery and conduct experiments in static electricity.

# *Dividing & Rationalizing*

## DEMONSTRATION 6.3

### Simplified Radicals

Conditions:
1. No fractions in radicand
2. No radicals in denominator

### Dividing Radicals

① $\frac{\sqrt{15}}{\sqrt{5}} = \sqrt{\frac{15}{5}} = \sqrt{3}$

② $\sqrt[3]{\frac{3}{8}} = \frac{\sqrt[3]{3}}{\sqrt[3]{8}} = \frac{\sqrt[3]{3}}{2}$

### Rationalizing

③ $\frac{3}{2\sqrt{5}}$

$\frac{3}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{10}$

④ $\sqrt{\frac{5}{n}}$

$\sqrt{\frac{5}{n}} = \frac{\sqrt{5}}{\sqrt{n}} \cdot \frac{\sqrt{n}}{\sqrt{n}} = \frac{\sqrt{5n}}{n}$

⑤ $\sqrt[3]{\frac{5}{3b}}$

$\sqrt[3]{\frac{5}{3b}} = \frac{\sqrt[3]{5}}{\sqrt[3]{3b}} \cdot \frac{\sqrt[3]{3^2b^2}}{\sqrt[3]{3^2b^2}} = \frac{\sqrt[3]{45b^2}}{3b}$

### Multiply By Conjugate

⑥ $\frac{3\sqrt{2}}{2-\sqrt{6}}$

$\frac{3\sqrt{2}}{2-\sqrt{6}} \cdot \frac{2+\sqrt{6}}{2+\sqrt{6}} = \frac{6\sqrt{2}+3\sqrt{12}}{4-6}$

$\frac{6\sqrt{2}+6\sqrt{3}}{-2} = -3\sqrt{2}-3\sqrt{3}$

⑦ $\frac{3-\sqrt{3}}{6+2\sqrt{3}}$

$\frac{3-\sqrt{3}}{6+2\sqrt{3}} \cdot \frac{6-2\sqrt{3}}{6-2\sqrt{3}} = \frac{18-6\sqrt{3}-6\sqrt{3}+6}{36-12}$

$\frac{24-12\sqrt{3}}{24} = \frac{2-\sqrt{3}}{2}$

### Radical Expressions

⑧ $\sqrt{\frac{1}{5}} + \sqrt{20} + \sqrt{75}$

$\frac{\sqrt{1}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} + 2\sqrt{5} + 5\sqrt{3}$

$\frac{\sqrt{5}}{5} + \frac{10\sqrt{5}}{5} + 5\sqrt{3}$

$\frac{11\sqrt{5}}{5} + 5\sqrt{3}$

# *Dividing & Rationalizing*

## PROBLEM SET 6.3

Rationalize and simplify:

1. $\dfrac{\sqrt{12}}{\sqrt{3}}$
2. $\dfrac{\sqrt{14}}{\sqrt{2}}$
3. $\dfrac{\sqrt[3]{81}}{\sqrt[3]{9}}$
4. $\dfrac{\sqrt[3]{54}}{\sqrt[3]{6}}$
5. $\sqrt{\dfrac{8}{9}}$
6. $\sqrt{\dfrac{21}{12}}$
7. $\sqrt[4]{\dfrac{5}{16}}$
8. $\sqrt[4]{\dfrac{7}{81}}$
9. $\sqrt[3]{\dfrac{5}{9p^2}}$
10. $\sqrt[3]{\dfrac{9}{4m^2}}$
11. $\sqrt[4]{\dfrac{2}{3}}$
12. $\sqrt[4]{\dfrac{3}{2}}$
13. $\dfrac{1}{3+\sqrt{5}}$
14. $\dfrac{3}{5-\sqrt{2}}$
15. $\dfrac{2}{3-\sqrt{5}}$
16. $\dfrac{7}{4-\sqrt{3}}$
17. $\dfrac{1+\sqrt{2}}{3-\sqrt{2}}$
18. $\dfrac{2+\sqrt{6}}{2-\sqrt{6}}$
19. $\dfrac{2-\sqrt{3}}{5+3\sqrt{3}}$
20. $\dfrac{3+4\sqrt{5}}{5+2\sqrt{5}}$
21. $\dfrac{\sqrt{x+1}}{\sqrt{x-1}}$
22. $\dfrac{m+\sqrt{a}}{2\sqrt{a}-p}$
23. $\sqrt{\dfrac{2}{5}}+\sqrt{40}+\sqrt{10}$
24. $\sqrt[3]{\dfrac{1}{4}}+\sqrt[3]{54}-\sqrt[3]{16}$

### Review Problems

Simplify:

25. $\sqrt[4]{32a^6b^7c^9}$
26. $\sqrt[3]{108x^5y^6z^4}$
27. $\sqrt{8}+\sqrt{2x^2y^4}$
28. $\sqrt[4]{12a^3b^3c^6}\cdot\sqrt[4]{12a^2bc^8}$

# *Radical Equations*

## DEMONSTRATION 6.4

If a variable appears in the radicand of a radical equation, you must check all possible solutions. Some will not be valid.

### One Radical, One Solution

① $3-\sqrt{x-2}=0$ Isolate radical

$3=\sqrt{x-2}$ Square both sides

$9=x-2$

$x=11$ Check solution

### No Solution

② $7+\sqrt{a-3}=1$

$\sqrt{a-3}=-6$ No solution

The principal root $\neq$ neg.

### Two Radicals in One Equation

③ $\sqrt{y+12}+1=\sqrt{y+21}$

Square both sides

$(y+12)+2\sqrt{y+12}+1=y+21$

$2\sqrt{y+12}=8$

$\sqrt{y+12}=4$ sq. both sides

$y+12=16$

$y=4$ Solution checks

### Cubed Root

④ $\sqrt[3]{3n-1}-2=0$

$\sqrt[3]{3n-1}=2$ cube both sides

$3n-1=8$

$3n=9 \longrightarrow n=3$

### Invalid Solution

⑤ $\sqrt{x+16}+x=14$

$\sqrt{x+16}=14-x$

$x+16=196-28x+x^2$

$x^2-29x+180=0$

$(x-9)(x-20)$

$x=9,\ \cancel{20}$ 20 does not check

### Multiple Variables

⑥ $r=\sqrt[3]{\dfrac{3w}{4\pi d}}$ cube both sides

Solve for d

$r^3=\dfrac{3w}{4\pi d}$ mult. by denom.

$4\pi d r^3=3w$ div. by coeff.

$d=\dfrac{3w}{4\pi r^3}$ for $r\neq 0$ qualify if needed

# *Radical Equations*

## PROBLEM SET 6.4

Solve each equation:

1. $\sqrt{2x+3} + 3 = 10$
2. $\sqrt{4a+8} + 5 = 7$
3. $\sqrt[3]{m+5} + 6 = 4$
4. $\sqrt[4]{2x+3} + 5 = 4$
5. $\sqrt{x+5} = \sqrt{2x-3}$
6. $\sqrt{m+12} - \sqrt{m} = 2$
7. $\sqrt{b+4} = \sqrt{b+20} - 2$
8. $\sqrt{x-4} - 6 = \sqrt{x+20}$
9. $\sqrt{3x-5} = 2 - \sqrt{x-1}$

Solve for the indicated variable:

10. $t = \sqrt{\dfrac{2s}{9}}$ solve for s
11. $T = \dfrac{1}{2}\sqrt{\dfrac{u}{g}}$ solve for g
12. $m^2 = \sqrt[3]{\dfrac{rp}{g^2}}$ solve for p

Review Problems

Simplify:

13. $\sqrt{n^2+10n+25}$
14. $\sqrt[4]{a^6 b^{10} c^5 d^7}$

Compute and simplify:

15. $\sqrt[4]{18a^3b^5c^{10}} \cdot \sqrt[4]{36a^2b^7c^4}$
16. $\dfrac{2+2\sqrt{6}}{4-2\sqrt{6}}$
17. $\sqrt[3]{\dfrac{1}{3}} + \sqrt[3]{72} - \sqrt[3]{9}$


Red Cloud's ultimate nightmare

# *Roots & Radicals*

## UNIT 6 REVIEW & PRACTICE

Simplify:

① $\sqrt[4]{64x^5y^6z^9}$

② $\sqrt{72a^5b^7c^6}$

③ $\sqrt[3]{-40n^7m^4}$

④ $\sqrt{n^2-10n+25}$

⑤ $\sqrt[3]{(3x-y)^5}$

Compute and simplify:

⑥ $\sqrt[4]{24a^2b^3c^5} \cdot \sqrt[4]{48a^3b^3c^6}$

⑦ $\sqrt{3mn} + \sqrt{27m^3n}$

⑧ $(4-\sqrt{3})(6-\sqrt{3})$

Rationalize and simplify:

⑨ $\sqrt[3]{\frac{2}{5n}}$

⑩ $\frac{2-\sqrt{2}}{4+2\sqrt{2}}$

⑪ $\sqrt{\frac{1}{3}} + \sqrt{75} - 2\sqrt{3}$

Solve each equation:

⑫ $\sqrt[3]{y+1} = 3$

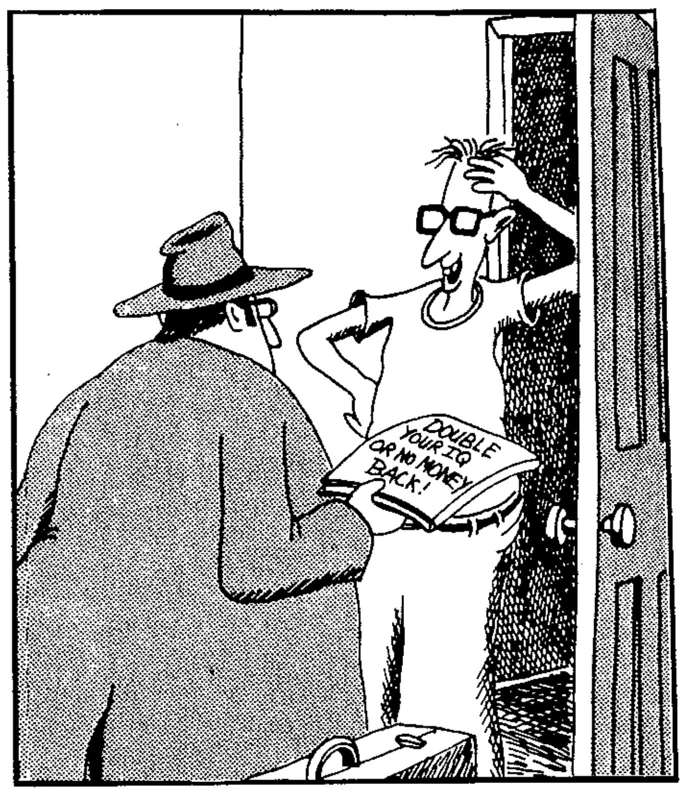
⑬ $\sqrt{y+12} + 1 = \sqrt{y+21}$

⑭ $\sqrt{5+2x} = x-5$

⑮ $\sqrt{x+11} - x = -9$

Solve for c:

⑯ $r = \sqrt[3]{\frac{2mM}{c}}$




"Well, I dunno . . . Okay, sounds good to me."

# UNIT 7

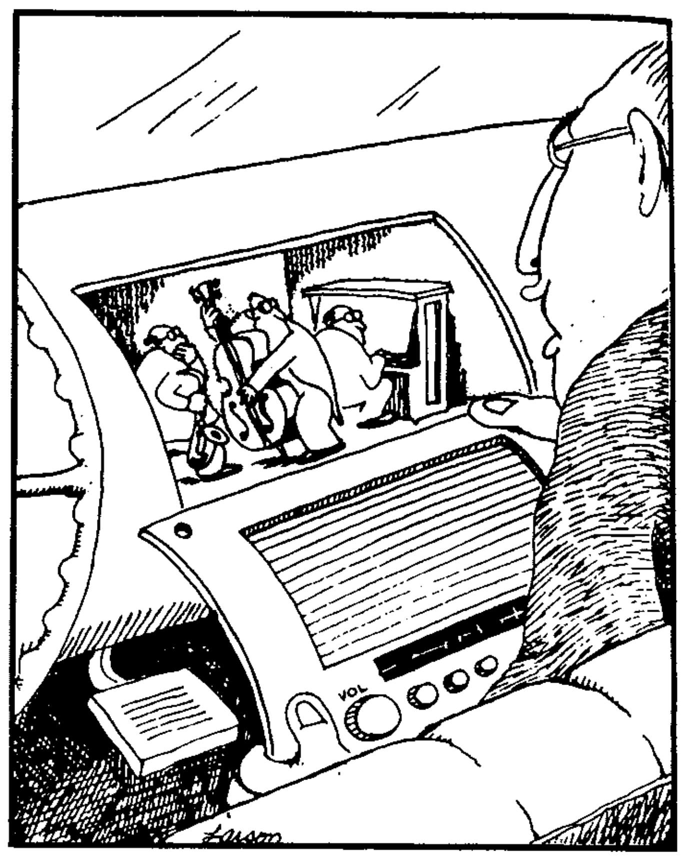
# *Rational Exponents & Complex Numbers*

7.1
*Rational Form & Radical Form*

7.2
*Negative Fractional Exponents*

7.3
*Pure Imaginary Numbers*

7.4
*Complex Numbers*




"Aha!"

# *Rational Form & Radical Form*

## DEMONSTRATION 7.1

### Fractional Exponents

$5^{\frac{2}{3}}$ ← exponent of radicand ← index; 5 ← radicand

$5^{\frac{2}{3}} = \sqrt[3]{5^2} = \sqrt[3]{25}$

### Evaluate

① $27^{\frac{1}{3}} = (3^3)^{\frac{1}{3}} = 3^{\frac{3}{3}} = 3$

② $16^{\frac{1}{4}} = (2^4)^{\frac{1}{4}} = 2^{\frac{4}{4}} = 2$

③ $27^{\frac{2}{3}} = (3^3)^{\frac{2}{3}} = 3^2 = 9$

### Express in Rational Form

④ $\sqrt{a^3bc^5d} = a^{3/2}b^{1/2}c^{5/2}d^{1/2}$

⑤ $\sqrt[3]{16x^6y^5} = \sqrt[3]{2^4x^6y^5} = 2^{\frac{4}{3}}x^2y^{\frac{5}{3}}$

⑥ $\sqrt[5]{(32x)^2} = \sqrt[5]{(2^5x)^2} = \sqrt[5]{2^{10}x^2}$

$2^{10/5}x^{2/5} = 2^2x^{2/5} = 4x^{2/5}$

### Express in Simplest Radical Form

⑦ $\sqrt[6]{25} = \sqrt[6]{5^2} = 5^{2/6}$

$5^{1/3} = \sqrt[3]{5}$

⑧ $3^{\frac{1}{2}}x^{\frac{2}{3}}y^{\frac{1}{6}}$

$3^{\frac{3}{6}}x^{\frac{4}{6}}y^{\frac{1}{6}} = \sqrt[6]{3^3x^4y} = \sqrt[6]{27x^4y}$

Note: In final form, the index should be as small as possible.

### Additional Practice

7a $\sqrt[4]{(64y^2z^2)^3}$

$\sqrt[4]{(2^6y^2z^2)^3} = \sqrt[4]{2^{18}y^6z^6}$

$2^{18/4}y^{6/4}z^{6/4} = 2^{9/2}y^{3/2}z^{3/2}$

$\sqrt{2^9y^3z^3}$

$16yz\sqrt{2yz}$

8a $4^{\frac{2}{3}}a^{\frac{1}{6}}b^{\frac{1}{2}}$

$4^{\frac{4}{6}}a^{\frac{1}{6}}b^{\frac{3}{6}} = \sqrt[6]{4^4ab^3}$

$\sqrt[6]{(2^2)^4ab^3} = \sqrt[6]{2^8ab^3}$

$2\sqrt[6]{4ab^3}$

# *Rational Form & Radical Form*

## PROBLEM SET 7.1

Express in exponential form:

1. $\sqrt{21}$
2. $\sqrt[3]{30}$
3. $\sqrt[6]{32}$
4. $\sqrt[4]{x}$
5. $\sqrt[3]{y}$
6. $\sqrt{25x^3y^4}$
7. $\sqrt[3]{8m^3r^6}$
8. $\sqrt[4]{8x^3y^5}$
9. $\sqrt[4]{27}$
10. $\sqrt[3]{16a^5b^7}$
11. $\sqrt[3]{n^2}$
12. $\sqrt[6]{b^3}$

Express in simplest radical form:

13. $64^{1/6}$
14. $5^{1/2}$
15. $6^{1/3}$
16. $x^{3/4}$
17. $a^{3/2}b^{5/2}$
18. $4^{1/3}x^{2/3}y^{4/3}$
19. $2^{5/3}x^{7/3}$
20. $(2x)^{1/2}x^{1/2}$
21. $5^{1/3}p^{2/3}q^{1/3}$
22. $(3m)^{2/5}n^{3/5}$
23. $r^{5/2}q^{3/4}$
24. $w^{4/7}y^{3/7}$
25. $x^{1/3}y^{1/2}$
26. $a^{5/6}b^{3/2}x^{7/3}$
27. $5^{2}b^{1/2}c^{1/4}$
28. $x^{3/4}y^{1/3}z^{5/6}$
29. $\sqrt[4]{9}$
30. $\sqrt[4]{49}$
31. $\sqrt[6]{8}$
32. $\sqrt[8]{16}$

Evaluate:

33. $121^{1/2}$
34. $\left(\frac{1}{32}\right)^{1/5}$
35. $\sqrt[3]{12^3}$
36. $\sqrt[4]{256}$
37. $\left(\frac{343}{64}\right)^{1/3}$
38. $\left(\frac{216}{729}\right)^{2/3}$
39. $(6^{2/3})^3$
40. $(9^{3/4})^{2/3}$
41. $(.125)^{2/3}$
42. $(.008)^{1/3}$
43. $(.027)^{1/3}$
44. $(.0016)^{1/4}$

Near Gettysburg, 1863: A reflective moment.

# *Negative Fractional Exponents*

## DEMONSTRATION 7.2

### Negative Fractional Exponents

① $5^{-\frac{2}{3}} = \frac{1}{5^{2/3}} = \frac{1}{\sqrt[3]{5^2}} \cdot \frac{\sqrt[3]{5}}{\sqrt[3]{5}} = \frac{\sqrt[3]{5}}{5}$

radical form $\boxed{\frac{\sqrt[3]{5}}{5}}$ rational form $\boxed{\frac{5^{1/3}}{5}}$

### Multiplying and Dividing

② $n^{\frac{1}{2}} n^{\frac{2}{3}} = n^{\frac{3}{6}} n^{\frac{4}{6}} = n^{\frac{7}{6}}$

radical form $\boxed{n\sqrt[6]{n}}$ rational form $\boxed{n^{\frac{7}{6}}}$

③ $\frac{4^{\frac{2}{3}}}{8^{\frac{1}{4}}} = \frac{(2^2)^{2/3}}{(2^3)^{1/4}} = \frac{2^{4/3}}{2^{3/4}} = 2^{\frac{4}{3} - \frac{3}{4}}$

$2^{\frac{16}{12} - \frac{9}{12}} = 2^{\frac{7}{12}}$

radical form $\boxed{\sqrt[12]{2^7}}$ rational form $\boxed{2^{7/12}}$

④ $\frac{3^{\frac{1}{2}}}{9^{-\frac{2}{3}}} = (3^{1/2})(9^{2/3}) = (3^{1/2})(3^2)^{2/3}$

$(3^{1/2})(3^{4/3}) = (3^{3/6})(3^{8/6}) = 3^{11/6}$

radical form $\boxed{3\sqrt[6]{3^5}}$ rational form $\boxed{3^{11/6}}$

⑤ $(n^{1/3})(n^{-1/3}) = n^0 = 1$

Conditions For Simplified Expressions

- No negative exponents
- No fractional exponents in the denominator
- No complex fractions
- Smallest index possible

### Rationalizing

⑥ $\frac{12}{3^{2/3}}$

$\frac{12}{3^{2/3}} \cdot \frac{3^{1/3}}{3^{1/3}} = \frac{12 \cdot 3^{1/3}}{3} = 4 \cdot 3^{1/3}$

⑦ $\frac{a^{1/2} + b^2}{a^{3/2}}$

$\frac{a^{1/2} + b^2}{a^{3/2}} \cdot \frac{a^{1/2}}{a^{1/2}}$

$\frac{a + a^{1/2} b^2}{a^2}$

continued

# *Negative Fractional Exponents*

## DEMONSTRATION 7.2

⑧ $\dfrac{a^{1/2}\,b}{a^{2/3}-a^{-1/3}}$

$$\frac{a^{1/2}\,b}{a^{2/3}-a^{-1/3}}\cdot\frac{a^{1/3}}{a^{1/3}}=\frac{a^{5/6}b}{a^{3/3}-a^{0}}=\frac{a^{5/6}b}{a-1}$$

Multiply By Conjugate

⑨ $\dfrac{a^{1/2}-b^{1/2}}{a^{1/2}+b^{1/2}}$

$$\frac{a^{1/2}-b^{1/2}}{a^{1/2}+b^{1/2}}\cdot\frac{a^{1/2}-b^{1/2}}{a^{1/2}-b^{1/2}}=\frac{a-2a^{\frac{1}{2}}b^{\frac{1}{2}}+b}{a-b}$$

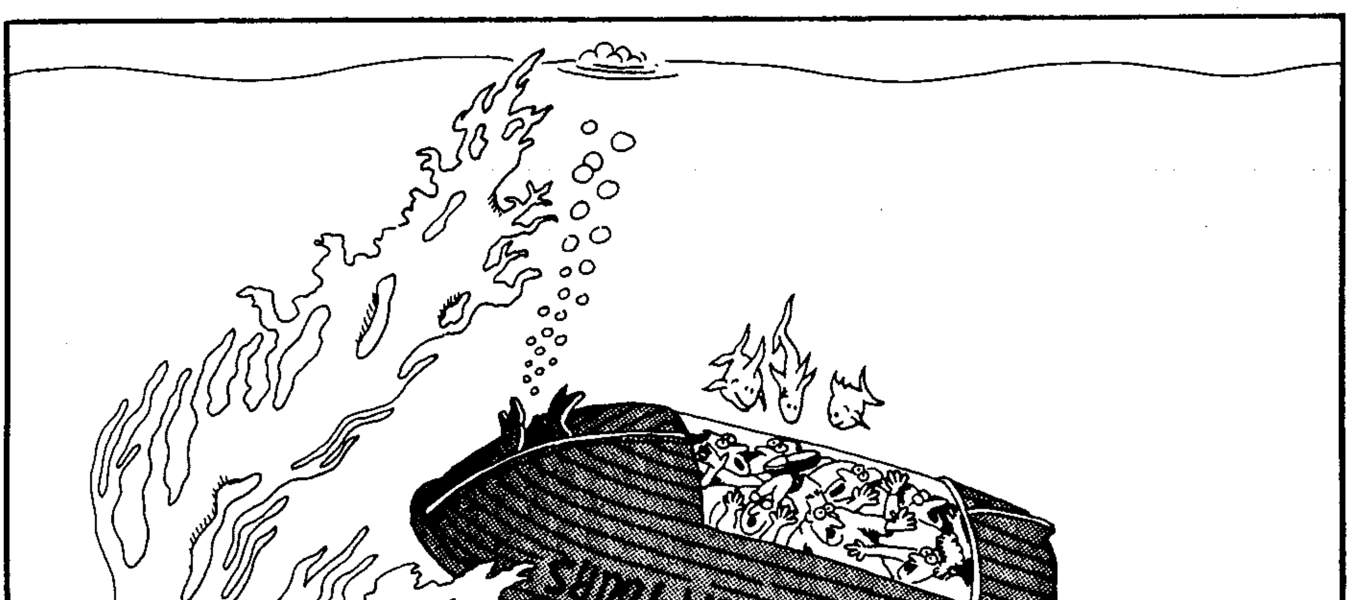
Simplify

⑩ $\left(\dfrac{a^{-1/2}}{3^{-2}a^{-3/2}}\right)^{1/2}$

$$\frac{a^{-1/4}}{3^{-1}a^{-3/4}}=\frac{3a^{3/4}}{a^{1/4}}$$

$$3a^{3/4-1/4}=3a^{2/4}$$

$$3a^{1/2}$$



# *Negative Fractional Exponents*

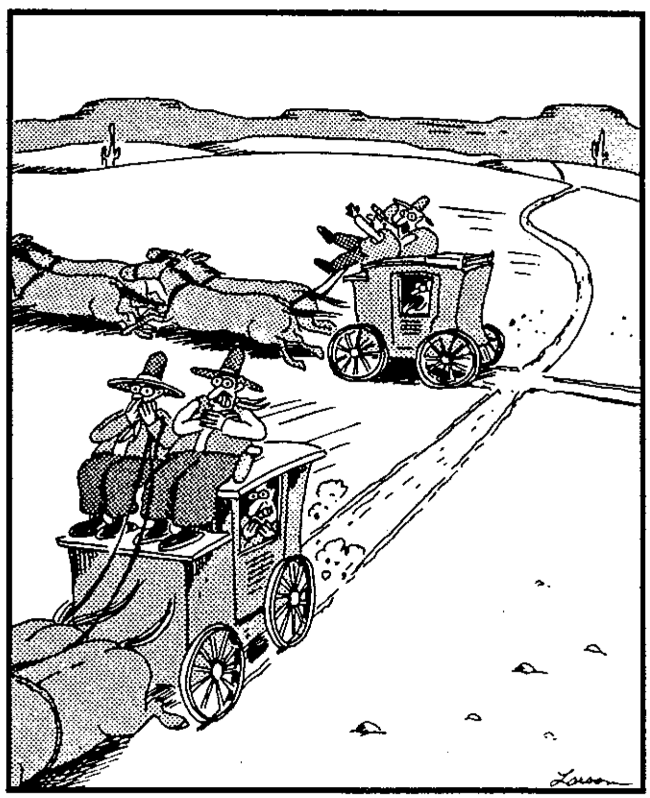
## PROBLEM SET 7.2

Evaluate in radical form:

1. $6^{-1/2}$
2. $32^{-1/4}$
3. $8^{-3/4}$
4. $8^{1/3} \cdot 8^{4/3}$
5. $9^{2/3}x^{4/3}x^{11/3}$
6. $(4n)^{2/5}n^{4/5}$
7. $x^{1/2}x^{3/4}x^{1/3}$
8. $\dfrac{36^{3/4}}{36^{1/4}}$
9. $\dfrac{8^{3/4}}{32^{-1/2}}$

Simplify in rational form:

10. $\dfrac{1}{y^{2/5}}$
11. $\dfrac{3}{r^{4/5}}$
12. $b^{-1/4}$
13. $m^{-5/6}$
14. $\dfrac{15}{5^{2/3}}$
15. $\dfrac{24}{6^{2/3}}$
16. $\dfrac{rm^{1/2}}{b^{3/2}}$
17. $\dfrac{pq}{a^{1/3}}$
18. $\dfrac{b^{3/2}+3b^{-1/2}}{b^{1/2}}$
19. $\dfrac{a^{5/3}m+3a^{-1/3}}{a^{2/3}}$
20. $\dfrac{3x+4x^2}{x^{-2/3}}$
21. $\dfrac{3m}{b^{-3/2}a^{1/3}}$
22. $(r^{-\frac{1}{6}})^{-2/3}$
23. $(y^{1/3})^{-3/4}$
24. $\dfrac{r^{3/2}}{r^{1/2}+2}$
25. $\dfrac{x^{1/2}+y^{1/2}}{x^{1/2}-y^{1/2}}$
26. $\dfrac{rs}{r^{1/2}+r^{3/2}}$
27. $\dfrac{x^{1/3}}{x^{2/3}-x^{-1/3}}$
28. $\dfrac{a^{-2/3}b^{1/2}}{b^{-3/2}a^{1/3}}$
29. $\left(\dfrac{x^{-2}y^{-6}}{9}\right)^{-1/2}$
30. $\left(\dfrac{z^{-2/3}}{5^{-1}z^{1/3}}\right)^{-2}$



Near misses of the Old West

# *Pure Imaginary Numbers*

## DEMONSTRATION 7.3

The imaginary unit ($i$) is defined as the solution to $\sqrt{-1}$. The product of $i$ and any real (non-zero) value is considered to be a pure imaginary number.

Powers of $i$

$i^1 = i$ $\quad i^5 = i$
$i^2 = -1$ $\quad i^6 = -1$
$i^3 = -i$ $\quad i^7 = -i$
$i^4 = 1$ $\quad i^8 = 1$

Simplify

1. $(3i)(5i)$

$(3i)(5i) = 15i^2 = (15)(-1) = -15$

2. $\sqrt{-16}$

$\sqrt{-16} = i\sqrt{16} = 4i$

3. $\sqrt{-24}$

$\sqrt{-24} = i\sqrt{24} = 2i\sqrt{6}$

Note: You must pull $i$ out of the radicals before multiplying

4. $(\sqrt{-3})(\sqrt{-12})$

$(\sqrt{-3})(\sqrt{-12}) = (i\sqrt{3})(i\sqrt{12}) = i^2\sqrt{36}$
$6i^2 = -6$

Solving Equations: Imaginary Solution

5. $x^2+5=0$

$x^2=-5$

$x=\pm\sqrt{-5}$

$x=\pm i\sqrt{5}$

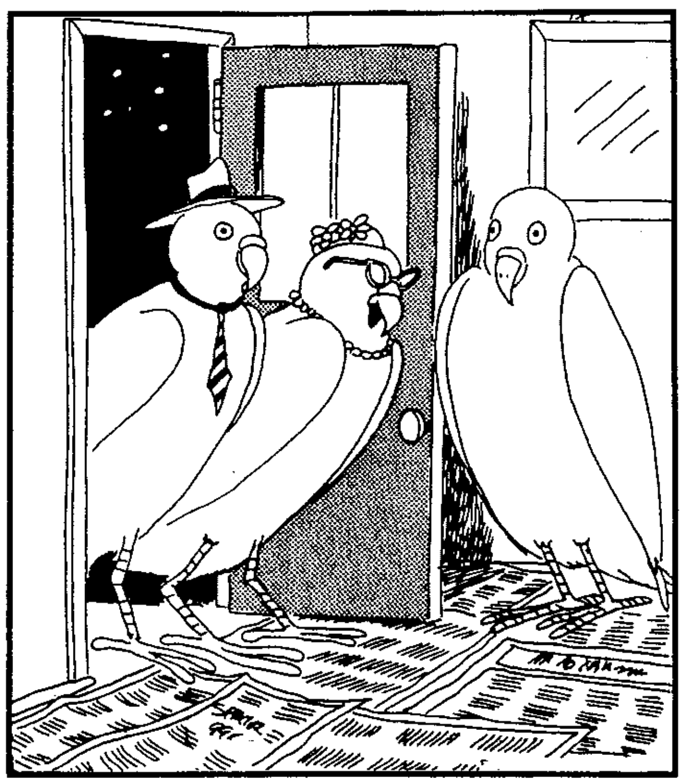
6. $4x^2+45=0$

$4x^2=-45$

$x^2=-45/4$

$x=\dfrac{\pm\sqrt{-45}}{2}$

$x=\dfrac{\pm 3i\sqrt{5}}{2}$



"What a lovely home, Edna! . . . And look at the fresh newspaper, Stanley!"

# *Pure Imaginary Numbers*

## PROBLEM SET 7.3

Simplify:

① $i^5$ ③ $i^{43}$

② $i^{10}$ ④ $i^{82}$

⑤ $(\sqrt{-4})^3$

⑥ $(-2\sqrt{-8})(3\sqrt{-2})$

⑦ $(4\sqrt{-12})(-2\sqrt{-3})$

⑧ $(6\sqrt{-24})(-3\sqrt{6})$

⑨ $(2\sqrt{15})(-3\sqrt{-15})$

⑩ $(2i)(3i)^2$

⑪ $(5i)(-2i)^2$

⑫ $(-6i^3)(4i^2)$

Solve each equation:

⑬ $2y^2+8=0$

⑭ $3b^2+18=0$

⑮ $5x^2+125=0$

⑯ $3z^2+24=0$

⑰ $4m^2+5=0$

⑱ $9k^2+32=0$

Review Problems

Express in radical form:

⑲ $a^{2/3}b^{5/2}c^{5/6}$ ㉑ $\dfrac{4^{3/4}}{8^{-2/3}}$

⑳ $2^{4/3}x^{1/2}$ ㉒ $16^{-3/4}$

Express in rational form:

㉓ $\sqrt[3]{8a^5b^7}$ ㉕ $\dfrac{a^{-2/3}-b^{1/2}}{a^{1/3}}$

㉔ $-(n^{1/2})^{-2/3}$ ㉖ $\dfrac{a^{3/2}}{a^{1/2}+b^{3/2}}$

# *Complex Numbers*

## DEMONSTRATION 7.4

A complex number is written in the form $a+bi$ where $a$ and $b$ are real numbers. To be equal, both the real and imaginary parts of two complex numbers must be equal.

### Adding / Subtracting

① $(3+6i)+(7-2i) = 10+4i$

② $(6-5i)-(3-2i) = 3-3i$

### Multiply with FOIL

③ $(6-7i)(4+3i)$
$(24+18i-28i-21i^2)$
$(24-10i-21i^2) = 45-10i$

④ $(9+2i)(5+i)(9-2i)$
$(45+9i+10i+2i^2)(9-2i)$
$(43+19i)(9-2i)$
$(387-86i+171i-38i^2)$
$425+85i$

### Determine x and y

⑤ $(x-y)+(x+y)i = 2-4i$

$$\begin{array}{rl} x-y &= 2 \\ x+y &= -4 \\ \hline 2x &= -2 \end{array}$$

$x=-1 \qquad y=-3$

### Multiplying Conjugates

⑥ $(3+5i)(3-5i)$

$3^2-(5i)^2 = 9-25i^2 = 34$

### Simplify

⑦ $\dfrac{3+7i}{2i}$ no $i$ term can be in denominator

$$\frac{3+7i}{2i}\cdot\frac{i}{i} = \frac{3i+7i^2}{2i^2} = \frac{-7+3i}{-2}$$

avoid negative denominators $\dfrac{7-3i}{2}$

⑧ $\dfrac{4+3i}{1-2i}$

$$\frac{4+3i}{1-2i}\cdot\frac{1+2i}{1+2i} = \frac{4+11i+6i^2}{1-4i^2} = \frac{-2+11i}{5}$$

⑨ Find the mult. inverse $(3-5i)$

$$\frac{1}{3-5i}\cdot\frac{3+5i}{3+5i} = \frac{3+5i}{9-25i^2} = \frac{3+5i}{34}$$

# *Complex Numbers*

## PROBLEM SET 7.4

Simplify:

① $(3+2i)+(4+5i)$

② $(2+6i)+(4+3i)$

③ $(9+6i)-(3+2i)$

④ $(11-\sqrt{-9})-(-4+\sqrt{-25})$

⑤ $(4+3i)(2-7i)(3+i)$

⑥ $(7-i)(4+2i)(5+2i)$

Determine x and y:

⑦ $(2x+y)+(x-y)i=7-i$

⑧ $(x+2y)+(2x-y)i=5+5i$

⑨ $(x+4y)+(2x-3y)i=13+4i$

Simplify:

⑩ $\dfrac{4-7i}{-3i}$

⑪ $\dfrac{5-6i}{-3i}$

⑫ $\dfrac{2+i}{5i}$

⑬ $\dfrac{3}{4-i}$

⑭ $\dfrac{2}{6+5i}$

⑮ $\dfrac{4}{\sqrt{3}+2i}$

⑯ $\dfrac{2+i\sqrt{3}}{2-i\sqrt{3}}$

⑰ $\dfrac{1+i\sqrt{2}}{1-i\sqrt{2}}$

Find the multiplicative inverse:

⑱ $7-3i$

⑲ $3+7i$

⑳ $\dfrac{4i}{3+i}$

㉑ $\dfrac{2i}{5-i}$


"Hello, Emily. This is Gladys Murphy up the street. Fine, thanks . . . Say, could you go to your window and describe what's in my front yard?"

# *Rational Exponents & Complex Numbers*

## UNIT 7 REVIEW & PRACTICE

Express in rational form:

① $\sqrt[3]{16x^5y^6z^8}$

② $\sqrt[4]{96a^5b^7c^8}$

Express in radical form:

③ $r^2s^{\frac{1}{3}}y^{\frac{1}{2}}$

④ $(3x)^{\frac{1}{2}}x^{\frac{1}{4}}$

⑤ $\sqrt[6]{27}$

⑥ $\sqrt[4]{64x^{10}}$

Evaluate:

⑦ $(25^{3/4})^{2/3}$

⑧ $(8^{\frac{2}{3}})(16^{-\frac{3}{4}})$

Simplify in rational form:

⑨ $\dfrac{x^{2/3}}{x^{2/3}-x^{-1/3}}$

⑩ $\dfrac{x^{1/2}-y^{1/2}}{x^{1/2}+y^{1/2}}$

⑪ $\left(\dfrac{a^{-2/3}}{2^{1/2}a^2}\right)^{-1/2}$

⑫ $\left(\dfrac{2^4a}{a^2b^{-1}}\right)^{-1/2}$

Simplify:

⑬ $(\sqrt{-8})(\sqrt{-12})$

⑭ $(\sqrt{-6})(\sqrt{-4})(\sqrt{-3})$

⑮ $(3i^3)(2i)^2$

⑯ $(-2i)^4(-4i^3)$

Solve:

⑰ $2n^2 = -27/8$

⑱ $4x^2 + 75 = 0$

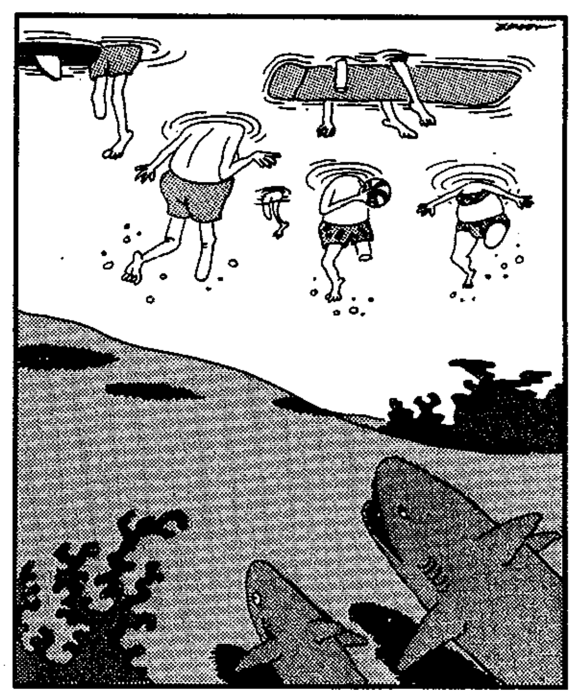
Simplify:

⑲ $(7+2i)(5-3i)$

⑳ $(3+8i)(3-8i)$

㉑ $\dfrac{4+3i}{1-2i}$

㉒ $\dfrac{2-2i}{2+2i}$



"This is it, son—my old chompin' grounds. ... Gosh, the memories."

# UNIT 8

# *Quadratics*




"By the way, we're playing cards with the Millers tonight . . . And Edna says if you promise not to use your X-ray vision, Warren promises not to bring his Kryptonite."

# *Completing The Square & Quadratic Formula*

## DEMONSTRATION 8.1

There are three algebraic methods that can be used to solve quadratic equations: factoring, completing the square, and the quadratic formula.

Factoring

① $3x^2 = 10x - 3$

$3x^2 - 10x + 3 = 0$

$3x^2 - 9x - x + 3 = 0$

$3x(x-3) - 1(x-3) = 0$

$(x-3)(3x-1) = 0$

$x = 3 \text{ or } 1/3$

Completing the Square

② $x^2 - 8x + 11 = 0$

$x^2 - 8x = -11$

$x^2 - 8x + 16 = -11 + 16$

$(x-4)^2 = 5$

$x - 4 = \pm\sqrt{5}$

$x = 4 \pm \sqrt{5}$

Add ½ middle term squared

Completing the Square: 1st Term Coefficient

③ $2m^2 - 8m + 3 = 0$

$m^2 - 4m = -3/2$

$m^2 - 4m + 4 = -3/2 + 4$

$(m-2)^2 = 5/2$

$m - 2 = \pm\sqrt{5/2}$

Divide by 1st term coeff.

continued

$$m - 2 = \frac{\pm\sqrt{5}}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\pm\sqrt{10}}{2}$$

$$m = 2 \pm \frac{\sqrt{10}}{2}$$

Completing the Square: Special Coefficients

④ $ax^2 + abx + ab^2 = 0$

$x^2 + bx = -b^2$

$x^2 + bx + b^2/4 = -b^2 + b^2/4$

$$\left(x + b/2\right)^2 = \frac{b^2 - 4b^2}{4}$$

$$x + \frac{b}{2} = \frac{\pm\sqrt{-3b^2}}{2} = \frac{\pm bi\sqrt{3}}{2}$$

$$x = \frac{-b \pm bi\sqrt{3}}{2}$$

Quadratic Formula: Imaginary Solution

⑤ $3x^2 - 5x + 9 = 0$

$a = 3$

$b = -5$

$c = 9$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

continued

# *Completing The Square & Quadratic Formula*

## DEMONSTRATION 8.1

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(3)(9)}}{2(3)}$$

$$x = \frac{5 \pm \sqrt{-83}}{6} = \frac{5 \pm i\sqrt{83}}{6}$$

Quadratic Formula:
Three Solutions

⑥ $x^3 = 27$

$x^3 - 27 = 0$

$(x-3)(x^2+3x+9) = 0$

$a = 1$
$b = 3$
$c = 9$

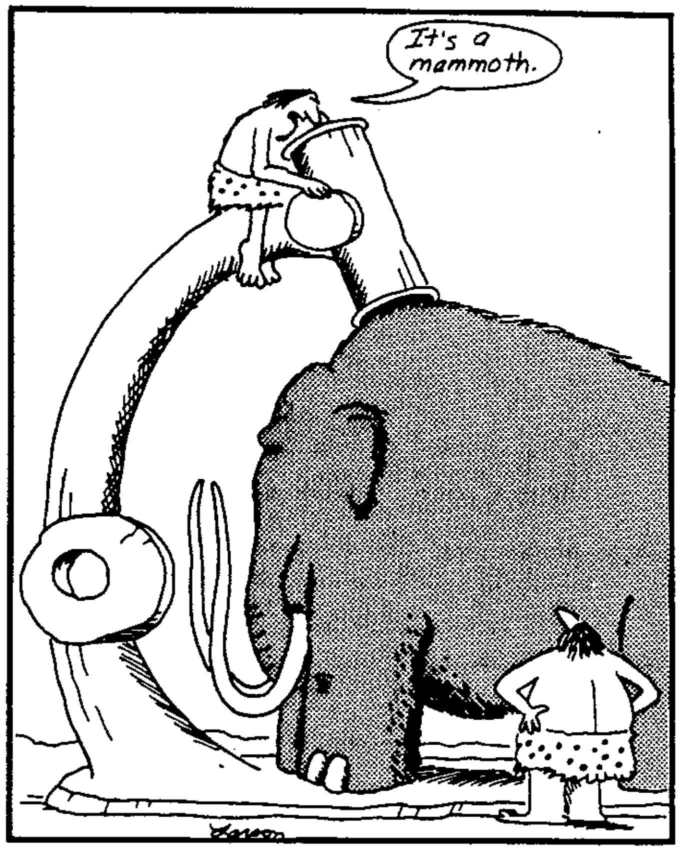
$$x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(9)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{-27}}{2}$$

$x = 3$

$$x = \frac{-3 \pm 3i\sqrt{3}}{2}$$

$$x = 3, \frac{-3 \pm 3i\sqrt{3}}{2}$$




Early microscope

# *Completing The Square & Quadratic Formula*

## PROBLEM SET 8.1

Solve each equation by factoring:

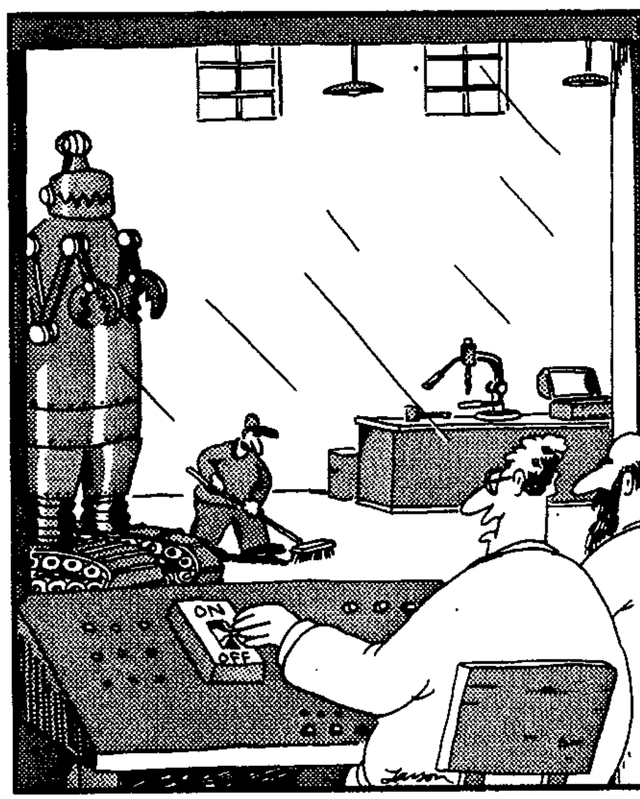
1. $y^2 - 4y - 21 = 0$
2. $m^2 + 6m = 27$
3. $6d^2 + 13d + 6 = 0$
4. $2y^2 + 11y - 21 = 0$

Solve each equation by completing the square:

5. $r^2 - 6r + 8 = 0$
6. $n^2 - 8n + 14 = 0$
7. $3c^2 - 14c + 8 = 0$
8. $3x^2 - 12x + 4 = 0$
9. $x^2 + ax + a = 0$
10. $ax^2 + bx + c = 0$

Solve each equation with the quadratic formula:

11. $4x^2 - 11x - 3 = 0$
12. $2x^2 + 3x + 3 = 0$
13. $x^3 + 8 = 0$
14. $a^3 = 125$
15. $ax^2 + b^2x - 3b = 0$




"Hey, who's that? ... Oh—Mitch, the janitor. Well, our first test run has just gotten a little more interesting."

# *The Discriminant & The Nature of Roots*

## DEMONSTRATION 8.2

The radicand of the quadratic formula ($b^2-4ac$) is called the discriminant. It can give information about the solutions (roots) of a quadratic equation.

| Discriminant Value | Nature of the Roots |
|---|---|
| Zero | 1 rational root |
| Positive (perfect square) | 2 rational roots |
| Positive (non per. sq.) | 2 irrational roots |
| Negative | 2 imaginary roots |

Use the discriminant to indicate the nature of the roots:

① $4x^2+20x+25=0$ $\quad b^2-4ac = (20)^2-4(4)(25)=0$ $\quad$ 1 rational root

② $a^2+a-12=0$ $\quad b^2-4ac = (1)^2-4(1)(-12) = 49$ $\quad$ 2 rational roots

③ $x^2+5x-3=0$ $\quad b^2-4ac = (5)^2-4(1)(-3) = 37$ $\quad$ 2 irrational roots

④ $3y^2+4y+5=0$ $\quad b^2-4ac = (4)^2-4(3)(5)=-44$ $\quad$ 2 imaginary roots

The sum and product of the roots can be used to build an equation.

Quadratic Equation

$$ax^2+bx+c=0$$

$$\text{Sum} = \frac{-b}{a}$$

$$\text{Product} = \frac{c}{a}$$

Determine the sum and product of the roots:

⑤ $3x^2-16x-12=0$

$$\text{Sum} = \frac{-b}{a} = \frac{16}{3} \qquad \text{prod.} = \frac{c}{a} = -4$$

# *The Discriminant & The Nature of Roots*

## DEMONSTRATION 8.2

Find a quadratic equation with the given roots:

⑥ $\frac{-2}{3}, 6$

sum: $\left(\frac{-2}{3}\right)+(6)=\frac{16}{3}$

prod: $\left(-\frac{2}{3}\right)(6) = -4$

$x^2-\frac{16}{3}x-4=0$

$3x^2-16x-12=0$

⑦ $5 \pm 2i$

sum: $(5+2i)+(5-2i)=10$

prod: $(5+2i)(5-2i)$

$25-4i^2 = 29$

$x^2-10x+29=0$

Be sure to include
$=0$
when writing equations

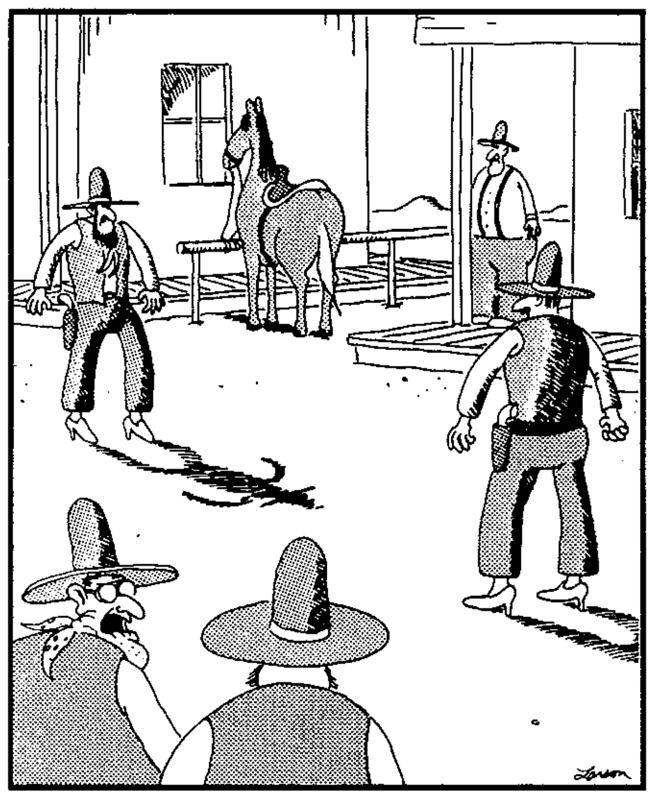
⑧ $\sqrt{5}, \frac{-3}{2}\sqrt{5}$

sum: $(\sqrt{5})+\left(\frac{-3}{2}\sqrt{5}\right) = \frac{-1}{2}\sqrt{5}$

prod: $(\sqrt{5})\left(\frac{-3}{2}\sqrt{5}\right) = \frac{-15}{2}$

$x^2+\frac{\sqrt{5}}{2}x-\frac{15}{2}=0$

$2x^2+x\sqrt{5}-15=0$



"I tell ya, Ben—no matter who wins this thing, Boot Hill ain't ever gonna be the same."

# *The Discriminant & The Nature of Roots*

## PROBLEM SET 8.2

Determine the discriminant and nature of the roots:

1. $a^2 + 12a + 32 = 0$
2. $y^2 - 4y + 4 = 0$
3. $m^2 - 6m + 4 = 0$
4. $m^2 - 2m + 5 = 0$
5. $3n^2 - 19n = -6$
6. $x^2 - x + 1 = 0$
7. $x^2 - 10x + 25 = 0$

Find the sum and product of the roots:

8. $y^2 + 5y + 6 = 0$
9. $x^2 - 3x + 1 = 0$
10. $2c^2 - 5c + 1 = 0$
11. $2x^2 - 6x + 5 = 0$
12. $9n^2 - 1 = 0$
13. $s^2 - 16 = 0$
14. $8m^2 + 6m = -1$

Find a quadratic equation with the given roots:

15. $6, 4$
16. $\frac{3}{4}, -4$
17. $\sqrt{3}, 2\sqrt{3}$
18. $5 \pm \sqrt{2}$
19. $3 \pm 7i$
20. $\frac{5 \pm 3i}{4}$

### Review Problems

Solve by completing the square:

21. $6m^2 + 7m - 3 = 0$

Solve with the quadratic formula:

22. $x^3 + 64 = 0$

Solve with either method:

23. $bx^2 + cx + 2c = 0$

# *The Quadratic Form*

## DEMONSTRATION 8.3

An equation in the quadratic form can be solved in the same manner as a quadratic equation.

① $x^4 - 13x^2 + 36 = 0$

$(x^2)^2 - 13x^2 + 36 = 0$

$(x^2-9)(x^2-4) = 0$

$(x+3)(x-3)(x+2)(x-2) = 0$

$x = \pm 3, \pm 2$

② $x^4 - 49 = 0$

$(x^2)^2 - 49 = 0$

$(x^2+7)(x^2-7) = 0$

$x^2 = -7 \qquad x^2 = 7$

$x = \pm\sqrt{-7} \qquad x = \pm\sqrt{7}$

$x = \pm i\sqrt{7}$

$x = \pm i\sqrt{7}, \pm\sqrt{7}$

③ $x - 7\sqrt{x} - 8 = 0$

$(\sqrt{x})^2 - 7\sqrt{x} - 8 = 0$

$(\sqrt{x}-8)(\sqrt{x}+1) = 0$

$\sqrt{x} = 8 \qquad \sqrt{x} = -1$

$x = 64 \qquad x = \cancel{1}$

$x = 1$ does not check. (When a variable is in the radicand, it must be checked.)

④ $n^6 - 9n^3 + 8 = 0$

$(n^3)^2 - 9n^3 + 8 = 0$

$(n^3-8)(n^3-1) = 0$

$(n-2)(n^2+2n+4)(n-1)(n^2+n+1) = 0$

$\dfrac{-2 \pm \sqrt{(2)^2 - 4(1)(4)}}{2(1)} \qquad \dfrac{-1 \pm \sqrt{(1)^2 - 4(1)(1)}}{2(1)}$

$\dfrac{-2 \pm \sqrt{-12}}{2} \qquad \dfrac{-1 \pm \sqrt{-3}}{2}$

$-1 \pm i\sqrt{3} \qquad \dfrac{-1 \pm i\sqrt{3}}{2}$

$n = 2 \qquad n = 1$

$n = 2, -1 \pm i\sqrt{3}, 1, (-1 \pm i\sqrt{3})/2$

⑤ $x^{2/3} - 16 = 0$

$(x^{1/3})^2 - 16 = 0$

$(x^{1/3}+4)(x^{1/3}-4) = 0$

$x^{1/3} = -4 \qquad x^{1/3} = 4$

$x = (-4)^3 \qquad x = 4^3$

$x = -64 \qquad x = 64$

$x = \pm 64$

Fractional exponents are like radicals. Both answers must be checked in this problem

# *The Quadratic Form*

## PROBLEM SET 8.3

Solve each equation:

① $x^4 - 25 = 0$

② $x^4 - 16 = 0$

③ $a^4 - 36 = 0$

④ $n^4 + 9n^2 + 18 = 0$

⑤ $c^4 - 2c^2 - 8 = 0$

⑥ $x^4 - 6x^2 + 8 = 0$

⑦ $n - 13\sqrt{n} + 36 = 0$

⑧ $x - 2\sqrt{x} + 1 = 0$

⑨ $y - 6\sqrt{y} - 16 = 0$

⑩ $m^6 - 64 = 0$

⑪ $x^6 - 1 = 0$

⑫ $n^6 - 7n^3 - 8 = 0$

⑬ $a^6 - 26a^3 - 27 = 0$

Review Problems

Complete the square:

⑭ $3y^2 + 36y + 12 = 0$

Write an equation with the given roots:

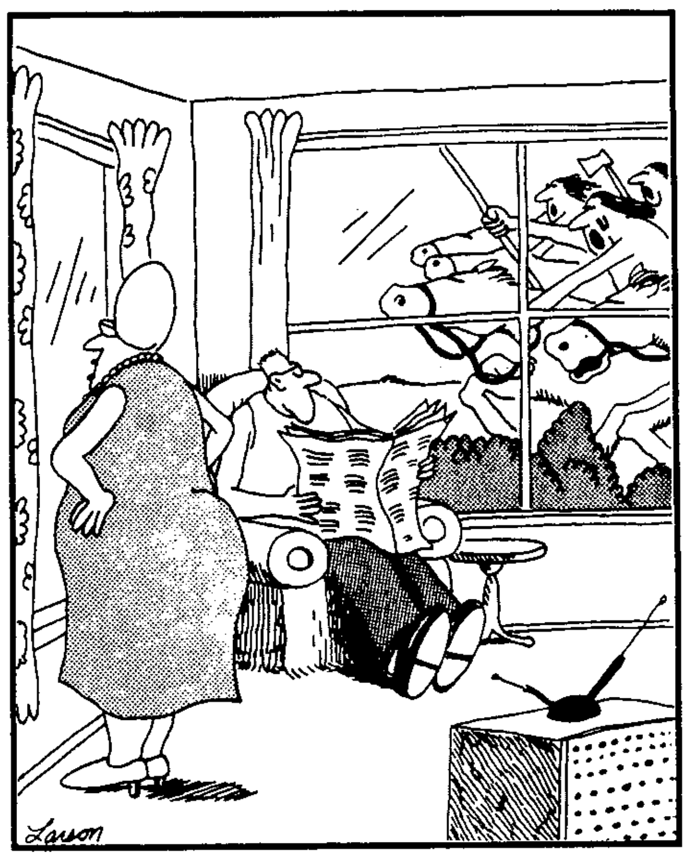
⑮ $\frac{2}{3} \pm \sqrt{6}$

Challenge Problems

Solve:

⑯ $m - 11m^{1/2} + 30 = 0$

⑰ $y^{-1} - 5y^{-1/2} + 6 = 0$




"How cute, Earl . . . The kids have built a little fort in the backyard."

# *Graphing Quadratic Functions*

## DEMONSTRATION 8.4

A quadratic function can be shown graphically as a parabola on the coordinate axis. If the coefficient of $x^2$ is positive, the parabola will open upward. The graph is symmetrical about the vertical axis (axis of symmetry) that runs through the turning point (vertex).

Name the axis of symmetry and the vertex. Draw the graph. Determine the roots:

① $y = x^2 - 2x + 7$

axis $x = \frac{-b}{2a}$ $\boxed{x=1}$

$(1)^2 - 2(1) + 7 = 6$ $\boxed{(1, 6)}$ min. pt.

| x | y | |
|---|---|---|
| 0 | 7 | $(0)^2 - 2(0) + 7 = 7$ |
| -1 | 10 | $(-1)^2 - 2(-1) + 7 = 10$ |
| -2 | 15 | $(-2)^2 - 2(-2) + 7 = 15$ |

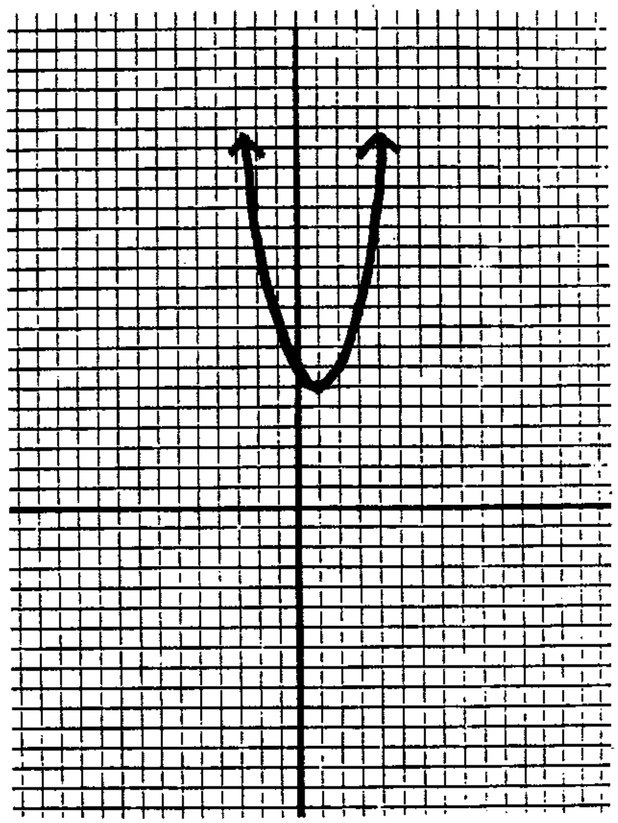
$x^2 - 2x + 7 = 0$

a=1
b=-2
c=7

$$\frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(7)}}{2(1)}$$

$$\frac{2 \pm \sqrt{-24}}{2} = \frac{2 \pm 2i\sqrt{6}}{2}$$

$\boxed{x = 1 \pm i\sqrt{6}}$ roots



If the parabola crosses the x-axis, the roots will be the x-intercepts. The parabola above has imaginary roots.

The roots can be determined by solving the related equation algebraically (factoring, formula, completing the square).

# *Graphing Quadratic Functions*

## DEMONSTRATION 8.4

The graph of a quadratic inequality is a shaded region inside or outside of the parabola.

② $y \geq x^2 - 4x - 6$

axis: $x = \frac{-b}{2a}$ $\boxed{x=2}$

vertex: $(2)^2 - 4(2) - 6 = -10$ $\boxed{(2,-10)}$ min. pt.

| x | y |
|---|---|
| 1 | -9 |
| 0 | -6 |
| -2 | 6 |

$(1)^2 - 4(1) - 6 = -9$
$(0)^2 - 4(0) - 6 = -6$
$(-2)^2 - 4(-2) - 6 = 6$

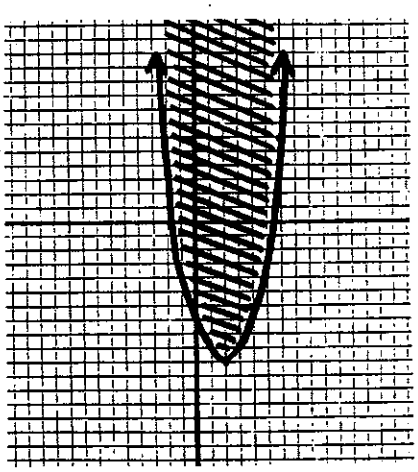
$x^2 - 4x - 6 = 0$

$a = 1$
$b = -4$
$c = -6$

$$\frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-6)}}{2(1)}$$

$$\frac{4 \pm 2\sqrt{10}}{2} = 2 \pm \sqrt{10}$$

$\boxed{2 - \sqrt{10} \leq x \leq 2 + \sqrt{10}}$



③ $y > -x^2 + 6x - 5$

axis: $x = \frac{-b}{2a}$ $\boxed{x=3}$

vertex: $-(3)^2 + 6(3) - 5 = 4$ $\boxed{(3,4)}$ max. pt.

| x | y |
|---|---|
| 2 | 3 |
| 1 | 0 |
| 0 | -5 |

$-(2)^2 + 6(2) - 5 = 3$
$-(1)^2 + 6(1) - 5 = 0$
$-(0)^2 + 6(0) - 5 = -5$

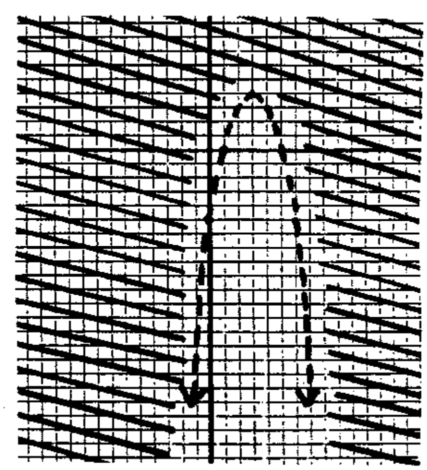
$-x^2 + 6x - 5 = 0$

$x^2 - 6x + 5 = 0$

$(x-5)(x-1) = 0$

$x = 5, 1$

$\boxed{x < 1 \text{ or } x > 5}$



# *Graphing Quadratic Functions*

## PROBLEM SET 8.4

For each quadratic function:

a) Identify the axis of symmetry
b) Indicate the maximum or minimum point
c) Draw the graph
d) Determine the roots of the related equation

① $y = x^2 + 6x + 2$

② $y = x^2 - 2x + 7$

③ $y = -2x^2 + 16x - 31$

④ $y = -\frac{1}{2}x^2 + 6x - 19$

For each quadratic function:

a) Identify the axis of symmetry
b) Indicate the maximum or minimum point
c) Draw the graph
d) Indicate the solution to the related inequality

⑤ $y \leq x^2 + 10x + 24$

⑥ $y > x^2 + 4x - 12$

⑦ $y > -x^2 + 6x - 8$

⑧ $y \leq -x^2 + 8x - 12$

"Blast! Up to now, the rhino was one of my prime suspects."

# *Quadratics*

## UNIT 8 REVIEW & PRACTICE

Use factoring:

① $6x^2+7x-3=0$

Complete the square:

② $2n^2+n-21=0$

③ $3x^2+4x+2=0$

Quadratic formula:

④ $2a^2-5a+4=0$

Any method:

⑤ $ax^2+bx+3b=0$

⑥ $bx^2+acx+c=0$

Use the discriminant to determine the nature of the roots:

⑦ $4x^2-40x+25=0$

⑧ $2y^2+6y+5=0$

Determine an equation with the given roots:

⑨ $\frac{3}{4}, \frac{1}{3}$ ⑩ $2\pm\sqrt{3}$ ⑪ $5\pm 3i$

Quadratic form:

⑫ $x-4\sqrt{x}-32=0$

⑬ $x^4-12x^2+27=0$

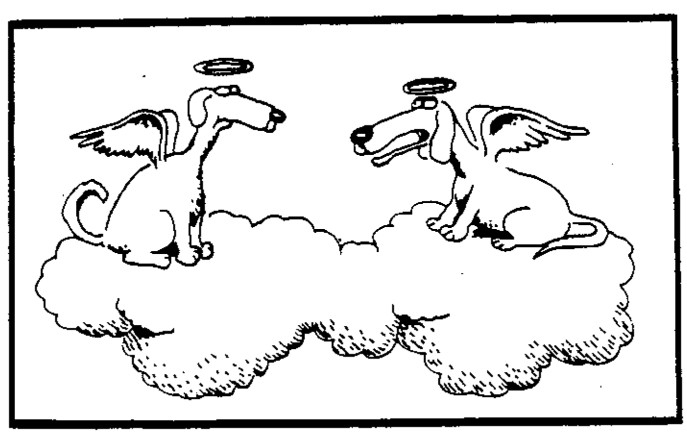
⑭ $x^{2/3}-9x^{1/3}+20=0$

⑮ $n^6+9n^3+8=0$

Indicate the axis and vertex, draw the graph, and determine the solution (as a union or intersection):

⑯ $y \le x^2+4x+3$

⑰ $y \le -x^2+6x+5$



"For twelve perfect years I was a car-chaser. Pontiacs, Fords, Chryslers . . . I took them all on . . . and yesterday my stupid owner backs over me in the driveway."

# UNIT 9

# *Rational Expressions*

9.1
*Rational Operations*

9.2
*Complex Fractions*

9.3
*Rational Equations*

9.4
*Inverse Variation*

**Historic note: Until his life's destiny was further clarified, Robin Hood spent several years robbing from the rich and giving to the porcupines.**

# *Rational Operations*

## DEMONSTRATION 9.1

This lesson reviews basic operations with rational expressions. For this unit, it will be assumed that all values are real and variables do not have to be qualified. (Denominators $\neq 0$)

### Simplifying Expressions

① $\dfrac{x^2+10x+25}{x^2+2x-15}$

$$\frac{\cancel{(x+5)}(x+5)}{\cancel{(x+5)}(x-3)} = \frac{x+5}{x-3}$$

### Multiplication

② $\dfrac{x^2-9}{x^2+x-12} \cdot \dfrac{x+2}{x+3}$

$$\frac{\cancel{(x+3)}\cancel{(x-3)}(x+2)}{(x+4)\cancel{(x-3)}\cancel{(x+3)}} = \frac{x+2}{x+4}$$

### Division

③ $\dfrac{x^2}{x^2-25y^2} \div \dfrac{x}{x+5y}$

$$\frac{x^2}{x^2-25y^2} \cdot \frac{x+5y}{x}$$

$$\frac{x^2\cancel{(x+5y)}}{\cancel{(x+5y)}(x-5y)x} = \frac{x}{x-5y}$$

### Addition

④ $\dfrac{5}{a-b} + \dfrac{3}{ab-a^2}$

$$\frac{5}{a-b} + \frac{3}{a(b-a)}$$

$$\frac{5}{a-b} - \frac{3}{a(a-b)} = \frac{5a-3}{a^2-ab}$$

### Subtraction

⑤ $\dfrac{x+4}{2x-8} - \dfrac{x+12}{4x-16}$

$$\frac{x+4}{2(x-4)} - \frac{x+12}{4(x-4)}$$

$$\frac{2(x+4)-(x+12)}{4(x-4)}$$

$$\frac{2x+8-x-12}{4(x-4)}$$

$$\frac{\cancel{(x-4)}}{4\cancel{(x-4)}} = \frac{1}{4}$$

# *Rational Operations*

## PROBLEM SET 9.1

Simplify:

(1) $\dfrac{6y^3-9y^2}{2y^2+5y-12}$

(2) $\dfrac{x^2-x-20}{x^2+7x+12}$

(3) $\dfrac{y^2+4y+4}{3y^2+5y-2}$

(4) $\dfrac{a^2+2a+1}{2a^2+3a+1}$

Multiply and simplify:

(5) $\dfrac{x^2-y^2}{y^2} \cdot \dfrac{y^3}{y-x}$

(6) $\dfrac{x^2-y^2}{x+y} \cdot \dfrac{11}{x-y}$

(7) $-\dfrac{x^2-y^2}{x+y} \cdot \dfrac{1}{x-y}$

(8) $\dfrac{x^2+3x-10}{x^2+8x+15} \cdot \dfrac{x^2+5x+6}{x^2+4x+4}$

Divide and simplify:

(9) $\dfrac{(x+y)^2}{a} \div \dfrac{x+y}{ab}$

(10) $\dfrac{a^2-b^2}{2a} \div \dfrac{a-b}{ab}$

(11) $\dfrac{3x-21}{x^2-49} \div \dfrac{3x}{x^2+7x}$

(12) $\dfrac{y^2-y}{w^2-y^2} \div \dfrac{y^2-2y+1}{1-y}$

Add/Subtract and simplify:

(13) $\dfrac{3x}{x-y} + \dfrac{4x}{y-x}$

(14) $\dfrac{3a+2}{a+b} + \dfrac{4}{2a+2b}$

(15) $\dfrac{2a}{3a-15} + \dfrac{-16a+20}{3a^2-12a-15}$

(16) $\dfrac{m^2+n^2}{m^2-n^2} + \dfrac{m}{n-m} + \dfrac{n}{m+n}$

(17) $\dfrac{7}{y-8} - \dfrac{6}{8-y}$

(18) $\dfrac{y}{y-9} - \dfrac{-9}{9-y}$

(19) $3m+1 - \dfrac{2m}{3m+1}$

(20) $\dfrac{x}{x^2+2x+1} - \dfrac{x+2}{x+1} + \dfrac{3x}{x+1}$

# *Complex Fractions*

## DEMONSTRATION 9.2

Simplifying a complex fraction combines all of the operations with rational expressions.

Simplify:

① $$\frac{\left(\frac{4-x^2}{6}\right)}{\left(\frac{2-x}{3}\right)}$$

$$\frac{(2+x)(2-x)}{6} \cdot \frac{3}{(2-x)} = \frac{2+x}{2}$$

② $$\frac{\left(\frac{a}{b}\right)}{\left(\frac{a}{3}\right)} - \frac{\left(\frac{b}{a}\right)}{\left(\frac{b}{2}\right)}$$

$$\frac{a}{b}\cdot\frac{3}{a} - \frac{b}{a}\cdot\frac{2}{b} = \frac{3}{b} - \frac{2}{a} = \frac{3a-2b}{ab}$$

③ $$\frac{\frac{1}{x}-\frac{1}{y}}{1+\frac{1}{x}}$$

$$\frac{\frac{y-x}{xy}}{\frac{x+1}{x}} = \frac{y-x}{xy}\cdot\frac{x}{x+1} = \frac{y-x}{xy+y}$$

④ $$\frac{y+2+\frac{10}{y-5}}{\frac{4}{y-5}+2}$$

$$\frac{\left(\frac{(y+2)(y-5)+10}{(y-5)}\right)}{\left(\frac{4+2(y-5)}{(y-5)}\right)} = \frac{\frac{(y^2-3y)}{(y-5)}}{\frac{(2y-6)}{(y-5)}}$$

$$\frac{(y^2-3y)}{(y-5)}\cdot\frac{(y-5)}{(2y-6)}$$

$$\frac{y^2-3y}{2y-6} = \frac{y(y-3)}{2(y-3)} = \frac{y}{2}$$

"Hey! Look what Zog do!"

# *Complex Fractions*

## PROBLEM SET 9.2

Simplify:

① $\dfrac{\left(\frac{x^2-y^2}{2}\right)}{\left(\frac{x-y}{4}\right)}$

② $\dfrac{w^2+2w+1}{\left(\frac{w+1}{3}\right)}$

③ $\dfrac{\left(\frac{5a^2-20}{2a+4}\right)}{\left(\frac{10a-20}{4a}\right)}$

④ $\dfrac{\left(\frac{c^2+2c-3}{3c+3}\right)}{\left(\frac{c^2+5c+6}{2c+2}\right)}$

⑤ $\dfrac{\left(\frac{p^2+7p}{3p}\right)}{\left(\frac{49-p^2}{3p-21}\right)}$

⑥ $\dfrac{\left(\frac{3m}{2m^2+7m-15}\right)}{\left(\frac{6}{2m^2-3m}\right)}$

⑦ $\dfrac{\left(\frac{x}{2}\right)}{\left(\frac{x}{3}\right)} - \dfrac{\left(\frac{x}{5}\right)}{\left(\frac{x}{6}\right)}$

⑧ $\dfrac{\left(\frac{2x}{ab}\right)}{\left(\frac{3x}{a}\right)} - \dfrac{\left(\frac{5x}{4}\right)}{\left(\frac{6x}{5}\right)}$

⑨ $\dfrac{3+\frac{5}{a+2}}{3-\frac{10}{a+7}}$

⑩ $\dfrac{\frac{2x}{2x+1}-1}{1+\frac{2x}{1-2x}}$

⑪ $\dfrac{\frac{5x}{x^2-16}}{\frac{10}{x-4}+\frac{10}{x+4}}$

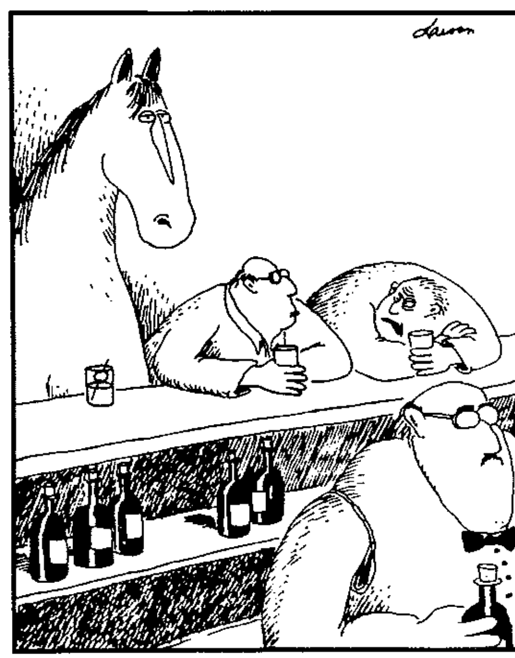
⑫ $\dfrac{\frac{x+4}{x-6}+\frac{x+1}{x+2}}{\frac{4}{x^2-4x-12}}$

⑬ $\dfrac{n+1-\frac{2}{n}}{n+4+\frac{4}{n}}$

⑭ $\dfrac{\frac{1}{x+5}+\frac{1}{x-3}}{\frac{2x^2-3x-5}{x^2+2x-15}}$

Challenge Problem:

⑮ $\dfrac{(a^2-5a+6)^{-1}}{(a-2)^{-2}} \div \dfrac{(a-3)^{-1}}{(a-2)^{-3}}$



"Sure — but can you make him drink?"

# *Rational Equations*

## DEMONSTRATION 9.3

An equation that contains one or more rational expressions is called a rational equation. To solve, multiply the entire equation by the least common denominator. Always check for excluded values (solutions that put 0 in the denominator).

Solve

① $\frac{c}{c-4} - \frac{6}{4-c} = c$

$(c-4)\left[\frac{c}{c-4} + \frac{6}{c-4} = c\right]$

$c+6 = c(c-4)$

$c+6 = c^2 - 4c$

$c^2 - 5c - 6 = 0$

$(c-6)(c+1) = 0$

$\boxed{c = 6, -1}$

② $\frac{3}{4} - \frac{3}{y+2} = \frac{9}{28}$

$(28)(y+2)\left[\frac{3}{4} - \frac{3}{y+2} = \frac{9}{28}\right]$

$3(7)(y+2) - 3(28) = 9(y+2)$

$21y + 42 - 84 = 9y + 18$

$12y = 60$

$\boxed{y = 5}$

③ $\frac{7}{x-3} = \frac{x+4}{x-3}$

$(x-3)\left[\frac{7}{x-3} = \frac{x+4}{x-3}\right]$

$7 = x + 4$

$x = 3$

3 is an excluded value

$\boxed{\text{No Solutions}}$

④ $w + \frac{w}{w-1} = \frac{4w-3}{w-1}$

$(w-1)\left[w + \frac{w}{w-1} = \frac{4w-3}{w-1}\right]$

$w(w-1) + w = 4w - 3$

$w^2 - w + w = 4w - 3$

$w^2 - 4w + 3 = 0$

$(w-3)(w-1) = 0$

$w = 3, \cancel{1}$

1 is an excluded value

$\boxed{w = 3}$

# *Rational Equations*

## PROBLEM SET 9.3

Solve each equation:

① $1 + \dfrac{3}{y-1} = \dfrac{4}{3}$

② $\dfrac{5}{2x} - \dfrac{3}{10} = \dfrac{1}{x}$

③ $\dfrac{1}{9} + \dfrac{1}{2a} = \dfrac{1}{a^2}$

④ $\dfrac{1}{x-1} + \dfrac{2}{x} = 0$

⑤ $\dfrac{4t}{3t-2} + \dfrac{2t}{3t+2} = 2$

⑥ $\dfrac{12}{x^2-16} - \dfrac{24}{x-4} = 3$

⑦ $\dfrac{5}{x-3} - \dfrac{x}{3-x} = x$

⑧ $\dfrac{x-3}{2x} = \dfrac{x-2}{2x+1} - \dfrac{1}{2}$

⑨ $\dfrac{2}{y+2} - \dfrac{y}{2-y} = \dfrac{y^2+4}{y^2-4}$

⑩ $\dfrac{t+4}{t} + \dfrac{3}{t-4} = \dfrac{-16}{t^2-4t}$

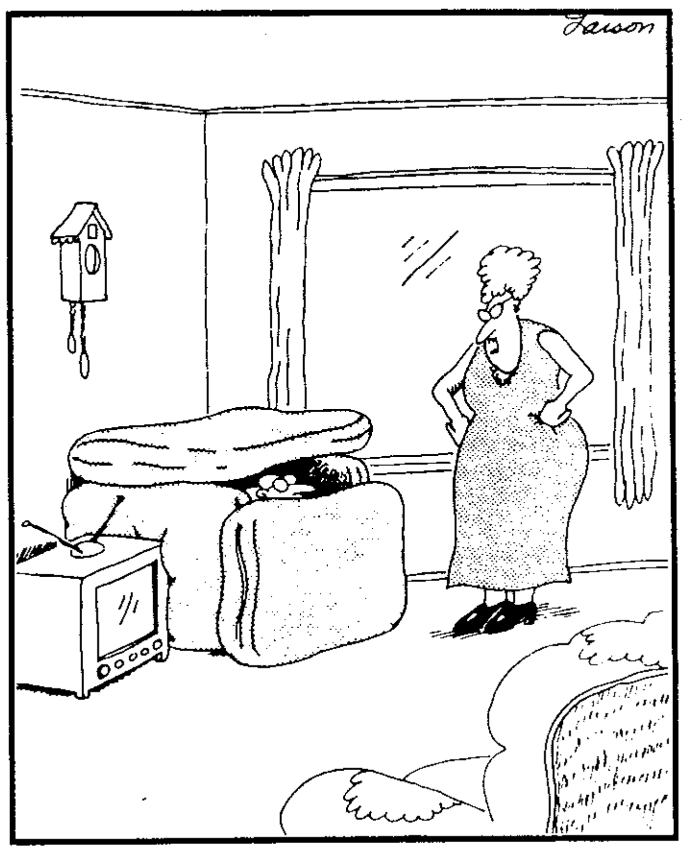
⑪ $\dfrac{y}{y-5} + \dfrac{17}{25-y^2} = \dfrac{1}{y+5}$

⑫ $\dfrac{x}{x^2-1} + \dfrac{2}{x+1} = \dfrac{1}{2x-2}$

<u>Review Problem</u>

Simplify:

⑬ $$\dfrac{\dfrac{2x}{y} + 1 - \dfrac{y}{x}}{\dfrac{2x}{y} + \dfrac{y}{x} - 3}$$



"Well, you can just rebuild the fort later, Harold . . . Phyllis and Shirley are coming over and I'll need the cushions."

# *Inverse Variation*

## DEMONSTRATION 9.4

In a standard proportion, the variables vary directly (one increases as the other increases). In a situation that calls for inverse variation, one variable increases while the other decreases.

Direct Variation

$$\frac{x_1}{x_2} = \frac{y_1}{y_2}$$

Inverse Variation

$$\frac{x_1}{x_2} = \frac{y_2}{y_1}$$

Solve:

① If $y$ varies directly as $x$, and $y = 8$ when $x = -3$, find $y$ when $x = 4$

Direct Variation

$\frac{x_1}{x_2} = \frac{y_1}{y_2}$ $\quad \frac{-3}{4} = \frac{8}{y}$ $\quad -3y = 32$, $y = \boxed{-32/3}$

② If $y$ varies inversely as $x$, and $y = 3$ when $x = 4$, find $y$ when $x = 18$

Inverse Variation

$\frac{x_1}{x_2} = \frac{y_2}{y_1}$ $\quad \frac{4}{18} = \frac{y}{3}$ $\quad 18y = 12$, $y = \boxed{2/3}$

③ The volume of gas varies inversely with its pressure as long as the temperature remains constant. If the volume is 1600 ml when the pressure is 25 cm of mercury, what is the volume at 40 cm?

Inverse Variation

$\frac{x_1}{x_2} = \frac{y_2}{y_1}$ $\quad \frac{1600}{x} = \frac{40}{25}$

$40x = 40{,}000$

$x = 1000$

$\boxed{1000\ ml}$

# *Inverse Variation*

## PROBLEM SET 9.4

Direct variation:

① If $y=8$, then $x=2$. Find $y$ when $x=9$.

② If $y=10$, then $x=-3$. Find $x$ when $y=4$.

③ If $y=11$, then $x=1/5$. Find $y$ when $x=2/5$.

Inverse variation:

④ If $y=1/5$, then $x=9$. Find $y$ when $x={}^-3$.

⑤ If $y=-2$, then $x=-8$. Find $x$ when $y=2/3$.

⑥ If $y=7$, then $x=-3$. Find $y$ when $x=4$.

Solve:

⑦ A 75-foot tree casts a 40-foot shadow. How tall is a tree that casts a 10-foot shadow at the same time of day?

⑧ When temperature remains constant, the volume of any gas varies inversely with its pressure. If the volume is 120 $ft^3$ under 6 lbs. of pressure, what is the volume at 8 lbs. of pressure?

⑨ The pressure of air required in an automobile tire varies inversely with its volume. If 30 lbs. of pressure is required for 140 $in^3$, find the pressure for 100 $in^3$.

⑩ A realtor made a commission of \$4800 on a sale of a \$90,000 house. At that rate, what is the commission on a house selling for \$129,000?

Review Problems

Add and simplify:

⑪ $\dfrac{8}{2y-16} - \dfrac{y}{8-y}$

Simplify:

⑫ $\dfrac{\dfrac{5}{y+7} + \dfrac{2}{y}}{\dfrac{2}{y} - \dfrac{10}{y^2+7y}}$

Solve:

⑬ $\dfrac{4x^2}{x^2-9} - \dfrac{2x}{x+3} = \dfrac{3}{x-3}$

# *Rational Expressions*

## UNIT 9 REVIEW & PRACTICE

Multiply / Divide and simplify:

① $\dfrac{a^3-b^3}{b^2-a^2} \cdot \dfrac{a+b}{a^2+ab+b^2}$

② $\dfrac{x^2-11x+24}{x^2-18x+80} \div \dfrac{x^2-9x+20}{x^2-15x+50}$

③ $\dfrac{x^2-2x+1}{y-5} \div \dfrac{(x-1)^2}{y^2-25}$

Add/Subtract and simplify:

④ $\dfrac{3}{m-2} + \dfrac{2}{2-m}$

⑤ $\dfrac{3x+2}{3x-6} - \dfrac{x+2}{x^2-4}$

⑥ $\dfrac{2n}{n^2-5n} - \dfrac{-3n}{n-5}$

Simplify complex fractions:

⑦ $\dfrac{\frac{x}{y} - \frac{y}{x}}{\frac{1}{x} + \frac{1}{y}}$

⑧ $\dfrac{\frac{m+4}{m} - \frac{3}{m+5}}{\frac{m-1}{m^2+5m} + \frac{3}{m+5}}$

⑨ $\dfrac{n+5+\frac{4}{n+1}}{n+3}$

Solve:

⑩ $\dfrac{3x}{2x-5} + \dfrac{2x}{5-2x} = \dfrac{x-1}{2x+5}$

⑪ $\dfrac{3}{x+2} + \dfrac{12}{x^2-4} = \dfrac{-1}{x-2}$

⑫ $\dfrac{x+3}{x+2} = 2 - \dfrac{3}{x^2+5x+6}$

Variation:

⑬ If y varies inversely as x, and y = 9 when x = 5/2, find y when x = −3/5.

⑭ If y varies directly as x, and y = −8 when x = −3, find x when y = 6.

⑮ If area remains constant, a triangle's base and height vary inversely. If the height is 6 and the base is 10, find the height when the base is 12.

# UNIT 10

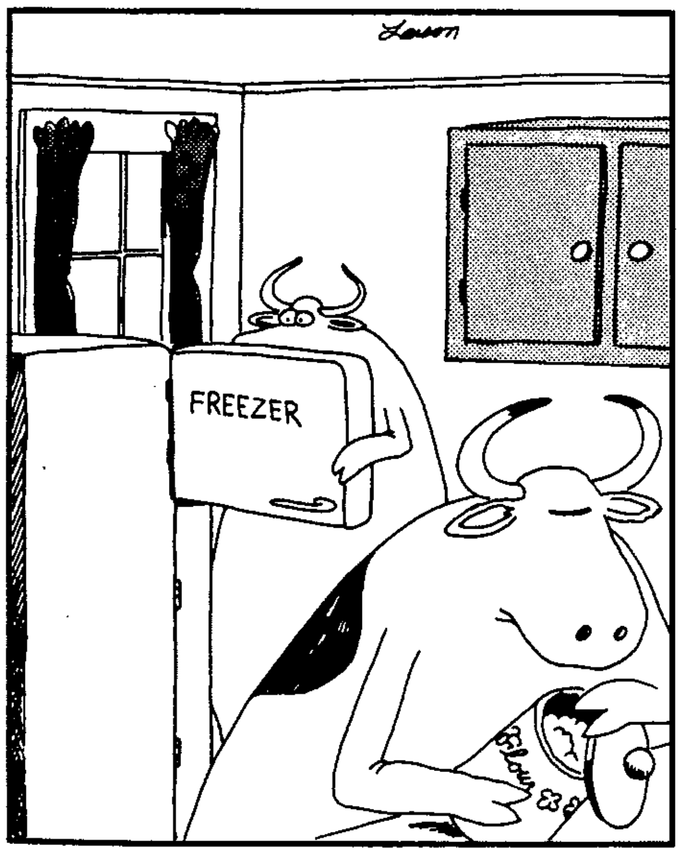
# *Problem Solving*

## Problem Types:

*Ratios*
*Fraction Problems*
*Rate, Time, & Distance*
*Average Speed*
*Work Problems*
*Area Problems*
*Lever Problems*

## Emphasis:

*Quadratic Equations*
*& Rational Equations*




While Farmer Brown was away, the cows got into the kitchen and were having the time of their lives — until Betsy's unwitting discovery.

# *Problem Solving*

## DEMONSTRATION 10.1

The following problems focus on ratios, number problems, area, rate, average speed, and work problems. Many of these involve quadratic equations or rational equations.

### Ratios

① Two numbers are in the ratio 6:11. If the first number is decreased by 4 and the second increased by 6, the resulting numbers are in the ratio 4:9. Find the original numbers.

$$\frac{6n}{11n} \rightarrow \frac{6n-4}{11n+6} = \frac{4}{9}$$

$$9(6n-4) = 4(11n+6)$$
$$54n - 36 = 44n + 24$$
$$10n = 60$$
$$n = 6$$

$$\frac{6n}{11n} = \frac{6(6)}{11(6)} = \frac{36}{66}$$

### Numbers

② The sum of a number's reciprocal and twice the square of the reciprocal is 15. Find the original number.

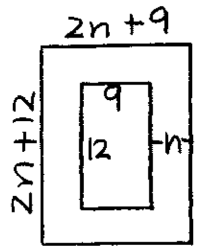
$$\frac{1}{n} + 2\left(\frac{1}{n}\right)^2 = 15$$

$$(n^2)\left[\frac{1}{n} + \frac{2}{n^2} = 15\right]$$

$$n + 2 = 15n^2$$
$$15n^2 - n - 2 = 0$$
$$(3n+1)(5n-2) = 0$$
$$n = -1/3 \text{ or } 2/5$$

### Area

③ Jean has a 9 by 12 ft. rug. Along the outside is a uniform strip of uncovered floor with an area of 270 ft.² How wide is the strip?




$$(2n+9)(2n+12) - (9)(12) = 270$$
$$4n^2 + 42n + 108 - 108 = 270$$
$$4n^2 + 42n - 270 = 0$$
$$2n^2 + 21n - 135 = 0$$

$$\frac{-21 \pm \sqrt{(21)^2 - 4(2)(-135)}}{2(2)}$$

$$\frac{-21 \pm 39}{4} \longrightarrow \cancel{-60/4} \text{ or } 18/4 \quad 4\tfrac{1}{2} \text{ ft.}$$

# *Problem Solving*

## DEMONSTRATION 10.1

Rate

④ A trip of 500 miles would take 2½ hours less if the speed is increased by 10 mph. Find the original speed.

| | R | · | T | = | D |
|---|---|---|---|---|---|
| orig. speed | $(r)$ | | $\left(\frac{500}{r}\right)$ | = | 500 |
| new speed | $(r+10)$ | | $\left(\frac{500}{r+10}\right)$ | = | 500 |

$$\frac{500}{r} - \frac{5}{2} = \frac{500}{r+10}$$

$$\frac{1000-5r}{2r} = \frac{500}{r+10}$$

$(r+10)(1000-5r) = (500)(2r)$
$1000r - 5r^2 + 10{,}000 - 50r = 1000r$
$5r^2 + 50r - 10{,}000 = 0$
$r^2 + 10r - 2000 = 0$
$(r+50)(r-40) = 0$
$r = \cancel{-50}$ or $40$    40 mph

Average Speed

⑤ Andrea drives to the camp ground at 48 mph. She drives 60 mph on the return trip. What was her average speed?

Formula:

$$\text{Average Speed} = \frac{\text{Total distance}}{\text{Total time}}$$

Option 1) Use "d" (distance)

| | R | · | T | = | D |
|---|---|---|---|---|---|
| to camp | 48 | · | $d/48$ | = | $d$ |
| return | 60 | · | $d/60$ | = | $d$ |

$$\frac{\text{total dist.}}{\text{total time}} \quad \frac{2d}{\frac{d}{48} + \frac{d}{60}}$$

$$\frac{2d}{\underset{2880}{\underline{60d+48d}}} = \frac{5760\cancel{d}}{108\cancel{d}} = 53\tfrac{1}{3}\text{ mph}$$

Option 2) Pick a distance

| | R | · | T | = | D |
|---|---|---|---|---|---|
| to camp | 48 | · | $240/48$ | = | 240 |
| return | 60 | · | $240/60$ | = | 240 |

$$\frac{\text{total dist.}}{\text{total time}} \quad \frac{480}{5+4} = \frac{480}{9}$$

$53\tfrac{1}{3}$ mph

# *Problem Solving*

## DEMONSTRATION 10.1

Work Problem

⑥ Sue can paint a room in 4 hours. Laura can do the job in 7 hours. If they do the job together with Laura working 3 times as long, how much time will each one work?

| | R | · | T | = | W |
|---|---|---|---|---|---|
| Sue | 1/4 | · | t | = | t/4 |
| Laura | 1/7 | · | 3t | = | 3t/7 |

$$28\left[\frac{t}{4} + \frac{3t}{7} = 1\right]$$

$7t + 12t = 28$

$19t = 28$

$t = 28/19$ Sue ≈ 1:28

$3t = 84/19$ Laura ≈ 4:25

⑦ If 8 adults can set up the campsite in 6 hours, and 6 children can do the same job in 10 hours, how long would it take a crew of 2 adults and 3 children?

Note: If 8 adults take 6 hours, 2 adults take 24 hours

Note: If 6 children take 10 hours, 3 children take 20 hours

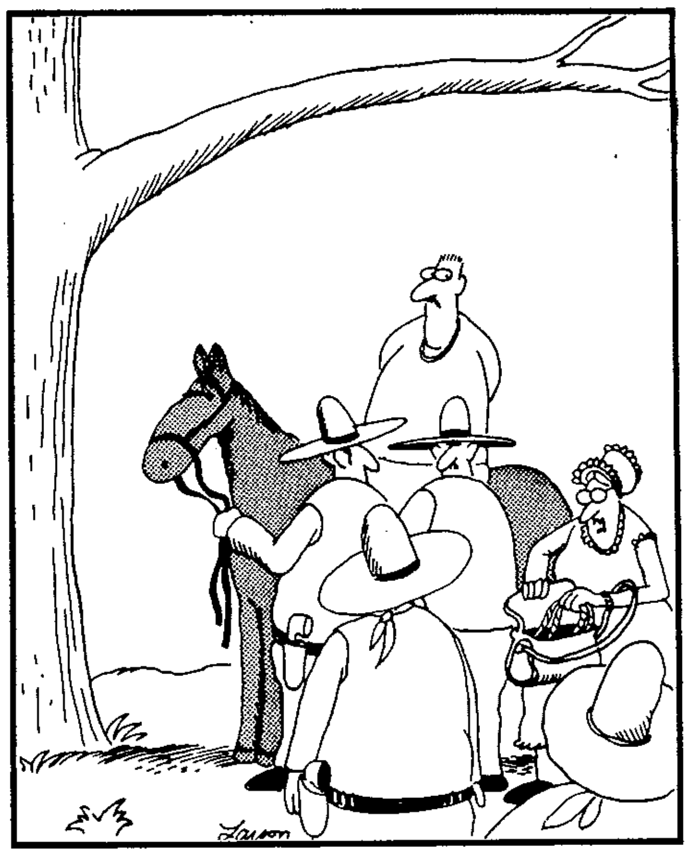
| | R | · | T | = | W |
|---|---|---|---|---|---|
| 2 adults | 1/24 | · | t | = | t/24 |
| 3 children | 1/20 | · | t | = | t/20 |

$$(120)\left[\frac{t}{24} + \frac{t}{20} = 1\right]$$

$5t + 6t = 120$

$11t = 120$

$t = 120/11 \approx 10{:}55$



"Well, good heavens! . . . I can't believe you men . . . I'VE got some rope!"

# Problem Solving

## PROBLEM SET 10.1

Solve:

① The ratio of 4 less than a number to 26 more than the number is 1 to 3. Find the number.

② Two numbers are in a ratio of 6 to 7. If the first number is increased by 2 and the second is increased by 1, the resulting numbers are in the ratio of 4 to 5. Find the original numbers.

③ The sum of a number and 6 times its reciprocal is 5. Find the number.

④ The sum of the reciprocal and twice the square of the reciprocal of a number is 3. Find the number.

⑤ The length of a garden is 6 feet more than its width. A walkway 3 feet wide surrounds the outside of the garden. The total area of the walkway is 288 $ft^2$. Find the dimensions of the garden.

⑥ A rectangular picture is 12 by 16 inches. If a frame of uniform width contains an area of 165 $in^2$, what is the uniform width of the frame?

⑦ A boat travels at a rate of 15 Km per hour in still water. It travels 60 km upstream in the same time that it travels 90 km downstream. What is the rate of the current?

⑧ Increasing the speed of the Pickerington Express Bus by 13 km/h resulted in a 260 km trip taking an hour less than before. What was the original speed of the bus?

⑨ A man drives from Phoenix to Tuscon at 40 mph. He makes the return trip at 60 mph. What is his average speed for the round trip?

⑩ A large pipe can fill a tank in 6 hours and a smaller pipe can fill it in 8 hours. A third pipe can empty the tank in 12 hours. How long would it take to fill the tank if all three pipes are open?

# *Problem Solving*

## PROBLEM SET 10.1

(11) Working alone, a mechanic can do a job in 6 hours. His helper needs 15 hours to do the job alone. When they do a job together, the helper works twice as long as the mechanic. How long will each work when they do a job together?

(12) If 10 students can complete a project in 2 hours and 4 teachers can complete the same job in 1 hour, how long would it take for 2 students and 2 teachers to complete the job together?

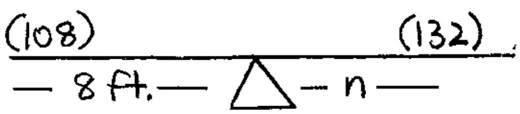
"... and then the second group comes in — 'row, row, row your boat' ..."

# *Problem Solving*

## DEMONSTRATION 10.2

In a lever problem, the product of the weight and its distance from the fulcrum must be equal on both sides for the lever to be in balance.

① The fulcrum of a 16-foot seesaw is placed in the middle. Jason, who weighs 108 pounds, is seated 8 feet from the fulcrum. How far from the fulcrum should Laura sit if she weighs 132 pounds?

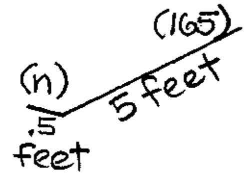



$(108)(8) = (132)(n)$

$864 = 132n$

$n = 6.\overline{54}$      $6.\overline{54}$ feet

② Dean wants to lift a large rock with a crowbar. The short end of the crowbar is 6 inches from the fulcrum and the long end is 5 feet from the fulcrum. If Dean weighs 165, what is the maximum weight he can lift?

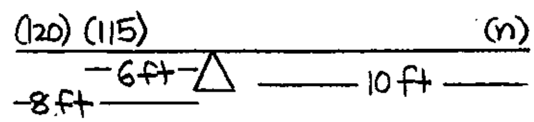



$(165)(5) = (.5)(n)$

$825 = .5n$

$n = 1650$ lbs.

③ Patti and James are seated on the same side of a seesaw. Patti is 6 feet from the fulcrum and weighs 115 pounds. James is 8 feet from the fulcrum and weighs 120 pounds. Jack is seated on the other side, 10 feet from the fulcrum. If the seesaw is balanced, how much does Jack weigh?




$(120)(8) + (115)(6) = (10)(n)$

$960 + 690 = 10n$

$1650 = 10n$

$n = 165$

165 pounds

## PROBLEM SET 10.2

Word Problems (continued):

(13) Mary Jo and Doug are 14 feet apart on opposite ends of a seesaw that is perfectly balanced. Mary Jo weighs 120 pounds and Doug weighs 160 pounds. How far is Mary Jo from the fulcrum?

(14) A lever has a 140 pound weight on one end and a 160 pound weight on the other end. The lever is balanced, and the 140 pound weight is exactly 1 foot farther from the fulcrum. How far from the fulcrum is the 160 pound weight?

(15) The denominator of a fraction is 1 more than twice the numerator. When the numerator is increased by 8 and the denominator is increased by 10, the resulting fraction equals $2/5$. Find the original fraction.

(16) The sum of the numerator and denominator of a fraction is 95. Reduced to lowest terms, the fraction equals $5/14$. Find the original fraction.

(17) The sum of the squares of two consecutive integers is 265. Find the integers.

(18) The denominator of a fraction is 1 less than twice the numerator. If 7 is added to both the numerator and denominator, the resulting fraction has a value of $7/10$. Find the


"Somebody better run fetch the sheriff."

# *Problem Solving*

## PROBLEM SET 10.2

original fraction.

(19) The Hillside Garden Club wants to double the area of its rectangular display of roses. If it is now 6 by 4 meters, by what equal amount must each dimension be increased?

(20) A 25 by 50 ft. rectangular garden is increased on all sides by the same amount. If the area increases by 400 ft,$^2$ how wide all the way around is the uniform addition to the garden?

(21) A plane has a speed of 120 mph in still air. On a certain day, the pilot flies 700 miles with a tailwind. On the return trip, the plane covers 5/7 of the distance in the same time. What is the speed of the wind?

(22) Lenny intends to drive his motorbike to a friend's house 200 miles away. If he increases his planned rate of speed by 10 miles per hour, he can decrease his travel time by 40 minutes. What is his planned rate of speed?

(23) On Monday, Sam drove 40 mph to his cousin's house. On Tuesday, he drove three times farther to his uncle's cabin. What was his rate of speed on Tuesday if he averaged 32 mph for the two days?

(24) A carpenter can build a shed in 6 hours, but his apprentice needs 16 hours to do the same job. When they work together to build the shed, the apprentice works 5 hours more than the carpenter. How long does each work?

(25) Janice can paint a room in 12 hours if she uses a brush and in 8 hours if she uses a roller. She begins to paint with a brush and then changes to a roller. If she finishes in 9 hours, how long did she work with a brush?

(26) If 6 girls can paint the bedroom in 90 minutes and 4 boys can do the same

job in 80 minutes, how long would it take for 3 girls and 2 boys to paint the bedroom ?

(27) Amy, who weighs 108 pounds, is seated 5 feet from the fulcrum of a seesaw. Barbara is seated on the same side of the seasaw, two feet farther from the fulcrum than Amy. Barbara weighs 96 pounds. The seesaw is balanced when Sue, who weighs 101 pounds, sits on the other side. How far is Sue from the fulcrum?

(28) To move a log, Norma places a rock 2 feet from the log to use as a fulcrum. She then uses a 12 foot plank as a lever. If Norma weighs 110 pounds, will she be able to lift the log that weighs 600 pounds?

Challenge Problems

(29) Pipe A can fill a tank in 4 hours and pipe B can fill the tank in 3 hours. With the tank empty, pipe A is turned on, and one hour later, pipe B is turned on as well. How long will pipe B be on before the tank is full? (Answer to the nearest minute).

(30) If a pyramid could be completed in 20 years by 100,000 slaves or by 75,000 paid workers, how long would it take a work force composed of 50,000 slaves and 18,750 paid workers to build the pyramid ?

Note: Despite large numbers, this is similar to #12 and #26.

"So, then . . . Would that be 'us the people' or 'we the people'?"

# *Problem Solving*

## PROBLEM SET 10.2

(31) The distance between two towns consists of two stretches of road whose lengths are consecutive integers. A man drives on the shorter stretch at 60 mph and the rest of the way at 46 mph so that his average speed for the whole trip is 52 mph. Find the total distance.

Note: Make a diagram and label the consecutive integer distances algebraically. In a chart, use rate and distance to determine time. To make an equation, use the formula for average speed. Be patient in working with a very complex fraction.

(32) Flying east between two cities, a plane's speed is 380 mph. On the return trip, it flies at 420 mph. Find the average speed for the round trip. (Note: No wind in this problem).

Note: Since no distance is given, you have two options.

Option 1) Define any distance you want in your chart.

Option 2) Use "d" as the distance.

In both cases, set up a complex fraction using the average speed formula. Simplify the fraction.

(33) Two candles are the same length. One burns up in 6 hours and the other in 9 hours. If they are both lighted at the same time, how long is it before one is twice as long as the other?

Note: Make a diagram of the two candles. Keep in mind that to determine the height of a candle, you must subtract the distance burned up from 1 (which represents an entire candle).

To prepare for the unit test: Redo selected problems from 1–33. There is no review and practice page for this unit.