Friendship Junior High School
Accelerated Math Program
Mr. Lavine (Room 102A)

A.T.I.M.

Advanced Topics In Mathematics

UNIT 4
Inequalities & Coordinate Graphing

UNIT 5
Linear Programming

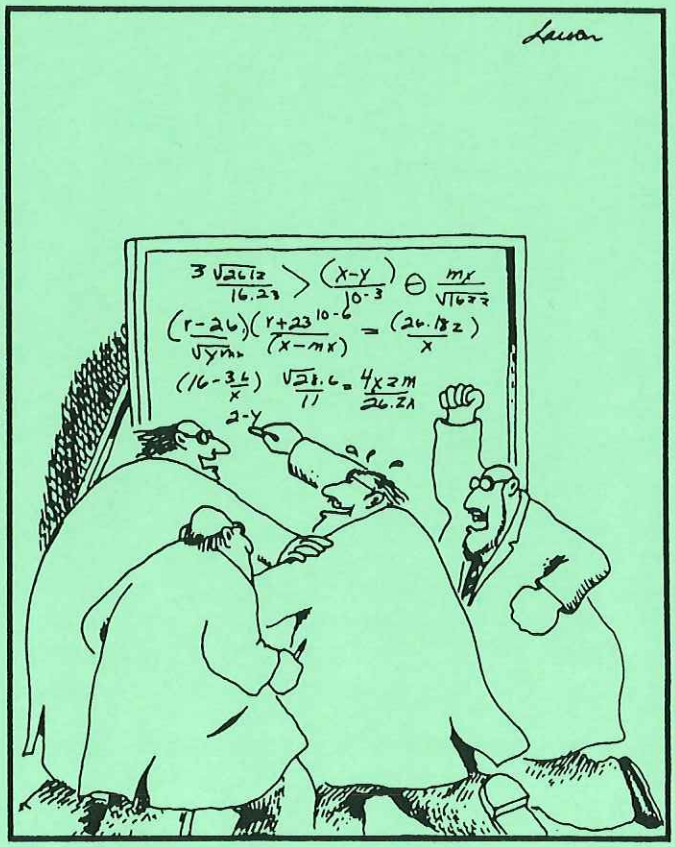
UNIT 6
Roots & Radicals

UNIT 7
Rational Exponents & Complex Numbers

UNIT 8
Quadratics

UNIT 9
Rational Expressions

UNIT 10
Problem Solving



"Go for it, Sidney! You've got it! You've got it! Good hands! Don't choke!"

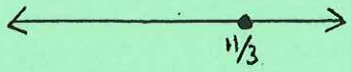
Inequalities & Absolute Value

ANSWER KEY 4.1

① $3(2n-5)=7$

$6n-15=7$

$6n=22$ $\boxed{n=11/3}$



② $3x-\frac{2x}{5}=3(x-1)$

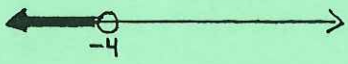
$3x-\frac{2x}{5}=3x-3$

$15x-2x=15x-15$

$-2x=-15$ $\boxed{x=15/2}$

③ $3n+1<n-7$

$2n<-8$ $\boxed{n<-4}$



④ $1+2(x+4)\geq 1+3(x+2)$

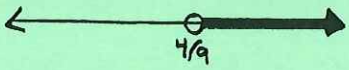
$1+2x+8\geq 1+3x+6$

$-x\geq -2$ $\boxed{x\leq 2}$

⑤ $2(m-5)-3(2m-5)<5m+1$

$2m-10-6m+15<5m+1$

$-9m<-4$ $\boxed{m>4/9}$



⑥ $2n-\frac{n}{4}\leq 2(3n+1)$

$2n-\frac{n}{4}\leq 6n+2$

$8n-n\leq 24n+8$

$-17n\leq 8$ $\boxed{n\geq -8/17}$

⑦ $3n-9>3(n-1)-6$

$3n-9>3n-3-6$

$-9>-9$ $\boxed{\text{no solutions}}$

⑧ $4(n-1)\leq 2n+2(n+3)$

$4n-4\leq 2n+2n+6$

$-4\leq 6$ $\boxed{\text{all solutions}}$

⑨ $\frac{3x-3}{5}<\frac{2(2x-1)}{6}$

$\frac{3x-3}{5}<\frac{4x-2}{6}$

$6(3x-3)<5(4x-2)$

$18x-18<20x-10$

$-2x<8$ $\boxed{x>-4}$

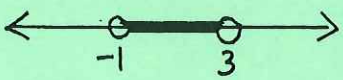
⑩ $-4<3x-1<8$

$-4<3x-1$ and $3x-1<8$

$-3<3x$ and $3x<9$

$-1<x$ and $x<3$

$\boxed{-1<x<3}$



⑪ $|n-4|\leq 8$

$n-4\leq 8$ and $n-4\geq -8$

$n\leq 12$ and $n\geq -4$

$\boxed{-4\leq n\leq 12}$

⑫ $|x+3|>17$

$x+3>17$ or $x+3<-17$

$\boxed{x>14 \text{ or } x<-20}$

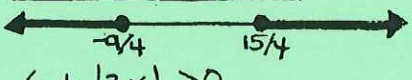
⑬ $|4x-3|+3\geq 15$

$|4x-3|\geq 12$

$4x-3\geq 12$ or $4x-3\leq -12$

$4x\geq 15$ or $4x\leq -9$

$\boxed{x\geq 15/4 \text{ or } x\leq -9/4}$



⑭ $6+|3x|>0$

$|3x|>-6$ $\boxed{\text{all solutions}}$

⑮ $|2n-5|<7$

$2n-5<7$ and $2n-5>-7$

$2n<12$ and $2n>-2$

$n<6$ and $n>-1$

$\boxed{-1<n<6}$

Inequalities & Absolute Value

ANSWER KEY 4.1

(16) $|6a+25|+14<6$

$|6a+25|<-8$ **no solutions**

(17) $|n-3|<5$ and $|n|\neq 1$

$(n-3<5$ and $n-3>-5)$ and $(n\neq 1$ and $n\neq -1)$

$(n<8$ and $n>-2)$

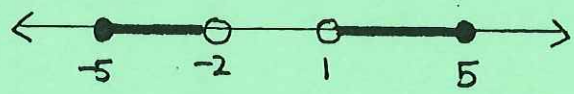
$(-2<n<8)$ and $(n\neq 1$ and $n\neq -1)$

(18) $|2n+1|>3$ and $|n|\leq 5$

$(2n+1>3$ or $2n+1<-3)$ and $(n\leq 5$ and $n\geq -5)$

$(2n>2$ or $2n<-4)$

$(n>1$ or $n<-2)$ and $(-5\leq n\leq 5)$



(19) $4<|2n-2|\leq 12$

$4<|2n-2|$ and $|2n-2|\leq 12$

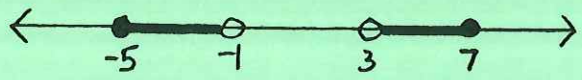
$|2n-2|>4$

$(2n-2>4$ or $2n-2<-4)$ and $(2n-2\leq 12$ and $2n-2\geq -12)$

$(2n>6$ or $2n<-2)$ and $(2n\leq 14$ and $2n\geq -10)$

$(n>3$ or $n<-1)$ and $(n\leq 7$ and $n\geq -5)$

$(n>3$ or $n<-1)$ and $(-5\leq n\leq 7)$



(20) $2\leq|4n+2|<18$

$2\leq|4n+2|$ and $|4n+2|<18$

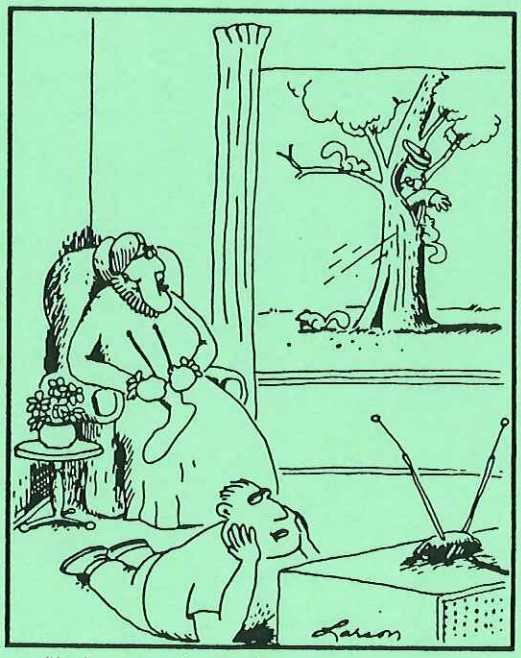
$|4n+2|\geq 2$

$(4n+2\geq 2$ or $4n+2\leq -2)$ and $(4n+2<18$ and $4n+2>18)$

$(4n\geq 0$ or $4n\leq -4)$ and $(4n<16$ and $4n>-20)$

$(n\geq 0$ or $n\leq -1)$ and $(n<4$ and $n>-5)$

$(n\geq 0$ or $n\leq -1)$ and $(-5<n<4)$



"Andrew, go out and get your grandfather . . . the squirrels have got him again."

Graphing Linear Equations

ANSWER KEY 4.2

① $(-6,4)(2,-3)$

$\frac{(4)-(-3)}{(-6)-(2)} = \frac{7}{-8}$

$y = \frac{-7}{8}x + b$

$(4) = \frac{-7}{8}(-6) + b$

$4 = \frac{21}{4} + b$

$b = -5/4$

$\boxed{y = -7/8x - 5/4}$

② $(5,-2)(1,-6)$

$\frac{(-2)-(-6)}{(5)-(1)} = \frac{4}{4} = 1$

$\frac{A}{-B} = 1$ $A=1$ $B=-1$

$x - y = C$

$(5)-(-2) = 7$

$\boxed{x - y = 7}$

③ $(4,9)(-5,-2)$

$\frac{(9)-(-2)}{(4)-(-5)} = \frac{11}{9}$

$\boxed{y - 9 = 11/9(x-4)}$

- or -

$\boxed{y + 2 = 11/9(x+5)}$

④ $y = 2/3x + 6$ parallel slope $= 2/3$

$\frac{A}{-B} = \frac{2}{3}$ $A=2$ $B=-3$

$2x - 3y = C$ $(4,-2)$

$2(4) - 3(-2) = 14$

$\boxed{2x - 3y = 14}$

⑤ $3x - 4y = 12$ parallel slope $= 3/4$

$y = 3/4x + b$ $(-6,3)$

$(3) = \frac{3}{4}(-6) + b$

$b = 15/2$

$\boxed{y = 3/4x + 15/2}$

⑥ $x - 5y = 6$ parallel slope $= 1/5$

$(6,4)$ $\boxed{y - 4 = \frac{1}{5}(x-6)}$

⑦ $y = 4/5x - 12$ perpendicular slope $= -5/4$

$\frac{-A}{B} = \frac{-5}{4}$ $A=5$ $B=4$

$Ax + By = C$ $(0,6)$

$5x + 4y = C$

$5(0) + 4(6) = 24$

$\boxed{5x + 4y = 24}$

⑧ $2x - y = 8$ perpendicular slope $= -1/2$

$y = -\frac{1}{2}x + b$ $(9,-4)$

$(-4) = -\frac{1}{2}(9) + b$

$b = 1/2$

$\boxed{y = -1/2x + 1/2}$

⑨ $3x - 2y = 8$ perpendicular slope $= -2/3$

$(-3,1)$ $\boxed{y - 1 = -2/3(x+3)}$

⑩ $x = 4$ is vertical

$y =$ __ is perpendicular (horizontal)

$(6,-7)$ $\boxed{y = -7}$

⑪ $y = -4x - 3$ parallel slope $= -4$

$y = -4x + b$

$b = 0$ for all lines through the origin

$\boxed{4x + y = 0}$ or $\boxed{y = -4x}$

⑫ $3x - y = 4$

$-y = -3x + 4$

$\boxed{y = 3x - 4}$

⑬ $x + 3y = 8$

$3y = -x + 8$

$\boxed{y = -1/3x + 8/3}$

⑭ $2x + 5y = 10$

$5y = -2x + 10$

$\boxed{y = -2/5x + 2}$

⑮ $5x - 3y = 9$

$-3y = -5x + 9$

$\boxed{y = 5/3x - 3}$

Graphing Linear Equations

ANSWER KEY 4.2

(16) $2x - 3y = 18$

slope = $-A/B$ → $2/3$

y-int = C/B → -6 → $(0,-6)$

x-int = C/A → 9 → $(9,0)$

(17) $y = -2/3x + 4$

slope = m → $-2/3$

y-int = b → 4 → $(0,4)$

x-int = $-b/m$ → 6 → $(6,0)$

(18) $y + 3 = 3/4(x-6)$

slope = $3/4$

Change to slope-int form

$y = 3/4(x-6) - 3$

$y = 3/4x - 15/2$

y-int = b → $-15/2$ → $(0,-15/2)$

x-int = $-b/m$ → 10 → $(10,0)$

(19) $3x + y = 7$

slope = $-A/B$ → -3

y-int = C/B → 7 → $(0,7)$

x-int = C/A → $7/3$ → $(7/3,0)$

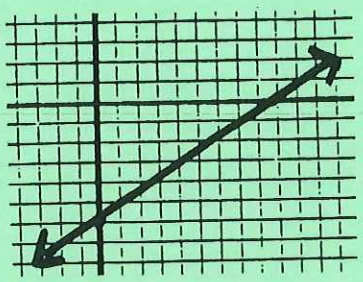
(20) $y = -\frac{1}{4}x - 2$

slope = m → $-1/4$

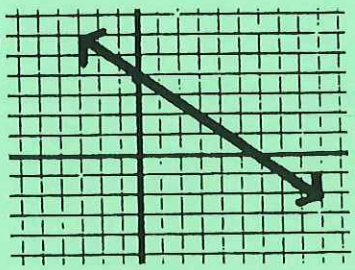
y-int = b → -2 → $(0,-2)$

x-int = $-b/m$ → -8 → $(-8,0)$

(16)



(17)



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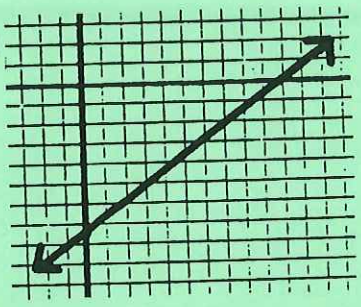

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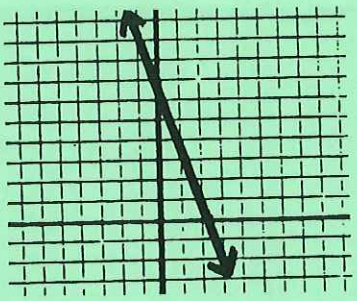
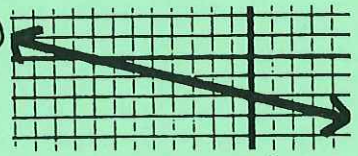


(20)



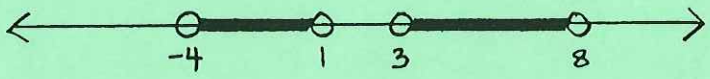
(21) $2 < |2x-4| < 12$

$|2x-4| > 2$ and $|2x-4| < 12$

$(2x-4 > 2$ or $2x-4 < -2)$ and $(2x-4 < 12$ and $2x-4 > -12)$

$(2x > 6$ or $2x < 2)$ $\quad (x < 8$ and $x > -4)$

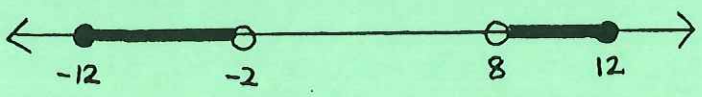
$(x > 3$ or $x < 1)$ and $(-4 < x < 8)$



(22) $|n-3| > 5$ and $|n| \leq 12$

$(n-3 > 5$ or $n-3 < -5)$ and $(n \leq 12$ and $n \geq -12)$

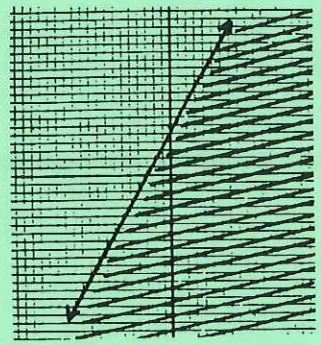
$(n > 8$ or $n < -2)$ and $(-12 \leq n \leq 12)$



Graphing Systems of Inequalities

ANSWER KEY 4.3

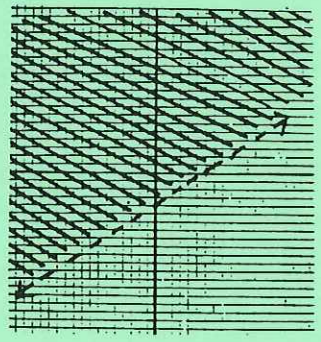
① $y \leq 2x+6$



② $2x-3y<12$

$-3y<-2x+12$

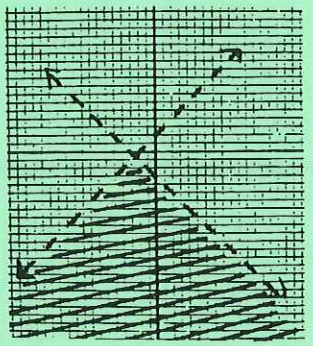
$y>\frac{2}{3}x-4$



③ $y \geq x-3$ $y \geq -x-3$

④ $y<-x-3$ $x+2>y$

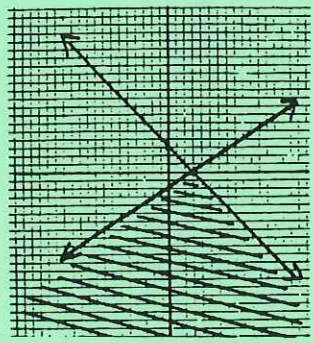
$y<x+2$



⑤ $x+y \leq 3$ $2x-3y \geq 6$

$y \leq -x+3$ $-3y \geq -2x+6$

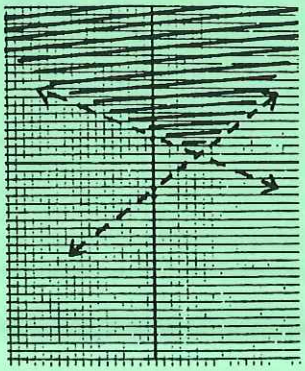
$y \leq \frac{2}{3}x-2$



⑥ $x+2y>8$ $3x-4y<12$

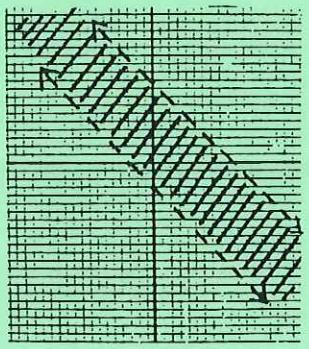
$2y>-x+8$ $-4y<-3x+12$

$y>-\frac{1}{2}x+4$ $y>\frac{3}{4}x-3$

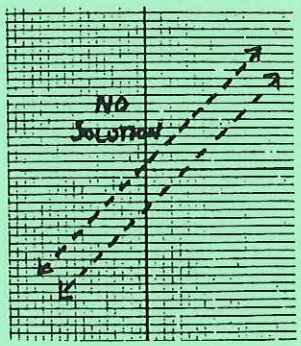


⑦ $x+y>-3$ $x+y<10$

$y>-x-3$ $y<-x+10$



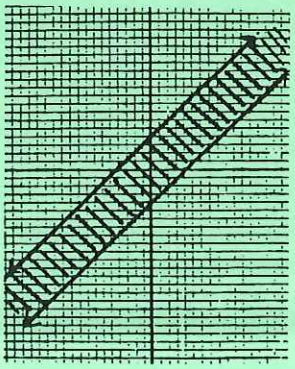
⑧ $y>x+1$ $y<x-5$



⑨ $|y-x| \leq 4$

$y-x \leq 4$ and $y-x \geq -4$

$y \leq x+4$ and $y \geq x-4$



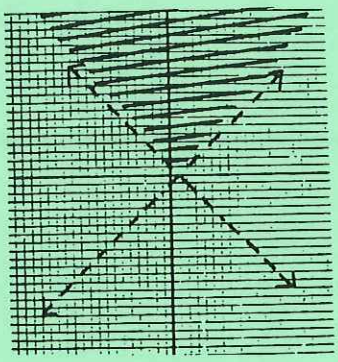
Graphing Systems of Inequalities

ANSWER KEY 4.3

⑩ $|x-1| < y$

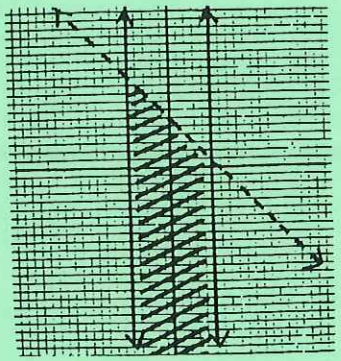
$x-1 < y$ and $x-1 > -y$

$y > x-1$ and $y > -x+1$



⑬ $|x| \leq 5 \quad x+y < 6$

$(x \leq 5$ and $x \geq -5)$ and $(y < -x+6)$



⑯ $|y-x| > 3$

$y-x > 3$ or

$y-x < -3$

$y > x+3$ or

$y < x-3$

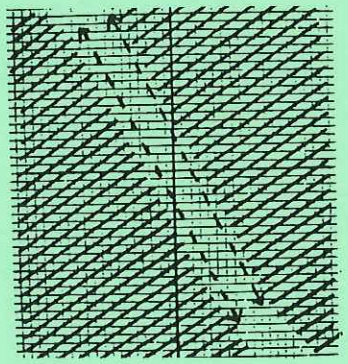
$|y| < 10$

$y < 10$ and $y > -10$

⑪ $|2x+y| > 4$

$2x+y > 4$ or $2x+y < -4$

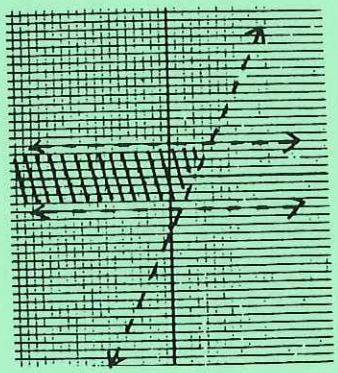
$y > -2x+4$ or $y < -2x-4$

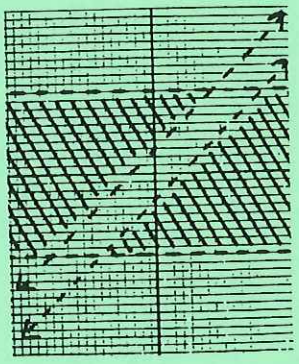


⑭ $|y| < 4 \quad 2x-y < 6$

$(y < 4$ and $y > -4)$ and $(-y < -2x+6)$

$(y > 2x-6)$

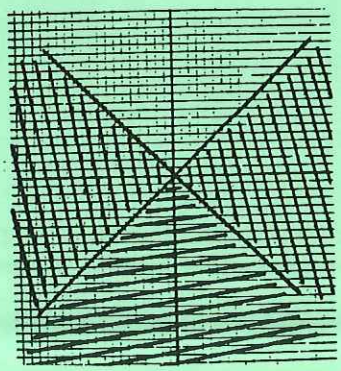




⑫ $|x| \geq y$

$x \geq y$ or $x \leq -y$

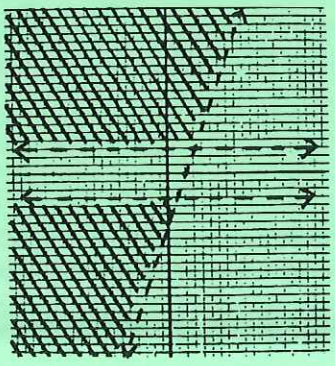
$y \leq x$ or $y \leq -x$



⑮ $|y| > 3 \quad 3x-y < 6$

$(y > 3$ or $y < -3)$ and $(-y < -3x+6)$

$(y > 3x-6)$



Inequalities & Coordinate Graphing

ANSWER KEY: UNIT 4 REVIEW & PRACTICE

① $2(x+4)+\frac{x}{3} < 4x-1$

$2x+8+\frac{x}{3} < 4x-1$

$6x+24+x < 12x-3$

$-5x < -27$ $\boxed{x > 27/5}$

② $\frac{x+3}{4} \geq \frac{2(x-3)}{3}$

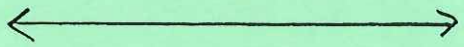
$3x+9 \geq 8x-24$

$-5x \geq -33$ $\boxed{x \leq 33/5}$

③ $3(2x-3) \geq 2(3x+2)$

$6x-9 \geq 6x+4$

$-9 \geq 4$ $\boxed{\text{no solutions}}$



④ $4(x-1) < 4x+5$

$4x-4 < 4x+5$

$-4 < 5$ $\boxed{\text{all solutions}}$

⑤ $-1 < n+4 < 9$

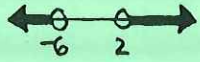
$\boxed{-5 < n < 5}$

⑥ $|3x+6| > 12$

$3x+6 > 12$ or $3x+6 < -12$

$3x > 6$ or $3x < -18$

$\boxed{x > 2 \text{ or } x < -6}$



⑦ $3 \leq |n-4| \leq 10$

$|n-4| \geq 3$ and $|n-4| \leq 10$

$(n-4 \geq 3$ or $n-4 \leq -3)$ and $(n-4 \leq 10$ and $n-4 \geq -10)$

$(n \geq 7$ or $n \leq 1)$ and $(n \leq 14$ and $n \geq -6)$

$\boxed{(n \geq 7 \text{ or } n \leq 1) \text{ and } (-6 \leq n \leq 14)}$

⑧ $|2x+1| > 5$ and $|x| < 10$

$(2x+1 > 5$ or $2x+1 < -5)$ and $(x < 10$ and $x > -10)$

$(2x > 4$ or $2x < -6)$ and $(x < 10$ and $x > -10)$

$\boxed{(x > 2 \text{ or } x < -3) \text{ and } (-10 < x < 10)}$

⑨ $5x-3y=4$ parallel slope $= 5/3$

$y = \frac{5}{3}x+b$ $(6,-2)$ $\boxed{y = \frac{5}{3}x-12}$

$(-2) = \frac{5}{3}(6)+b$

$b = -12$

y-int $\boxed{(0,-12)}$

x-int $\boxed{(36/5,0)}$

⑩ $y = -3/4x+7$ perpendicular slope $= 4/3$

$\frac{4}{3} = \frac{A}{-B}$ $A=4$ $B=-3$ $Ax+By=C$ $(2,5)$

$4x-3y=C$

$4(2)-3(5) = -7$

y-int $\boxed{(0,7/3)}$

x-int $\boxed{(-7/4,0)}$

$\boxed{4x-3y=-7}$

⑪ $(-2,6)$ $(10,-1)$

$\frac{(6)-(-1)}{(-2)-(10)} = \frac{7}{-12}$

$\boxed{y-6 = \frac{-7}{12}(x+2)}$ -or-

$\boxed{y+1 = -7/12(x-10)}$

⑫ $2x+3y=-4$

$3y = -2x-4$

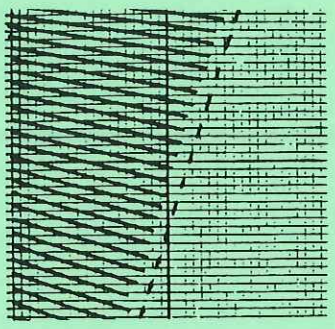
$\boxed{y = -2/3x-4/3}$

Inequalities & Coordinate Graphing

ANSWER KEY: UNIT 4 REVIEW & PRACTICE

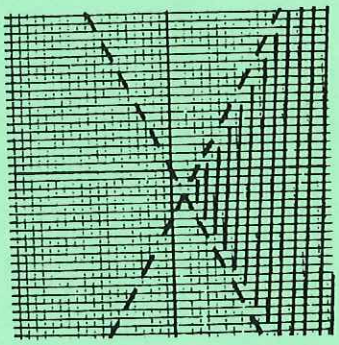
(13) $3x-y<8$

$-y<-3x+8 \rightarrow y>3x-8$



(14) $2x-y>6 \rightarrow y<2x-6$

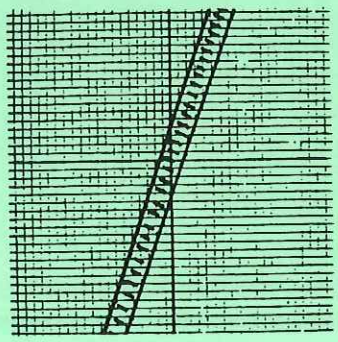
$y>-2x$



(15) $|3x-y|\leq 4$

$3x-y\leq 4$ and $3x-y\geq -4$

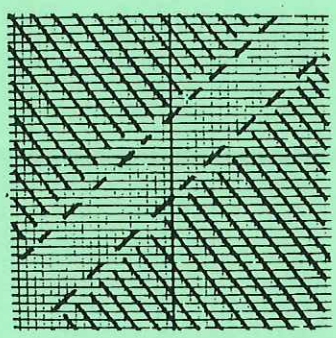
$y\geq 3x-4$ and $y\leq 3x+4$



(16) $|x-y|>5$

$x-y>5$ or $x-y<-5$

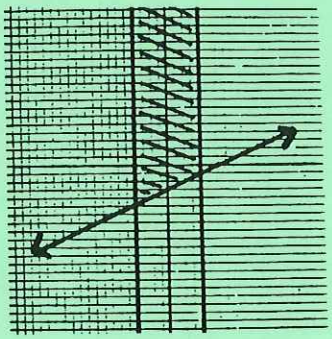
$y<x-5$ or $y>x+5$



(17) $|x|\leq 4$ and $x-2y\leq 6$

$(x\leq 4$ and $x\geq -4)$ and $(-2y\leq -x+6)$

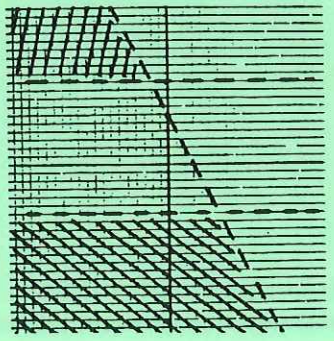
$(-4\leq x\leq 4)$ and $(y\geq \frac{1}{2}x-3)$



(18) $|y-2|>8$ and $2x+y<6$

$(y-2>8$ or $y-2<8)$ and $(y<-2x+6)$

$(y>10$ or $y<-6)$ and $(y<-2x+6)$



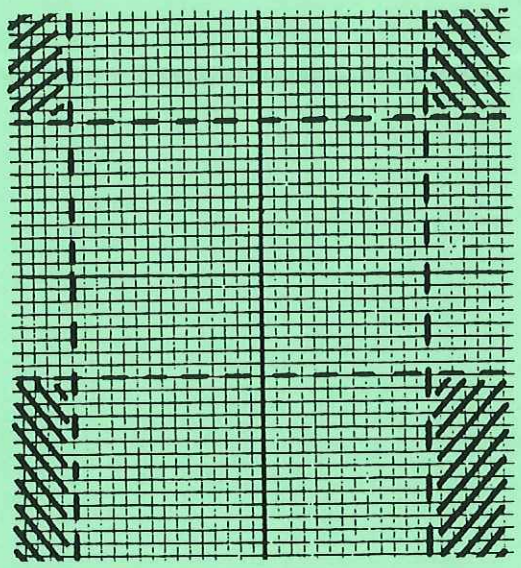
Inequalities & Coordinate Graphing

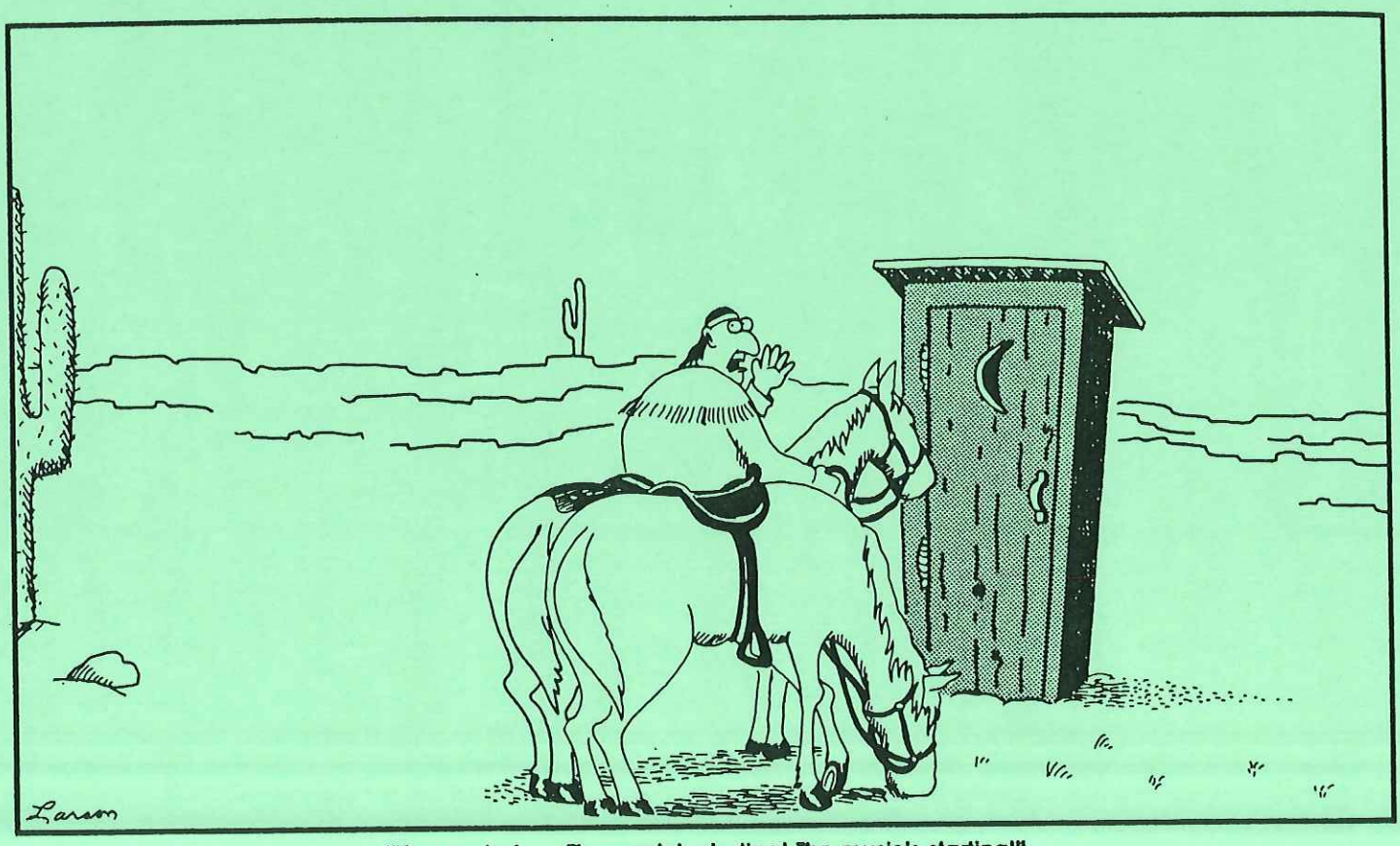
ANSWER KEY: UNIT 4 REVIEW & PRACTICE

(19) $|x+1| > 14$ and $|y-2| > 10$

$(x+1 > 14$ or $x+1 < -14)$ and $(y-2 > 10$ or $y-2 < -10)$

$(x > 13$ or $x < -15)$ and $(y > 12$ or $y < -8)$





"Kemosabe! . . . The music's starting! The music's starting!"

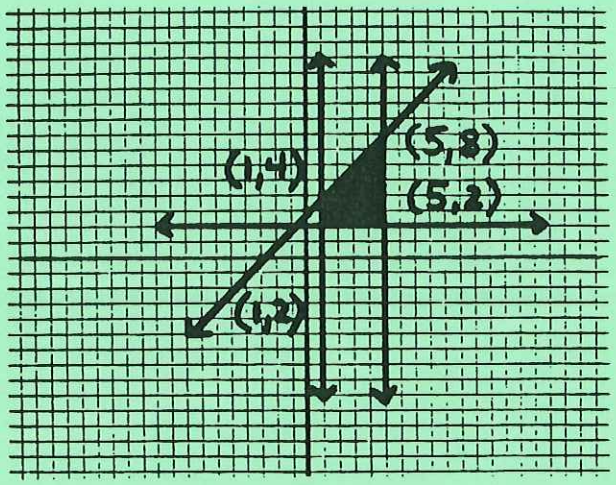
Graphing Systems

ANSWER KEY 5.1

① $y \geq 2$
$1 \leq x \leq 5$
$y \leq x+3$

$f(x,y) = 3x-2y$

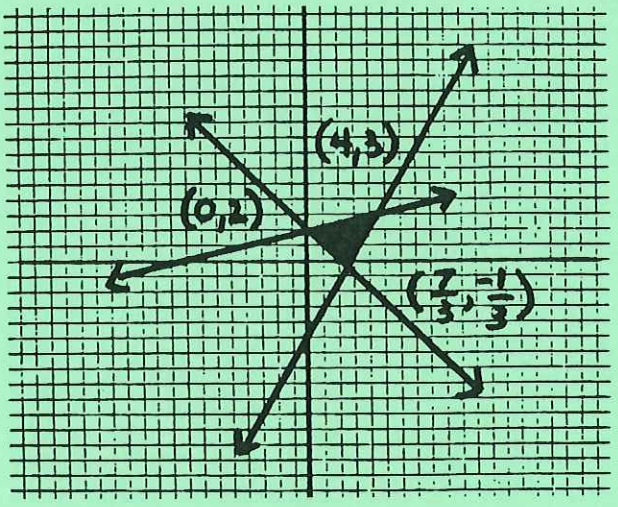
$(1,2)$	$3(1)-2(2) = -1$	
$\boxed{(1,4)}$	$3(1)-2(4) = \boxed{-5}$	min
$(5,8)$	$3(5)-2(8) = -1$	
$\boxed{(5,2)}$	$3(5)-2(2) = \boxed{11}$	max



② $x+y \geq 2 \rightarrow y \geq -x+2$
$4y \leq x+8 \rightarrow y \leq \frac{1}{4}x+2$
$y \geq 2x-5$

$f(x,y) = 4x+3y$

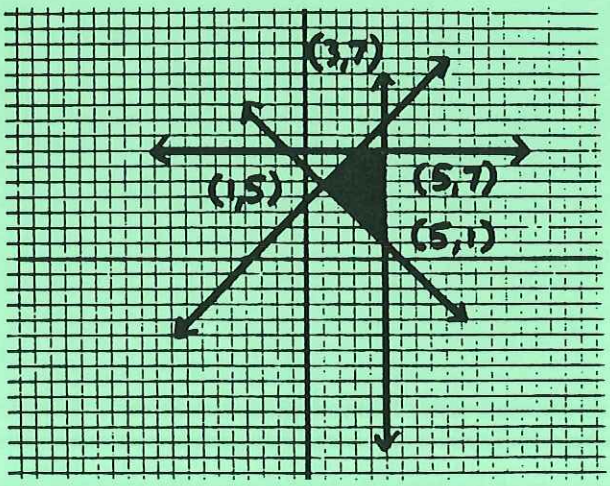
$\boxed{(0,2)}$	$4(0)+3(2) = \boxed{6}$	min
$\boxed{(4,3)}$	$4(4)+3(3) = \boxed{25}$	max
$(\frac{7}{3}, -\frac{1}{3})$	$4(\frac{7}{3})+3(-\frac{1}{3}) = 25/3$	



③ $y \leq 7$
$y \leq x+4$
$y \geq -x+6$
$x \leq 5$

$f(x,y) = 2x-3y$

$(1,5)$	$= 2(1)-3(5) = -13$	
$\boxed{(3,7)}$	$= 2(3)-3(7) = \boxed{-15}$	min
$(5,7)$	$= 2(5)-3(7) = -11$	
$\boxed{(5,1)}$	$= 2(5)-3(1) = \boxed{7}$	max



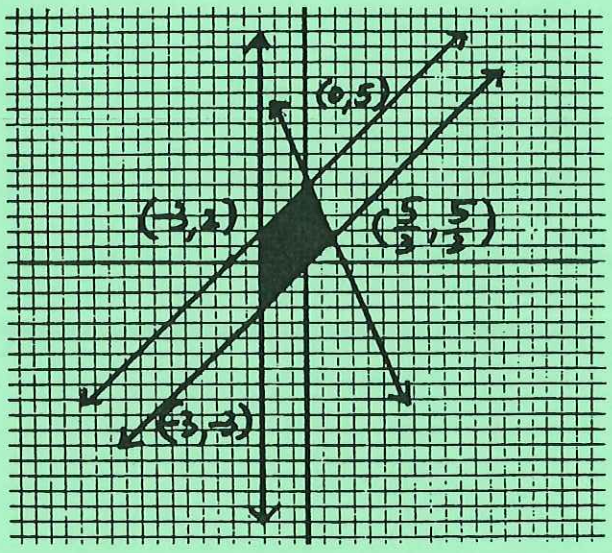
Graphing Systems

ANSWER KEY 5.1

④ $y \le x+5$
$y \ge x$
$x \ge -3$
$y+2x \le 5 \rightarrow y \le -2x+5$

$f(x,y) = x-2y$

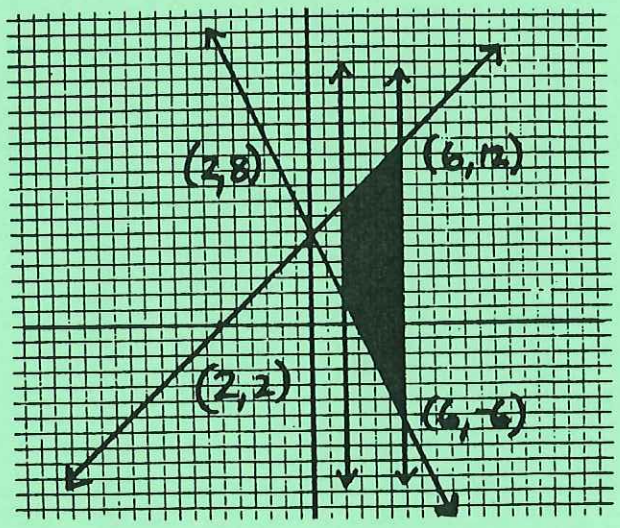
$\boxed{(0,5)}$	$(0)-2(5) =$	$\boxed{-10}$	min
$(-3,2)$	$(-3)-2(2) =$	-7	
$\boxed{(-3,-3)}$	$(-3)-2(-3) =$	$\boxed{3}$	max
$(5/3, 5/3)$	$(5/3)-2(5/3) =$	$-5/3$	



⑤ $y \le x+6$
$y+2x \ge 6 \rightarrow y \ge -2x+6$
$2 \le x \le 6$

$f(x,y) = 3x+y$

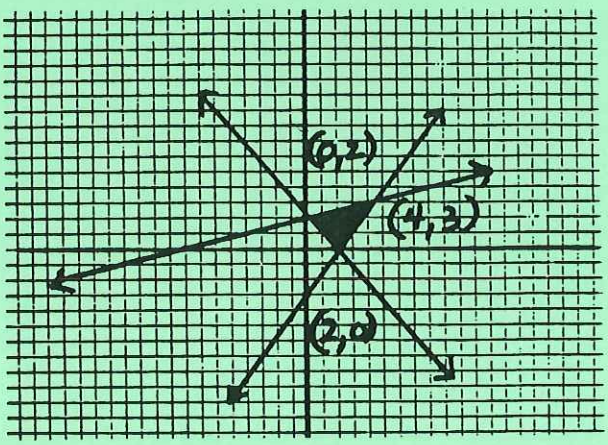
$(2,8)$	$3(2)+(8) =$	14	
$\boxed{(6,12)}$	$3(6)+(12) =$	$\boxed{30}$	max
$\boxed{(2,2)}$	$3(2)+(2) =$	$\boxed{8}$	min
$(6,-6)$	$3(6)+(-6) =$	12	



⑥ $x+y \ge 2 \rightarrow y \ge -x+2$
$4y \le x+8 \rightarrow y \le \frac{1}{4}x+2$
$2y \ge 3x-6 \rightarrow y \ge \frac{3}{2}x-3$

$f(x,y) = 3y+x$

$\boxed{(4,3)}$	$3(3)+(4) =$	$\boxed{13}$	max
$\boxed{(2,0)}$	$3(0)+(2) =$	$\boxed{2}$	min
$(0,2)$	$3(2)+(0) =$	6	

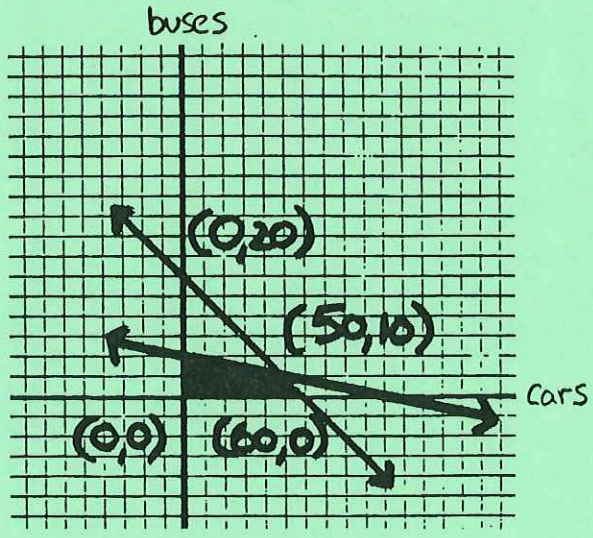


Problem Solving

ANSWER KEY 5.2

① x = cars
y = buses

$x \geq 0$
$y \geq 0$
$x + y \leq 60 \rightarrow y \leq -x + 60$
$6x + 30y \leq 600 \rightarrow y \leq \frac{-1}{5}x + 20$



$f(x,y) = 2.5x + 7.5y$

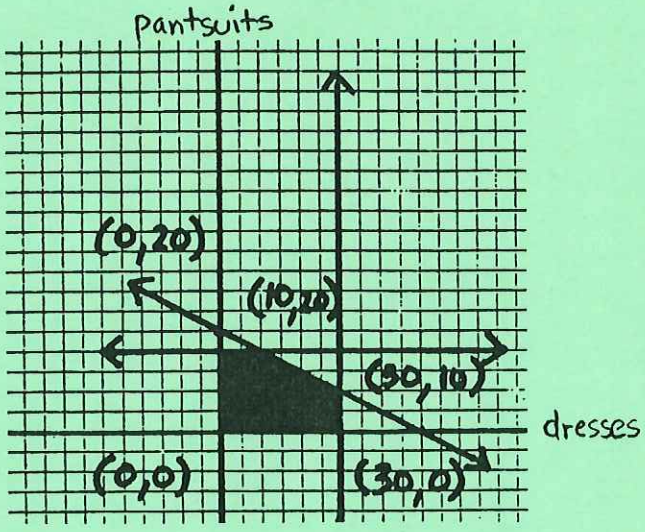
(0,0)	$2.5(0)+7.5(0)$	$= 0$
(60,0)	$2.5(60)+7.5(0)$	$= 150$
(50,10)	$2.5(50)+7.5(10)$	$= \boxed{200}$
(0,20)	$2.5(0)+7.5(20)$	$= 150$

50 cars
10 buses

$200 profit

② x = dresses
y = pantsuits

$x \leq 30$
$y \leq 20$
$10x + 20y \leq 500 \rightarrow y \leq \frac{-1}{2}x + 25$



a) $f(x,y) = 40x + 40y$ b) $f(x,y) = 15x + 45y$

	$40(0)+40(0)$	$= 0$	(0,0)	$15(0)+45(0) = 0$
	$40(30)+40(0)$	$= 1200$	(30,0)	$15(30)+45(0) = 450$
	$40(30)+40(10)$	$= \boxed{1600}$	(30,10)	$15(30)+45(10) = 900$
	$40(10)+40(20)$	$= 1200$	(10,20)	$15(10)+45(20) = \boxed{1050}$
	$40(0)+40(20)$	$= 800$	(0,20)	$15(0)+45(20) = 900$

a) **30 dresses**
10 pantsuits

$1600 profit

b) **10 dresses**
20 pantsuits

$1050 profit

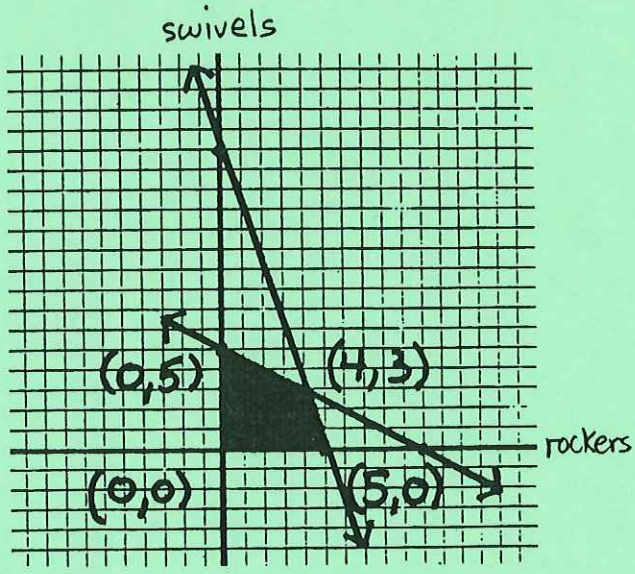
Problem Solving

ANSWER KEY 5.2

③ x = rockers
y = swivels

$x \geq 0$
$y \geq 0$

Op A) $2x + 4y \leq 20 \rightarrow y \leq -\frac{1}{2}x + 5$
Op B) $3x + y \leq 15 \rightarrow y \leq -3x + 15$



$f(x,y) = 12x + 10y$

(0,0)	$12(0)+10(0)$	= 0
(0,5)	$12(0)+10(5)$	= 50
(4,3)	$12(4)+10(3)$	= **78**
(5,0)	$12(5)+10(0)$	= 60

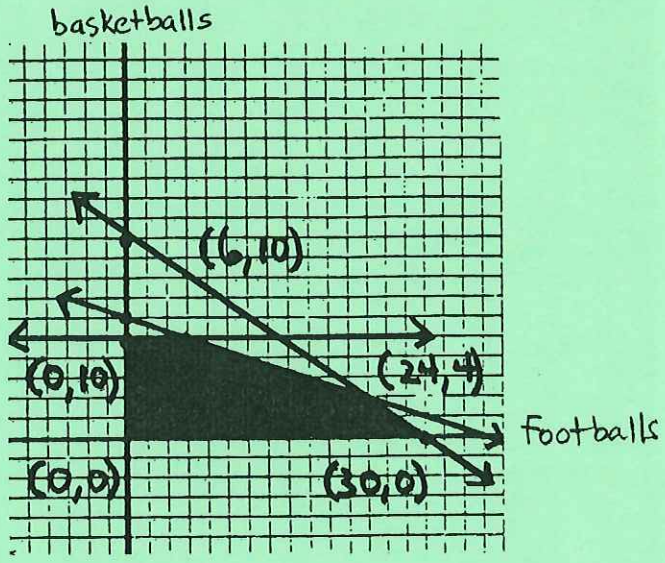
4 rockers
3 swivels

$78 profit

④ x = footballs
y = basketballs

$x \geq 0$ & $y \geq 0$

Op A) $4x + 6y \leq 120 \rightarrow y \leq \frac{2}{3}x + 20$
Op B) $2x + 6y \leq 72 \rightarrow y \leq -\frac{1}{3}x + 12$
Op C) $y \leq 10$



$f(x,y) = 3x + 2y$

(0,0)	$3(0)+2(0)$	= 0
(0,10)	$3(0)+2(10)$	= 20
(6,10)	$3(6)+2(10)$	= 38
(24,4)	$3(24)+2(4)$	= 80
(30,0)	$3(30)+2(0)$	= **90**

30 footballs, 0 basketballs

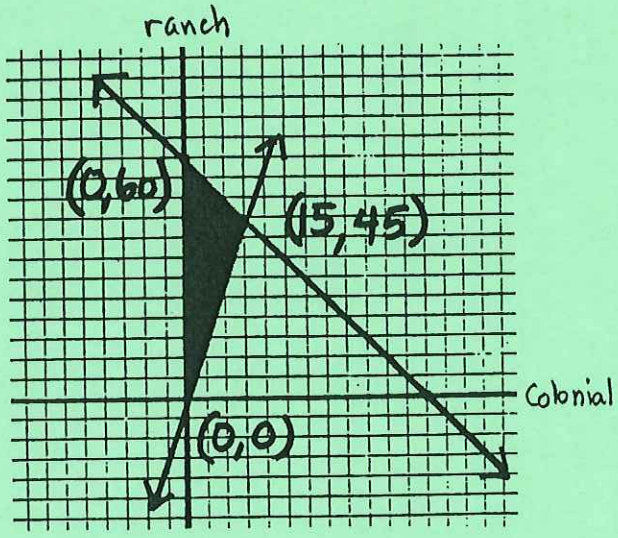
$90 profit

Problem Solving

ANSWER KEY 5.2

⑤ x = colonial houses
y = ranch houses

$x \geq 0$
$y \geq 0$
$x + y \leq 60 \rightarrow y \leq -x + 60$
$y \geq 3x$



$f(x,y) = 5000x + 4500y$

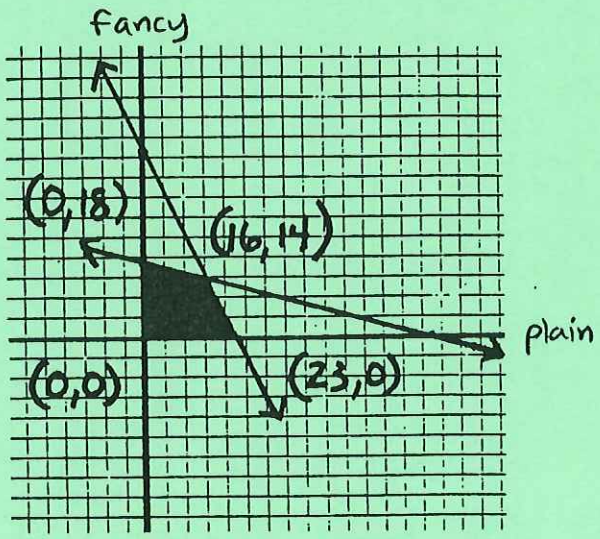
(0,0)	$5000(0) + 4500(0)$	$= 0$
(0,60)	$5000(0) + 4500(60)$	$= 270{,}000$
(15,45)	$5000(15) + 4500(45)$	$= \mathbf{277{,}500}$

15 colonial houses
45 ranch houses

$277,500 profit

⑥ x = plain legs
y = fancy legs

$x \geq 0$
$y \geq 0$
lathe) $2x + y \leq 46 \rightarrow y \leq -2x + 46$
sand) $x + 4y \leq 72 \rightarrow y \leq -\frac{1}{4}x + 18$



$f(x,y) = 3x + 5y$

(0,0)	$3(0) + 5(0)$	$= 0$
(0,18)	$3(0) + 5(18)$	$= 90$
(16,14)	$3(16) + 5(14)$	$= \mathbf{118}$
(23,0)	$3(23) + 5(0)$	$= 69$

16 plain legs
14 fancy legs

$118 profit

Linear Programming

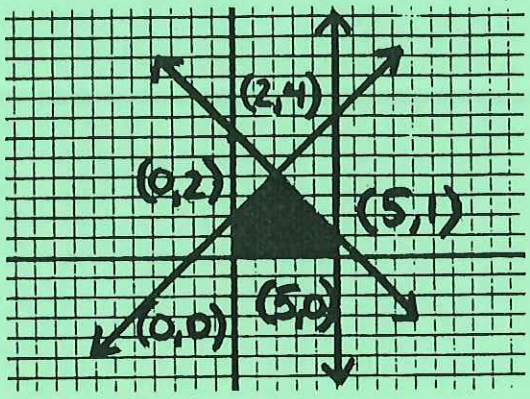
ANSWER KEY: UNIT 5 REVIEW & PRACTICE

① $y \geq 0$

$0 \leq x \leq 5$

$-x + y \leq 2 \rightarrow y \leq x + 2$

$x + y \leq 6 \rightarrow y \leq -x + 6$



$f(x,y) = 5x - 3y$

$(0,0) = 0$ $\quad (2,4) = -2$

$\boxed{(5,0)} = \boxed{25}$ max $\quad \boxed{(0,2)} = \boxed{-6}$ min

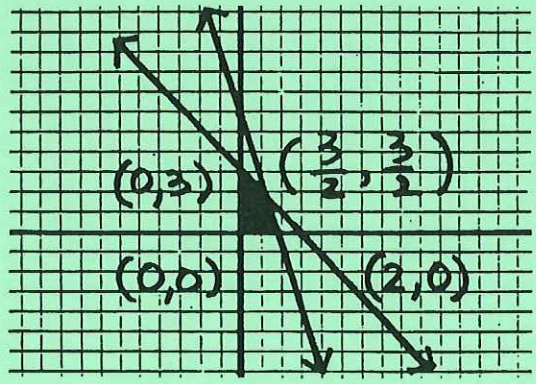
$(5,1) = 22$

② $x \geq 0$

$y \geq 0$

$x + y \leq 3 \rightarrow y \leq -x + 3$

$3x + y \leq 6 \rightarrow y \leq -3x + 6$



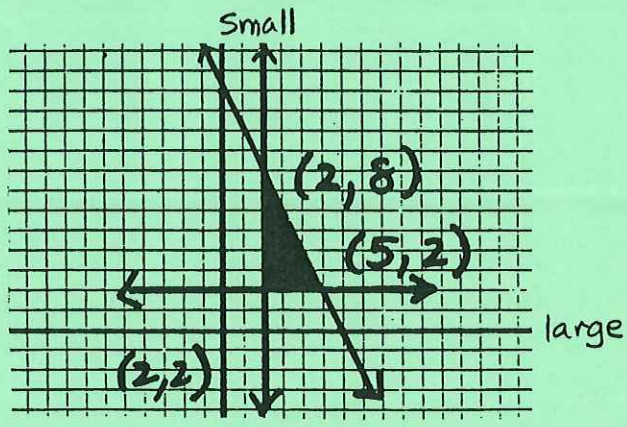
$f(x,y) = 2x + 4y$

$\boxed{(0,0)} = \boxed{0}$ min $\quad (\frac{3}{2}, \frac{3}{2}) = 9$

$\boxed{(0,3)} = \boxed{12}$ max $\quad (2,0) = 4$

③ x = large $\quad x \geq 2$ and $y \geq 2$

y = small $\quad 4x + 2y \leq 24 \rightarrow y \leq -2x + 12$



$f(x,y) = 50x + 20y$

$(2,2)$ $50(2) + 20(2) = 140$

$(2,8)$ $50(2) + 20(8) = 260$

$\boxed{(5,2)}$ $50(5) + 20(2) = \boxed{290}$

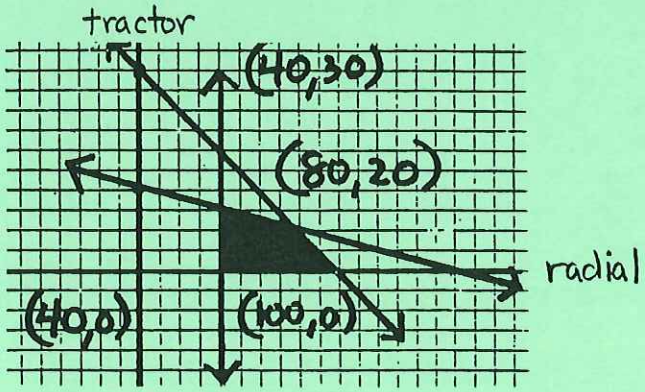
5 large
2 small
$290

④ x = radial tires $\quad x \geq 40$

y = tractor tires $\quad y \geq 0$

Rubber) $5x + 20y \leq 800 \rightarrow y \leq \frac{-1}{4}x + 40$

Labor) $12x + 12y \leq 1200 \rightarrow y \leq -x + 100$



$f(x,y) = 10x + 25y$

$(40,0)$ $10(40) + 25(0) = 400$

$(100,0)$ $10(100) + 25(0) = 1000$

$(40,30)$ $10(40) + 25(30) = 1150$

$\boxed{(80,20)}$ $10(80) + 25(20) = \boxed{1300}$

80 radial
20 tractor
$1300

Simplifying Radicals

ANSWER KEY 6.1

(1) $\sqrt{169} = 13$

(2) $\sqrt{225} = 15$

(3) $\sqrt{121n^2} = 11|n|$

(4) $-\sqrt{144x^6} = -12|x^3|$

(5) $-\sqrt{64a^2b^4} = -8|a|b^2$

(6) $\sqrt{121b^2c^6} = 11|bc^3|$

(7) $\sqrt{80a^4b^3} = 4a^2b\sqrt{5b}$

(8) $\sqrt{60x^3y^2z^4} = 2x|y|z^2\sqrt{15x}$

(9) $-\sqrt{48a^8b^7c^5} = -4a^4|b^3|c^2\sqrt{3bc}$

(10) $\sqrt{12a^9b^7c^4} = 2a^4|b^3|c^2\sqrt{3ab}$

(11) $\sqrt[3]{-8b^3m^3} = -2bm$

(12) $\sqrt[3]{-27n^3s^6} = -3ns^2$

(13) $\sqrt[3]{64a^7b^4} = 4a^2b\sqrt[3]{ab}$

(14) $\sqrt[3]{80x^4y^9} = 2xy^3\sqrt[3]{10x}$

(15) $\sqrt[4]{48a^4b^5} = 2|a|b\sqrt[4]{3b}$

(16) $\sqrt[4]{162a^6b^9c^5} = 3|ac|b^2\sqrt[4]{2a^2bc}$

(17) $\sqrt{(x+y)^2} = |x+y|$

(18) $\sqrt{(3a+b)^2} = |3a+b|$

(19) $\sqrt[3]{(x+y)^3} = x+y$

(20) $\sqrt{(x+3)^4} = (x+3)^2$

(21) $\sqrt[5]{(2n-3)^5} = 2n-3$

(22) $\sqrt{x^2+10x+25} = |x+5|$

(23) $\sqrt{n^2+6n+9} = |n+3|$

(24) $\sqrt[3]{.008n^4} = .2n\sqrt[3]{n}$

(25) $\sqrt[5]{-32} = -2$

(26) $\sqrt[4]{a^6b^5c^3d^2} = |ab|\sqrt[4]{a^2bc^3d^2}$

(27) $\sqrt[3]{x^4y^5z^7} = xyz^2\sqrt[3]{xy^2z}$

(28) $\sqrt[6]{a^7b^9c^{11}} = abc\sqrt[6]{ab^3c^5}$

(29) $\sqrt[7]{-x^{10}y^{15}} = -xy^2\sqrt[7]{x^3y}$

(30) $-\sqrt{9x^8y^6} = -3x^4|y^3|$

(31) $\sqrt{-1} = -1$

(32) $\sqrt[4]{a^4b^4} = |ab|$

(33) $\sqrt[4]{x^4y^8z^5} = |x|y^2z\sqrt[4]{z}$

(34) $\sqrt[4]{n^9m^{10}s^5} = n^2m^2|s|\sqrt[4]{nm^2s}$

(35) $\sqrt[6]{x^9y^{13}z^{12}} = |x|y^2z^2\sqrt[6]{x^3y}$

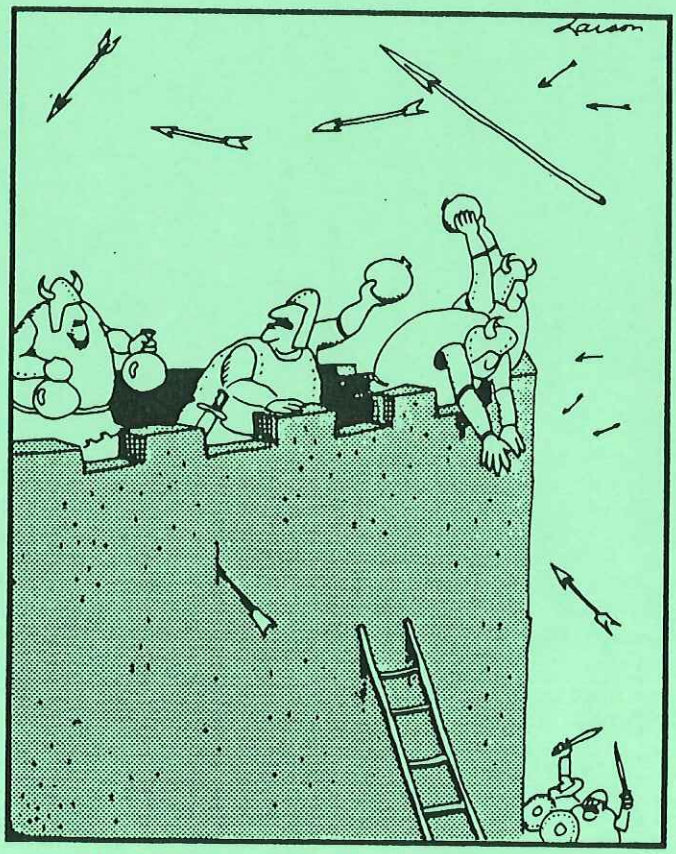
(36) $\sqrt[3]{a^4b^7c^{12}} = ab^2c^4\sqrt[3]{ab}$

(37) $\sqrt[4]{x^5y^8z^{13}} = xy^2z^3\sqrt[4]{xz}$

(38) $\sqrt[5]{a^8b^6c} = ab\sqrt[5]{a^3bc}$

(39) $\sqrt[4]{a^{10}b^{13}c^9} = a^2|b^3|c^2\sqrt[4]{a^2bc}$

(40) $\sqrt{a^3b^6c^2d^5} = |ab^3c|d^2\sqrt{ad}$



"Hot oil! We need hot oil! . . . Forget the water balloons!"

Radical Operations

ANSWER KEY 6.2

① $\sqrt[4]{5m^3b^5} \cdot \sqrt[4]{125m^2b^3}$

$\sqrt[4]{5^4m^5b^8} = \boxed{5mb^2\sqrt[4]{m}}$

② $\sqrt[4]{3b^6r^7} \cdot \sqrt[4]{81b^2r^2}$

$\sqrt[4]{3^4b^8r^9} = \boxed{3b^2r^2\sqrt[4]{3r}}$

③ $\sqrt[3]{54r^4s^3} \cdot \sqrt[3]{16rs}$

$\sqrt[3]{2^5 \cdot 3^3 \cdot r^5s^4} = \boxed{6rs\sqrt[3]{4r^2s}}$

④ $\sqrt{125m^2n} \cdot \sqrt{32m^4n^6}$

$\sqrt{5^3 \cdot 2^5 \cdot m^6n^7} = \boxed{20|m^3|n^3\sqrt{10n}}$

⑤ $\sqrt[4]{32a^5b^3} \cdot \sqrt[4]{162a^3b^2}$

$\sqrt[4]{2^6 \cdot 3^4 \cdot a^8b^5} = \boxed{6a^2b\sqrt[4]{4b}}$

⑥ $\sqrt{r}(\sqrt{r} + r\sqrt{5}) = \boxed{r + r\sqrt{r5}}$

⑦ $\sqrt{b}(b + a\sqrt{b}) = \boxed{b\sqrt{b} + ab}$

⑧ $\sqrt{m}(\sqrt{p} + \sqrt{mq}) = \boxed{\sqrt{mp} + m\sqrt{q}}$

⑨ $8\sqrt[3]{2a} + 3\sqrt[3]{2a} - 8\sqrt[3]{2a}$

$\boxed{3\sqrt[3]{2a}}$

⑩ $\sqrt[3]{54} - \sqrt[3]{128x^3}$

$\sqrt[3]{3^3 \cdot 2} - \sqrt[3]{2^7 \cdot x^3}$

$3\sqrt[3]{2} - 4x\sqrt[3]{2} = \boxed{(3-4x)\sqrt[3]{2}}$

must have parenthesis

⑪ $\sqrt[4]{x^2} + \sqrt[4]{x^6}$

$\sqrt[4]{x^2} + |x|\sqrt[4]{x^2} = \boxed{(1 + |x|)\sqrt[4]{x^2}}$

must have parenthesis

⑫ $-\sqrt{2x^2y^4} + \sqrt{8x^2y^4}$

$-|x|y^2\sqrt{2} + 2|x|y^2\sqrt{2} = \boxed{|x|y^2\sqrt{2}}$

⑬ $(7 + \sqrt{11p})(7 - \sqrt{11p}) = \boxed{49 - 11p}$

⑭ $(\sqrt{3} + \sqrt{5})(\sqrt{3} - \sqrt{5}) = 3 - 5 = \boxed{-2}$

The two problems above show mult. of conjugates.

⑮ $(3 - \sqrt[3]{4})(\sqrt[3]{2} + \sqrt[3]{16})$

$3\sqrt[3]{2} + 3\sqrt[3]{2^4} - \sqrt[3]{2^3} - \sqrt[3]{2^6}$

$3\sqrt[3]{2} + 6\sqrt[3]{2} - 2 - 4 = \boxed{9\sqrt[3]{2} - 6}$

⑯ $(4 - \sqrt[3]{9})(\sqrt[3]{3} + \sqrt[3]{81})$

$4\sqrt[3]{3} + 4\sqrt[3]{3^4} - \sqrt[3]{3^3} - \sqrt[3]{3^6}$

$4\sqrt[3]{3} + 12\sqrt[3]{3} - 3 - 9$

$\boxed{16\sqrt[3]{3} - 12}$

⑰ $(y + \sqrt[3]{4})(y^2 - y\sqrt[3]{4} + \sqrt[3]{16})$

$y^3 - y^2\sqrt[3]{4} + y\sqrt[3]{16}$

$+ y^2\sqrt[3]{4} - y^3\sqrt{16} + \sqrt[3]{2^6}$

$y^3 \qquad + \sqrt[3]{2^6}$

$\boxed{y^3 + 4}$

Radical Operations

ANSWER KEY 6.2

(18) $(x - \sqrt[3]{3})(x^2 + x\sqrt[3]{3} + \sqrt[3]{9})$

$$x^3 + x^2\sqrt[3]{3} + x\sqrt[3]{9} - x^2\sqrt[3]{3} - x\sqrt[3]{9} - \sqrt[3]{27}$$

$x^3 \qquad\qquad -\sqrt[3]{3^3}$

$\boxed{x^3 - 3}$

(19) $(m + \sqrt[3]{a})(m^2 - m\sqrt[3]{a} + \sqrt[3]{a^2})$

$$m^3 - m^2\sqrt[3]{a} + m\sqrt[3]{a^2} + m^2\sqrt[3]{a} - m\sqrt[3]{a^2} + \sqrt[3]{a^3}$$

$m^3 \qquad\qquad +\sqrt[3]{a^3}$

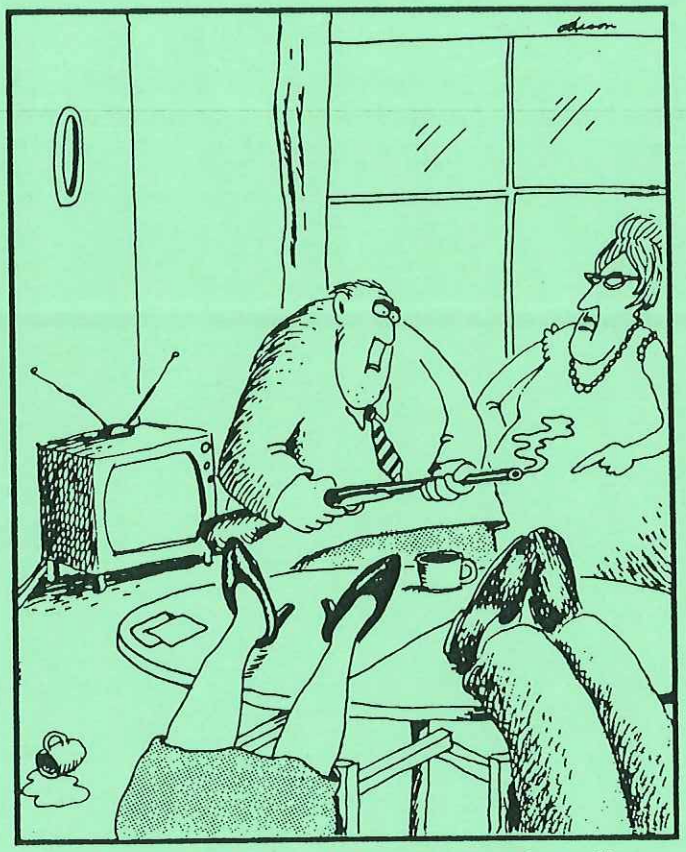
$\boxed{m^3 + a}$

(20) $(2 + \sqrt[3]{k})(4 - 2\sqrt[3]{k} + \sqrt[3]{k^2})$

$$8 - 4\sqrt[3]{k} + 2\sqrt[3]{k^2} + 4\sqrt[3]{k} - 2\sqrt[3]{k^2} + \sqrt[3]{k^3}$$

$8 \qquad\qquad +\sqrt[3]{k^3}$

$\boxed{8 + k}$



"That settles it, Carl! . . . From now on, you're getting only decaffeinated coffee!"

Dividing & Rationalizing

ANSWER KEY 6.3

① $\frac{\sqrt{12}}{\sqrt{3}} = \sqrt{4} = \boxed{2}$

② $\frac{\sqrt{14}}{\sqrt{2}} = \boxed{\sqrt{7}}$

③ $\frac{\sqrt[3]{81}}{\sqrt[3]{9}} = \frac{\sqrt[3]{3^4}}{\sqrt[3]{3^2}} = \frac{3\sqrt[3]{3}}{\sqrt[3]{3^2}} \cdot \frac{\sqrt[3]{3}}{\sqrt[3]{3}} = \frac{3\sqrt[3]{9}}{3} = \boxed{\sqrt[3]{9}}$

④ $\frac{\sqrt[3]{54}}{\sqrt[3]{6}} = \frac{3\sqrt[3]{2}}{\sqrt[3]{6}} \cdot \frac{\sqrt[3]{6^2}}{\sqrt[3]{6^2}} = \frac{3\sqrt[3]{72}}{6} = \frac{6\sqrt[3]{9}}{6} = \boxed{\sqrt[3]{9}}$

⑤ $\sqrt{\frac{8}{9}} = \frac{\sqrt{8}}{\sqrt{9}} = \boxed{\frac{2\sqrt{2}}{3}}$

⑥ $\sqrt{\frac{21}{12}} = \sqrt{\frac{7}{4}} = \frac{\sqrt{7}}{\sqrt{4}} = \boxed{\frac{\sqrt{7}}{2}}$

⑦ $\sqrt[4]{\frac{5}{16}} = \frac{\sqrt[4]{5}}{\sqrt[4]{16}} = \boxed{\frac{\sqrt[4]{5}}{2}}$

⑧ $\sqrt[4]{\frac{7}{81}} = \frac{\sqrt[4]{7}}{\sqrt[4]{81}} = \boxed{\frac{\sqrt[4]{7}}{3}}$

⑨ $\sqrt[3]{\frac{5}{9p^2}} = \frac{\sqrt[3]{5}}{\sqrt[3]{9p^2}} \cdot \frac{\sqrt[3]{3p}}{\sqrt[3]{3p}} = \boxed{\frac{\sqrt[3]{15p}}{3p}}$

⑩ $\sqrt[3]{\frac{9}{4m^2}} = \frac{\sqrt[3]{9}}{\sqrt[3]{4m^2}} \cdot \frac{\sqrt[3]{2m}}{\sqrt[3]{2m}} = \boxed{\frac{\sqrt[3]{18m}}{2m}}$

⑪ $\sqrt[4]{\frac{2}{3}} = \frac{\sqrt[4]{2}}{\sqrt[4]{3}} \cdot \frac{\sqrt[4]{3^3}}{\sqrt[4]{3^3}} = \boxed{\frac{\sqrt[4]{54}}{3}}$

⑫ $\sqrt[4]{\frac{3}{2}} = \frac{\sqrt[4]{3}}{\sqrt[4]{2}} \cdot \frac{\sqrt[4]{2^3}}{\sqrt[4]{2^3}} = \boxed{\frac{\sqrt[4]{24}}{2}}$

⑬ $\frac{1}{3+\sqrt{5}} \cdot \frac{3-\sqrt{5}}{3-\sqrt{5}} = \frac{3-\sqrt{5}}{9-5} = \boxed{\frac{3-\sqrt{5}}{4}}$

⑭ $\frac{3}{5-\sqrt{2}} \cdot \frac{5+\sqrt{2}}{5+\sqrt{2}} = \frac{15+3\sqrt{2}}{25-2} = \boxed{\frac{15+3\sqrt{2}}{23}}$

⑮ $\frac{2}{3-\sqrt{5}} \cdot \frac{3+\sqrt{5}}{3+\sqrt{5}} = \frac{6+2\sqrt{5}}{9-5} = \frac{6+2\sqrt{5}}{4}$

reduce $\rightarrow \boxed{\frac{3+\sqrt{5}}{2}}$

⑯ $\frac{7}{4-\sqrt{3}} \cdot \frac{4+\sqrt{3}}{4+\sqrt{3}} = \frac{28+7\sqrt{3}}{16-3} = \boxed{\frac{28+7\sqrt{3}}{13}}$

⑰ $\frac{1+\sqrt{2}}{3-\sqrt{2}} \cdot \frac{3+\sqrt{2}}{3+\sqrt{2}} = \frac{3+\sqrt{2}+3\sqrt{2}+2}{9-2}$

$\boxed{\frac{5+4\sqrt{2}}{7}}$

⑱ $\frac{2+\sqrt{6}}{2-\sqrt{6}} \cdot \frac{2+\sqrt{6}}{2+\sqrt{6}} = \frac{4+4\sqrt{6}+6}{4-6}$

$\frac{10+4\sqrt{6}}{-2} = \boxed{-5-2\sqrt{6}}$

⑲ $\frac{2-\sqrt{3}}{5+3\sqrt{3}} \cdot \frac{5-3\sqrt{3}}{5-3\sqrt{3}} = \frac{10-6\sqrt{3}-5\sqrt{3}+9}{25-27}$

$\frac{19-11\sqrt{3}}{-2} = \boxed{\frac{-19+11\sqrt{3}}{2}}$

Dividing & Rationalizing

ANSWER KEY 6.3

(20) $\frac{3+4\sqrt{5}}{5+2\sqrt{5}} \cdot \frac{5-2\sqrt{5}}{5-2\sqrt{5}} = \frac{15-6\sqrt{5}+20\sqrt{5}-40}{25-20}$

$$\boxed{\frac{-25+14\sqrt{5}}{5}}$$

(21) $\frac{\sqrt{x+1}}{\sqrt{x-1}} \cdot \frac{\sqrt{x-1}}{\sqrt{x-1}} = \boxed{\frac{\sqrt{x^2-1}}{x-1}}$

(22) $\frac{m+\sqrt{a}}{2\sqrt{a}-p} \cdot \frac{2\sqrt{a}+p}{2\sqrt{a}+p} = \frac{2m\sqrt{a}+mp+2a+p\sqrt{a}}{4a-p^2}$

$$\boxed{\frac{(2m+p)\sqrt{a}+mp+2a}{4a-p^2}}$$

(23) $\sqrt{\frac{2}{5}} + \sqrt{40} + \sqrt{10}$

$\frac{\sqrt{2}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} + 2\sqrt{10} + \sqrt{10}$

$\frac{\sqrt{10}}{5} + \frac{10\sqrt{10}}{5} + \frac{5\sqrt{10}}{5} = \boxed{\frac{16\sqrt{10}}{5}}$

(24) $\sqrt[3]{\frac{1}{4}} + \sqrt[3]{54} - \sqrt[3]{16}$

$\frac{\sqrt[3]{1}}{\sqrt[3]{2^2}} \cdot \frac{\sqrt[3]{2}}{\sqrt[3]{2}} + 3\sqrt[3]{2} - 2\sqrt[3]{2}$

$\frac{\sqrt[3]{2}}{2} + \frac{6\sqrt[3]{2}}{2} - \frac{4\sqrt[3]{2}}{2} = \boxed{\frac{3\sqrt[3]{2}}{2}}$

(25) $\sqrt[4]{32a^6b^7c^9} = \boxed{2|ab|c^2\sqrt[4]{2a^2b^3c}}$

(26) $\sqrt[3]{108x^5y^6z^4} = \boxed{3xy^2z\sqrt[3]{4x^2z}}$

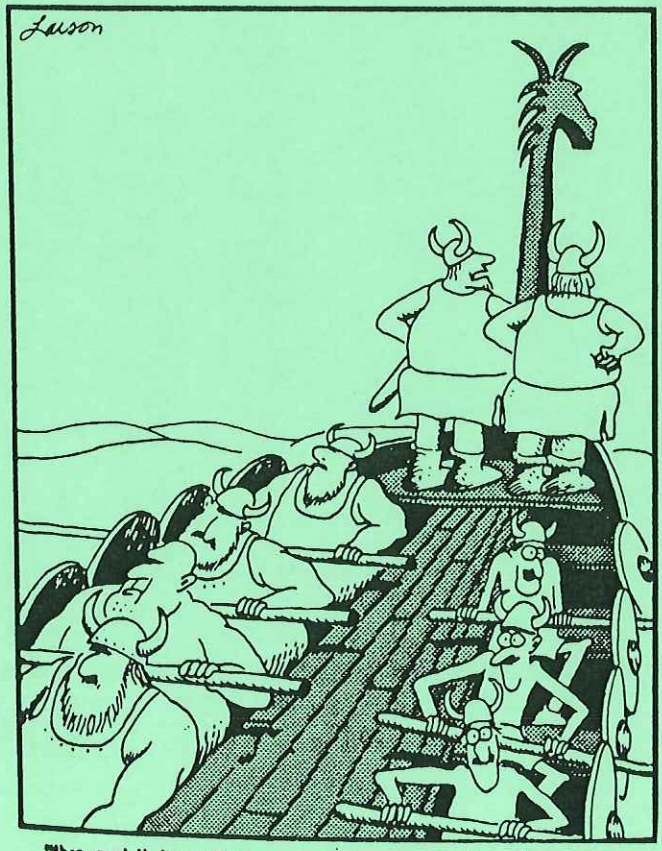
(27) $\sqrt{8} + \sqrt{2x^2y^4}$

$2\sqrt{2} + |x|y^2\sqrt{2} = \boxed{(2+|x|y^2)\sqrt{2}}$

(28) $\sqrt[4]{12a^3b^3c^6} \cdot \sqrt[4]{12a^2bc^8}$

$\sqrt[4]{2^4 \cdot 3^2 \cdot a^5b^4c^{14}}$

$$\boxed{2ab|c^3|\sqrt[4]{9ac^2}}$$



"I've got it, too, Omar . . . a strange feeling like we've just been going in circles."

Radical Equations

ANSWER KEY 6.4

① $\sqrt{2x+3}+3=10$

$\sqrt{2x+3}=7$ square both sides

$2x+3=49$

$2x=46$ $\boxed{x=23}$

② $\sqrt{4a+8}+5=7$

$\sqrt{4a+8}=2$ square both sides

$4a+8=4$

$4a=-4$ $\boxed{a=-1}$

③ $\sqrt[3]{m+5}+6=4$

$\sqrt[3]{m+5}=-2$ cube both sides

$m+5=-8$

$\boxed{m=-13}$

④ $\sqrt[4]{2x+3}+5=4$

$\sqrt[4]{2x+3}=-1$

since the index (4) is even, the principal root cannot be negative

$\boxed{\text{no solutions}}$

⑤ $\sqrt{x+5}=\sqrt{2x-3}$ sq. both sides

$x+5=2x-3$

$-x=-8$ $\boxed{x=8}$

⑥ $\sqrt{m+12}-\sqrt{m}=2$

$\sqrt{m+12}=\sqrt{m}+2$ sq. both sides

$m+12=m+4\sqrt{m}+4$

$8=4\sqrt{m}$

$2=\sqrt{m}$ sq. both sides

$m=4$

$\boxed{m=4}$

⑦ $\sqrt{b+4}=\sqrt{b+20}-2$ sq. both sides

$b+4=b+20-4\sqrt{b+20}+4$

$-20=-4\sqrt{b+20}$

$5=\sqrt{b+20}$ sq. both sides

$b+20=25$

$\boxed{b=5}$

⑧ $\sqrt{x-4}-6=\sqrt{x+20}$ sq. both sides

$x-4-12\sqrt{x-4}+36=x+20$

$-12\sqrt{x-4}=-12$

$\sqrt{x-4}=1$ sq. both sides

$x-4=1$

$x=\cancel{5}$ 5 does not check! $\boxed{\text{no solutions}}$

⑨ $\sqrt{3x-5}=2-\sqrt{x-1}$ sq. both sides

$3x-5=4-4\sqrt{x-1}+x-1$

$2x-8=-4\sqrt{x-1}$

$\frac{x-4}{-2}=\sqrt{x-1}$ sq. both sides

$\frac{x^2-8x+16}{4}=x-1$

$x^2-8x+16=4x-4$

$x^2-12x+20=0$

$(x-10)(x-2)=0$

$x=\cancel{10}$ or $\boxed{2}$

↳ 10 does not check!

⑩ $t=\sqrt{\frac{2s}{g}}$ sq. both sides

$t^2=\frac{2s}{g}$

$t^2g=2s$ $\boxed{s=\frac{t^2g}{2}}$

Radical Equations

ANSWER KEY 6.4

(11) $T = \frac{1}{2}\sqrt{\frac{u}{g}}$

$2T = \sqrt{\frac{u}{g}}$ sq. both sides

$4T^2 = \frac{u}{g}$

$4T^2 g = u$

$\boxed{g = \frac{u}{4T^2} \text{ for } T \neq 0}$

(12) $m^2 = \sqrt[3]{\frac{rp}{g^2}}$ cube both sides

$m^6 = \frac{rp}{g^2}$

$m^6 g^2 = rp$

$\boxed{p = \frac{m^6 g^2}{r} \text{ for } r \neq 0}$

(13) $\sqrt{n^2+10n+25}$

$\sqrt{(n+5)^2} = \boxed{|n+5|}$

(14) $\sqrt[4]{a^6 b^{10} c^5 d^7} = \boxed{|a|\, b^2 c d \sqrt[4]{a^2 b^2 c d^3}}$

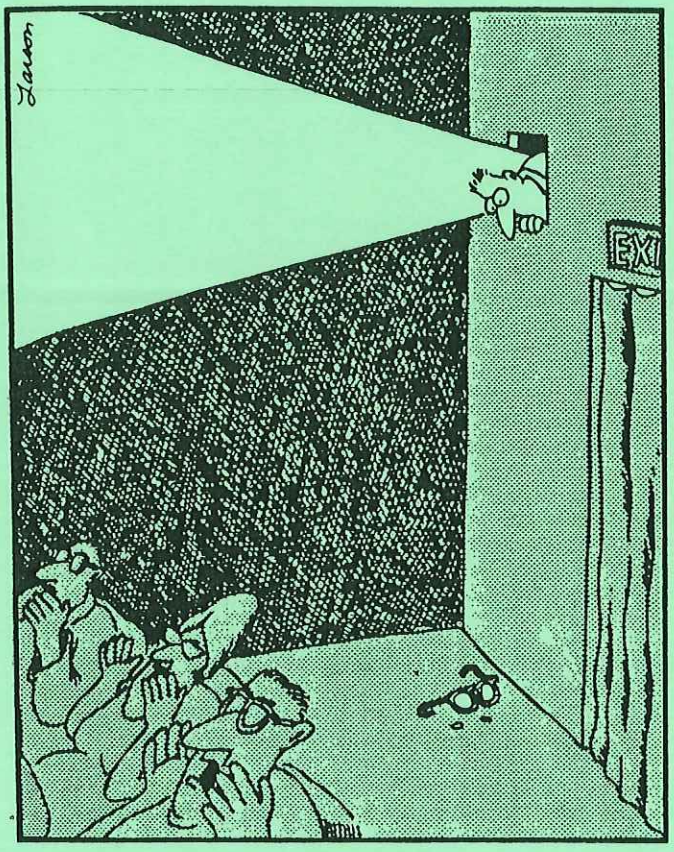
(15) $\sqrt[4]{18a^3b^5c^{10}} \cdot \sqrt[4]{36a^2b^7c^4}$

$\sqrt[4]{2^3 \cdot 3^4 \cdot a^5 b^{12} c^{14}}$

$\boxed{3ab^3|c^3|\sqrt[4]{8ac^2}}$

(16) $\frac{2+2\sqrt{6}}{4-2\sqrt{6}} \cdot \frac{4+2\sqrt{6}}{4+2\sqrt{6}} = \frac{8+4\sqrt{6}+8\sqrt{6}+24}{16-24}$

$\frac{32+12\sqrt{6}}{-8} = \boxed{\frac{-8-3\sqrt{6}}{2}}$



"Focus! . . . Focus!"

(17) $\sqrt[3]{\frac{1}{3}} + \sqrt[3]{72} - \sqrt[3]{9}$

$\frac{\sqrt[3]{1}}{\sqrt[3]{3}} \cdot \frac{\sqrt[3]{9}}{\sqrt[3]{9}} + 2\sqrt[3]{9} - \sqrt[3]{9}$

$\frac{\sqrt[3]{9}}{3} + \sqrt[3]{9}$

$\frac{\sqrt[3]{9}}{3} + \frac{3\sqrt[3]{9}}{3} = \boxed{\frac{4\sqrt[3]{9}}{3}}$

Roots & Radicals

ANSWER KEY: UNIT 6 REVIEW & PRACTICE

① $\sqrt[4]{64x^5y^6z^9} = \boxed{2|xy|z^2\sqrt[4]{4xy^2z}}$

② $\sqrt{72a^5b^7c^6}$ $\boxed{6a^2|b^3c^3|\sqrt{2ab}}$

③ $\sqrt[3]{-40n^7m^4}$ $\boxed{-2n^2m\sqrt[3]{5nm}}$

④ $\sqrt{n^2-10n+25} = \boxed{|n-5|}$

⑤ $\sqrt[3]{(3x-y)^5} = \boxed{(3x-y)\sqrt[3]{(3x-y)^2}}$

must use ↗ parenthesis

⑥ $\sqrt[4]{24a^2b^3c^5} \cdot \sqrt[4]{48a^3b^3c^6}$

$\sqrt[4]{2^7\cdot3^2\cdot a^5b^6c^{11}} = \boxed{2abc^2\sqrt[4]{72ab^3c^3}}$

no ab. value for a or b. both are + or both −

⑦ $\sqrt{3mn} + \sqrt{27m^3n}$

$\sqrt{3mn} + 3|m|\sqrt{3mn} = \boxed{(1+3|m|)\sqrt{3mn}}$

must have parenthesis ↗

⑧ $(4-\sqrt{3})(6-\sqrt{3})$

$24-4\sqrt{3}-6\sqrt{3}+3 = \boxed{27-10\sqrt{3}}$

⑨ $\sqrt[3]{\frac{2}{5n}} = \frac{\sqrt[3]{2}}{\sqrt[3]{5n}} \cdot \frac{\sqrt[3]{25n^2}}{\sqrt[3]{25n^2}} = \boxed{\frac{\sqrt[3]{50n^2}}{5n}}$

⑩ $\frac{2-\sqrt{2}}{4+2\sqrt{2}} \cdot \frac{4-2\sqrt{2}}{4-2\sqrt{2}} = \frac{8-4\sqrt{2}-4\sqrt{2}+4}{16-8}$

$\frac{12-8\sqrt{2}}{8} = \boxed{\frac{3-2\sqrt{2}}{2}}$

⑪ $\sqrt{\frac{1}{3}} + \sqrt{75} - 2\sqrt{3}$

$\frac{\sqrt{1}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}} + 5\sqrt{3} - 2\sqrt{3}$

continued

$\frac{\sqrt{3}}{3} + 3\sqrt{3} = \frac{\sqrt{3}}{3} + \frac{9\sqrt{3}}{3} = \boxed{\frac{10\sqrt{3}}{3}}$

⑫ $\sqrt[3]{y+1} = 3$ cube both sides

$y+1 = 27$ $\boxed{y=26}$

⑬ $\sqrt{y+12}+1 = \sqrt{y+21}$ sq. both sides

$(y+12)+2\sqrt{y+12}+1 = y+21$

$2\sqrt{y+12} = 8$

$\sqrt{y+12} = 4$ sq. both sides

$y+12 = 16$ $\boxed{y=4}$

⑭ $\sqrt{5+2x} = x-5$ sq. both sides

$5+2x = x^2-10x+25$

$x^2-12x+20=0$

$(x-10)(x-2)=0$

$\boxed{x=10}$ or ~~x~~ 2 does not check

⑮ $\sqrt{x+11}-x=-9$

$\sqrt{x+11} = x-9$ sq. both sides

$x+11 = x^2-18x+81$

$x^2-19x+70=0$

$(x-14)(x-5)=0$

$\boxed{x=14}$ or ~~5~~ 5 does not check

⑯ $r = \sqrt[3]{\frac{2mM}{c}}$ cube both sides

$r^3 = \frac{2mM}{c}$

$r^3c = 2mM$

$\boxed{c = \frac{2mM}{r^3} \text{ for } r \neq 0}$

Rational Form & Radical Form

ANSWER KEY 7.1

① $\sqrt{21} = \boxed{21^{1/2}}$

② $\sqrt[3]{30} = \boxed{30^{1/3}}$

③ $\sqrt[6]{32} = \sqrt[6]{2^5} = \boxed{2^{5/6}}$

④ $\sqrt[4]{x} = \boxed{x^{1/4}}$

⑤ $\sqrt[3]{y} = \boxed{y^{1/3}}$

⑥ $\sqrt{25x^3y^4} = \boxed{5x^{3/2}y^2}$

⑦ $\sqrt[3]{8m^3r^6} = \boxed{2mr^2}$

⑧ $\sqrt[4]{8x^3y^5} = \boxed{2^{3/4}x^{3/4}y^{5/4}}$

⑨ $\sqrt[4]{27} = \sqrt[4]{3^3} = \boxed{3^{3/4}}$

⑩ $\sqrt[3]{16a^5b^7} = \boxed{2^{4/3}a^{5/3}b^{7/3}}$

⑪ $\sqrt[3]{n^2} = \boxed{n^{2/3}}$

⑫ $\sqrt[6]{b^3} = b^{3/6} = \boxed{b^{1/2}}$

⑬ $64^{1/6} = (2^6)^{1/6} = 2^{6/6} = \boxed{2}$

⑭ $5^{1/2} = \boxed{\sqrt{5}}$

⑮ $6^{1/3} = \boxed{\sqrt[3]{6}}$

⑯ $x^{3/4} = \boxed{\sqrt[4]{x^3}}$

⑰ $a^{3/2}b^{5/2} = \sqrt{a^3b^5}$

$\boxed{ab^2\sqrt{ab}}$

⑱ $4^{1/3}x^{2/3}y^{1/3}$

$\sqrt[3]{4x^2y^4} = \boxed{y\sqrt[3]{4x^2y}}$

⑲ $2^{5/3}x^{7/3} = \sqrt[3]{2^5x^7}$

$\boxed{2x^2\sqrt[3]{4x}}$

⑳ $(2x)^{1/2}x^{1/2} = 2^{1/2}x^{1/2}x^{1/2}$

$2^{1/2}x = \boxed{x\sqrt{2}}$

㉑ $5^{1/3}p^{2/3}q^{1/3}$

$\boxed{\sqrt[3]{5p^2q}}$

㉒ $(3m)^{2/5}n^{3/5}$

$\boxed{\sqrt[5]{9m^2n^3}}$

㉓ $r^{5/2}q^{3/4} = r^{10/4}q^{3/4}$

$\sqrt[4]{r^{10}q^3} = \boxed{r^2\sqrt[4]{r^2q^3}}$

㉔ $w^{4/7}y^{3/7} = \boxed{\sqrt[7]{w^4y^3}}$

㉕ $x^{1/3}y^{1/2} = x^{2/6}y^{3/6}$

$\boxed{\sqrt[6]{x^2y^3}}$

㉖ $a^{5/6}b^{3/2}x^{7/3}$

$a^{5/6}b^{9/6}x^{14/6}$

$\sqrt[6]{a^5b^9x^{14}}$

$\boxed{bx^2\sqrt[6]{a^5b^3x^2}}$

㉗ $5^2b^{1/2}c^{1/4} = 25b^{2/4}c^{1/4}$

$\boxed{25\sqrt[4]{b^2c}}$

㉘ $x^{3/4}y^{1/3}z^{5/6}$

$x^{9/12}y^{4/12}z^{10/12}$

$\boxed{\sqrt[12]{x^9y^4z^{10}}}$

㉙ $\sqrt[4]{9} = \sqrt[4]{3^2} = 3^{2/4} = 3^{1/2} = \boxed{\sqrt{3}}$

㉚ $\sqrt[4]{49} = \sqrt[4]{7^2} = 7^{2/4} = 7^{1/2} = \boxed{\sqrt{7}}$

㉛ $\sqrt[6]{8} = \sqrt[6]{2^3} = 2^{3/6} = 2^{1/2} = \boxed{\sqrt{2}}$

㉜ $\sqrt[8]{16} = \sqrt[8]{2^4} = 2^{4/8} = 2^{1/2} = \boxed{\sqrt{2}}$

㉝ $121^{1/2} = \sqrt{121} = \boxed{11}$

㉞ $(1/32)^{1/5} = \frac{1^{1/5}}{32^{1/5}} = \frac{1}{\sqrt[5]{32}} = \boxed{\frac{1}{2}}$

㉟ $\sqrt[3]{12^3} = 12^{3/3} = \boxed{12}$

㊱ $\sqrt[4]{256} = \sqrt[4]{2^8} = 2^{8/4} = 2^2 = \boxed{4}$

㊲ $\left(\frac{343}{64}\right)^{1/3} = \left(\frac{7^3}{2^6}\right)^{1/3} = \frac{7}{2^2} = \boxed{\frac{7}{4}}$

㊳ $\left(\frac{216}{729}\right)^{\frac{2}{3}} = \left(\frac{6^3}{3^6}\right)^{2/3} = \frac{6^2}{3^4}$

$\frac{36}{81} = \boxed{\frac{4}{9}}$

㊴ $(6^{2/3})^3 = 6^{6/3} = 6^2 = \boxed{36}$

㊵ $(9^{3/4})^{2/3} = \left[(3^2)^{3/4}\right]^{2/3} = 3^{12/12} = \boxed{3}$

㊶ $(.125)^{2/3} = \left[(.5)^3\right]^{2/3} = .5^2$

$\boxed{.25}$

㊷ $(.008)^{1/3} = (.2^3)^{1/3} = .2^{3/3}$

$\boxed{.2}$

㊸ $(.027)^{1/3} = (.3^3)^{1/3} = .3^{3/3}$

$\boxed{.3}$

㊹ $(.0016)^{1/4} = (.2^4)^{1/4} = .2^{4/4}$

$\boxed{.2}$

Negative Fractional Exponents

ANSWER KEY 7.2

① $6^{-1/2} = \frac{1}{6^{1/2}} = \frac{1}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \boxed{\frac{\sqrt{6}}{6}}$

② $32^{-1/4} = (2^5)^{-1/4} = 2^{-5/4} = \frac{1}{2^{5/4}}$

$\frac{1}{\sqrt[4]{2^5}} = \frac{1}{2\sqrt[4]{2}} \cdot \frac{\sqrt[4]{2^3}}{\sqrt[4]{2^3}} = \boxed{\frac{\sqrt[4]{8}}{4}}$

③ $8^{-3/4} = (2^3)^{-3/4} = 2^{-9/4} = \frac{1}{2^{9/4}}$

$\frac{1}{\sqrt[4]{2^9}} = \frac{1}{4\sqrt[4]{2}} \cdot \frac{\sqrt[4]{2^3}}{\sqrt[4]{2^3}} = \boxed{\frac{\sqrt[4]{2^3}}{8}}$

④ $8^{\frac{1}{3}} \cdot 8^{\frac{4}{3}} = 8^{\frac{5}{3}} = (2^3)^{5/3} = 2^5 = \boxed{32}$

⑤ $9^{\frac{2}{3}} \times 3^{\frac{4}{3}} \times 3^{\frac{11}{3}} = (3^2)^{2/3} \times 3^{15/3}$

$3^{4/3} \times 3^5 = 3^5 \sqrt[3]{3^4} = \boxed{3x^5\sqrt[3]{3}}$

⑥ $(4n)^{\frac{2}{5}} n^{\frac{4}{5}} = 4^{\frac{2}{5}} n^{\frac{2}{5}} n^{\frac{4}{5}} = (2^2)^{\frac{2}{5}} n^{\frac{6}{5}}$

$2^{\frac{4}{5}} n^{\frac{6}{5}} = \sqrt[5]{2^4 n^6} = \boxed{n\sqrt[5]{16n}}$

⑦ $x^{\frac{1}{2}} \times x^{\frac{3}{4}} \times x^{\frac{1}{3}} = x^{\frac{6}{12}} \times x^{\frac{9}{12}} \times x^{\frac{4}{12}} = x^{\frac{19}{12}}$

$\sqrt[12]{x^{19}} = \boxed{x\sqrt[12]{x^7}}$

⑧ $\frac{36^{3/4}}{36^{1/4}} = 36^{\frac{3}{4}-\frac{1}{4}} = 36^{\frac{1}{2}} = \sqrt{36} = \boxed{6}$

⑨ $\frac{8^{3/4}}{32^{-1/2}} = (2^3)^{3/4}(2^5)^{1/2} = 2^{\frac{9}{4}} \cdot 2^{\frac{5}{2}}$

$2^{\frac{9}{4}} \cdot 2^{\frac{10}{4}} = 2^{\frac{19}{4}} = \sqrt[4]{2^{19}} = \boxed{16\sqrt[4]{2^3}}$

⑩ $\frac{1}{y^{2/5}} \cdot \frac{y^{3/5}}{y^{3/5}} = \boxed{\frac{y^{3/5}}{y}}$

⑪ $\frac{3}{r^{4/5}} \cdot \frac{r^{1/5}}{r^{1/5}} = \boxed{\frac{3r^{1/5}}{r}}$

⑫ $b^{-1/4} = \frac{1}{b^{1/4}} \cdot \frac{b^{3/4}}{b^{3/4}} = \boxed{\frac{b^{3/4}}{b}}$

⑬ $m^{-5/6} = \frac{1}{m^{5/6}} \cdot \frac{m^{1/6}}{m^{1/6}} = \boxed{\frac{m^{1/6}}{m}}$

⑭ $\frac{15}{5^{2/3}} \cdot \frac{5^{1/3}}{5^{1/3}} = \frac{15 \cdot 5^{1/3}}{5} = \boxed{3 \cdot 5^{1/3}}$

⑮ $\frac{24}{6^{2/3}} \cdot \frac{6^{1/3}}{6^{1/3}} = \frac{24 \cdot 6^{1/3}}{6} = \boxed{4 \cdot 6^{1/3}}$

⑯ $\frac{rm^{1/2}}{b^{3/2}} \cdot \frac{b^{1/2}}{b^{1/2}} = \boxed{\frac{rm^{1/2}b^{1/2}}{b^2}}$

⑰ $\frac{pq}{a^{1/3}} \cdot \frac{a^{2/3}}{a^{2/3}} = \boxed{\frac{pqa^{2/3}}{a}}$

⑱ $\frac{b^{\frac{3}{2}} + 3b^{-1/2}}{b^{1/2}} \cdot \frac{b^{1/2}}{b^{1/2}} = \boxed{\frac{b^2+3}{b}}$

⑲ $\frac{a^{5/3}m + 3a^{-1/3}}{a^{2/3}} \cdot \frac{a^{1/3}}{a^{1/3}} = \boxed{\frac{a^2m+3}{a}}$

⑳ $\frac{3x + 4x^2}{x^{-2/3}} \cdot \frac{x^{2/3}}{x^{2/3}} = \boxed{3x^{5/3} + 4x^{8/3}}$

㉑ $\frac{3m}{b^{-3/2}a^{1/3}} \cdot \frac{b^{3/2}a^{2/3}}{b^{3/2}a^{2/3}} = \boxed{\frac{3mb^{3/2}a^{2/3}}{a}}$

Negative Fractional Exponents

ANSWER KEY 7.2

(22) $(r^{-1/6})^{-2/3} = \boxed{r^{1/4}}$

(23) $(y^{1/3})^{-3/4} = y^{-1/4} = \frac{1}{y^{1/4}} \cdot \frac{y^{3/4}}{y^{3/4}} = \boxed{\frac{y^{3/4}}{y}}$

(24) $\frac{r^{3/2}}{r^{1/2}+2} \cdot \frac{r^{1/2}-2}{r^{1/2}-2} = \boxed{\frac{r^2-2r^{3/2}}{r-4}}$

(25) $\frac{x^{1/2}+y^{1/2}}{x^{1/2}-y^{1/2}} \cdot \frac{x^{1/2}+y^{1/2}}{x^{1/2}+y^{1/2}} = \boxed{\frac{x+2x^{\frac{1}{2}}y^{\frac{1}{2}}+y}{x-y}}$

(26) $\frac{rs}{r^{\frac{1}{2}}+r^{\frac{3}{2}}} \cdot \frac{r^{-1/2}}{r^{-1/2}} = \boxed{\frac{r^{1/2}s}{1+r}}$

(27) $\frac{x^{1/3}}{x^{\frac{2}{3}}-x^{-1/3}} \cdot \frac{x^{\frac{1}{3}}}{x^{\frac{1}{3}}} = \boxed{\frac{x^{\frac{2}{3}}}{x-1}}$

(28) $\frac{a^{-3/2}b^{1/2}}{b^{-3/2}a^{1/3}} = \frac{b^{1/2}b^{3/2}}{a^{2/3}a^{1/3}} = \boxed{\frac{b^2}{a}}$

(29) $\left(\frac{x^{-2}y^{-6}}{q}\right)^{-1/2} = \frac{xy^3}{q^{-1/2}} = xy^3q^{1/2}$

$\sqrt{q}\ xy^3 = \boxed{3xy^3}$

(30) $\left(\frac{z^{-2/3}}{5^{-1}z^{1/3}}\right)^{-2} = \frac{z^{4/3}}{5^2z^{-2/3}} = \frac{z^{4/3}z^{2/3}}{5^2}$

$\boxed{\frac{z^2}{25}}$

Pure Imaginary Numbers

ANSWER KEY 7.3

① $i^5 = \boxed{i}$

② $i^{10} = i^2 = \boxed{-1}$

③ $i^{43} = i^3 = \boxed{-i}$

④ $i^{82} = i^2 = \boxed{-1}$

⑤ $(\sqrt{-4})^3 = (2i)^3 = 8i^3 = \boxed{-8i}$

⑥ $(-2\sqrt{-8})(3\sqrt{-2}) = (-2i\sqrt{8})(3i\sqrt{2})$

$(-4i\sqrt{2})(3i\sqrt{2}) = -24i^2 = \boxed{24}$

⑦ $(4\sqrt{-12})(-2\sqrt{-3}) = (4i\sqrt{12})(-2i\sqrt{3})$

$(8i\sqrt{3})(-2i\sqrt{3}) = -48i^2 = \boxed{48}$

⑧ $(6\sqrt{-24})(-3\sqrt{6}) = (6i\sqrt{24})(-3\sqrt{6})$

$(12i\sqrt{6})(-3\sqrt{6}) = \boxed{-216i}$

⑨ $(2\sqrt{15})(-3\sqrt{-15}) = (2\sqrt{15})(-3i\sqrt{15}) = \boxed{-90i}$

⑩ $(2i)(3i)^2 = (2i)(9i^2) = 18i^3 = \boxed{-18i}$

⑪ $(5i)(-2i)^2 = (5i)(4i^2) = 20i^3 = \boxed{-20i}$

⑫ $(-6i^3)(4i^2) = -24i^5 = \boxed{-24i}$

⑬ $2y^2 + 8 = 0$

$2y^2 = -8$

$y^2 = -4$

$y = \pm i\sqrt{4}$

$\boxed{y = \pm 2i}$

⑭ $3b^2 + 18 = 0$

$3b^2 = -18$

$b^2 = -6$

$\boxed{b = \pm i\sqrt{6}}$

⑮ $5x^2 + 125 = 0$

$5x^2 = -125$

$x^2 = -25$

$x = \pm i\sqrt{25}$

$\boxed{x = \pm 5i}$

⑯ $3z^2 + 24 = 0$

$3z^2 = -24$

$z^2 = -8$

$z = \pm i\sqrt{8}$

$z = \boxed{\pm 2i\sqrt{2}}$

⑰ $4m^2 + 5 = 0$

$4m^2 = -5$

$m^2 = -5/4$

$m = \pm \frac{\sqrt{-5}}{2}$

$\boxed{m = \pm \frac{i\sqrt{5}}{2}}$

⑱ $9k^2 + 32 = 0$

$9k^2 = -32$

$k^2 = -32/9$

$k = \pm \frac{i\sqrt{32}}{3}$

$\boxed{k = \pm \frac{4i\sqrt{2}}{3}}$

⑲ $a^{2/3} b^{5/2} c^{5/6} = a^{4/6} b^{15/6} c^{5/6}$

$\sqrt[6]{a^4 b^{15} c^5} = \boxed{b^2 \sqrt[6]{a^4 b^3 c^5}}$

⑳ $2^{4/3} x^{1/2} = 2^{8/6} x^{3/6} = \sqrt[6]{2^8 x^3}$

$\boxed{2\sqrt[6]{4x^3}}$

㉑ $\frac{4^{3/4}}{8^{-2/3}} = \frac{(2^2)^{3/4}}{(2^3)^{-2/3}} = \frac{2^{3/2}}{2^{-2}} = (2^{3/2})(2^2) = 2^{7/2}$

$\sqrt{2^7} = \boxed{8\sqrt{2}}$

㉒ $16^{-3/4} = (2^4)^{-3/4} = 2^{-3} = \frac{1}{2^3} = \boxed{\frac{1}{8}}$

㉓ $\sqrt[3]{8a^5 b^7} = \boxed{2a^{5/3} b^{7/3}}$

㉔ $-(n^{1/2})^{-2/3} = -n^{-1/3} = \frac{-1}{n^{1/3}} \cdot \frac{n^{2/3}}{n^{2/3}} = \boxed{\frac{-n^{2/3}}{n}}$

㉕ $\frac{a^{-2/3} - b^{1/2}}{a^{1/3}} \cdot \frac{a^{2/3}}{a^{2/3}} = \boxed{\frac{1 - a^{2/3} b^{1/2}}{a}}$

㉖ $\frac{a^{3/2}}{a^{1/2} + b^{3/2}} \cdot \frac{a^{1/2} - b^{3/2}}{a^{1/2} - b^{3/2}} = \boxed{\frac{a^2 - a^{3/2} b^{3/2}}{a - b^3}}$

Complex Numbers

ANSWER KEY 7.4

① $(3+2i)+(4+5i) = \boxed{7+7i}$

② $(2+6i)+(4+3i) = \boxed{6+9i}$

③ $(9+6i)-(3+2i) = \boxed{6+4i}$

④ $(11-\sqrt{-9})-(4+\sqrt{-25})$

$(11-3i)-(4+5i) = \boxed{15-8i}$

⑤ $(4+3i)(2-7i)(3+i)$

$(8-28i+6i-21i^2)(3+i)$

$(29-22i)(3+i)$

$(87+29i-66i-22i^2) = \boxed{109-37i}$

⑥ $(7-i)(4+2i)(5+2i)$

$(28+14i-4i-2i^2)(5+2i)$

$(30+10i)(5+2i)$

$(150+60i+50i+20i^2) = \boxed{130+110i}$

⑦ $(2x+y)+(x-y)i = 7-i$

$2x+y=7$

$x-y=-1$

$3x = 6$

$\boxed{x=2}$

$\boxed{y=3}$

⑧ $(x+2y)+(2x-y)i = 5+5i$

$x+2y=5 \quad 2x-y=5$

$4x-2y=10$

$5x = 15$

$\boxed{x=3}$

$\boxed{y=1}$

⑨ $(x+4y)+(2x-3y)i = 13+4i$

$x+4y=13 \quad 2x-3y=4$

$-2x-8y=-26$

$-11y=-22$

$\boxed{y=2}$

$\boxed{x=5}$

⑩ $\frac{4-7i}{-3i} \cdot \frac{i}{i} = \frac{4i-7i^2}{-3i^2} = \boxed{\frac{7+4i}{3}}$

⑪ $\frac{5-6i}{-3i} \cdot \frac{i}{i} = \frac{5i-6i^2}{-3i^2} = \boxed{\frac{6+5i}{3}}$

⑫ $\frac{2+i}{5i} \cdot \frac{i}{i} = \frac{2i+i^2}{5i^2} = \frac{-1+2i}{-5}$ or $\boxed{\frac{1-2i}{5}}$

⑬ $\frac{3}{4-i} \cdot \frac{4+i}{4+i} = \frac{12+3i}{16-i^2} = \boxed{\frac{12+3i}{17}}$

⑭ $\frac{2}{6+5i} \cdot \frac{6-5i}{6-5i} = \frac{12-10i}{36-25i^2} = \boxed{\frac{12-10i}{61}}$

⑮ $\frac{4}{\sqrt{3}+2i} \cdot \frac{\sqrt{3}-2i}{\sqrt{3}-2i} = \frac{4\sqrt{3}-8i}{3-4i^2} = \boxed{\frac{4\sqrt{3}-8i}{7}}$

⑯ $\frac{2+i\sqrt{3}}{2-i\sqrt{3}} \cdot \frac{2+i\sqrt{3}}{2+i\sqrt{3}} = \frac{4+4i\sqrt{3}+3i^2}{4-3i^2}$

$\boxed{\frac{1+4i\sqrt{3}}{7}}$

continued

Complex Numbers

ANSWER KEY 7.4

(17) $\frac{1+i\sqrt{2}}{1-i\sqrt{2}} \cdot \frac{1+i\sqrt{2}}{1+i\sqrt{2}} = \frac{1+2i\sqrt{2}+2i^2}{1-2i^2} = \boxed{\frac{-1+2i\sqrt{2}}{3}}$

(18) mult. inverse

$\frac{1}{7-3i} \cdot \frac{7+3i}{7+3i} = \frac{7+3i}{49-9i^2} = \boxed{\frac{7+3i}{58}}$

(19) mult. inverse

$\frac{1}{3+7i} \cdot \frac{3-7i}{3-7i} = \frac{3-7i}{9-49i^2} = \boxed{\frac{3-7i}{58}}$

(20) mult. inverse

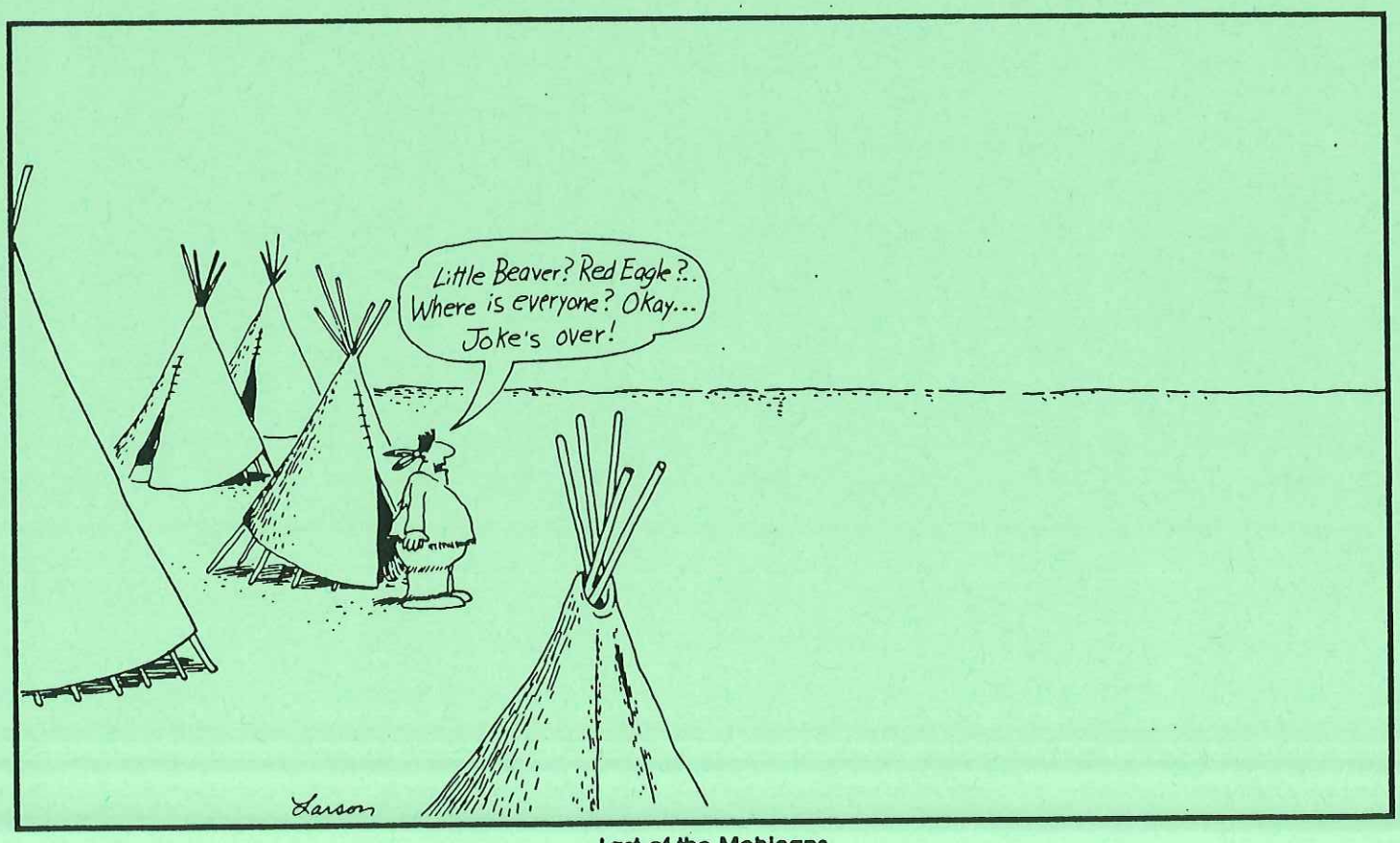
$\frac{3+i}{4i} \cdot \frac{i}{i} = \frac{3i+i^2}{4i^2} = \frac{-1+3i}{-4}$ or $\boxed{\frac{1-3i}{4}}$

(21) mult. inverse

$\frac{5-i}{2i} \cdot \frac{i}{i} = \frac{5i-i^2}{2i^2}$

$\frac{1+5i}{-2}$

$\boxed{\frac{-1-5i}{2}}$



Last of the Mohicans

Rational Exponents & Complex Numbers

ANSWER KEY: UNIT 7 REVIEW & PRACTICE

① $\sqrt[3]{16x^5y^6z^8} = \sqrt[3]{2^4x^5y^6z^8}$

$\boxed{2^{4/3}x^{5/3}y^2z^{8/3}}$

② $\sqrt[4]{96a^5b^7c^8} = \sqrt[4]{2^5\cdot 3\cdot a^5b^7c^8}$

$\boxed{2^{5/4}\cdot 3^{1/4}a^{5/4}b^{7/4}c^2}$

③ $r^2s^{1/3}y^{1/2} = r^2s^{2/6}y^{3/6} = \boxed{r^2\sqrt[6]{s^2y^3}}$

④ $(3x)^{1/2}x^{1/4} = 3^{1/2}x^{1/2}x^{1/4}$

$3^{1/2}x^{3/4} = 3^{2/4}x^{3/4} = \boxed{\sqrt[4]{9x^3}}$

⑤ $\sqrt[6]{27} = \sqrt[6]{3^3} = 3^{3/6} = 3^{1/2} = \boxed{\sqrt{3}}$

⑥ $\sqrt[4]{64x^{10}} = \sqrt[4]{2^6x^{10}} = 2^{6/4}x^{10/4}$

$2^{3/2}x^{5/2} = \sqrt{2^3x^5} = \boxed{2x^2\sqrt{2x}}$

⑦ $(25^{3/4})^{2/3} = 25^{1/2} = (5^2)^{1/2} = \boxed{5}$

⑧ $(8^{2/3})(16^{-3/4}) = (2^3)^{2/3}(2^4)^{-3/4}$

$(2^2)(2^{-3}) = 2^{-1} = \boxed{1/2}$

⑨ $\dfrac{x^{2/3}}{x^{2/3}-x^{-1/3}}\cdot\dfrac{x^{1/3}}{x^{1/3}} = \boxed{\dfrac{x}{x-1}}$

⑩ $\dfrac{x^{1/2}-y^{1/2}}{x^{1/2}+y^{1/2}}\cdot\dfrac{x^{1/2}-y^{1/2}}{x^{1/2}-y^{1/2}} = \boxed{\dfrac{x-2x^{1/2}y^{1/2}+y}{x-y}}$

⑪ $\left(\dfrac{a^{-2/3}}{2^{1/2}a^2}\right)^{-1/2} = \dfrac{a^{1/3}}{2^{-1/4}a^{-1}} = \boxed{2^{1/4}a^{4/3}}$

⑫ $\left(\dfrac{2^4a}{a^2b^{-1}}\right)^{-1/2} = \dfrac{2^{-2}a^{-1/2}}{a^{-1}b^{1/2}} = \dfrac{a^{1/2}}{4b^{1/2}}$

$\dfrac{a^{1/2}}{4b^{1/2}}\cdot\dfrac{b^{1/2}}{b^{1/2}} = \boxed{\dfrac{a^{1/2}b^{1/2}}{4b}}$

⑬ $(\sqrt{-8})(\sqrt{-12}) = (2i\sqrt{2})(2i\sqrt{3})$

$4i^2\sqrt{6} = \boxed{-4\sqrt{6}}$

⑭ $(\sqrt{-6})(\sqrt{-4})(\sqrt{-3}) = (i\sqrt{6})(2i)(i\sqrt{3})$

$2i^3\sqrt{18} = 6i^3\sqrt{2} = \boxed{-6i\sqrt{2}}$

⑮ $(3i^3)(2i)^2 = (3i^3)(4i^2)$

$12i^5 = \boxed{12i}$

⑯ $(-2i)^4(-4i^3) = (16i^4)(-4i^3)$

$-64i^7 = \boxed{64i}$

⑰ $2n^2 = -27/8$

$n^2 = -27/16$

$n = \pm\dfrac{\sqrt{-27}}{4}$

$\boxed{n = \pm\dfrac{3i\sqrt{3}}{4}}$

⑱ $4x^2+75=0$

$x^2 = -75/4$

$x = \pm\dfrac{\sqrt{-75}}{2}$

$\boxed{x = \pm\dfrac{5i\sqrt{3}}{2}}$

⑲ $(7+2i)(5-3i)$

$35-21i+10i-6i^2$

$35-11i-6i^2 = \boxed{41-11i}$

Rational Exponents & Complex Numbers

ANSWER KEY: UNIT 7 REVIEW & PRACTICE

(20) $(3+8i)(3-8i)$ conjugates

$(3)^2 - (8i)^2$

$9 - 64i^2$

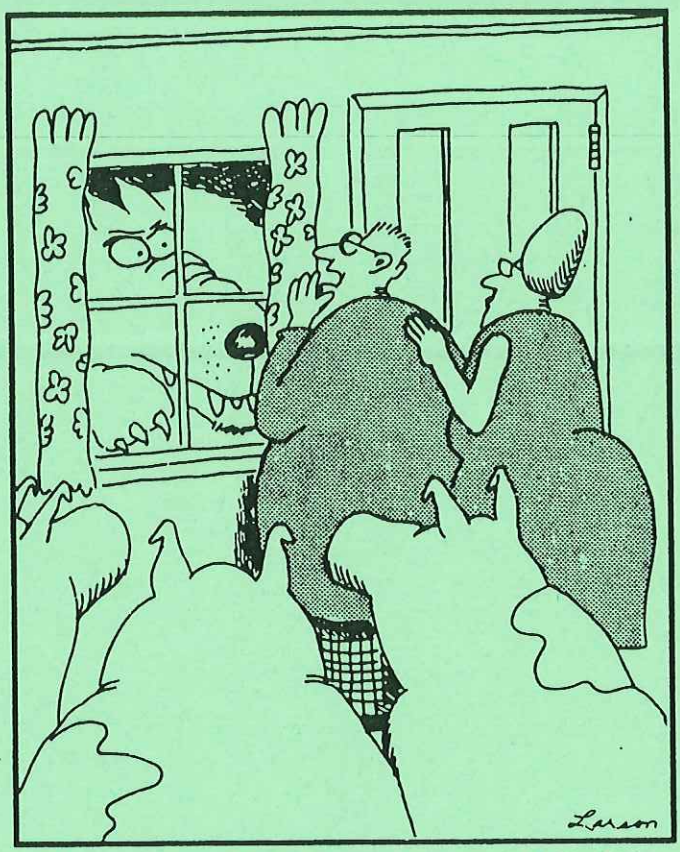
$9 - (-64) = \boxed{73}$

(21) $\frac{4+3i}{1-2i} \cdot \frac{1+2i}{1+2i} = \frac{4+8i+3i+6i^2}{1-4i^2}$

$\frac{4+11i+6i^2}{1-(-4)} = \boxed{\frac{-2+11i}{5}}$

(22) $\frac{2-2i}{2+2i} \cdot \frac{2-2i}{2-2i} = \frac{4-8i+4i^2}{4-4i^2}$

$\frac{-8i}{8} = \boxed{-i}$



"Listen out there! We're George and Harriet Miller! We just dropped in on the pigs for coffee! We're coming out! . . . We don't want trouble!"

Completing The Square & Quadratic Formula

ANSWER KEY 8.1

① $y^2-4y-21=0$
$(y-7)(y+3)=0$
$\boxed{y=7 \text{ or } -3}$

② $m^2+6m=27$
$m^2+6m-27=0$
$(m+9)(m-3)=0$
$\boxed{m=-9 \text{ or } 3}$

③ $6d^2+13d+6=0$
$6d^2+9d+4d+6=0$
$3d(2d+3)+2(2d+3)=0$
$(2d+3)(3d+2)=0$
$\boxed{d=-3/2 \text{ or } -2/3}$

④ $2y^2+11y-21=0$
$2y^2+14y-3y-21=0$
$2y(y+7)-3(y+7)=0$
$(y+7)(2y-3)=0$
$\boxed{y=-7 \text{ or } 3/2}$

⑤ $r^2-6r+8=0$
$r^2-6r=-8$
$r^2-6r+9=-8+9$
$(r-3)^2=1$
$r-3=\pm\sqrt{1}$
$r=3\pm1$
$\boxed{r=2 \text{ or } 4}$

⑥ $n^2-8n+14=0$
$n^2-8n=-14$
$n^2-8n+16=-14+16$
$(n-4)^2=2$
$n-4=\pm\sqrt{2}$
$\boxed{n=4\pm\sqrt{2}}$

⑦ $3c^2-14c+8=0$
$c^2-14/3c=-8/3$
$c^2-\frac{14}{3}c+\frac{49}{9}=-\frac{8}{3}+\frac{49}{9}$
$(c-7/3)^2=25/9$
$c-7/3=\pm5/3$
$\boxed{c=4 \text{ or } 2/3}$

⑧ $3x^2-12x+4=0$
$x^2-4x=-4/3$
$x^2-4x+4=-4/3+4$
$(x-2)^2=8/3$
$x-2=\pm\sqrt{8/3}$
$x-2=\pm\frac{\sqrt{8}}{\sqrt{3}}\cdot\frac{\sqrt{3}}{\sqrt{3}}=\frac{2\sqrt{6}}{3}$
$\boxed{x=2\pm\frac{2\sqrt{6}}{3} \text{ or } x=\frac{6\pm2\sqrt{6}}{3}}$

⑨ $x^2+ax+a=0$
$x^2+ax=-a$
$x^2+ax+\frac{a^2}{4}=-a+\frac{a^2}{4}$
$(x+\frac{a}{2})^2=\frac{a^2-4a}{4}$
$x+\frac{a}{2}=\frac{\pm\sqrt{a^2-4a}}{2}$
$\boxed{x=\frac{-a\pm\sqrt{a^2-4a}}{2}}$

⑩ $ax^2+bx+c=0$
$x^2+\frac{b}{a}x=-\frac{c}{a}$
$x^2+\frac{b}{a}x+\frac{b^2}{4a^2}=-\frac{c}{a}+\frac{b^2}{4a^2}$
$(x+b/2a)^2=\frac{b^2-4ac}{4a^2}$
$x+b/2a=\frac{\pm\sqrt{b^2-4ac}}{2a}$
$\boxed{x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}}$ ← quadratic formula

Completing The Square & Quadratic Formula

ANSWER KEY 8.1

(11) $4x^2-11x-3=0$

$a=4$, $b=-11$, $c=-3$

$$x=\frac{-(-11)\pm\sqrt{(-11)^2-4(4)(-3)}}{2(4)}$$

$$x=\frac{11\pm 13}{8}$$

$$\boxed{x=3 \text{ or } -1/4}$$

(12) $2x^2+3x+3=0$

$a=2$, $b=3$, $c=3$

$$x=\frac{-(3)\pm\sqrt{(3)^2-4(2)(3)}}{2(2)}$$

$$x=\frac{-3\pm\sqrt{-15}}{4}$$

$$\boxed{x=\frac{-3\pm i\sqrt{15}}{4}}$$

(13) $x^3+8=0$

$(x+2)(x^2-2x+4)=0$

$x=-2$

$a=1$, $b=-2$, $c=4$

$$x=\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(4)}}{2(1)}$$

$$x=\frac{2\pm\sqrt{-12}}{2}=\frac{2\pm 2i\sqrt{3}}{2}$$

$$\boxed{x=-2,\ 1\pm i\sqrt{3}}$$

(14) $a^3=125$

$a^3-125=0$

$(a-5)(a^2+5a+25)=0$

$a=5$

$a=1$, $b=5$, $c=25$

$$a=\frac{-(5)\pm\sqrt{(5)^2-4(1)(25)}}{2(1)}$$

$$a=\frac{-5\pm\sqrt{-75}}{2}=\frac{-5\pm 5i\sqrt{3}}{2}$$

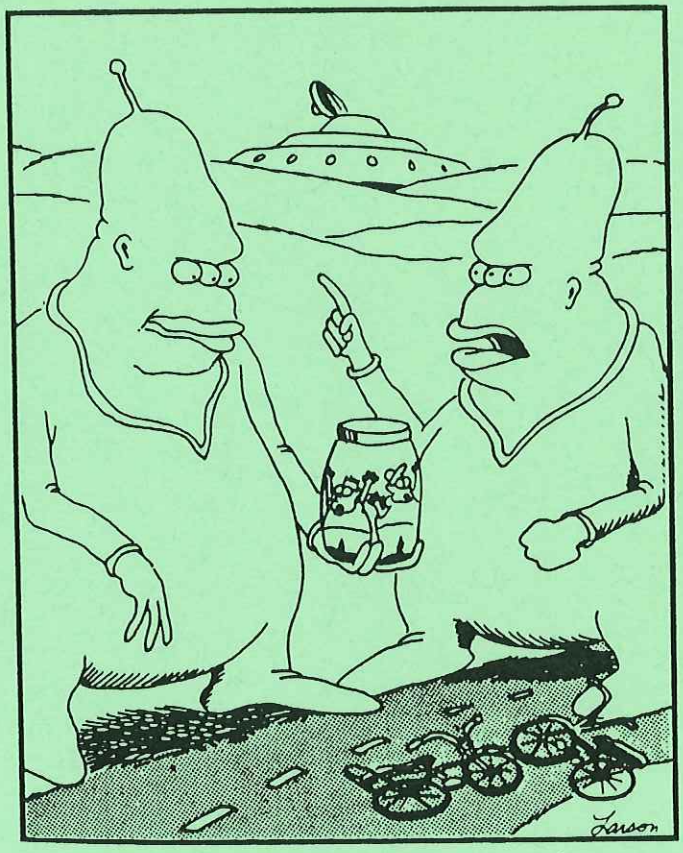
$$\boxed{a=5,\ \frac{-5\pm 5i\sqrt{3}}{2}}$$

(15) $ax^2+b^2x-3b=0$

$a=a$, $b=b^2$, $c=-3b$

$$x=\frac{-(b^2)\pm\sqrt{(b^2)^2-4(a)(-3b)}}{2(a)}$$

$$\boxed{x=\frac{-b^2\pm\sqrt{b^4+12ab}}{2a}}$$



"Now don't forget, Gorok! . . . THIS time punch some holes in the lid!"

The Discriminant & The Nature of Roots

ANSWER KEY 8.2

① $a^2+12a+32=0$
$b^2-4ac = (12)^2-4(1)(32) = \boxed{16}$
perfect square
2 rational roots

② $y^2-4y+4=0$
$b^2-4ac = (-4)^2-4(1)(4) = \boxed{0}$
1 rational root

③ $m^2-6m+4=0$
$b^2-4ac = (-6)^2-4(1)(4) = \boxed{20}$
positive, non perfect square
2 irrational roots

④ $m^2-2m+5=0$
$b^2-4ac = (-2)^2-4(1)(5) = \boxed{-16}$
negative discriminant
2 imaginary roots

⑤ $3n^2-19n=-6$
$3n^2-19n+6=0$
$b^2-4ac = (-19)^2-4(3)(6) = \boxed{289}$
perfect square
2 rational roots

⑥ $x^2-x+1=0$
$b^2-4ac = (-1)^2-4(1)(1) = \boxed{-3}$
negative discriminant
2 imaginary roots

⑦ $x^2-10x+25=0$
$b^2-4ac = (-10)^2-4(1)(25) = \boxed{0}$
zero discriminant
1 rational root

⑧ $y^2+5+6=0$
sum $= -5/1$ $\boxed{-5}$
prod $= 6/1$ $\boxed{6}$

⑨ $x^2-3x+1=0$
sum $= 3/1$ $\boxed{3}$
prod $= 1/1$ $\boxed{1}$

⑩ $2c^2-5c+1=0$
sum $= 5/2$ $\boxed{5/2}$
prod $= 1/2$ $\boxed{1/2}$

⑪ $2x^2-6x+5=0$
sum $= 6/2$ $\boxed{3}$
prod $= 5/2$ $\boxed{5/2}$

⑫ $9n^2-1=0$
sum $= 0/9$ $\boxed{0}$
prod $= -1/9$ $\boxed{-1/9}$

⑬ $s^2-16=0$
sum $= 0/1$ $\boxed{0}$
prod $= -16/1$ $\boxed{-16}$

⑭ $8m^2+6m+1=0$
sum $= -6/8$ $\boxed{-3/4}$
prod $= 1/8$ $\boxed{1/8}$

⑮ sum: $6+4=10$ prod: $(6)(4)=24$
$\boxed{x^2-10x+24=0}$

⑯ sum: $\frac{3}{4}+(-4)=\frac{-13}{4}$ prod: $(\frac{3}{4})(-4)=-3$
$x^2+\frac{13}{4}x-3=0$
$\boxed{4x^2+13x-12=0}$

⑰ sum: $\sqrt{3}+2\sqrt{3}=3\sqrt{3}$ prod: $(\sqrt{3})(2\sqrt{3})=6$
$\boxed{x^2-3x\sqrt{3}+6=0}$

⑱ sum: $5\pm\sqrt{2}=10$ prod: $(5+\sqrt{2})(5-\sqrt{2})=23$
$\boxed{x^2-10x+23=0}$

⑲ sum: $3\pm 7i=6$ prod: $(3+7i)(3-7i)=58$
$\boxed{x^2-6x+58=0}$

The Discriminant & The Nature of Roots

ANSWER KEY 8.2

(20) sum: $\frac{5\pm 3i}{4} = 5/2$ prod: $\left(\frac{5+3i}{4}\right)\left(\frac{5-3i}{4}\right) = 17/8$

$x^2 - 5/2\,x + 17/8 = 0$

$\boxed{8x^2 - 20x + 17 = 0}$

(21) $6m^2 + 7m - 3 = 0$

$m^2 + \frac{7}{6}m = \frac{3}{6}$

$m^2 + \frac{7}{6}m + \frac{49}{144} = \frac{1}{2} + \frac{49}{144}$

$(m + 7/12)^2 = \frac{121}{144}$

$m + 7/12 = \pm \frac{11}{12}$

$m = \frac{-7 \pm 11}{12}$ $\boxed{m = \frac{1}{3} \text{ or } \frac{-3}{2}}$

(22) $x^3 + 64 = 0$

$(x+4)(x^2 - 4x + 16) = 0$

$a=1$, $b=-4$, $c=16$

$$\frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(16)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{-48}}{2} = \frac{4 \pm 4i\sqrt{3}}{2}$$

$x = -4$ $x = 2 \pm 2i\sqrt{3}$

$\boxed{x = -4,\ 2 \pm 2i\sqrt{3}}$

(23) Completing the square

$bx^2 + cx + 2c = 0$

$x^2 + \frac{c}{b}x = \frac{-2c}{b}$

$x^2 + \frac{c}{b}x + \frac{c^2}{4b^2} = \frac{-2c}{b} + \frac{c^2}{4b^2}$

$\left(x + \frac{c}{2b}\right)^2 = \frac{c^2 - 8bc}{4b^2}$

$x + \frac{c}{2b} = \pm \frac{\sqrt{c^2 - 8bc}}{2b}$

$$\boxed{x = \frac{-c \pm \sqrt{c^2 - 8bc}}{2b}}$$

Quadratic formula

$bx^2 + cx + 2c = 0$

$a = b$, $b = c$, $c = 2c$

$$x = \frac{-(c) \pm \sqrt{(c)^2 - 4(b)(2c)}}{2(b)}$$

$$\boxed{x = \frac{-c \pm \sqrt{c^2 - 8bc}}{2b}}$$

The Quadratic Form

ANSWER KEY 8.3

① $x^4-25=0$

$(x^2+5)(x^2-5)=0$

$x^2=-5 \quad x^2=5$

$x=\pm\sqrt{5} \quad x=\pm\sqrt{5}$

$x=\pm i\sqrt{5}$

$\boxed{x=\pm i\sqrt{5}, \pm\sqrt{5}}$

② $x^4-16=0$

$(x^2+4)(x^2-4)=0$

$x^2=-4 \quad x^2=4$

$x=\pm\sqrt{-4} \quad x=\pm 2$

$x=\pm 2i$

$\boxed{x=\pm 2i, \pm 2}$

③ $a^4-36=0$

$(a^2+6)(a^2-6)=0$

$a^2=-6 \quad a^2=6$

$a=\pm\sqrt{-6} \quad a=\pm\sqrt{6}$

$a=\pm i\sqrt{6}$

$\boxed{a=\pm i\sqrt{6}, \pm\sqrt{6}}$

④ $n^4+9n^2+18=0$

$(n^2+6)(n^2+3)=0$

$n^2=-6 \quad n^2=-3$

$n=\pm\sqrt{-6} \quad n=\pm\sqrt{-3}$

$n=\pm i\sqrt{6} \quad n=\pm i\sqrt{3}$

$\boxed{n=\pm i\sqrt{6}, \pm i\sqrt{3}}$

⑤ $c^4-2c^2-8=0$

$(c^2-4)(c^2+2)=0$

$(c+2)(c-2)(c^2+2)=0$

$c=\pm 2, \quad c^2=-2$

$c=\pm i\sqrt{2}$

$\boxed{c=\pm 2, \pm i\sqrt{2}}$

⑥ $x^4-6x^2+8=0$

$(x^2-4)(x^2-2)=0$

$(x+2)(x-2)(x^2-2)=0$

$x=\pm 2, \quad x^2=2$

$x=\pm\sqrt{2}$

$\boxed{x=\pm 2, \pm\sqrt{2}}$

⑦ $n-13\sqrt{n}+36=0$

$(\sqrt{n})^2-13\sqrt{n}+36=0$

$(\sqrt{n}-9)(\sqrt{n}-4)=0$

$\sqrt{n}=9, \quad \sqrt{n}=4$

$\boxed{n=81, 16}$

⑧ $x-2\sqrt{x}+1=0$

$(\sqrt{x})^2-2\sqrt{x}+1=0$

$(\sqrt{x}-1)^2=0$

$\sqrt{x}=1$

$\boxed{x=1}$

⑨ $y-6\sqrt{y}-16=0$

$(\sqrt{y})^2-6\sqrt{y}-16=0$

$(\sqrt{y}-8)(\sqrt{y}+2)=0$

$\sqrt{y}=8, \quad \sqrt{y}=-2$

$y=64, \quad \cancel{4}$

$\boxed{y=64}$

note: $y=4$ does not check out

⑩ $m^6-64=0$

$(m^3)^2-8^2=0$

$(m^3+8)(m^3-8)=0$

$(m+2)(m^2-2m+4)(m-2)(m^2+2m+4)=0$

$\boxed{m=\pm 2, \pm 1\pm i\sqrt{3}}$

$\downarrow \quad m=-2$

$\frac{-(-2)\pm\sqrt{(-2)^2-4(1)(4)}}{2(1)} \quad \downarrow \quad m=2$

$\frac{-(2)\pm\sqrt{(2)^2-4(1)(4)}}{2(1)}$

$\frac{2\pm\sqrt{-12}}{2}=1\pm i\sqrt{3}$

$\frac{-2\pm\sqrt{-12}}{2}=-1\pm i\sqrt{3}$

The Quadratic Form

ANSWER KEY 8.3

(11) $x^6-1=0$ $\quad \boxed{x=\pm 1, \frac{\pm 1 \pm i\sqrt{3}}{2}}$

$(x^3)^2-1=0$

$(x^3+1)(x^3-1)=0$

$(x+1)(x^2-x+1)(x-1)(x^2+x+1)=0$

$x=-1$: $\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(1)}}{2(1)}$ $\quad$ $x=1$: $\frac{-(1)\pm\sqrt{(1)^2-4(1)(1)}}{2(1)}$

$\frac{1\pm\sqrt{-3}}{2}=\frac{1\pm i\sqrt{3}}{2}$ $\quad$ $\frac{-1\pm\sqrt{-3}}{2}=\frac{-1\pm i\sqrt{3}}{2}$

(12) $n^6-7n^3-8=0$ $\quad \boxed{n=3,-1,\frac{-1\pm i\sqrt{3}}{2},\frac{1\pm i\sqrt{3}}{2}}$

$(n^3-8)(n^3+1)=0$

$(n-2)(n^2+2n+4)(n+1)(n^2-n+1)=0$

$n=2$: $\frac{-(2)\pm\sqrt{(2)^2-4(1)(4)}}{2(1)}$ $\quad$ $n=-1$: $\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(1)}}{2(1)}$

$\frac{-2\pm\sqrt{-12}}{2}=-1\pm i\sqrt{3}$ $\quad$ $\frac{1\pm\sqrt{-3}}{2}=\frac{1\pm i\sqrt{3}}{2}$

(13) $a^6-26a^3-27=0$ $\quad \boxed{a=3,-1,\frac{-3\pm 3i\sqrt{3}}{2},\frac{1\pm i\sqrt{3}}{2}}$

$(a^3-27)(a^3+1)=0$

$(a-3)(a^2+3a+9)(a+1)(a^2-a+1)=0$

$a=3$: $\frac{-3\pm\sqrt{3^2-4(1)(9)}}{2(1)}$ $\quad$ $a=-1$: $\frac{-(-1)\pm\sqrt{(-1)^2-4(1)(1)}}{2(1)}$

$\frac{-3\pm\sqrt{-27}}{2}=\frac{-3\pm 3i\sqrt{3}}{2}$ $\quad$ $\frac{1\pm\sqrt{-3}}{2}=\frac{1\pm i\sqrt{3}}{2}$

Note: Problems 10-13 all have a total of 6 solutions

(14) $3y^2+36y+12=0$

$y^2+12y=-4$

$y^2+12y+36=-4+36$

$(y+6)^2=32$

$y+6=\pm\sqrt{32}$

$y=-6\pm\sqrt{32}=-6\pm 4\sqrt{2}$

$\boxed{y=-6\pm 4\sqrt{2}}$

(15) roots $\frac{2}{3}\pm\sqrt{6}$

$\text{sum}=\left(\frac{2}{3}+\sqrt{6}\right)+\left(\frac{2}{3}-\sqrt{6}\right)=\frac{4}{3}$

$\text{prod}=\left(\frac{2}{3}+\sqrt{6}\right)\left(\frac{2}{3}-\sqrt{6}\right)=\frac{4}{9}-6=\frac{-50}{9}$

$x^2-\frac{4}{3}x-\frac{50}{9}=0$ $\quad \boxed{9x^2-12x-50=0}$

(16) $m-11m^{1/2}+30=0$

$(m^{1/2})^2-11m^{1/2}+30=0$

$(m^{1/2}-6)(m^{1/2}-5)=0$

$m^{1/2}=6$ $\quad m^{1/2}=5$

$m=36$ $\quad m=25$

$\boxed{m=36,25}$ both answers check

(17) $y^{-1}-5y^{-1/2}+6=0$

$(y^{-1/2})^2-5y^{-1/2}+6=0$

$(y^{-1/2}-3)(y^{-1/2}-2)=0$

$y^{-1/2}=3$ $\quad y^{-1/2}=2$

$y^{-1}=9$ $\quad y^{-1}=4$ $\leftarrow$ sq. both sides

$y=\frac{1}{9}$ $\quad y=\frac{1}{4}$ $\quad \boxed{y=1/9,1/4}$

Graphing Quadratic Functions

ANSWER KEY 8.4

① $y = x^2 + 6x + 2$

axis $x = \frac{-b}{2a}$ $\boxed{x = -3}$

$(-3)^2 + 6(-3) + 2 = -7$ $\boxed{(-3, -7)}$ min. pt.

x	y	
-2	-6	$(-2)^2 + 6(-2) + 2 = -6$
0	2	$(0)^2 + 6(0) + 2 = 2$
1	9	$(1)^2 + 6(1) + 2 = 9$

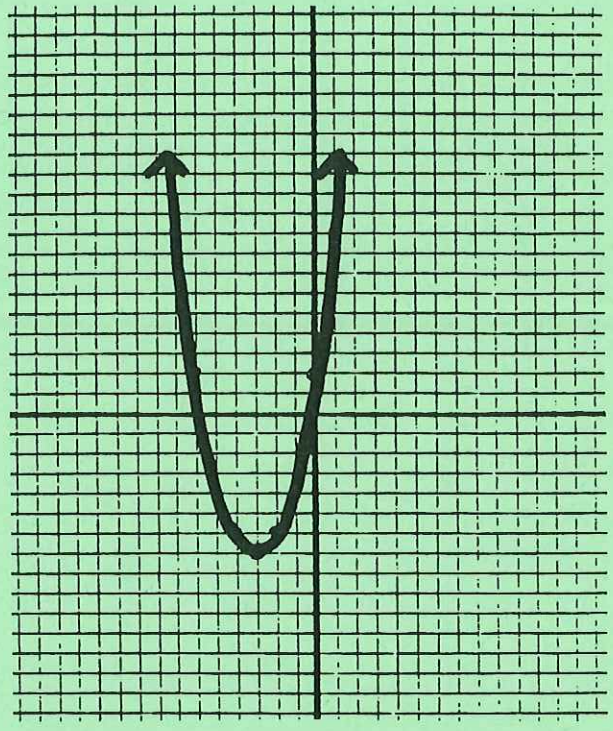
$x^2 + 6x + 2 = 0$

$a = 1$, $b = 6$, $c = 2$

$$\frac{-6 \pm \sqrt{(6)^2 - 4(1)(2)}}{2(1)}$$

roots

$$\frac{-6 \pm 2\sqrt{7}}{2} = \boxed{-3 \pm \sqrt{7}}$$



② $y = x^2 - 2x + 7$

axis $x = \frac{-b}{2a}$ $\boxed{x = 1}$

min. pt.

$(1)^2 - 2(1) + 7 = 6$ $\boxed{(1, 6)}$

x	y	
0	7	$(0)^2 - 2(0) + 7 = 7$
-1	10	$(-1)^2 - 2(-1) + 7 = 10$
-2	15	$(-2)^2 - 2(-2) + 7 = 15$

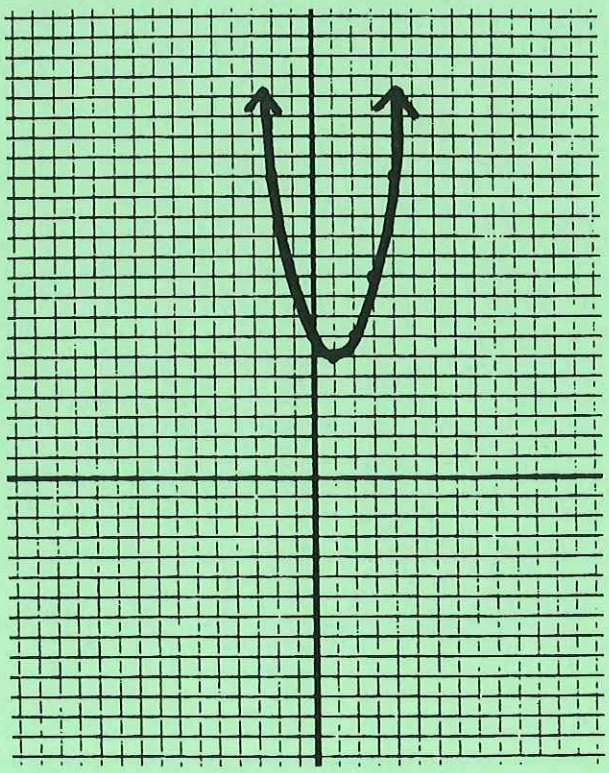
$x^2 - 2x + 7 = 0$

$a = 1$, $b = -2$, $c = 7$

$$\frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(7)}}{2(1)}$$

roots

$$\frac{2 \pm \sqrt{-24}}{2} = \boxed{1 \pm i\sqrt{6}}$$



Graphing Quadratic Functions

ANSWER KEY 8.4

③ $y = -2x^2 + 16x - 31$

axis $x = \frac{-b}{2a}$ $\boxed{x = 4}$

max. pt.

$-2(4)^2 + 16(4) - 31 = 1$ $\boxed{(4, 1)}$

x	y	
3	-1	$-2(3)^2 + 16(3) - 31 = -1$
2	-7	$-2(2)^2 + 16(2) - 31 = -7$
1	-17	$-2(1)^2 + 16(1) - 31 = -17$

$-2x^2 + 16x - 31 = 0$

$a = -2$

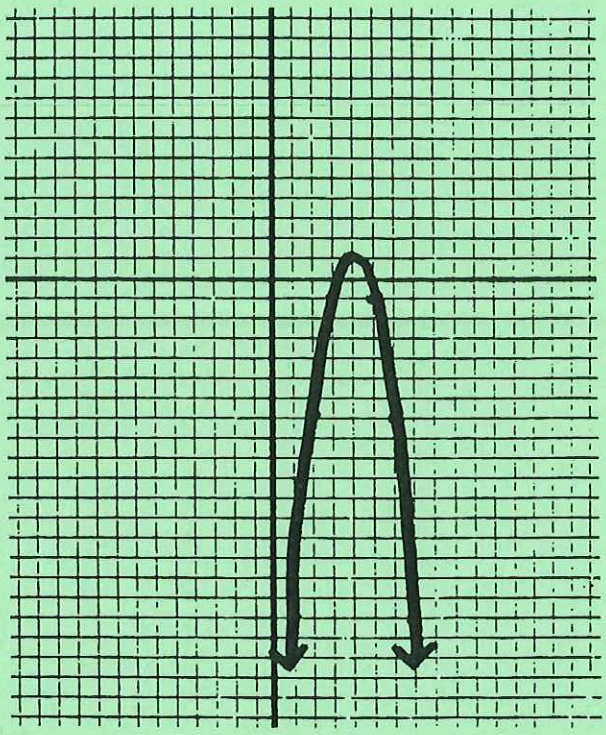
$b = 16$

$c = -31$

$$\frac{-(16) \pm \sqrt{(16)^2 - 4(-2)(-31)}}{2(-2)}$$

$$\frac{-16 \pm \sqrt{8}}{-4} = \frac{-16 \pm 2\sqrt{2}}{-4}$$

roots $\boxed{\frac{8 \pm \sqrt{2}}{2}}$



④ $y = \frac{-1}{2}x^2 + 6x - 19$

axis $x = \frac{-b}{2a}$ $\boxed{x = 6}$

max. pt.

$\frac{-1}{2}(6)^2 + 6(6) - 19 = -1$ $\boxed{(6, -1)}$

x	y	
5	$-1\frac{1}{2}$	$\frac{-1}{2}(5)^2 + 6(5) - 19 = -1\frac{1}{2}$
4	-3	$\frac{-1}{2}(4)^2 + 6(4) - 19 = -3$
3	$-5\frac{1}{2}$	$\frac{-1}{2}(3)^2 + 6(3) - 19 = -5\frac{1}{2}$

$\frac{-1}{2}x^2 + 6x - 19 = 0$

$a = \frac{-1}{2}$

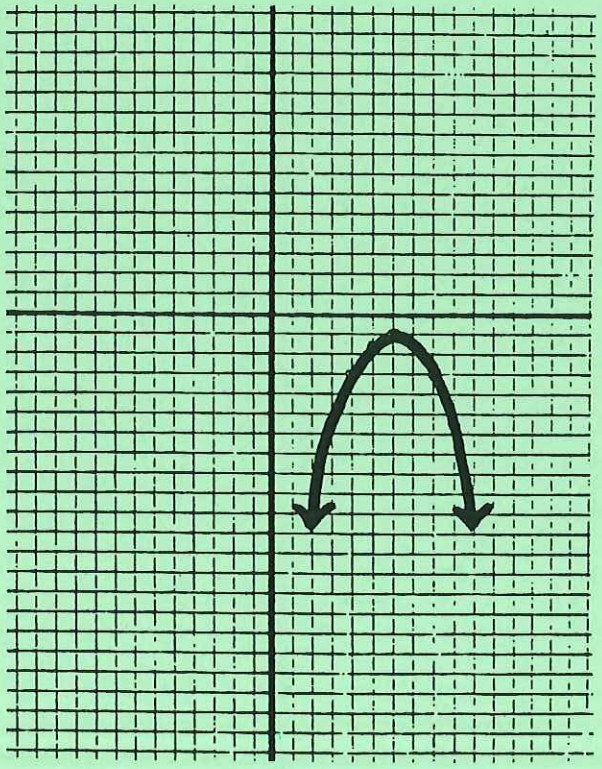
$b = 6$

$c = -19$

$$\frac{-6 \pm \sqrt{(6)^2 - 4(\frac{-1}{2})(-19)}}{2(-\frac{1}{2})}$$

$$\frac{-6 \pm \sqrt{-2}}{-1} = \boxed{6 \pm i\sqrt{2}}$$

roots



Graphing Quadratic Functions

ANSWER KEY 8.4

⑤ $y \leq x^2+10x+24$

axis $x = \frac{-b}{2a}$ $\boxed{x=-5}$

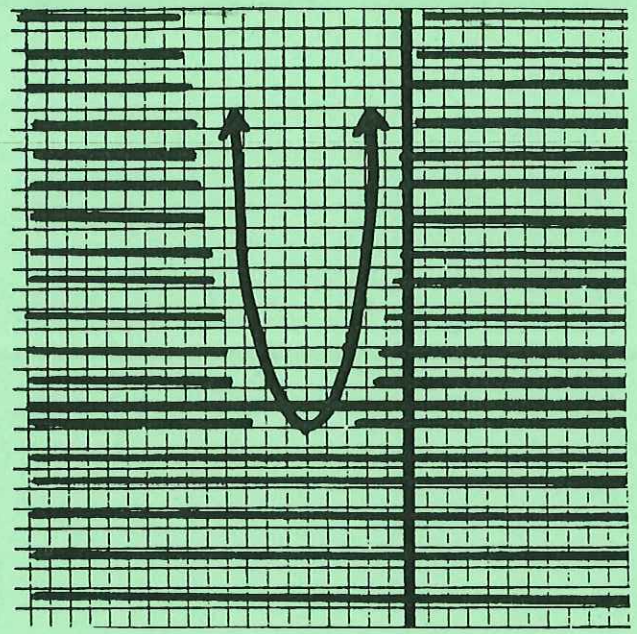
$(-5)^2+10(-5)+24 = -1$ $\boxed{(-5,-1)}$ min. pt.

x	y	
-4	0	$(-4)^2+10(-4)+24 = 0$
-3	3	$(-3)^2+10(-3)+24 = 3$
-2	8	$(-2)^2+10(-2)+24 = 8$

$x^2+10x+24=0$

$(x+6)(x+4)=0$

$x=-6,-4$ $\boxed{x \leq -6 \text{ or } x \geq -4}$



⑥ $y > x^2+4x-12$

axis $x = \frac{-b}{2a}$ $\boxed{x=-2}$

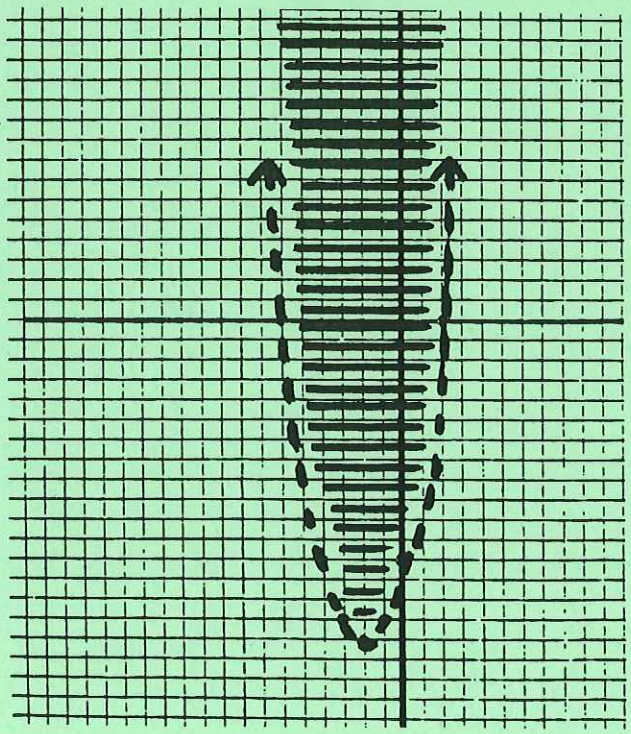
$(-2)^2+4(-2)-12 = -16$ $\boxed{(-2,-16)}$ min. pt.

x	y	
-1	-15	$(-1)^2+4(-1)-12 = -15$
0	-12	$(0)^2+4(0)-12 = -12$
2	0	$(2)^2+4(2)-12 = 0$

$x^2+4x-12=0$

$(x+6)(x-2)=0$

$x=-6,2$ $\boxed{-6<x<2}$



Graphing Quadratic Functions

ANSWER KEY 8.4

⑦ $y > -x^2+6x-8$

axis $x = \frac{-b}{2a}$ $\boxed{x=3}$

$-(3)^2+6(3)-8 = 1$ $\boxed{(3,1)}$ max. pt.

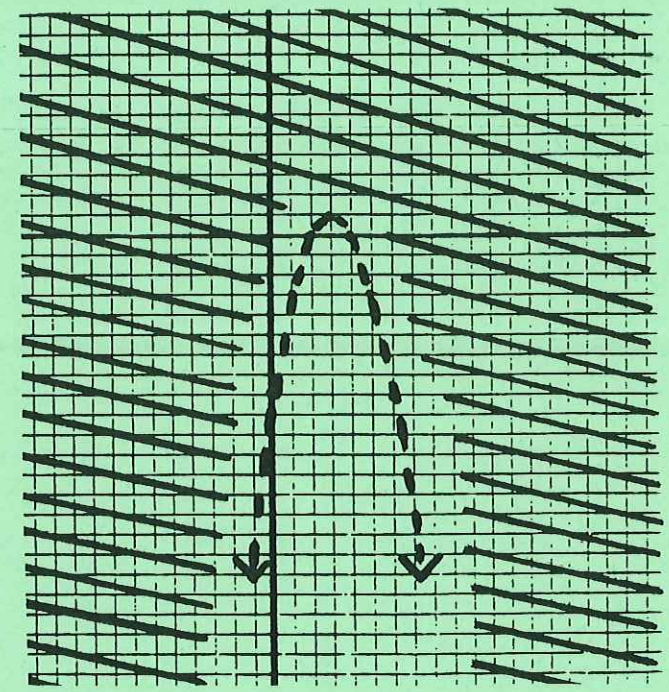
x	y	
2	0	$-(2)^2+6(2)-8 = 0$
1	-3	$-(1)^2+6(1)-8 = -3$
0	-8	$-(0)^2+6(0)-8 = -8$

$-x^2+6x-8=0$

$x^2-6x+8=0$

$(x-4)(x-2)=0$

$x = 4, 2$ $\boxed{x>4 \text{ or } x<2}$



⑧ $y \leq -x^2+8x-12$

axis $x = \frac{-b}{2a}$ $\boxed{x=4}$

$-(4)^2+8(4)-12 = 4$ $\boxed{(4,4)}$ max. pt.

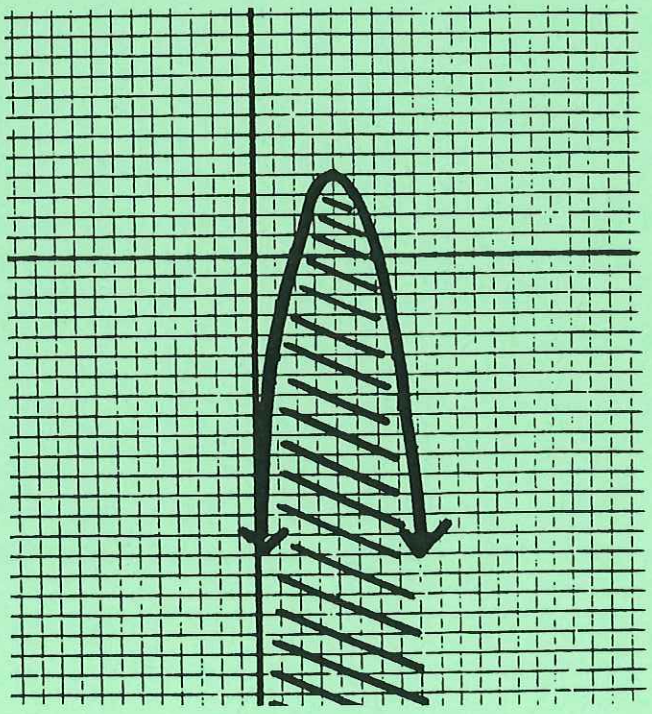
x	y	
3	3	$-(3)^2+8(3)-12 = 3$
2	0	$-(2)^2+8(2)-12 = 0$
1	-5	$-(1)^2+8(1)-12 = -5$

$-x^2+8x-12=0$

$x^2-8x+12=0$

$(x-6)(x-2)=0$

$x = 6, 2$ $\boxed{2 \leq x \leq 6}$



Quadratics

ANSWER KEY: UNIT 8 REVIEW & PRACTICE

① $6x^2+7x-3=0$

$6x^2+9x-2x-3=0$

$3x(2x+3)-1(2x+3)=0$

$(2x+3)(3x-1)=0$ $\boxed{x=-3/2,\ 1/3}$

② $2n^2+n-21=0$

$n^2+\frac{1}{2}n = \frac{21}{2}$

$n^2+\frac{1}{2}n+\frac{1}{16} = \frac{21}{2}+\frac{1}{16}$

$(n+\frac{1}{4})^2 = \frac{169}{16}$

$n+\frac{1}{4} = \pm\frac{13}{4}$

$n = \frac{-1\pm 13}{4}$ $\boxed{n=\frac{-7}{2},\ 3}$

③ $3x^2+4x+2=0$

$x^2+\frac{4}{3}x = -\frac{2}{3}$

$x^2+\frac{4}{3}x+\frac{4}{9} = -\frac{2}{3}+\frac{4}{9}$

$(x+\frac{2}{3})^2 = \frac{-2}{9}$

$x+\frac{2}{3} = \frac{\pm\sqrt{-2}}{3}$

$\boxed{x=\frac{-2\pm i\sqrt{2}}{3}}$

④ $2a^2-5a+4=0$

$a=2$, $b=-5$, $c=4$

$\frac{-(-5)\pm\sqrt{(-5)^2-4(2)(4)}}{2(2)}$

$\frac{5\pm\sqrt{-7}}{4} = \boxed{x=\frac{5\pm i\sqrt{7}}{4}}$

⑤ $ax^2+bx+3b=0$

$a=a$, $b=b$, $c=3b$

$\frac{-b\pm\sqrt{b^2-4a(3b)}}{2a}$

$\boxed{x=\frac{-b\pm\sqrt{b^2-12ab}}{2a}}$

⑥ $bx^2+acx+c=0$

$a=b$, $b=ac$, $c=c$

$\frac{-ac\pm\sqrt{(ac)^2-4(b)(c)}}{2(b)}$

$\boxed{x=\frac{-ac\pm\sqrt{a^2c^2-4bc}}{2b}}$

⑦ $4x^2-40x+25=0$

$b^2-4ac = (-40)^2-4(4)(25) = \boxed{1200}$

$\boxed{\text{2 irrational roots}}$

⑧ $2y^2+6y+5=0$

$b^2-4ac = (6)^2-4(2)(5) = \boxed{-4}$

$\boxed{\text{2 imaginary roots}}$

⑨ sum: $(\frac{3}{4})+(\frac{1}{3}) = \frac{13}{12}$ $x^2-\frac{13}{12}x+\frac{1}{4}=0$

prod: $(\frac{3}{4})(\frac{1}{3}) = \frac{1}{4}$ $\boxed{12x^2-13x+3=0}$

⑩ sum: $(2+\sqrt{3})+(2-\sqrt{3}) = 4$

prod: $(2+\sqrt{3})(2-\sqrt{3}) = 4-3 = 1$

$\boxed{x^2-4x+1=0}$

⑪ sum: $(5+3i)+(5-3i) = 10$

prod: $(5+3i)(5-3i) = 25-9i^2 = 34$

$\boxed{x^2-10x+34=0}$

⑫ $x-4\sqrt{x}-32=0$

$(\sqrt{x}-8)(\sqrt{x}+4)=0$

$\sqrt{x}=8$ $\sqrt{x}=-4$

$x=64$ $x=16$

$\boxed{x=64}$

16 does not check

⑬ $x^4-12x^2+27=0$

$(x^2-9)(x^2-3)=0$

$x^2=3$

$x=\pm\sqrt{3}$

$(x+3)(x-3)$

$x=\pm 3$

$\boxed{x=\pm 3,\pm\sqrt{3}}$

Quadratics

ANSWER KEY: UNIT 8 REVIEW & PRACTICE

(14) $x^{2/3} - 9x^{1/3} + 20 = 0$

$(x^{1/3})^2 - 9x^{1/3} + 20 = 0$

$(x^{1/3} - 5)(x^{1/3} - 4) = 0$

$x^{1/3} = 5 \quad x^{1/3} = 4$

$x = 125 \quad x = 64$

$\boxed{x = 125, 64}$

(15) $n^6 + 9n^3 + 8 = 0$

$(n^3)^2 + 9n^3 + 8 = 0$

$(n^3 + 8)(n^3 + 1) = 0$

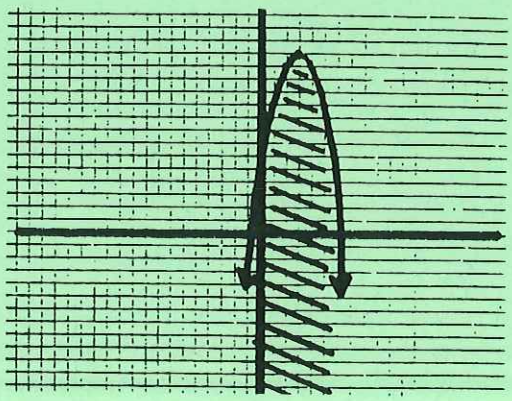
$(n+2)(n^2 - 2n + 4)(n+1)(n^2 - n + 1) = 0$

$n = -2$ ↓ $\quad \frac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(4)}}{2(1)}$ ↓ $n = -1$ $\quad \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(1)}}{2(1)}$

$\frac{2 \pm \sqrt{-12}}{2} = \frac{2 \pm 2i\sqrt{3}}{2} \quad \frac{1 \pm \sqrt{-3}}{2}$

$1 \pm i\sqrt{3} \quad \frac{1 \pm i\sqrt{3}}{2}$

$\boxed{n = -2, -1, 1 \pm i\sqrt{3}, \frac{1 \pm i\sqrt{3}}{2}}$



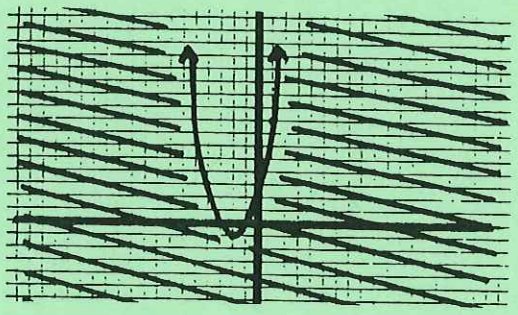
(16) $y \leq x^2 + 4x + 3 \quad x = \frac{-b}{2a} \quad \boxed{x = -2}$ axis

$(-2)^2 + 4(-2) + 3 = -1 \quad \boxed{(-2, -1)}$ min. pt.

x	y	
-1	0	$(-1)^2 + 4(-1) + 3 = 0$
0	3	$(0)^2 + 4(0) + 3 = 3$
1	8	$(1)^2 + 4(1) + 3 = 8$

$x^2 + 4x + 3 = 0$

$(x+3)(x+1) = 0 \quad \boxed{x \leq -3 \text{ or } x \geq -1}$



(17) $y \leq -x^2 + 6x + 5 \quad x = \frac{-b}{2a} \quad \boxed{x = 3}$ axis

$-(3)^2 + 6(3) + 5 = 14 \quad \boxed{(3, 14)}$ max. pt.

x	y	
2	13	$-(2)^2 + 6(2) + 5 = 13$
1	10	$-(1)^2 + 6(1) + 5 = 10$
0	5	$-(0)^2 + 6(0) + 5 = 5$

$-x^2 + 6x + 5 = 0$

$x^2 - 6x - 5 = 0$

$\frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(-5)}}{2(1)}$

$\frac{6 \pm \sqrt{56}}{2} = \frac{6 \pm 2\sqrt{14}}{2} \quad x = 3 \pm \sqrt{14}$

$\boxed{3 - \sqrt{14} \leq x \leq 3 + \sqrt{14}}$

← graph

Rational Operations

ANSWER KEY 9.1

① $\frac{6y^3-9y^2}{2y^2+5y-12} = \frac{3y^2(2y-3)}{(y+4)(2y-3)} = \boxed{\frac{3y^2}{y+4}}$

② $\frac{x^2-x-20}{x^2+7x+12} = \frac{(x-5)(x+4)}{(x+3)(x+4)} = \boxed{\frac{x-5}{x+3}}$

③ $\frac{y^2+4y+4}{3y^2+5y-2} = \frac{(y+2)(y+2)}{(y+2)(3y-1)} = \boxed{\frac{y+2}{3y-1}}$

④ $\frac{a^2+2a+1}{2a^2+3a+1} = \frac{(a+1)(a+1)}{(2a+1)(a+1)} = \boxed{\frac{a+1}{2a+1}}$

⑤ $\frac{x^2-y^2}{y^2} \cdot \frac{y^3}{y-x} = \frac{(x-y)(x+y)\,y^3}{-(x-y)\,y^2}$

$\boxed{-y(x+y)}$

⑥ $\frac{x^2-y^2}{x+y} \cdot \frac{11}{x-y} = \frac{(x+y)(x-y)(11)}{(x+y)(x-y)} = \boxed{11}$

⑦ $-\frac{x^2-y^2}{x+y} \cdot \frac{1}{x-y} = \frac{-(x+y)(x-y)(1)}{(x+y)(x-y)} = \boxed{-1}$

⑧ $\frac{x^2+3x-10}{x^2+8x+15} \cdot \frac{x^2+5x+6}{x^2+4x+4}$

$\frac{(x+5)(x-2)(x+3)(x+2)}{(x+5)(x+3)(x+2)(x+2)} = \boxed{\frac{x-2}{x+2}}$

⑨ $\frac{(x+y)^2}{a} \div \frac{x+y}{ab} = \frac{(x+y)(x+y)(ab)}{(a)(x+y)}$

$\boxed{b(x+y)}$

⑩ $\frac{a^2-b^2}{2a} \div \frac{a-b}{ab}$

$\frac{(a+b)(a-b)(ab)}{(2a)(a-b)} = \boxed{\frac{b(a+b)}{2}}$

⑪ $\frac{3x-21}{x^2-49} \div \frac{3x}{x^2+7x}$

$\frac{3(x-7)(x)(x+7)}{(x+7)(x-7)(3x)} = \boxed{1}$

⑫ $\frac{y^2-y}{w^2-y^2} \div \frac{y^2-2y+1}{1-y}$

$\frac{y(y-1)(-1)(y-1)}{(w-y)(w+y)(y-1)(y-1)} = \boxed{\frac{-y}{w^2-y^2}}$

⑬ $\frac{3x}{x-y} + \frac{4x}{y-x} = \frac{3x}{x-y} - \frac{4x}{x-y} = \boxed{\frac{-x}{x-y}}$

⑭ $\left[\frac{3a+2}{a+b} \div \frac{4}{2a+2b}\right]$

$\frac{(6a+4)+4}{2(a+b)} = \frac{6a+8}{2(a+b)} = \boxed{\frac{3a+4}{a+b}}$

⑮ $\frac{2a}{3a-15} + \frac{-16a+20}{3a^2-12a-15}$

$\left[\frac{2a}{3(a-5)} + \frac{-16a+20}{3(a+1)(a-5)}\right]$

$\frac{2a(a+1)-16a+20}{(3)(a+1)(a-5)} = \frac{2(a-5)(a-2)}{3(a+1)(a-5)} = \boxed{\frac{2(a-2)}{3(a+1)}}$

Rational Operations

ANSWER KEY 9.1

(16) $\dfrac{m^2+n^2}{m^2-n^2} + \dfrac{m}{n-m} + \dfrac{n}{m+n}$

$\left[\dfrac{m^2+n^2}{m^2-n^2} - \dfrac{m}{m-n} + \dfrac{n}{m+n}\right]$

$\dfrac{(m^2+n^2) - m(m+n) + n(m-n)}{m^2-n^2} = \dfrac{0}{m^2-n^2} = \boxed{0}$

(17) $\dfrac{7}{y-8} - \dfrac{6}{8-y} = \dfrac{7}{y-8} + \dfrac{6}{y-8} = \boxed{\dfrac{13}{y-8}}$

(18) $\dfrac{y}{y-9} - \dfrac{-9}{9-y} = \dfrac{y}{y-9} + \dfrac{-9}{y-9} = \dfrac{y-9}{y-9} = \boxed{1}$

(19) $3m+1 - \dfrac{2m}{3m+1} = \dfrac{(3m+1)(3m+1) - 2m}{3m+1}$

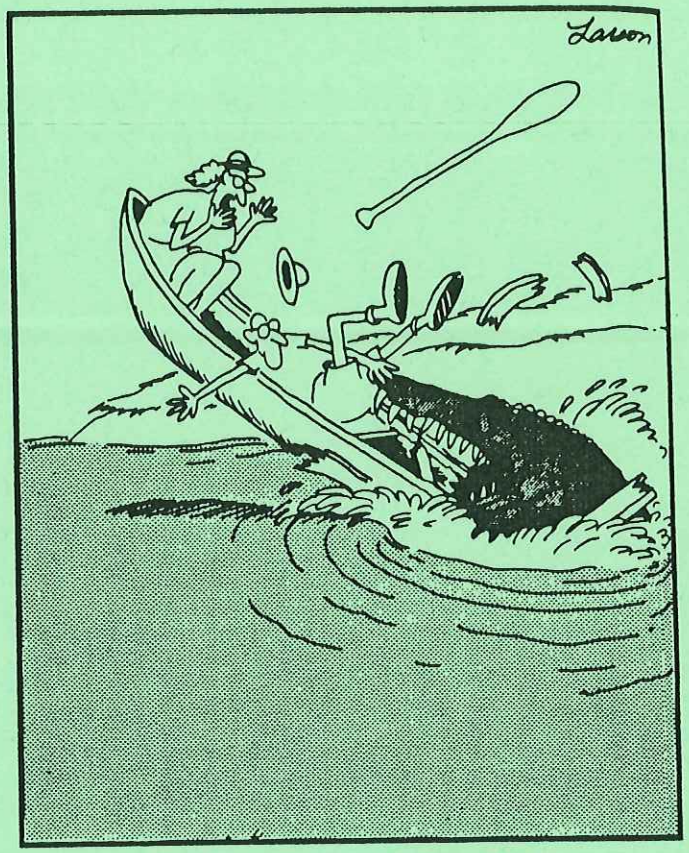
$\boxed{\dfrac{9m^2+4m+1}{3m+1}}$

(20) $\dfrac{x}{x^2+2x+1} - \dfrac{x+2}{x+1} + \dfrac{3x}{x+1}$

$\dfrac{x - (x+2)(x+1) + 3x(x+1)}{(x+1)^2}$

$\dfrac{x - (x^2+3x+2) + (3x^2+3x)}{(x+1)^2}$

$\dfrac{x - x^2 - 3x - 2 + 3x^2 + 3x}{(x+1)^2} = \boxed{\dfrac{2x^2+x-2}{(x+1)^2}}$



"Rub his belly, Ernie! Rub his belly!"

Complex Fractions

ANSWER KEY 9.2

① $\dfrac{\left(\frac{x^2-y^2}{2}\right)}{\left(\frac{x-y}{4}\right)} \rightarrow \dfrac{(x+y)(x-y)}{2} \cdot \dfrac{4}{(x-y)} = \boxed{2(x+y)}$

② $\dfrac{w^2+2w+1}{\left(\frac{w+1}{3}\right)} \rightarrow (w+1)(w+1) \cdot \dfrac{3}{(w+1)} = \boxed{3(w+1)}$

③ $\dfrac{\left(\frac{5a^2-20}{2a+4}\right)}{\left(\frac{10a-20}{4a}\right)} \rightarrow \dfrac{5(a+2)(a-2)}{2(a+2)} \cdot \dfrac{4a}{10(a-2)} = \boxed{a}$

④ $\dfrac{\left(\frac{c^2+2c-3}{3c+3}\right)}{\left(\frac{c^2+5c+6}{2c+2}\right)} \rightarrow \dfrac{(c+3)(c-1)}{3(c+1)} \cdot \dfrac{2(c+1)}{(c+3)(c+2)} = \boxed{\dfrac{2(c-1)}{3(c+2)}}$

⑤ $\dfrac{\left(\frac{p^2+7p}{3p}\right)}{\left(\frac{49-p^2}{3p-21}\right)} \rightarrow \dfrac{p(p+7)}{3p} \cdot \dfrac{3(p-7)}{(7+p)(7-p)}$

$\dfrac{(p-7)}{(7-p)} = \dfrac{-(7-p)}{(7-p)} = \boxed{-1}$

⑥ $\dfrac{\left(\frac{3m}{2m^2+7m-15}\right)}{\left(\frac{6}{2m^2-3m}\right)} \rightarrow \dfrac{3m}{(m+5)(2m-3)} \cdot \dfrac{m(2m-3)}{6}$

$\boxed{\dfrac{m^2}{2(m+5)}}$

⑦ $\dfrac{\left(\frac{x}{2}\right)}{\left(\frac{x}{3}\right)} - \dfrac{\left(\frac{x}{5}\right)}{\left(\frac{x}{6}\right)} \rightarrow \dfrac{3x}{2x} - \dfrac{6x}{5x}$

$\dfrac{3}{2} - \dfrac{6}{5} = \dfrac{15}{10} - \dfrac{12}{10} = \boxed{\dfrac{3}{10}}$

⑧ $\dfrac{\left(\frac{2x}{ab}\right)}{\left(\frac{3x}{a}\right)} - \dfrac{\left(\frac{5x}{4}\right)}{\left(\frac{6x}{5}\right)} \rightarrow \dfrac{2ax}{3abx} - \dfrac{25x}{24x}$

$\dfrac{2}{3b} - \dfrac{25}{24} = \dfrac{16}{24b} - \dfrac{25b}{24b} = \boxed{\dfrac{16-25b}{24b}}$

⑨ $\dfrac{3+\frac{5}{a+2}}{3-\frac{10}{a+7}} = \dfrac{\left(\frac{3(a+2)+5}{a+2}\right)}{\left(\frac{3(a+7)-10}{a+7}\right)} = \dfrac{\frac{3a+11}{a+2}}{\frac{3a+11}{a+7}}$

$\dfrac{3a+11}{a+2} \cdot \dfrac{a+7}{3a+11} = \boxed{\dfrac{a+7}{a+2}}$

⑩ $\dfrac{\frac{2x}{2x+1}-1}{1+\frac{2x}{1-2x}} = \dfrac{\left(\frac{2x-1(2x+1)}{2x+1}\right)}{\left(\frac{1(1-2x)+2x}{1-2x}\right)} = \dfrac{\left(\frac{-1}{2x+1}\right)}{\left(\frac{1}{1-2x}\right)}$

$\dfrac{-1}{2x+1} \cdot \dfrac{1-2x}{1} = \boxed{\dfrac{2x-1}{2x+1}}$

Complex Fractions

ANSWER KEY 9.2

(11) $$\frac{\frac{5x}{x^2-16}}{\frac{10}{x-4}+\frac{10}{x+4}} = \frac{\frac{5x}{x^2-16}}{\frac{10(x+4)+10(x-4)}{x^2-16}} = \frac{\frac{5x}{x^2-16}}{\frac{20x}{x^2-16}}$$

$$\frac{5x}{(x^2-16)} \cdot \frac{(x^2-16)}{20x} = \boxed{\frac{1}{4}}$$

(12) $$\frac{\frac{x+4}{x-6}+\frac{x+1}{x+2}}{\frac{4}{x^2-4x-12}} = \frac{\frac{(x+2)(x+4)+(x-6)(x+1)}{(x-6)(x+2)}}{\frac{4}{(x-6)(x+2)}}$$

$$(x^2+6x+8)+(x^2-5x-6) = 2x^2+x+2$$

$$\frac{\left(\frac{2x^2+x+2}{(x-6)(x+2)}\right)}{\frac{4}{(x-6)(x+2)}} \rightarrow \frac{2x^2+x+2}{(x-6)(x+2)} \cdot \frac{(x-6)(x+2)}{4}$$

$$\boxed{\frac{2x^2+x+2}{4}}$$

(13) $$\frac{n+1-\frac{2}{n}}{n+4+\frac{4}{n}} = \frac{\frac{n(n+1)-2}{n}}{\frac{n(n+4)+4}{n}}$$

$$\frac{\left(\frac{n^2+n-2}{n}\right)}{\left(\frac{n^2+4n+4}{n}\right)} = \frac{(n+2)(n-1)}{n} \cdot \frac{n}{(n+2)(n+2)}$$

$$\boxed{\frac{n-1}{n+2}}$$

(14) $$\frac{\frac{1}{x+5}+\frac{1}{x-3}}{\frac{2x^2-3x-5}{x^2+2x-15}} = \frac{\frac{1(x-3)+1(x+5)}{(x+5)(x-3)}}{\frac{(x+1)(2x-5)}{(x+5)(x-3)}}$$

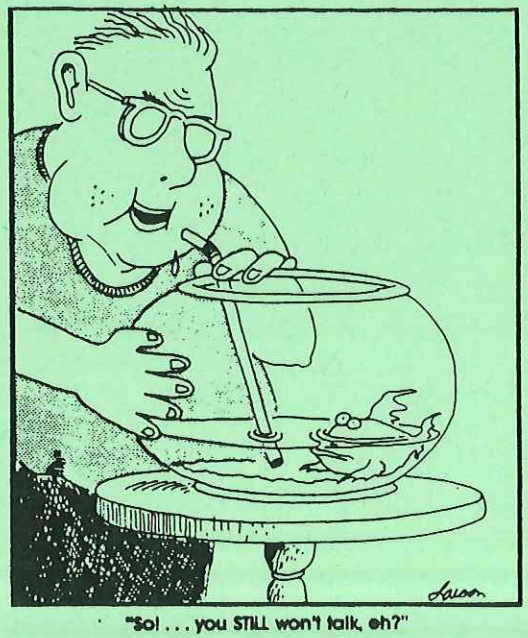
$$\frac{2(x+1)}{(x+5)(x-3)} \cdot \frac{(x+5)(x-3)}{(x+1)(2x-5)} = \boxed{\frac{2}{2x-5}}$$

(15) $$\frac{(a^2-5a+6)^{-1}}{(a-2)^{-2}} \div \frac{(a-3)^{-1}}{(a-2)^{-3}}$$

$$\frac{(a-2)^2}{(a^2-5a+6)} \div \frac{(a-2)^3}{(a-3)}$$

$$\frac{(a-2)(a-2)}{(a-3)(a-2)} \cdot \frac{(a-3)}{(a-2)(a-2)(a-2)}$$

$$\boxed{\frac{1}{a^2-4a+4}}$$



"So! . . . you STILL won't talk, eh?"

Rational Equations

ANSWER KEY 9.3

① $\left[1+\frac{3}{y-1}=\frac{4}{3}\right](3)(y-1)$

$3(y-1)+3(3)=4(y-1)$

$3y-3+9=4y-4$

$-y=-10$ $\boxed{y=10}$

② $\left[\frac{5}{2x}-\frac{3}{10}=\frac{1}{x}\right](2x)(10)$

$5(10)-3(2x)=1(2)(10)$

$50-6x=20$

$-6x=-30$ $\boxed{x=5}$

③ $\left[\frac{1}{9}+\frac{1}{2a}=\frac{1}{a^2}\right](9)(2)(a^2)$

$1(2)(a^2)+1(9)(a)=1(9)(2)$

$2a^2+9a=18$

$2a^2+9a-18=0$

$(2a-3)(a+6)=0$ $\boxed{a=3/2,-6}$

④ $\left[\frac{1}{x-1}+\frac{2}{x}=0\right](x-1)(x)$

$1(x)+2(x-1)=0(x-1)(x)$

$x+2x-2=0$

$3x=2$ $\boxed{x=2/3}$

⑤ $\left[\frac{4t}{3t-2}+\frac{2t}{3t+2}=2\right](3t-2)(3t+2)$

$4t(3t+2)+2t(3t-2)=2(3t-2)(3t+2)$

$12t^2+8t+6t^2-4t=18t^2-8$

$4t=-8$ $\boxed{t=-2}$

⑥ $\left[\frac{12}{x^2-16}-\frac{24}{x-4}=3\right](x^2-16)$

$12-24(x+4)=3(x^2-16)$

$12-24x-96=3x^2-48$

$3x^2+24x+36=0$

$3(x^2+8x+12)=0$

$(x+6)(x+2)=0$ $\boxed{x=-6,-2}$

⑦ $\frac{5}{x-3}-\frac{x}{3-x}=x$

$\left[\frac{5}{x-3}+\frac{x}{x-3}=x\right](x-3)$

$5+x=x(x-3)$

$5+x=x^2-3x$

$x^2-4x-5=0$

$(x-5)(x+1)=0$ $\boxed{x=5,-1}$

⑧ $\left[\frac{x-3}{2x}=\frac{x-2}{2x+1}-\frac{1}{2}\right](2)(x)(2x+1)$

$(x-3)(2x+1)=(x-2)(2x)-1(x)(2x+1)$

$2x^2-5x-3=2x^2-4x-2x^2-x$

$2x^2=3$

$x^2=3/2$

$x=\pm\frac{\sqrt{3}}{\sqrt{2}}\cdot\frac{\sqrt{2}}{\sqrt{2}}=\frac{\pm\sqrt{6}}{2}$ $\boxed{x=\frac{\pm\sqrt{6}}{2}}$

⑨ $\frac{2}{y+2}-\frac{y}{2-y}=\frac{y^2+4}{y^2-4}$

$\left[\frac{2}{y+2}+\frac{y}{y-2}=\frac{y^2+4}{y^2-4}\right](y^2-4)$

continued

Rational Equations

ANSWER KEY 9.3

$2(y-2) + y(y+2) = y^2+4$

$2y - 4 + y^2 + 2y = y^2 + 4$

$4y = 8$

$y = \cancel{2}$ 2 is an excluded value

no solutions

(10) $\left[\frac{t+4}{t} + \frac{3}{t-4} = \frac{-16}{t^2-4t}\right](t^2-4t)$

$(t+4)(t-4) + 3(t) = -16$

$t^2 - 16 + 3t = -16$

$t^2 + 3t = 0$

$t(t+3) = 0 \quad t = \cancel{0}, -3$

0 is an excluded value

$t = -3$

(11) $\frac{y}{y-5} + \frac{17}{25-y^2} = \frac{1}{y+5}$

$\left[\frac{y}{y-5} - \frac{17}{y^2-25} = \frac{1}{y+5}\right](y^2-25)$

$y(y+5) - 17 = 1(y-5)$

$y^2 + 5y - 17 = y - 5$

$y^2 + 4y - 12 = 0$

$(y+6)(y-2) = 0$ **$y = -6, 2$**

(12) $\frac{x}{x^2-1} + \frac{2}{x+1} = \frac{1}{2x-2}$

$\left[\frac{x}{x^2-1} + \frac{2}{x+1} = \frac{1}{2(x-1)}\right](2)(x^2-1)$

$x(2) + 2(2)(x-1) = 1(x+1)$

$2x + 4x - 4 = x + 1$

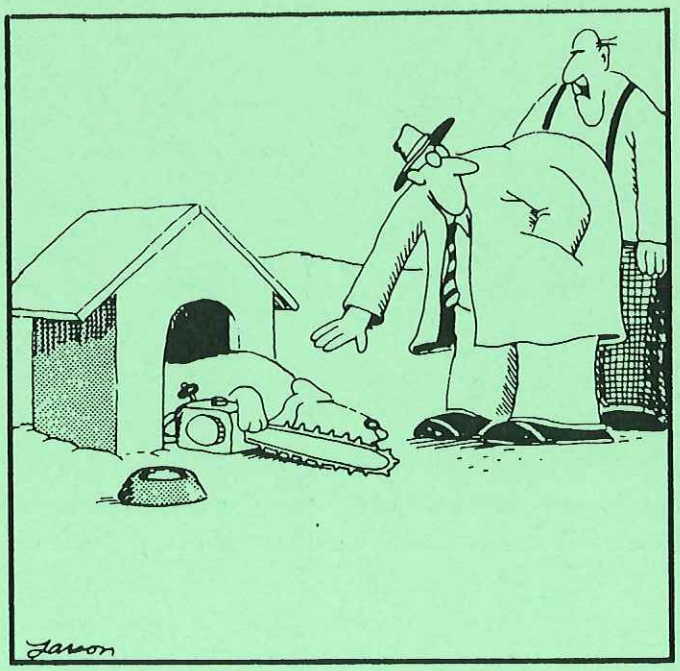
$5x = 5$

$x = \cancel{1}$ 1 is an excluded value

no solutions

(13) $\dfrac{\frac{2x}{y} + 1 - \frac{y}{x}}{\frac{2x}{y} + \frac{y}{x} - 3} = \dfrac{\frac{2x^2+xy-y^2}{xy}}{\frac{2x^2+y^2-3xy}{xy}}$

$\frac{(2x-y)(x+y)}{(x-y)} \cdot \frac{(x-y)}{(2x-y)(x-y)} = \boxed{\frac{x+y}{x-y}}$



"I wouldn't do that, mister . . . Old Zeek's liable to fire that sucker up."

Inverse Variation

ANSWER KEY 9.4

① $\frac{2}{9} = \frac{8}{y}$ $2y = 72$ $\boxed{y = 36}$

② $\frac{-3}{x} = \frac{10}{4}$ $10x = -12$ $\boxed{x = -6/5}$

③ $\frac{1/5}{2/5} = \frac{11}{y}$ $y/5 = 22/5$ $\boxed{y = 22}$

④ $\frac{9}{-3} = \frac{y}{1/5}$ $-3y = 9/5$ $\boxed{y = -3/5}$

⑤ $\frac{-8}{x} = \frac{2/3}{-2}$ $2x/3 = 16$ $\boxed{x = 24}$

⑥ $\frac{-3}{4} = \frac{y}{7}$ $4y = -21$ $\boxed{y = -21/4}$

⑦ shadow / tree $\frac{40}{75} = \frac{10}{n}$ $40n = 750$ $n = 18.75$

$\boxed{18.75 \text{ feet high}}$

⑧ $\frac{120}{n} = \frac{8}{6}$ $8n = 720$ $n = 90$

$\boxed{90 \text{ ft}^3}$

⑨ $\frac{30}{n} = \frac{100}{140}$ $100n = 4200$ $n = 42$ $\boxed{42 \text{ lbs}}$

⑩ commission / sale $\frac{4800}{90{,}000} = \frac{n}{129{,}000}$

$90{,}000n = 619{,}200{,}000$

$n = \boxed{\$6880}$

⑪ $\frac{8}{2y-16} - \frac{y}{8-y} \rightarrow \frac{8}{2(y-8)} + \frac{y}{y-8}$

$\frac{8 + y(2)}{2(y-8)} = \frac{2(y+4)}{2(y-8)} = \boxed{\frac{y+4}{y-8}}$

⑫ $\dfrac{\frac{5}{y+7} + \frac{2}{y}}{\frac{2}{y} - \frac{10}{y^2+7y}} = \dfrac{\frac{5y+2(y+7)}{y(y+7)}}{\frac{2(y+7)-10}{y(y+7)}}$

$\frac{7(y+2)}{y(y+7)} \cdot \frac{y(y+7)}{2(y+2)} = \boxed{\frac{7}{2}}$

⑬ $\left[\frac{4x^2}{x^2-9} - \frac{2x}{x+3} = \frac{3}{x-3}\right](x^2-9)$

$4x^2 - 2x(x-3) = 3(x+3)$

$4x^2 - 2x^2 + 6x = 3x + 9$

$2x^2 + 3x - 9 = 0$

$2x^2 + 6x - 3x - 9 = 0$

$2x(x+3) - 3(x+3) = 0$

$(2x-3)(x+3) = 0$

$x = 3/2, -3$ -3 is an excluded value

$\boxed{x = 3/2}$

Rational Expressions

ANSWER KEY: UNIT 9 REVIEW & PRACTICE

① $\frac{a^3-b^3}{b^2-a^2} \cdot \frac{a+b}{a^2+ab+b^2}$

$\frac{(a-b)(a^2+ab+b^2)(a+b)}{(b-a)(b+a)(a^2+ab+b^2)} = \frac{(a-b)}{-(a-b)} = \boxed{-1}$

② $\frac{x^2-11x+24}{x^2-18x+80} \div \frac{x^2-9x+20}{x^2-15x+50}$

$\frac{(x-8)(x-3)}{(x-8)(x-10)} \cdot \frac{(x-5)(x-10)}{(x-5)(x-4)} = \boxed{\frac{x-3}{x-4}}$

③ $\frac{x^2-2x+1}{y-5} \div \frac{(x-1)^2}{y^2-25}$

$\frac{(x-1)(x-1)}{(y-5)} \cdot \frac{(y+5)(y-5)}{(x-1)^2} = \boxed{y+5}$

④ $\frac{3}{m-2} + \frac{2}{2-m} = \frac{3}{m-2} - \frac{2}{m-2} = \boxed{\frac{1}{m-2}}$

⑤ $\frac{3x+2}{3x-6} - \frac{x+2}{x^2-4} = \frac{3x+2}{3(x-2)} - \frac{x+2}{x^2-4}$

$\frac{(3x+2)(x+2) - (x+2)(3)}{3(x^2-4)}$

$\frac{(3x^2+8x+4) - (3x+6)}{3(x^2-4)}$

$\frac{3x^2+5x-2}{3(x^2-4)} = \frac{(3x-1)(x+2)}{3(x-2)(x+2)} = \boxed{\frac{3x-1}{3x-6}}$

⑥ $\frac{2n}{n^2-5n} - \frac{-3n}{n-5}$

$\frac{2n-(-3n)(n)}{n(n-5)} = \frac{2n+3n^2}{n(n-5)} = \frac{n(2+3n)}{n(n-5)} = \boxed{\frac{2+3n}{n-5}}$

⑦ $\frac{\frac{x}{y} - \frac{y}{x}}{\frac{1}{x} + \frac{1}{y}} = \frac{\frac{x^2-y^2}{xy}}{\frac{y+x}{xy}}$

$\frac{(x-y)(x+y)}{xy} \cdot \frac{xy}{(x+y)} = \boxed{x-y}$

⑧ $\frac{\frac{m+4}{m} - \frac{3}{m+5}}{\frac{m-1}{m^2+5m} + \frac{3}{m+5}} = \frac{\frac{(m+4)(m+5)-3(m)}{m(m+5)}}{\frac{(m-1)+3(m)}{m^2+5m}}$

$\frac{\frac{(m^2+9m+20)-3m}{m^2+5m}}{\frac{4m-1}{m^2+5m}}$

$\frac{m^2+6m+20}{(m^2+5m)} \cdot \frac{(m^2+5m)}{4m-1} = \boxed{\frac{m^2+6m+20}{4m-1}}$

⑨ $\frac{n+5+\frac{4}{n+1}}{n+3} = \frac{\frac{(n+5)(n+1)+4}{n+1}}{n+3}$

$\frac{\left(\frac{n^2+6n+9}{n+1}\right)}{(n+3)} = \frac{(n+3)^2}{n+1} \cdot \frac{1}{(n+3)} = \boxed{\frac{n+3}{n+1}}$

Rational Expressions

ANSWER KEY: UNIT 9 REVIEW & PRACTICE

(10) $\frac{3x}{2x-5} + \frac{2x}{5-2x} = \frac{x-1}{2x+5}$

$\left[\frac{3x}{2x-5} - \frac{2x}{2x-5} = \frac{x-1}{2x+5}\right](2x-5)(2x+5)$

$3x(2x+5) - 2x(2x+5) = (x-1)(2x-5)$

$6x^2 + 15x - 4x^2 - 10x = 2x^2 - 7x + 5$

$12x = 5$

$\boxed{x = 5/12}$

(11) $\left[\frac{3}{x+2} + \frac{12}{x^2-4} = \frac{-1}{x-2}\right](x^2-4)$

$3(x-2) + 12 = -1(x+2)$

$3x - 6 + 12 = -x - 2$

$4x = -8$

$x = -2$ $\boxed{\text{no solutions}}$

-2 is an excluded value

(12) $\left[\frac{x+3}{x+2} = 2 - \frac{3}{x^2+5x+6}\right](x+2)(x+3)$

$(x+3)(x+3) = 2(x+3)(x+2) - 3$

$x^2 + 6x + 9 = 2x^2 + 10x + 12 - 3$

$x^2 + 4x = 0$

$x(x+4) = 0$ $\boxed{x = 0, -4}$

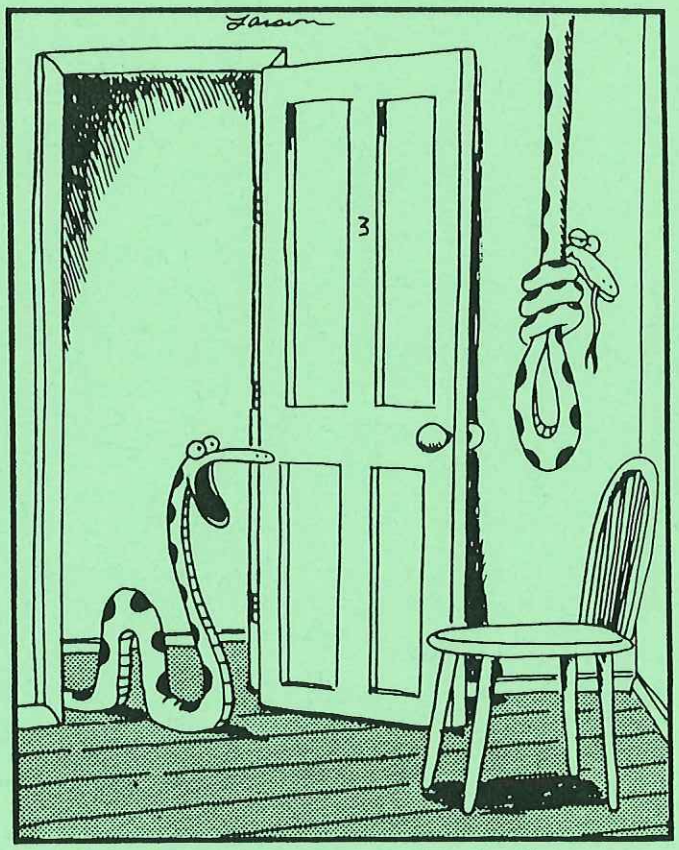
(13) $\frac{x_1}{x_2} = \frac{y_2}{y_1}$ $\frac{5/2}{-3/5} = \frac{y}{9}$

$-\frac{3}{5}y = \frac{45}{2}$ $\boxed{y = -75/2}$

(14) $\frac{x_1}{x_2} = \frac{y_1}{y_2}$ $\frac{-3}{x} = \frac{-8}{6}$ $-8x = -18$ $\boxed{x = 9/4}$

(15) $\frac{x_1}{x_2} = \frac{y_2}{y_1}$ $\frac{6}{x} = \frac{12}{10}$ $12x = 60$ $x = 5$

$\boxed{\text{ht. is } 5}$



"Oh no, Elliott! Why? . . . Why? . . ."

Problem Solving

ANSWER KEY 10.1

① $\frac{n-4}{n+26} = \frac{1}{3}$

$3(n-4) = 1(n+26)$

$3n-12 = n+26$

$2n = 38$

$n = 19$ **19**

② $\frac{6x+2}{7x+1} = \frac{4}{5}$

$5(6x+2) = 4(7x+1)$

$30x+10 = 28x+4$

$2x = -6$

$x = -3$

$6x = \boxed{-18}$

$7x = \boxed{-21}$

③ $n + 6\left(\frac{1}{n}\right) = 5$

$\left[n + \frac{6}{n} = 5\right](n)$

$n^2 + 6 = 5n$

$n^2 - 5n + 6 = 0$

$(n-3)(n-2) = 0$

$n = \boxed{3 \text{ or } 2}$

④ $\frac{1}{n} + 2\left(\frac{1}{n}\right)^2 = 3$

$\left[\frac{1}{n} + \frac{2}{n^2} = 3\right](n^2)$

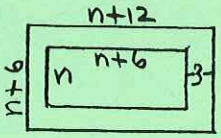
$n + 2 = 3n^2$

$3n^2 - n - 2 = 0$

$(n-1)(3n+2) = 0$

$n = \boxed{1 \text{ or } -2/3}$

⑤



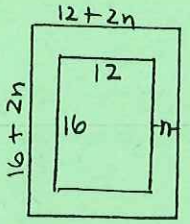
$(n+6)(n+12) - (n)(n+6) = 288$

$(n^2+18n+72) - (n^2+6n) = 288$

$12n = 216$

$n = 18$ **18 by 24 ft**

⑥



$(2n+16)(2n+12) - (12)(16) = 165$

$(4n^2+56n+192) - 192 = 165$

$4n^2+56n-165 = 0$

$\frac{-56 \pm \sqrt{(56)^2 - 4(4)(-165)}}{2(4)}$

$\frac{-56 \pm 76}{8} = \frac{20}{8}$ or $\frac{-132}{8}$

$2\frac{1}{2}$ in.

⑦

	$r \cdot t = d$	
upstream	$15-c \cdot 60/15-c = 60$	
downstream	$15+c \cdot 90/15+c = 90$	

$\frac{60}{15-c} = \frac{90}{15+c}$

$60(15+c) = 90(15-c)$

$900+60c = 1350-90c$

$150c = 450$

$c = 3$

3 Kph

⑧

	$r \cdot t = d$	
org. speed	$r \cdot 260/r = 260$	
new speed	$r+13 \cdot 260/r+13 = 260$	

$\frac{260}{r} - 1 = \frac{260}{r+13}$

$\frac{260-r}{r} = \frac{260}{r+13}$

$(260-r)(r+13) = 260r$

$260r + 3380 - r^2 - 13r = 260r$

$r^2 + 13r - 3380 = 0$

$\frac{-13 \pm \sqrt{(13)^2 - 4(1)(-3380)}}{2(1)} = \frac{-13 \pm 117}{2}$

52 or -65

52 Kph

Problem Solving

ANSWER KEY 10.1

⑨

	r	· t	= d
trip	40	· $d/40$	= d
return	60	· $d/60$	= d

$\frac{\text{total dist.}}{\text{total time}}$ = avg speed

$$\frac{2d}{\frac{d}{40}+\frac{d}{60}} = \frac{2d}{\frac{60d+40d}{2400}} = \frac{4800d}{100d}$$

48 mph

⑩

	r	· t	= w
large pipe	1/6	· t	= t/6
small pipe	1/8	· t	= t/8
drain pipe	1/12	· t	= t/12

$$\left[\frac{t}{6}+\frac{t}{8}-\frac{t}{12}=1\right](24)$$

$4t+3t-2t=24$

$5t=24 \quad t=24/5$ hrs. **4:48**

⑪

	r	· t	= w
mechanic	1/6	· t	= t/6
helper	1/15	· 2t	= 2t/15

$$\left[\frac{t}{6}+\frac{2t}{15}=1\right](30)$$

$5t+4t=30$

$9t=30$

$t = 30/9$ hrs **3:20** mechanic

$2t = 60/9$ hrs **6:40** helper

⑫ If 10 students need 2 hrs, then 2 students need 10 hrs.

If 4 teachers need 1 hr, then 2 teachers need 2 hrs.

	r	· t	= w
2 students	1/10	· t	= t/10
2 teachers	1/2	· t	= t/2

$$\left[\frac{t}{10}+\frac{t}{2}=1\right](10)$$

$t+5t=10$

$6t=10 \quad t=10/6$ **1:40**

⑬ (120) —n— △ —14−n— (160)

$(120)(n) = (160)(14-n)$

$120n = 2240 - 160n$

$280n = 2240$

$n=8$ Mary Jo is **8 ft.** from the fulcrum

⑭ (140) —n+1— △ —n— (160)

$(140)(n+1) = (160)(n)$

$140n+140 = 160n$

$-20n = -140$

$n=7$ The 160 lb weight is **7 ft.** from the fulcrum

Problem Solving

ANSWER KEY 10.1 / 10.2

(15) $\frac{n}{2n+1} \rightarrow \frac{n+8}{2n+11} = \frac{2}{5}$

$5(n+8) = 2(2n+11)$

$5n+40 = 4n+22$

$n = -18$

$\frac{n}{2n+1} = \boxed{\frac{-18}{-35}}$

(16) $\frac{n}{95-n} = \frac{5}{14}$

$14n = 5(95-n)$

$14n = 475-5n$

$19n = 475$

$n = 25$

$\boxed{\frac{25}{70}}$

(17) $n^2 + (n+1)^2 = 265$

$n^2 + (n^2+2n+1) = 265$

$2n^2+2n-264 = 0$

$n^2+n-132 = 0$

$(n+12)(n-11) = 0$

$n = -12$ or 11

n	-12	11
$n+1$	-11	12

(18) $\frac{n}{2n-1} \rightarrow \frac{n+7}{2n+6} = \frac{7}{10}$

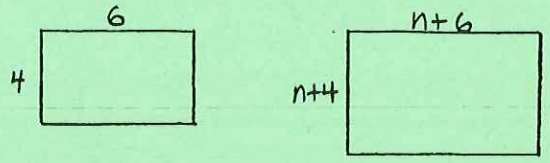
$10(n+7) = 7(2n+6)$

$10n+70 = 14n+42$

$-4n = -28$

$n = 7$ $\quad \frac{n}{2n-1} = \boxed{\frac{7}{13}}$

(19)



$(n+4)(n+6) = 2(4)(6)$

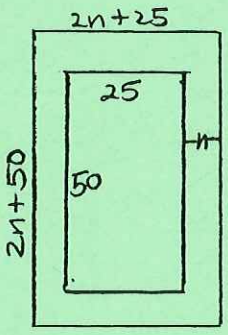
$n^2+10n+24 = 48$

$n^2+10n-24 = 0$

$(n+12)(n-2) = 0$

$n = -12$ or 2 $\quad \boxed{2 \text{ m}}$

(20)



$(2n+25)(2n+50) - (25)(50) = 400$

$(4n^2+150n+1250) - 1250 = 400$

$4n^2+150n-400 = 0$

$2n^2+75n-200 = 0$

$\frac{-75 \pm \sqrt{(75)^2 - 4(2)(-200)}}{2(2)}$

$\frac{-75 \pm 85}{4} = \frac{-160}{4}$ or $\frac{10}{4}$

$\boxed{2\frac{1}{2} \text{ ft.}}$

(21)

	r	$\cdot$ t	$= d$
with	$120+w$	$700/120+w$	700
against	$120-w$	$500/120-w$	500

$\frac{700}{120+w} = \frac{500}{120-w}$

$700(120-w) = 500(120+w)$

$84{,}000 - 700w = 60{,}000 + 500w$

$-1200w = -24{,}000$

$w = 20$ $\quad \boxed{20 \text{ mph}}$

Problem Solving

ANSWER KEY 10.2

(22)

	r	· t	= d
org speed	r	200/r	200
new speed	r+10	200/r+10	200

$$\frac{200}{r} - \frac{2}{3} = \frac{200}{r+10}$$

$$\frac{600-2r}{3r} = \frac{200}{r+10}$$

$$(600-2r)(r+10) = 3r(200)$$

$$600r + 6000 - 2r^2 - 20r = 600r$$

$$2r^2 + 20r - 6000 = 0$$

$$r^2 + 10r - 3000 = 0$$

$$\frac{-10 \pm \sqrt{(10)^2 - 4(1)(-3000)}}{2(1)} = \frac{-10 \pm 110}{2}$$

50 mph

(23)

	r	· t	= d
Monday	40	d/40	d
Tuesday	r	3d/r	3d

$$\frac{\text{tot. dist.}}{\text{tot. time}} = \text{avg. sp}$$

$$\frac{4d}{\frac{d}{40} + \frac{3d}{r}} = 32$$

an alternate method would be to pick any distance for "d"

$$\frac{4d}{\frac{dr+120d}{40r}} = \frac{160\cancel{d}r}{\cancel{d}(r+120)} = 32$$

$$160r = 32r + 3840$$

$$128r = 3840$$

$$r = 30$$

30 mph

(24)

	r	· t	= w
carpenter	1/6	t	t/6
apprentice	1/16	t+5	t+5/16

$$\left[\frac{t}{6} + \frac{t+5}{16} = 1\right](48)$$

$$8t + 3(t+5) = 48$$

$$8t + 3t + 15 = 48$$

$$11t = 33 \quad t = 3$$

3 hrs – carpenter
8 hrs – apprentice

(25)

	r	· t	= w
brush	1/12	t	t/12
roller	1/8	9-t	9-t/8

$$\left[\frac{t}{12} + \frac{9-t}{8} = 1\right](24)$$

$$2t + 3(9-t) = 24$$

$$2t + 27 - 3t = 24$$

$$-t = -3 \quad t = 3$$

3 hours

(26) If 6 girls take 90 min, 3 girls take 3 hrs.

If 4 boys take 80 min, 2 boys take $2\frac{2}{3}$ hrs.

	r	· t	= w
3 girls	1/3	t	t/3
2 boys	(3/8)	t	3t/8

$\frac{1}{2\frac{2}{3}} = \frac{3}{8}$

$$\left[\frac{t}{3} + \frac{3t}{8} = 1\right](24)$$

$$8t + 9t = 24 \quad t = 24/17$$

≈ 1:25

Problem Solving

ANSWER KEY 10.2

(27) (96) (108) (101)

$-5-\Delta$ — n —

— 7 —

$(96)(7) + (108)(5) = (101)(n)$

$672 + 540 = 101n$

$1212 = 101n$

$n = 12$ **12 feet**

(28) (110)

(600) -10 ← $12-2=10$

$-2-\Delta$

$(600)(2) = (110)(10)$

$1200 > 1100$ **NO**

(29)

	r	$\cdot$	t	$=$	w
pipe A	$1/4$	$\cdot$	t	$=$	$t/4$
pipe B	$1/3$	$\cdot$	$t-1$	$=$	$t-1/3$

$\left[\frac{t}{4} + \frac{t-1}{3} = 1\right](12)$

$3t + 4t - 4 = 12$

$7t = 16 \quad t = 16/7 \quad t-1 = 9/7$

$9/7$ hrs **$\approx 1:17$**

(30) If 100,000 slaves take 20 yrs,
50,000 slaves need 40 yrs.

If 75,000 workers take 20 yrs,
18,750 workers need 80 yrs.

	r	$\cdot$	t	$=$	w
50,000 slaves	$1/40$	$\cdot$	t	$=$	$t/40$
18,750 workers	$1/80$	$\cdot$	t	$=$	$t/80$

$\left[\frac{t}{40} + \frac{t}{80} = 1\right](80)$ $\quad 2t + t = 80$

$3t = 80$

$t = 80/3$ yrs.

$26\frac{2}{3}$ years

(31) n — $n+1$

	r	$\cdot$	t	$=$	d
shorter	60		$n/60$		n
longer	46		$n+1/46$		$n+1$

$\frac{\text{total dist.}}{\text{total time}} = \text{avg. speed}$

$\frac{2n+1}{\frac{n}{60} + \frac{n+1}{46}} = 52$

$\frac{2n+1}{\frac{46n+60n+60}{2760}} = \frac{5520n+2760}{106n+60} = 52$

$5520n + 2760 = 5512n + 3120$

$8n = 360$

$n = 45$

$n+1 = 46$

$45+46 =$ **91 miles**

Continued

Problem Solving

ANSWER KEY 10.2

(32)

	r	· t	= d
east	380	· $d/380$	= d
west	420	· $d/420$	= d

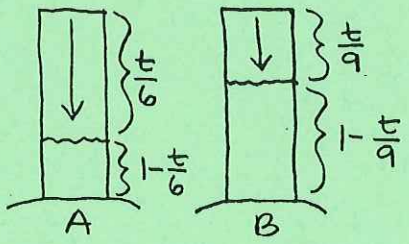
$$\frac{\text{total dist.}}{\text{total time}} = \text{avg. speed}$$

$$\frac{2d}{\frac{d}{380}+\frac{d}{420}} = \frac{2d}{\frac{800d}{159{,}600}}$$

$$\frac{319{,}200\cancel{d}}{800\cancel{d}} = \boxed{399 \text{ mph}}$$

"Calm down, Edna . . . Yes, it's some giant, hideous insect . . . but it could be some giant, hideous insect in need of help."

(33)



	r	· t	= d
A	1/6	t	t/6
B	1/9	t	t/9

$$2\left(1-\frac{t}{6}\right) = 1-\frac{t}{9}$$

$$2\left(\frac{6-t}{6}\right) = \frac{9-t}{9}$$

$$\frac{12-2t}{6} = \frac{9-t}{9}$$

$$9(12-2t) = 6(9-t)$$

$$108-18t = 54-6t$$

$$-12t = -54$$

$$t = 4\tfrac{1}{2}$$

$\boxed{4:30}$